University of Oklahoma
Center for Reservoir Characterization

DOE/BC/14869--3

GYPSY FIELD PROJECT IN RESERVOIR CHARACTERIZATION

Contract Number:	DE–FG22–95BC14869
Contractor:	University of Oklahoma Center for Reservoir Characterization Norman, Oklahoma 73019
Contract Date:	April 6, 1995
Anticipated Completion Date:	April 5, 1997
Government Award	$350,000 (Current Year)
Principal Investigator:	Daniel J. O'Meara Jr.
Project Manager	Robert E. Lemmon Bartlesville Project Office
Reporting Period	October 1 – December 31, 1995
Submitted to:	Pittsburgh Energy Technology Center U.S. Department of Energy Wallace Road, Building 921 (Overnight Mail) P.O. Box 10940, MS 921–118 Pittsburgh, Pennsylvania 15236 (412) 892–4862 (Telephone) Document Control AD–20

MASTER

100 East Boyd, Room 510, Norman, Oklahoma 73019
Telephone: (405) 325–3821 Fax: (405) 325–3180

GYPSY FIELD PROJECT
IN RESERVOIR CHARACTERIZATION

Objectives

The overall objective of this project is to use the extensive Gypsy Field laboratory and data set as a focus for developing and testing reservoir characterization methods that are targeted at improved recovery of conventional oil.

The Gypsy Field laboratory, as described by Doyle, O'Meara, and Witterholt (1992), consists of coupled outcrop and subsurface sites which have been characterized to a degree of detail not possible in a production operation. Data from these sites entail geological descriptions, core measurements, well logs, vertical seismic surveys, a 3D seismic survey, crosswell seismic surveys, and pressure transient well tests.

The overall project consists of four interdisciplinary sub–projects which are closely interlinked:

1. Modeling depositional environments.
2. Upscaling.
3. Sweep efficiency.
4. Tracer testing.

The first of these aims at improving our ability to model complex depositional environments which trap movable oil. The second entails testing the usefulness of current methods for upscaling from complex geological models to models which are more tractable for standard reservoir simulators. The third investigates the usefulness of numerical techniques for identifying unswept oil through rapid calculation of sweep efficiency in large reservoir models. The fourth explores what can be learned from tracer tests in complex depositional environments, particularly those which are fluvial dominated.

Summary of Technical Progress

During this quarter, the main activities involved the "Modeling depositional environments" Project", for which the progress is reported below:

1. Introduction. We study the determination of possibly discontinuous reservoir parameter functions from sparse pointwise measurements supplemented with measurements of a nonlinear function of the parameter. The specific application we have in mind is that of determining a permeability function or tensor from core measurements and pressure data, cf [3]. In a previous report we describe our efforts for models with two spatial dimensions, see also [7]. However, here the focus is on models with three spatial dimensions.

Our approach which is described in [7] involves two steps. The first is to detect the discontinuous behavior, and the second is to isolate and refine the region containing it. For the first step we use a regularized output least squares procedure in which the reservoir mapping is approximated by linear combinations of bicubic B-splines. The regularization used is the H^1 seminorm that is related to the potential energy functional of an elastic membrane. This regularization gives sufficient compactness to obtain the existence of a solution to the associated minimization problem while implying minimal additional smoothing. Moreover, it seems to be well suited for the detection of the discontinuities and sudden changes so often exhibited by geological mappings [2,5]. Having as least detected an anomaly, we next attempt to isolate it by estimating its magnitude and a region containing it. The result of the procedure is to obtain a discontinuous function. We then essentially subtract this function from the model coefficient thereby, at least intuitively, reducing the discontinuous behavior. Again we consider the detection step to test for further discontinuous behavior. The procedure is repeated for further refinements.

The regularized output least squares estimation procedure along with its differentiablility and resolution properties have been described in previous reports and papers [6,7]. we present the results of numerical experiments for the three dimensional case.

To formulate the problem suppose that Ω is a domain in $\mathbf{R}^3$ that is a unit cube for the purposes of our discussion. We also assume that there are data available that we indicate along segments (wells) 1-5

$$\{(x_i, y_i, z) : 0 < z < 1\},$$

see Figure 1. Consider the equation in Ω

$$-\nabla \cdot (K \nabla a) = -(k_1 p_x)_x - (k_2 p_y)_y - (k_3 p_z)_z = f \tag{1.1}$$

where

$$K = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}$$

and $k_i = k_i(x, y, z)$. We assume that the boundary condition

$$\cdot K \nabla p \cdot n = 0.$$

The function f models injection injection along 1

$$f(x, y, z) = \mathcal{F} \delta \tag{1.2}$$

where δ represents a Dirac-delta function with mass distributed along the well 1. Note it is not necessary to for the injection well to run the full depth of the cube or for that matter to be a straight line segment. We also assume that the pressure at well 3 is given by

$$p(x_3, y_3, z) = 0. \tag{1.3}$$

Under suitable conditions, it is well-known that (1.1)-(1.3) possesses a unique solution, [1].

To specify the estimation problem, we wish to determine the coefficients k_1, k_2, and k_3 from measurements $k_j^{(0)}(x_i, y_i, z)$ of k_j at the x-y locations (x_i, y_i) for $i = 1, ..., 5$ with $j = 1, 2, 3$ and $0 << 1$. In addition we also assume that measurements of p are available at these locations as well. Although dependence of the data has been represented as continuous, in fact the use of data at the more realistic case of discrete z-points makes no difference in the formulation and solution of the problem. The general technique by which we formulate the estimation problem in terms of the minimization of a fit-to-data functional that includes the squared difference between model values and data values along with a regularization term is referred to as a regularized output least squares technique. This procedure we use to detect a discontinuity.

