NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT No. 707

THE ADDITIONAL-MASS EFFECT OF PLATES AS DETERMINED BY EXPERIMENTS

By WILLIAM GRACEY

1941

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AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Abbreviation	Unit	Abbreviation
Length	l	meter	m	foot (or mile)	ft (or mi)
Time	t	second	s	second (or hour)	sec (or hr)
Force	F	weight of 1 kilogram	kg	weight of 1 pound	lb
Power	P	horsepower (metric)		horsepower	hp
Speed	V	kilometers per hour	kph	miles per hour	mph
		meters per second	mps	feet per second	fps

2. GENERAL SYMBOLS

W Weight $=mg$

g Standard acceleration of gravity $=9.80665$ m/s^2 or 32.1740 ft/sec^2

m Mass $=\frac{W}{g}$

I Moment of inertia $=mk^2$. (Indicate axis of radius of gyration k by proper subscript.)

μ Coefficient of viscosity

ν Kinematic viscosity

ρ Density (mass per unit volume)

Standard density of dry air, 0.12497 kg-m^{-4}-s^2 at 15° C and 760 mm; or 0.002378 lb-ft^{-4} sec^2

Specific weight of "standard" air, 1.2255 kg/m^3 or 0.07651 lb/cu ft

3. AERODYNAMIC SYMBOLS

S Area

S_w Area of wing

G Gap

b Span

c Chord

A Aspect ratio, $\frac{b^2}{S}$

V True air speed

q Dynamic pressure, $\frac{1}{2}\rho V^2$

L Lift, absolute coefficient $C_L=\frac{L}{qS}$

D Drag, absolute coefficient $C_D=\frac{D}{qS}$

D_0 Profile drag, absolute coefficient $C_{D_0}=\frac{D_0}{qS}$

D_i Induced drag, absolute coefficient $C_{D_i}=\frac{D_i}{qS}$

D_p Parasite drag, absolute coefficient $C_{D_p}=\frac{D_p}{qS}$

C Cross-wind force, absolute coefficient $C_C=\frac{C}{qS}$

i_w Angle of setting of wings (relative to thrust line)

i_t Angle of stabilizer setting (relative to thrust line)

Q Resultant moment

Ω Resultant angular velocity

R Reynolds number, $\rho\frac{Vl}{\mu}$ where l is a linear dimension (e.g., for an airfoil of 1.0 ft chord, 100 mph, standard pressure at 15° C, the corresponding Reynolds number is 935,400; or for an airfoil of 1.0 m chord, 100 mps, the corresponding Reynolds number is 6,865,000)

α Angle of attack

ϵ Angle of downwash

α_0 Angle of attack, infinite aspect ratio

α_i Angle of attack, induced

α_a Angle of attack, absolute (measured from zero-lift position)

γ Flight-path angle

2626°

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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By WILLIAM GRACEY

SUMMARY

The apparent increase in the inertia properties of a body moving in a fluid medium has been called the additional-mass effect. This report presents a résumé of test procedures and results of experimental determinations of the additional-mass effect of flat plates. In addition to data obtained from various foreign sources and from an NACA investigation in 1933, the results of tests recently conducted by the National Advisory Committee for Aeronautics are included. In the recent NACA tests, the additional-mass effect of rectangular plates of varying aspect ratio was redetermined, and the additional-mass effect of plates having tapered plan forms was investigated for the first time.

A test procedure is described by means of which values of additional mass are obtained as the difference between the moments of inertia of the plates experimentally determined in air and in vacuum.

The results of the present NACA tests, believed to be more accurate than data obtained in the older investigations, fall a little above the data obtained by the NACA in 1933 and somewhat below the values published in Germany in 1937. The German values appear erroneously high on the basis of theoretical considerations.

INTRODUCTION

That the mass of a moving body is apparently greater in a fluid medium than in a vacuum was noted as early as 1836 (reference 1). The apparent increase in mass can be attributed to the additional energy required to establish the field of flow about the moving body. Inasmuch as the motion of the body may be defined by considering its mass as equal to the actual mass of the body plus a fictitious mass, the effect of the inertia forces of the fluid may be represented as an apparent additional mass; this additional mass, in turn, may be considered as the product of an imaginary volume and the density of the fluid. The effect of the surrounding fluid has accordingly been called the *additional-mass effect.* The magnitude of this effect depends on the density of the fluid and the size and the shape of the body normal to the direction of motion.

Theoretical values of the additional mass of a number of bodies of infinite length and of ellipsoids or elliptic plates of finite dimensions have been previously derived (references 2 and 3). The verification of these values and the establishment of values for bodies of finite dimensions not covered by the theory, for example, rectangular plates, have provided the basis for experimental research on the phenomenon. Results from experimental determinations of the additional-mass effect have been reported from the aeronautical laboratories of the United States, England, Russia, and Germany (references 4 to 8). Although these tests were primarily conducted for the purpose of correcting the experimentally determined moments of inertia of airplanes, the results obtained are of importance in other aerodynamic problems. Because of the widespread interest in the problem and because of the lack of agreement in the results from the various laboratories, a compilation and an analysis of all the available data on the subject seemed desirable.