2. Estimation of the Discontinuity. Having detected a discontinuity by means of the regularized output least squares method (actually by any procedure), our next step is to isolate and obtain some estimate of it. We proceed by considering an example in which the admissible permeability functions $k_i, i = 1, 2, 3$ have a discontinuity determined by two regions within Ω parameterized by 2 real numbers a and b. We denote these two regions by $\Omega(a, b)$ and $\Omega \setminus \Omega(a, b)$. An admissible permeability function for $i = 1, 2, 3$ k_i is also parameterized constants k_{i1} and k_{i2} modelling the magnitude of the discontinuity between the regions. Hence, k_i takes the form

$$k_i(x, y, z) = k_{i0}(x, y, z) + k_{ie}(x, y, z) \tag{2.1}$$

where

$$k_{ie}(x, y) = k_{i1} \text{ if } (x, y) \in \Omega(a, b), \text{ and } k_{i2} \text{ otherwise.}$$

It is assumed that the function k_{i0} is known. Introducing the characteristic function Ξ of the set $\Omega(a, b)$, we may write

$$k_{ie}(x, y) = (k_{i1} - k_{i2})\Xi(x, y) + k_{i2}. \tag{2.2}$$

To fix ideas let us suppose that $\Omega(a, b)$ is a rectangular solid of the form $(x_0 - a, x_0 + a)$x$(y_0 - b, y_0 + b)$x$(z_0 - c, z_0 + c)$. However, we assume that in fact c is known from core data. The stiffness matrices are now given as

$$(G_l)_{ij} = \int_\Omega (k_{l0} + k_{l2}) \nabla\alpha_i \cdot \nabla\alpha_j dxdydz + (k_{l1} - k_{l2}) \int_{z_0-c}^{z_0+c} \int_{x_0-a}^{x_0+a} \int_{y_0-b}^{y_0+b} \nabla\alpha_i \cdot \nabla\alpha_j dxdydz$$

for $l = 1, 2, 3$ where $\{\alpha_j\}_{j=1}^N$ form a set of basis functions for the finite element approximation of (1.1)-(1.3). Setting

$$(G_0)_{ij} = \int_\Omega \nabla\alpha_i \cdot \nabla\alpha_j dxdydz,$$

$$(G_{l1})_{ij} = \int_\Omega k_{l0} \nabla\alpha_i \cdot \nabla\alpha_j dxdydz,$$

and

$$(G_{l2})_{ij}(a,b) = \int_{z_0-c}^{z_0+c} \int_{x_0-a}^{x_0+a} \int_{y_0-b}^{y_0+b} \nabla\alpha_i \cdot \nabla\alpha_j dxdy, \tag{2.3}$$

we have

$$G = \sum_{l=1}^{3} G_{l1} + (k_{l1} - k_{l2}) G_{l2}(a,b) + k_{l2} G_0. \tag{2.4}$$

Thus, we obtain an equation analogous to (2.16) given by

$$Gc = \rho. \tag{2.5}$$

with the approximating solution u expressed as $u = \sum_{i=1}^N c_i\alpha_i$.

Define the criterion

$$J(a,b,K_1,K_2) = \sum_{j=1}^{N_o} (\langle \Phi_j, u\rangle \; - \; z_j)^2$$

and the functional

$$\mathcal{N}(a,b,K_1,K_2) \;=\; \sum_{j=1}^{N_K} (\langle \phi_j, K\rangle \; - \; K_j)^2.$$

Under discretization, the functional J takes the form

$$J(a,b,K_1,K_2) = c^* H_P c - 2\zeta^* c + c_z$$

Note that $\mathcal{N}(a,b,K_1,K_2)$ is a discontinuous function and is, in fact, a piecewise constant function of a and b. Hence, we look for a, b, K_1, and K_2 that minimizes J while keeping $\mathcal{N}(a,b,K_1,K_2)$ at its minimum value, call it $\mathcal{N}_0$. That is, we seek

$$a,\ b,\ K_1, \text{ and } K_2 \text{ minimizing } J(a,b,K_1,K_2) \text{ subject to } \mathcal{N} = \mathcal{N}_0.$$

3. A numerical example. We consider a problem in which we specify a coefficient K(x,y) and generate pressure data based on that function by solving the problem

(1.1)-(1.3)for p with a specific forcing function f by finite elements. Using this data we then attempt to recover K. Let $\Omega = (0,1)\text{x}(0,1)$ and suppose that measurements of pressure and permeability can be made at locations (0.175,0.175,z), (0.835,0.175,z), (0.5,0.5,z), (0.175,0.835,z), and (0.835,0.835,z). For a test permeability function we use the following

$$k_{1test}(x,y,z) = \begin{cases} 5 + sin\pi(x+y+z) \text{ for } (x,y,z) \in \Omega/\Omega_0, \\ 10 + 2sin\pi(x+y+z), \text{ for}(x,y,z) \in \Omega_0, \end{cases}$$

$$k_{2test}(x,y,z) = \begin{cases} 6 + cos\pi(x+y+z) \text{ for } (x,y,z) \in \Omega/\Omega_0, \\ 11 + 2cos\pi(x+y+z), \text{ for}(x,y,z) \in \Omega_0, \end{cases}$$

$$k_{3test}(x,y,z) = \begin{cases} 7 + sin\pi(x+y+z) \text{ for } (x,y,z) \in \Omega/\Omega_0, \\ 12 + 2sin\pi(x+y+z), \text{ for}(x,y,z) \in \Omega_0, \end{cases}$$

shown in Figures 2-4. In the plots we have graphed all functions as functions of x and y on (0,1)x(0,1) with z = 1/2. Further, we suppose that $p = 0$ at the point (0.835,0.835) and that fluid is injected at the point (0.175,0.175). For the approximations to the pressure, we use tensor products of cubic B-splines [5] defined on a uniform mesh determined by subdividing (0,1) into 5 subintervals. Since imposing Neumann boundary conditions improves accuracy, we use 125 basis functions for approximating pressure adjusted to incorporate the Neumann boundary condition. For approximating the parameter, we again use tensor products of cubic B-splines but defined on a mesh determined by subdividing (0,1) into 3 equal subintervals. We use 64 basis functions to approximate the parameter. Using data at the observation points, we apply the regularized output least squares method as a detection procedure resulting in Figures 5-7. Based on this result, we search for a coefficient of the form

$$k_i(x,y) = \begin{cases} k_{i1} \text{ if } (x,y) \in (0,a) \text{ x } (0,b), \\ k_{i2} \text{ otherwise} \end{cases}$$

using the technique discussed in [7]. We then apply further detection by again using the regularized output least squares method to estimate the coefficient k_i where the permeability has the form

$$k(x,y) = k_{i1}(x,y) \; + \; k_{i2}(x,y).$$

The results are portrayed in Figures 8-10. Based on these computations, we again can use the procedure for detecting further discontinuities coefficients using a regularized output least squares procedure followed by a discontinuous searching procedure. These techniques may be alternated until it is determined that only background is being estimated.

4. Conclusions. We have applied the output least square estimation technique as a detection tool for the estimation of a discontinuous permeability tensor using core data and pressure data In addition we introduced a method to estimate the location and magnitude of a jump discontinuity. We also presented a numerical example for the location of discontinuities in a permeability function in the presence of a background. By alternating detection and discontinuity estimation procedures, it seems to be possible to construct coefficients with discontinuities in the presence of a background function.

REFERENCES

[1] Ladyzhenskaya, O.A. and N. Ural'tseva, Linear and Quasilinear Elliptic Equations, Academic Press, New York, 1968.

[2] Mitasova, H. and L. Mitas, Interpolation by Regularized Splines with Tension: I. Theory and Implementation. Math. Geology, v. 25, p. 641-655.

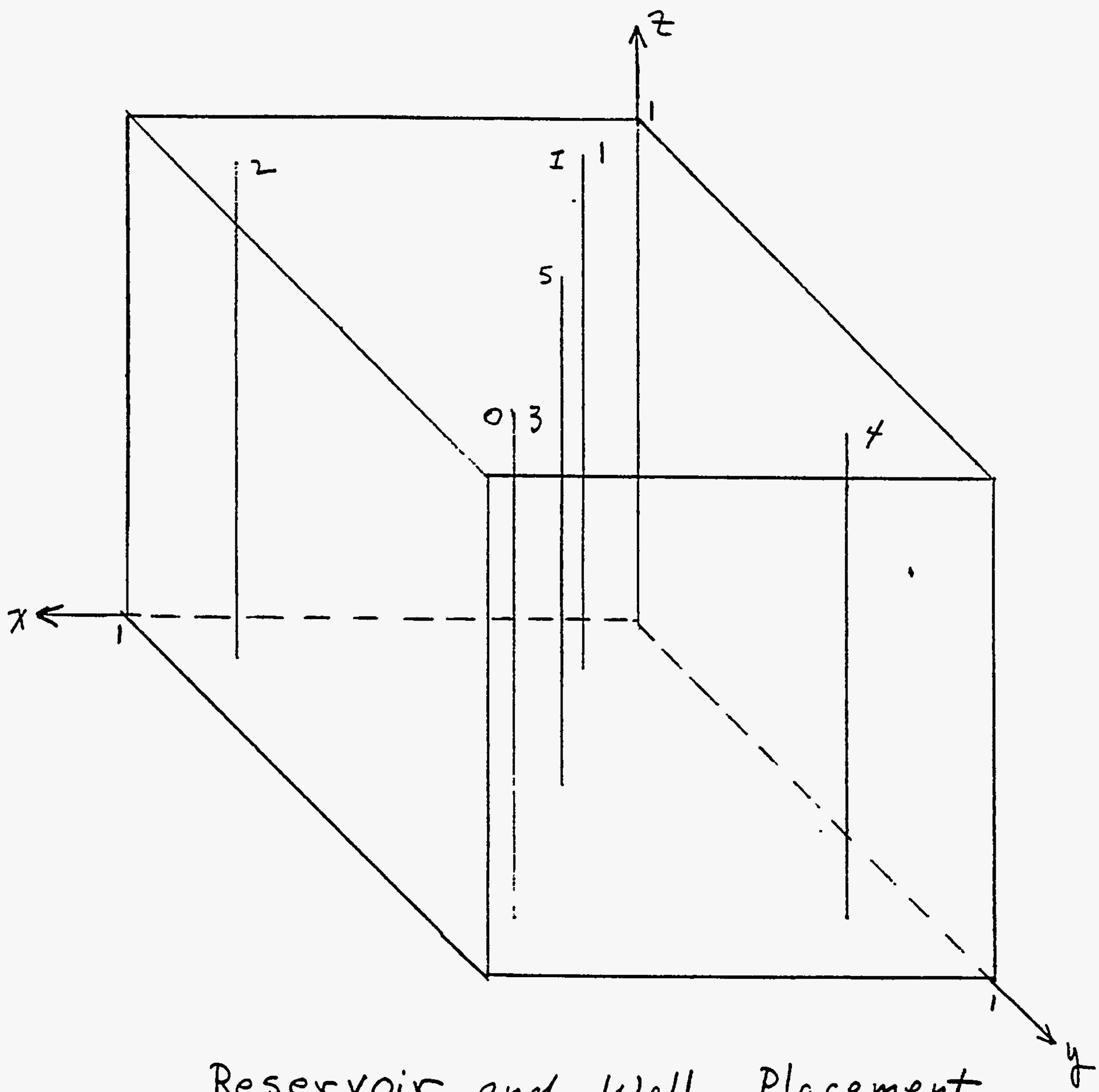
[3] Parker,R.L., Geophysical Inverse Theory, Princeton University Press, Princeton, New Jersey.