Extensive test programs on the additional-mass effect were conducted in the United States in 1933 and in Germany in 1937; it is with these tests that the present report is principally concerned. In an attempt to explain the discrepancies between the results of these two investigations, the National Advisory Committee for Aeronautics has repeated certain of its original tests, making use of improved apparatus and a different test procedure. The present tests consisted in a redetermination of the coefficients of additional mass and of additional moment of inertia for rectangular plan forms. In addition, a new aspect of the problem, the effect of taper ratio on the additional moments of inertia, was investigated.

SYMBOLS

For ready reference, the symbols used repeatedly throughout the report are collected in the following list. The dimensions of the plate are called chord c and span b, rather than length and breadth, to permit easy application to an airplane wing.

- c chord of plate
- b span of plate
- v over-all volume of plate
- l distance from center of plate to axis of oscillation
- V linear velocity
- ω angular velocity
- ρ mass density of fluid in which plate is immersed

m_e mass of fluid entrapped within plate
m_a additional mass
I_v virtual moment of inertia about midchord of plate
I_0 moment of inertia of structure of plate about its midchord
I_e moment of inertia of entrapped fluid about midchord of plate
I_a additional moment of inertia about midchord of plate
I_{a_l} additional moment of inertia about an axis removed a distance l from the midchord
W_a weight of plate in a fluid of density ρ
W_0 weight of plate in vacuum
T_a period of oscillation in fluid of density ρ
T_0 period of oscillation in vacuum
k coefficient of additional mass
k' coefficient of additional moment of inertia

THEORETICAL DISCUSSION

Although the theoretical aspects of the problem of the additional-mass effect have been fully treated in previous papers, the theory will be briefly reviewed as an introduction to the experimental work.

Numerical measures of the additional-mass effect are obtained from a consideration of the momentum imparted to the air by moving plates. For a thin flat plate of infinite span moving in a perfect fluid at constant velocity along the normal to its surface, the momentum imparted to the air per unit span is given by aerodynamic theory as

$$\frac{\rho c^2 \pi V}{4} \tag{1}$$

For plates of finite span, this expression must be corrected by the introduction of coefficients whose values depend on the dimensions of the plate. The additional mass for translation of a plate of span b is thus determined from the equation of linear momentum

$$m_a V = \frac{k\pi\rho c^2 b V}{4}$$

so that

$$m_a = \frac{k\pi\rho c^2 b}{4} \tag{2}$$

where k is the coefficient of additional mass.

Similarly, the additional moment of inertia for rotation about the midchord, that is, the chord at the semispan, of a plate of span b is determined from the equation of angular momentum

$$I_a\omega = m_a\frac{b^2}{12}\omega = \frac{k'\pi\rho c^2 b^3}{48}\omega$$

so that

$$I_a = \frac{k'\pi\rho c^2 b^3}{48} \tag{3}$$

where k' is the coefficient of additional moment of inertia.

The coefficients k and k' are both functions of the span-chord ratio b/c.

For rotation about an axis in the plane of the plate and parallel to the chord, equation (3) becomes

$$I_{a_l} = I_a + m_a l^2 = \frac{k'\pi\rho c^2 b^3}{48} + \frac{k\pi\rho c^2 b\, l^2}{4} \tag{4}$$

Likewise, for rotation about an axis in the plane of the plate and parallel to the span, the additional moment of inertia about the axis of rotation is

$$I_{a_l} = I_a + m_a l^2 = \frac{k'\pi\rho b^2 c^3}{48} + \frac{k\pi\rho c^2 b\, l^2}{4} \tag{5}$$

where k' is the coefficient of the additional moment of inertia that applies to the ratio c/b. When c/b is sufficiently small, the first term of this expression may be neglected so that equation (5) may be approximated as

$$I_{a_l} = m_a l^2 = \frac{k\pi\rho c^2 b\, l^2}{4} \tag{6}$$

Experimental values of the coefficients k and k' are obtained from determinations of m_a and I_a, which are usually obtained by swinging flat plates in a fluid medium. The additional mass and the additional moment of inertia are thus determined by deducting the moment of inertia of the structure of the plate from its virtual moment of inertia in the fluid. Whether I_a or m_a is determined depends on the choice of the axis about which the plate oscillates.

Values of I_a can be obtained from a single determination of the moment of inertia of the plate in air by swinging the plate in one of three ways:

1. As a compound pendulum about an axis through the midchord (the center of gravity being displaced below the midchord by properly weighting the plate)
2. As a compound pendulum about an axis parallel to the midchord outside the plane of the plate
3. As a torsional pendulum about an axis through the midchord

In each case, the coefficient of the additional moment of inertia is found from equation (3):

$$k' = \frac{48 I_a}{\pi\rho c^2 b^3}$$

Values of m_a may be directly found by vibrating the plate along the normal to the surface of the plate by springs. If the additional mass is determined by swinging tests, there is superimposed on the translatory motion to be measured a rotational component that must also be evaluated. Values of m_a, therefore, cannot be found from a single swinging experiment. If the plate is swung about an axis in the plane of the plate, parallel to the chord, and at a distance l from the center of the plate, equation (4) can be applied and m_a be determined by the elimination of I_a. The elimination of I_a may be accomplished either by substituting values of I_a determined in previous experiments or by the simultaneous solution of two expressions of equation (4) obtained by swinging the plate at two suspension lengths. The coefficient of additional mass is then found from the expression

$$k=\frac{4m_a}{\pi\rho c^2 b}$$

As a close approximation, the additional mass may be found from a single swinging test if the axis of rotation is parallel to the span in the plane of the plate. Then, if c/b is sufficiently small, m_a can be determined from equation (6).

SUMMARY OF PREVIOUS TESTS

In order to form an adequate basis for a discussion of the results of the various investigations, the nature of the different experimental procedures and the scope of the various test programs will first be briefly outlined.

In the German experiments of 1930 (reference 4), small plates were fixed to one end of a vertical tube

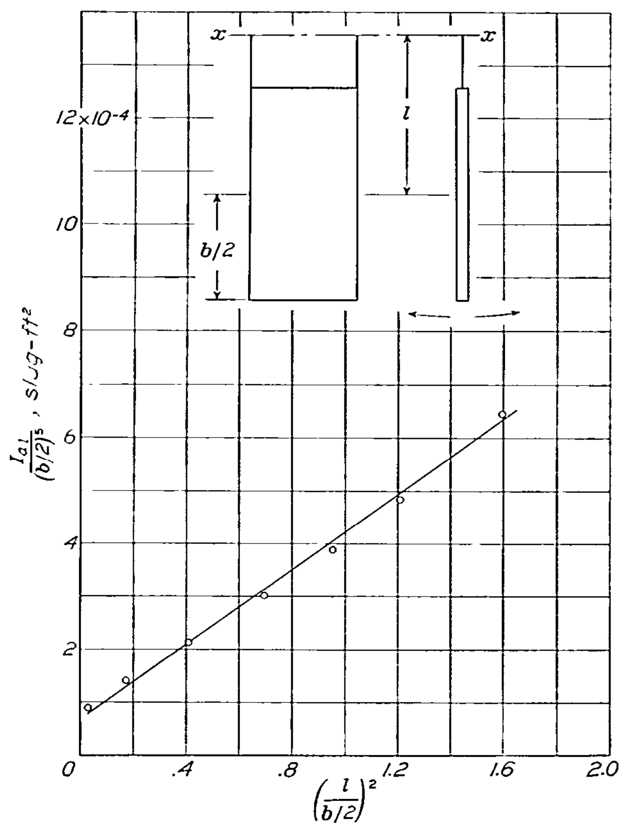

FIGURE 1.—Variation of additional moment of inertia with suspension length. From British tests (reference 5).

normal to the surface of the plates; the other end of the tube was secured to two flat steel springs in such a manner that the entire system was capable of vibrating in a vertical plane. The system was deflected about 0.2 millimeter and released; the resulting damped vibration was recorded by a scratch-recording device. The period of vibration was determined from measurements of the vibration record (upon which timing marks were also recorded) by the use of a micrometer microscope. Determinations of the additional mass of translation were obtained as the difference between the total mass as measured in air and under water. Four plates having dimensions in centimeters of 10 by 10, 10 by 20, 10 by 30, and 10 by 40 were tested.

The British tests (reference 5) were conducted primarily to find the additional-mass effect of a complete

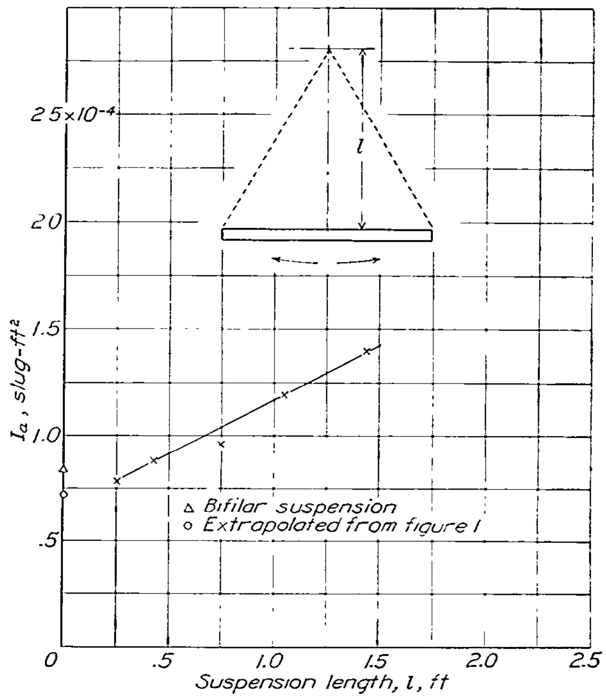

FIGURE 2.—Variation of additional moment of inertia with pendulum length. From British tests (reference 5).