[4] Schultz, M. Spline Analysis, Prentice Hall, Englewood Cliffs, New Jersey, 1973.

[5] White,L.W., Y.Jin, and D.J.O'Meara, Jr., Determination of Geological Mappings, submitted.

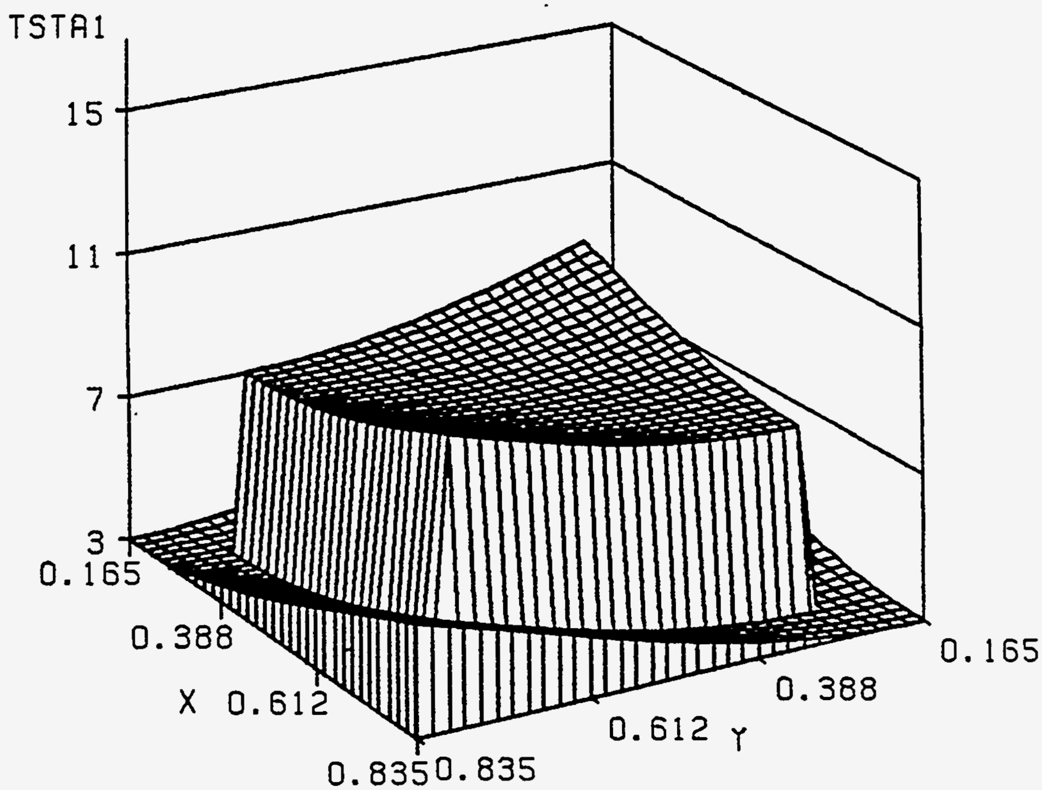
[6] White,L. and J. Zhou, Continuity and Uniqueness of Regularized Output Least Squares Optimal Estimations, J. of Math Analysis and Applications, to appear.

[7] White,L., Y.Jin, and D.J.O'Meara, Estimation of Discontinuous Elliptic Coefficients," J.Comp.and Applied Math., to appear.