1/20-scale balsa model of a Bristol fighter. The model was tested as a compound pendulum in an altitude chamber of two air densities. As a matter of interest, the test program was extended to include tests of a 2-foot balsa plate of aspect ratio 7. This plate was swung as a compound pendulum with the axis of rotation (1) parallel to the chord in the plane of the plate, (2) parallel to the chord in the plane of symmetry, and (3) parallel to the span in the plane of symmetry. The plate was suspended by fine threads when hung horizontally and by metal points set in the plate when hung vertically. The additional moments of inertia of the plate were found by deducting the computed moment of the structure from the experimental value obtained in air at normal density.

For the plate vertical the additional moment of inertia about the axis of rotation is plotted in figure 1 against $\left(\frac{l}{b/2}\right)^2$. The additional moment of inertia about the midchord is found by extrapolating this curve to $l=0$. This value is compared in figure 2 with the data obtained by swinging the plate with its plane horizontal. The value of I_a for $l=0$ for this curve was obtained by means of a bifilar suspension.

The Russian experiments (reference 6) were made for the purpose of obtaining experimental checks of theoretically derived formulas for the additional mass and moments of inertia of elliptic plates. For elliptic plates of span-chord ratio of 1, that is, for circular plates, these formulas reduce to

$$m_{a_z}=\frac{8}{3}\rho r^3 \tag{14}$$

$$I_{a_x}=I_{a_y}=\frac{16}{45}\rho r^5 \tag{15}$$

where m_{a_z} is the additional mass along the axis perpendicular to the plate, I_{a_x} and I_{a_y} are the additional moments of inertia about the axes of the plate, and r is the radius of the plate. These formulas may be applied to elliptic plates by substituting for r the semimajor axis and applying suitable correction factors. A plot of these calculated coefficients (C_z, the coefficient of additional mass, C_x and C_y, the coefficients of additional moments of inertia) is shown in figure 3 as a function of c/b. For plates of nonelliptic shape, the assumption is made that the moment of inertia would be that of an elliptic plate with the same axes increased in the ratio of the areas. This ratio would be $16/3\pi$ or 1.7 for a rectangle.

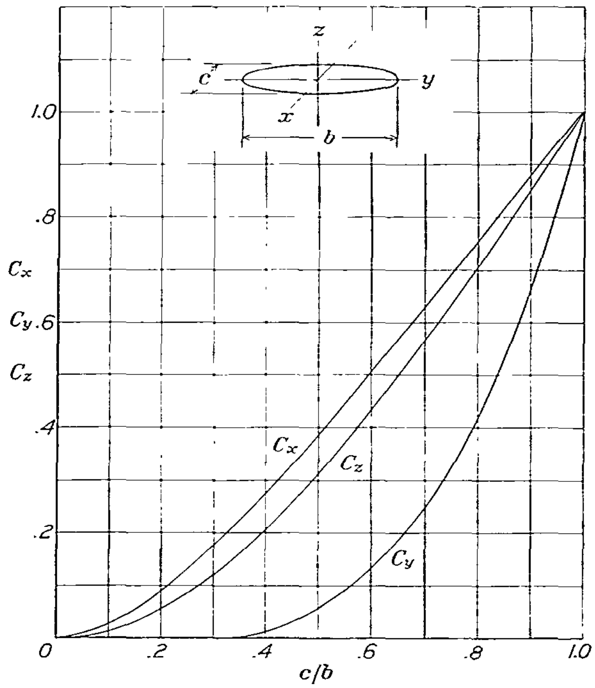

FIGURE 3.—Computed values for the coefficients of additional mass and of the additional moments of inertia for elliptical plates. From Russian report (reference 6).

The Russian tests were conducted on small cardboard frames to both sides of which paper was glued. The models tested included (dimensions in cm): three ellipses 28 by 28, 28 by 19, and 28 by 9; three rectangles 28 by 19, 28 by 17, and 28 by 9; and two rectangles with rounded corners 28 by 17, and 28 by 9. The moments of inertia about the two axes of the plate were found by swinging the models in air by means of a bifilar suspension. A tetrafilar suspension was used to find the moment of inertia about the axis perpendicular to the plate (about which the additional moment of inertia should be zero). The moments of inertia of the structures of the plates were computed on the assumption that the material was homogeneous and that the density was the same throughout.

Tests made at the laboratories of the NACA in 1933 (reference 7) were conducted on four light wooden frameworks covered on both sides with paper. The plates, the b/c ratio of which varied from 2 to 8, had a span of 4 feet and a thickness of one-fourth inch. Each plate was swung at four suspension lengths (1, 1½, 2, and 2½ times the chord) about an axis parallel to the midchord and outside the plane of the plate. The additional moments of inertia were found by deducting the computed moments of inertia of the structures of the plates and of the entrapped air from the virtual moments of inertia determined in air of normal density.

The additional-mass curve given in figure 3 of reference 7 was obtained from the German experiments of 1930. The curve was extrapolated to $b/c=10$ by the approximate empirical formula

$$k=1-\frac{0.537}{b/c}$$

An experimental check for $b/c=4$ was made at the NACA laboratories by swinging a 5- by 20-foot plate constructed of a wooden framework covered with doped fabric. The virtual moment of inertia of the covered plate was determined by swinging it with its plane vertical about an axis parallel to the span and at 1½-chord lengths from the center of the plate. The moment of inertia of the structure was found by swinging the uncovered frame and adding the moment of inertia of the fabric as obtained by computations.

A later German investigation in 1937 (reference 8) consisted in swinging as compound pendulums two rectangular frameworks 0.75 by 3.0 meters, one constructed of aluminum tubing and the other constructed of steel tubing. The b/c ratio of the plates was varied from 0.25 to 8 by the corresponding partial covering of the frame. The frames were tested in air of normal density with and without the covering. Equal mass distribution for the covering was obtained in the uncovered frame by placing thin wires inside the tubes. The experimentally determined moments of inertia of the frame were reported to compare favorably with the computed values.

For the tests of the additional moment of inertia, the frames oscillated about knife edges or ball bearings at the midchord; weights were added to bring the center of gravity below the axis of rotation. The axis of oscillation for the additional-mass tests was in the plane of the plate displaced 1.8 meters from the center of the plate.

PRESENT NACA TESTS

EXPERIMENTAL PROCEDURE

The additional-mass effect was determined in the present tests by swinging covered frameworks as compound pendulums. For the tests of additional moment of inertia, the axis of rotation was outside the plane of the plate; for the additional-mass tests, the axis of rotation was within the plane of the plate. In both cases, the axis was parallel to and at a distance l from the midchord. Each plate was tested at only one suspension length, values of the additional mass being found from equation (4) by substitution of values of I_a previously determined. Some of the frameworks were first covered on only one side and then on both sides to study the effect of the cross members on the additional mass.

A modification of the equation for the compound pendulum to apply to bodies of small density was developed in reference 7. The application of this equation to the determination of the additional-mass effect of flat plates required a further modification to account for the air entrapped within the structure of the plates covered on both sides. The resulting equation is

$$I_v - I_e = \frac{W_a l T_a^2}{4\pi^2} - \left(\frac{W_a}{g} + r\rho + m_a\right) l^2 - m_e \frac{b^2}{12} \qquad (7)$$

It may be noted in passing that $I_v = I_0 + I_a + I_e$ (reference 7).

Equation (7) is applicable when the weight and the moments of inertia of the gear supporting the plate can be neglected. It is obvious that for solid plates $I_e = 0$. Furthermore, if the buoyancy of the structure of hollow plates is negligible, m_e may be taken as equal to $r\rho$.

Equation (7) shows that T_a^2 should vary directly with ρ. For vacuum conditions, equation (7) would then become

$$I_0 = \frac{W_0 l T_0^2}{4\pi^2} - \frac{W_0}{g} l^2 \qquad (8)$$

Because of the impracticability of attaining a perfect vacuum, T_0 cannot be directly measured. If swinging tests are conducted at a number of air densities between atmospheric pressure and vacuum, however, T_0 can be determined by extrapolating the curve of ρ against T_a to zero density. For a test of this nature to be valid, the weight and the suspension length must remain constant for different air densities in order that the period be the only pendulum characteristic to vary with air density.

APPARATUS

The present tests were performed in a vacuum tank, the inside diameter of which was 54 inches (fig. 4). Absolute pressures within the tank, varying from 27 to 4 inches of mercury, were determined as the difference between the gage pressure (measured with a mercury manometer) and the atmospheric pressure (measured with a barometer). Temperatures within the tank were also measured to determine the air density.