Reservoir and Well Placement

Figure 1



Z=0.5

Figure 2

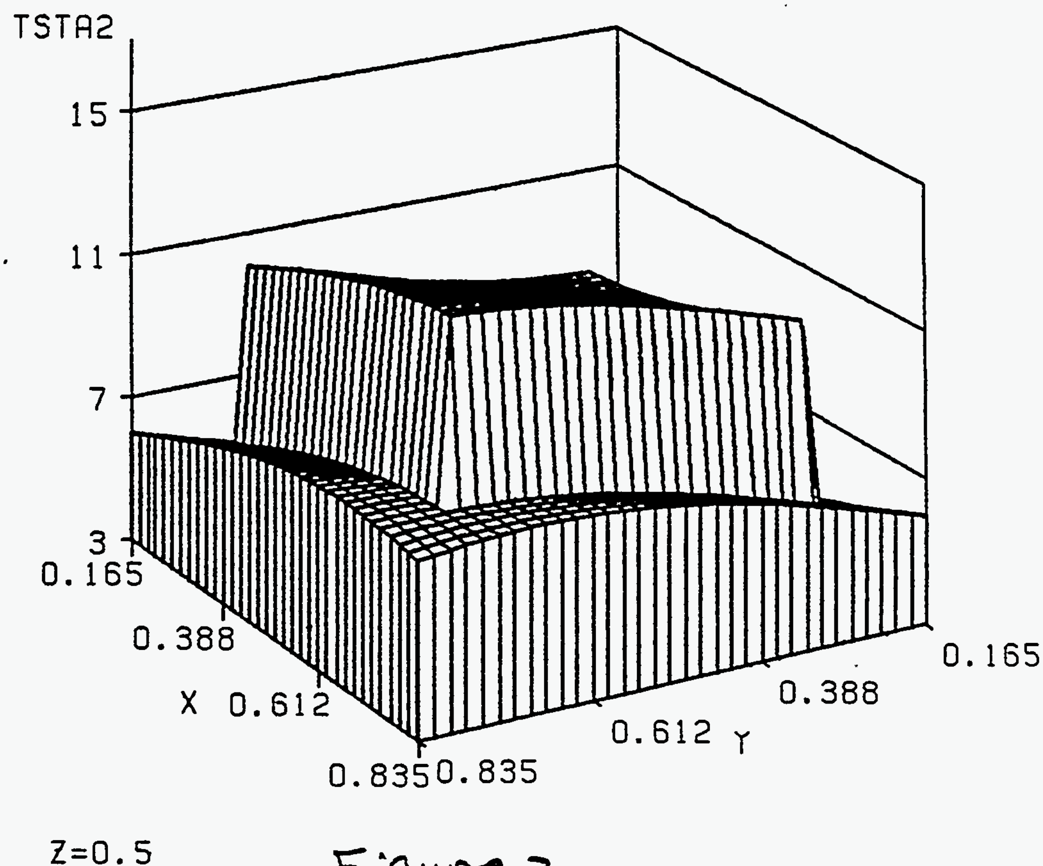


Figure 3

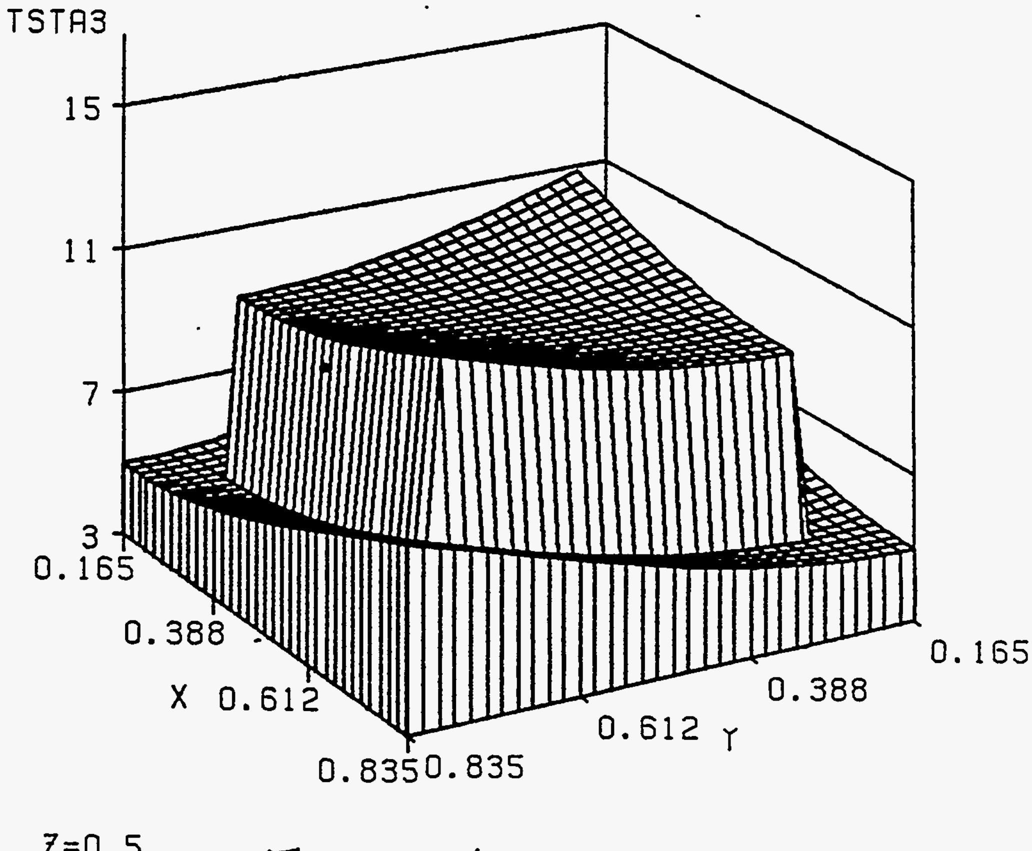
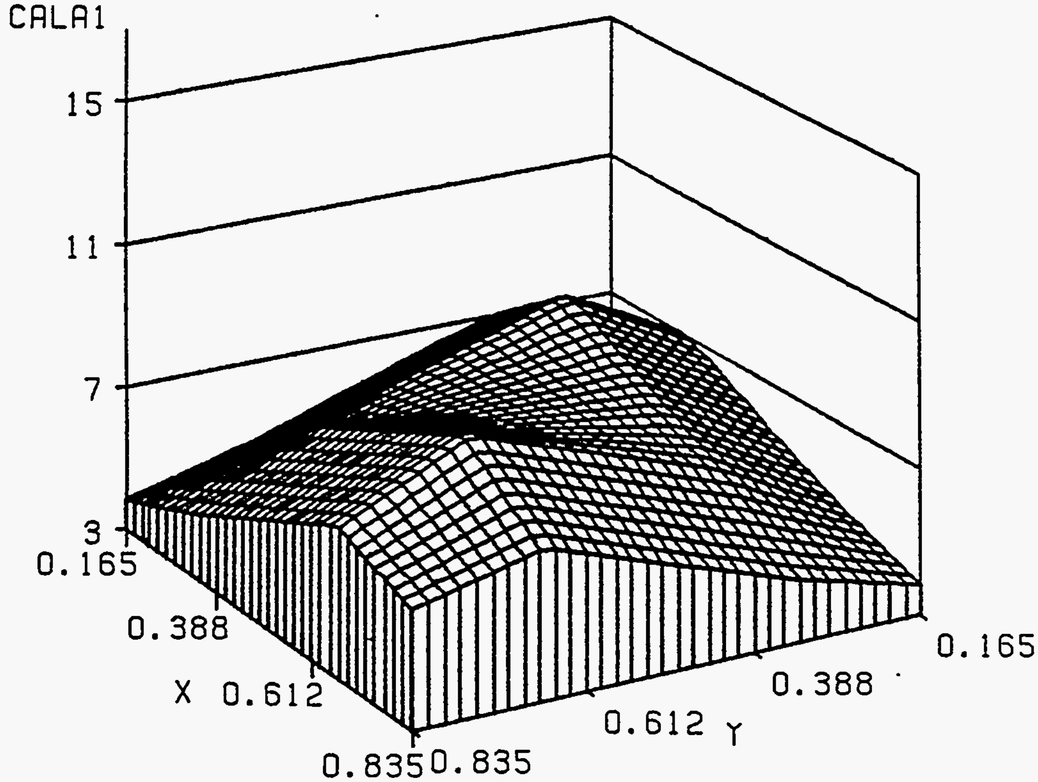