The plates used for the tests were all constructed of a framework of aluminum tubing of 0.125-inch outside diameter, covered with 0.001-inch aluminum foil. This type of plate was chosen because its weight remains the same at different air pressures. Balsa plates were first tested but were discarded because of their weight variations with air pressure and humidity. All plates tested

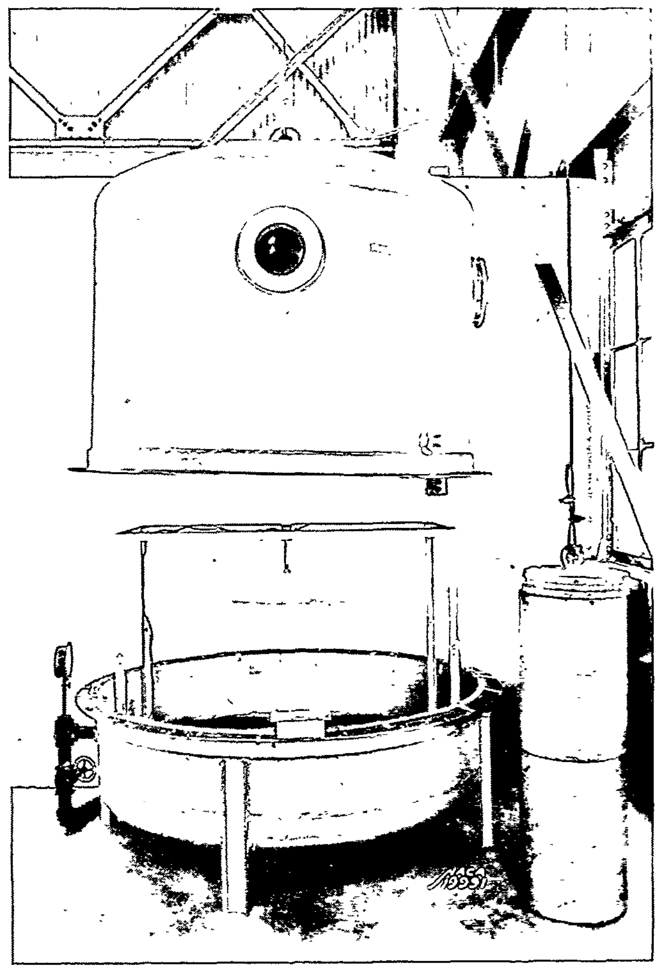

FIGURE 4.—Vacuum tank, showing plate suspended for test of the additional moment of inertia.

had a span of 20 inches with three aluminum-tubing cross members equally spaced along the span.

Tests of the additional moment of inertia were conducted on four rectangular plates of b/c ratio 2, 4, 6, and 8 and on two tapered plates (fig. 5) of aspect ratio 4 and taper ratios 2.5:1 and 5:1. Tests of the additional mass were conducted on two rectangular plates of aspect ratio 4 and 6.

For tests of the additional moment of inertia with the axis of rotation outside the plane of the plate, the plate was suspended from knife edges by 0.003-inch copper wire (fig. 6). The mass and the moments of inertia of these suspension wires were found to be negligible.

For tests of the additional mass with the axis of rotation within the plane of the plate, the plates were fitted with small knife edges about which rotation took place (fig. 7).

When the plates were covered on only one side, the volume of the tubes was found to be sufficiently small that the terms $V\rho l^2$ and $\frac{m_e b^2}{12}$ in equation (7) could be neglected. The plates covered on both sides were vented with small holes in the covering in order that the density of the air entrapped by the plate would at all times be the same as that of the surrounding air. The weight of the plate as measured in air could thus be applied for all air densities.

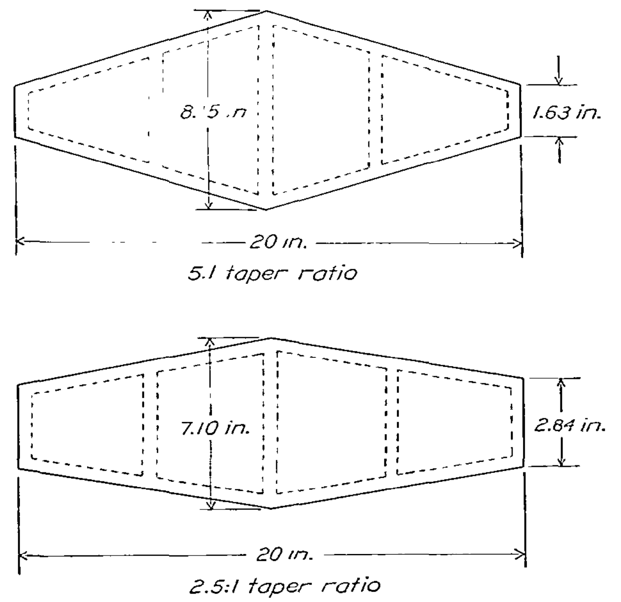

FIGURE 5.—Plan forms of the tapered plates tested in the NACA investigation.