Figure 4



Z=0.5

Figure 5

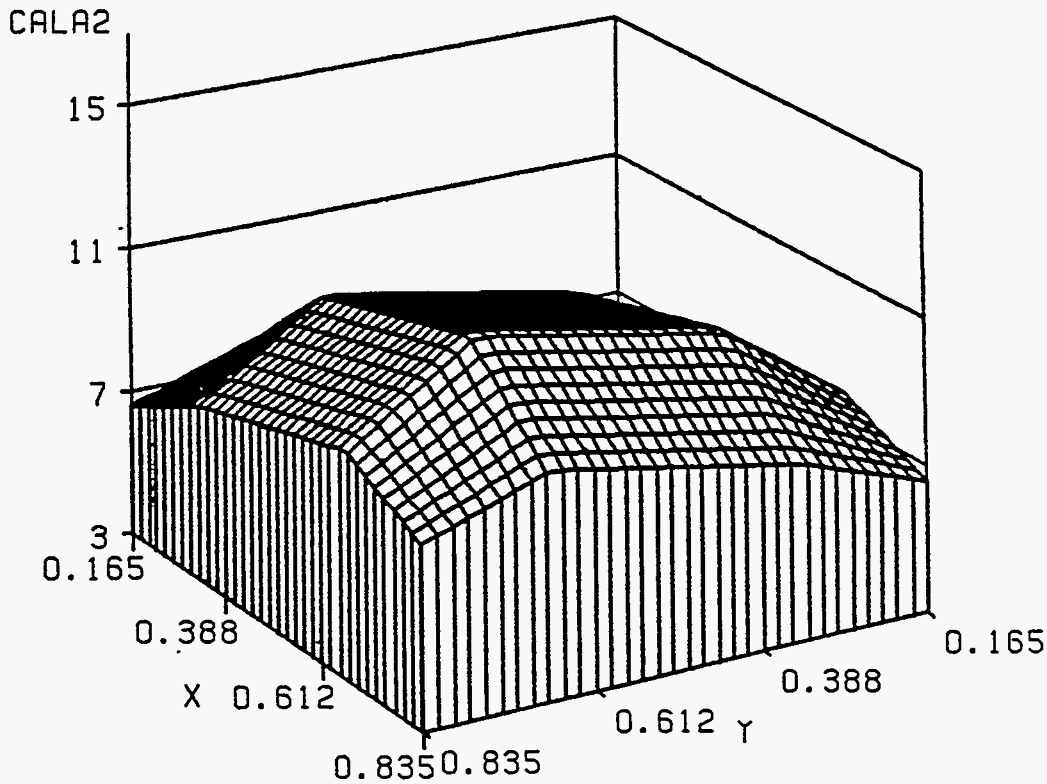


Figure 6

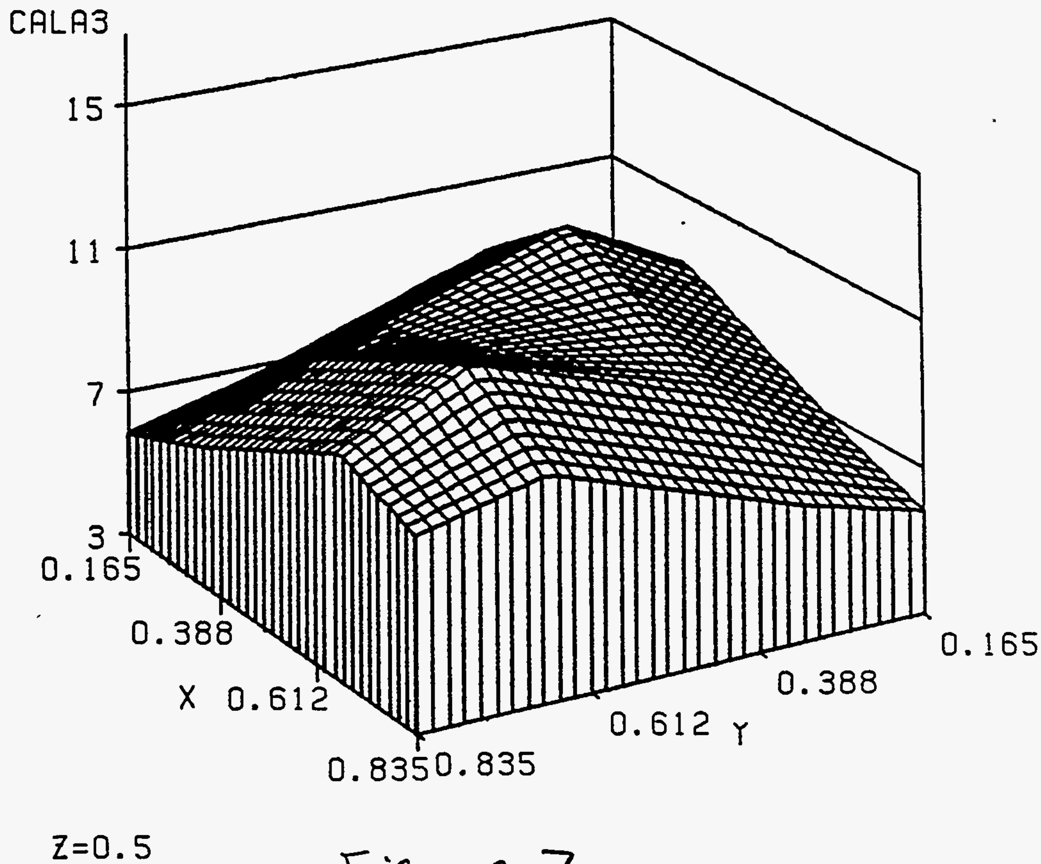
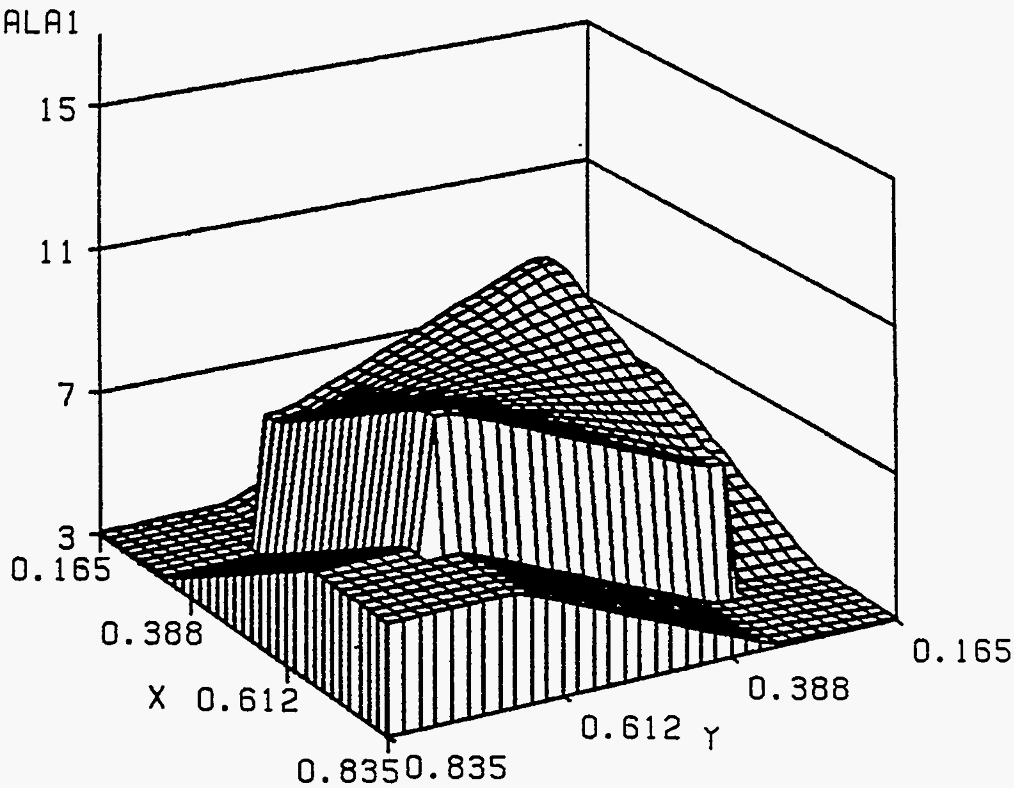