The characteristics of the compound pendulum necessary for the solution of equation (7) are the weight, the suspension length, and the period of oscillation. The weight and the suspension length, both of which remained constant as the tank pressure varied, were determined in accordance with conventional laboratory practice. Because of the limited number of oscillations obtainable at pressures near atmospheric, a timing device more accurate than the ordinary stop watch was employed. The period at each of a number of tank pressures was determined as the average of 20 to 100 oscillations, the number of oscillations depending on the air density.

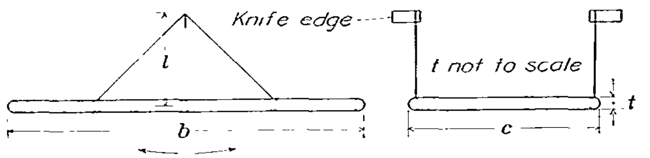

FIGURE 6.—Diagram showing method of suspending plates for tests of the additional moment of inertia.

RESULTS

The results of the present NACA tests are tabulated in table I. The precision, as based on a comparison of the computed and the experimental values of the moments of inertia of the plates in vacuum, is seen to be within 3.5 percent.

The values of the coefficient of the additional moment of inertia obtained by the present investigation are compared in figure 8 with those of previous tests. The present NACA results do not agree with any of the previous data but fall between the German and the original NACA curves. Although the evidence is not conclusive, the present tests indicate that plates covered on only one side give results which are erroneously high.

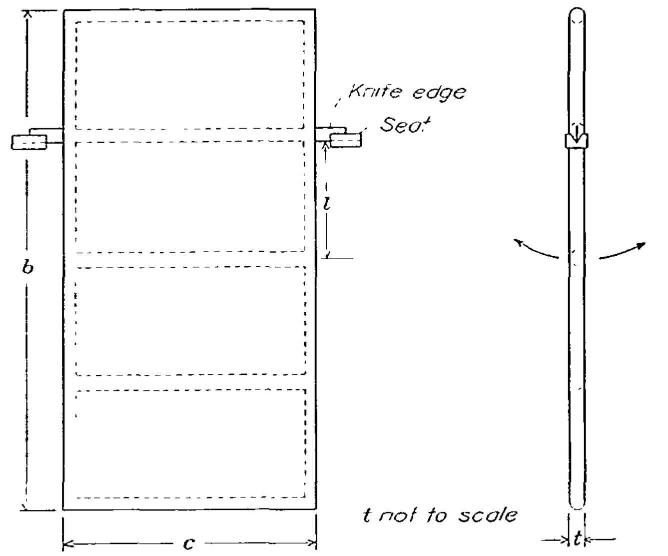

FIGURE 7.—Diagram showing method of suspending plates for tests of the additional mass.

This result might be expected because some air is probably entrapped between the cross members of the frame.

It should be noted that the dispersion in the test points at each value of b/c for the 1933 NACA tests may be due to the fact that each point was obtained for a different suspension length.

The size of the tubes of the frameworks used in the German tests is not reported nor is mention made as to whether the plates were covered on one or both sides. Thus, although no statement can be made as to the additional mass contributed by the uncovered frame, the practice of entirely neglecting the interference effects between the component parts of the partly covered frame is questionable.

The Russian results shown in figure 8 fall below all the other curves. Although good precision is reported for the tests, it should be remembered that the moments of inertia of the plates were computed on the assumption of a uniform density of the plates.

The coefficients of additional mass obtained by the various investigations are presented in figure 9. The German results of 1937 are seen to be considerably higher than those obtained from both the German tests of 1930 and the 1933 NACA tests. If, as already pointed out, the plates covered on only one side yield results that are too high, the present NACA tests may be seen to check reasonably well the results of the 1930 German tests and the 1933 NACA results. The British test point checks neither of the curves although, if the result were corrected for buoyancy (which the British author apparently did not do), the corrected value of k would be of the order of 1.0.

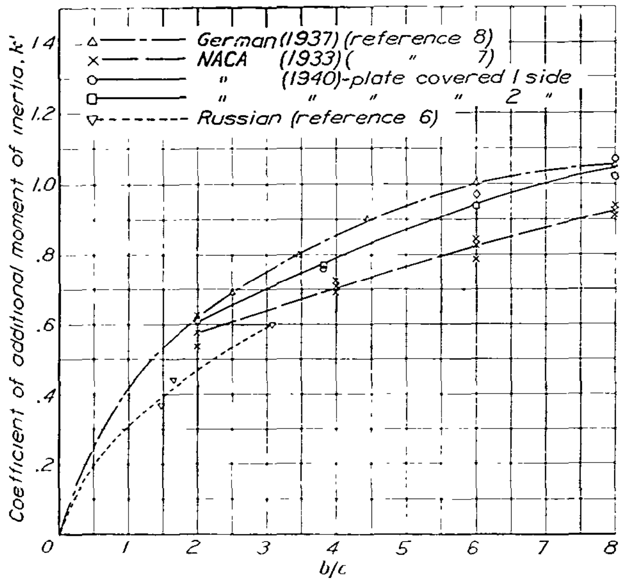

FIGURE 8.—Coefficients of additional moment of inertia for rectangular plates.