Figure 7

Z=0.5



UNKNOWN GEO., PERTURB BY REG

Figure 8

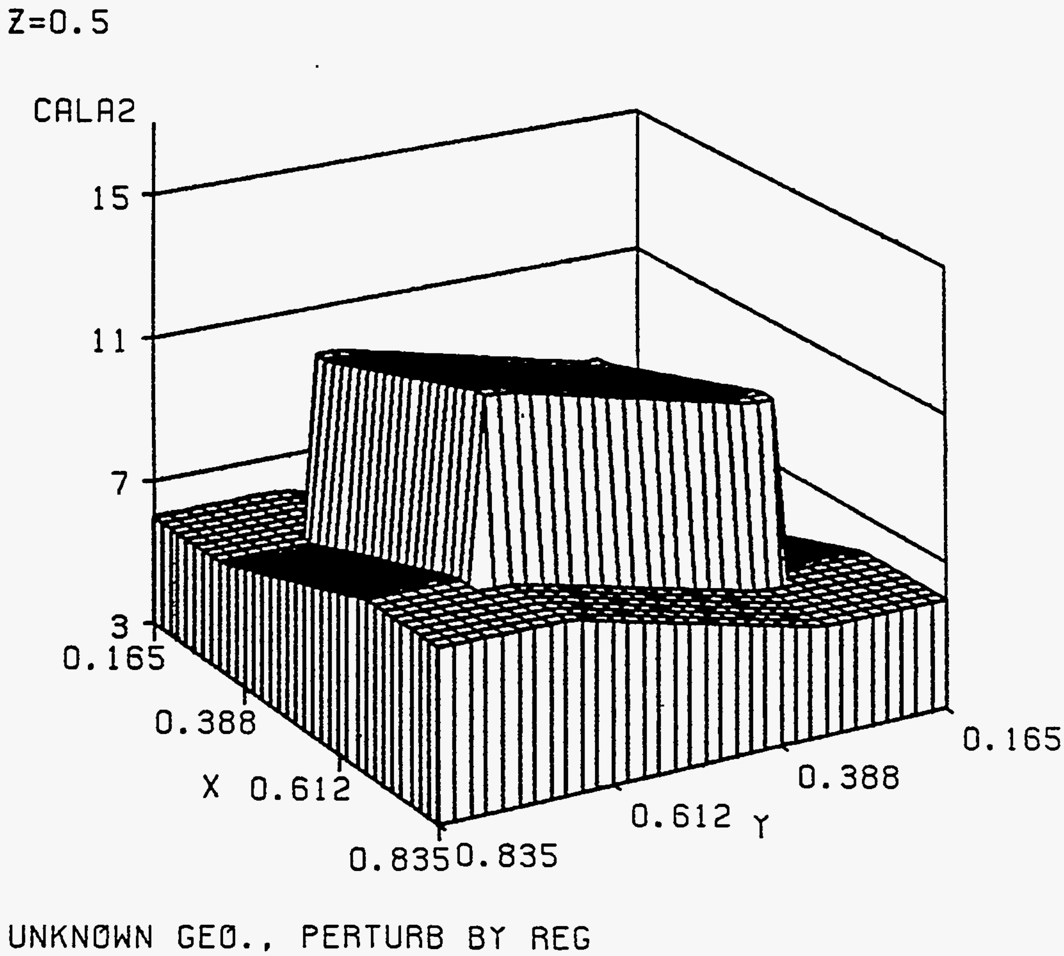
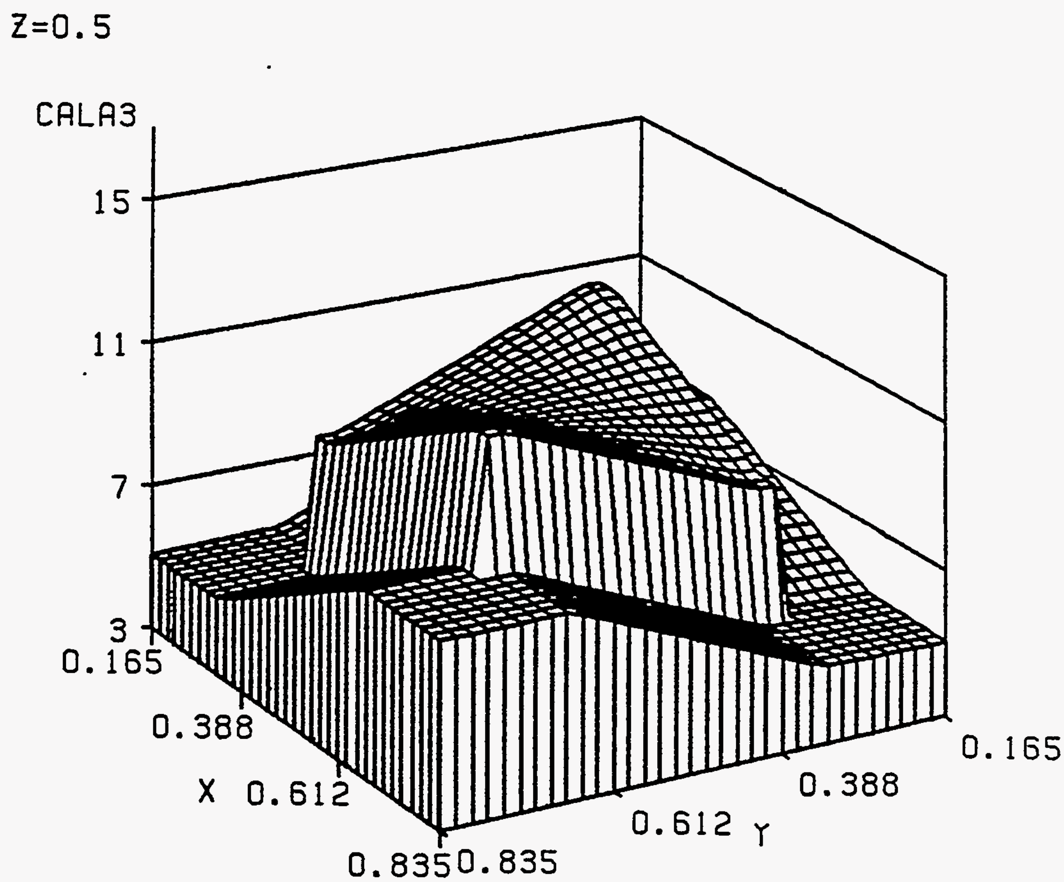

Z=0.5
CALA2
15
11
7
3
0.165
0.388
X 0.612
0.835
0.835
0.612 Y
0.388
0.165
UNKNOWN GEO., PERTURB BY REG

Figure 9



UNKNOWN GEO., PERTURB BY REG

Figure 10