A theoretical curve from reference 2 for the additional mass of elliptic plates is included in figure 9. The fact that the German data of reference 8 are as much as 20 percent above this curve (and 18 percent above 1.0) at the higher values of b/c indicates the probability that these experimental curves may be in error. It may be noted in passing that the same values of the additional mass of elliptic plates may be obtained by the use of either the theoretical values of k on this figure or the values of the coefficient C_z of figure 3.

The additional moment of inertia should theoretically be independent of the distance from the axis of rotation to the plane of the plate because displacement of the axis should result only in an additional component of motion parallel to the plane of the plate. Experimental confirmation of this assumption was obtained from the 1933 NACA tests; although the values of the additional moment of inertia were found to vary somewhat with suspension length, the variations were inconsistent and were within the experimental error. The values obtained in the British and the 1937 German investigations, on the other hand, are shown in figures 2 and 10 to increase consistently with suspension length. Pleines concluded that the dependence of I_a on l is negligible only for values of l/b less than one-half. In order to eliminate this source of error, the present NACA tests were made with values of l/b of about one-quarter.

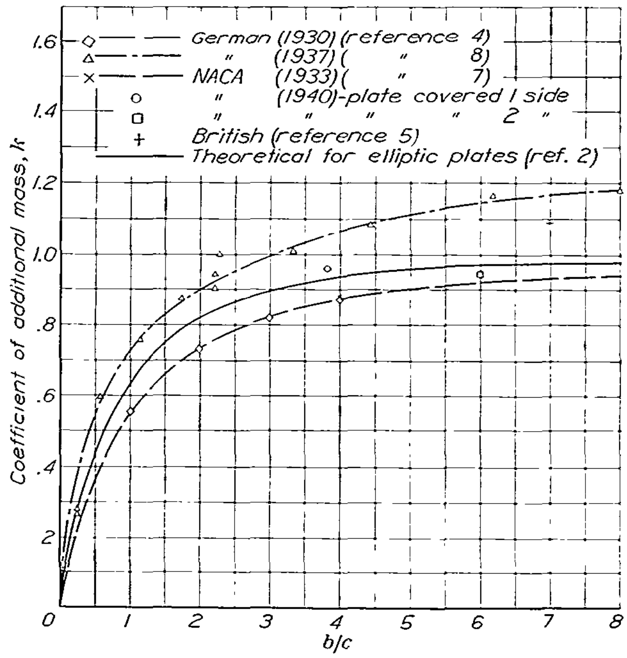

FIGURE 9.—Coefficients of additional mass for rectangular and elliptic plates.

DISCUSSION

The preceding discussion of the different test procedures gives some indication of the variety of methods by which the additional-mass effect may be determined. Variations may arise from differences in methods of suspending the plates, in the choice of the axis of rotation, and in the make-up of the plates themselves.

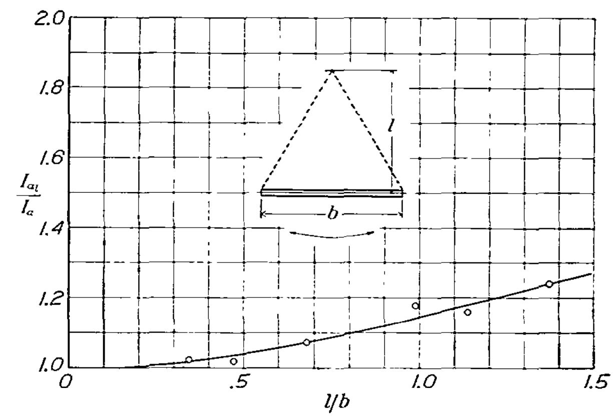

FIGURE 10.—Variation of the additional moment of inertia with suspension length (German tests of 1937; reference 8.)

Equally important are the differences in the methods of determining the moments of inertia of the structures of the plates. The methods of determining I_0 may be summarized as follows:

1. By computation
2. By swinging the uncovered frame in air and adding the moment of inertia of the covering either by com-

putation or by properly accounting for its mass distribution

3. By swinging the plate in a number of air densities

The uncertainty of obtaining accurate results by computation applies both to solid plates, for which a constant density must be assumed, and to frameworks, for which the moment of inertia is found as the summation of the moments of inertia of numerous elements. Method 2 introduces the unknown effects of mutual interference and additional mass of the component parts. If these effects are negligible, however, this method is advantageous because the frame may be tested with and without covering under otherwise comparable conditions. The third method is most easily applied if the pendulum weight and the suspension length remain constant for different air densities. The use of this

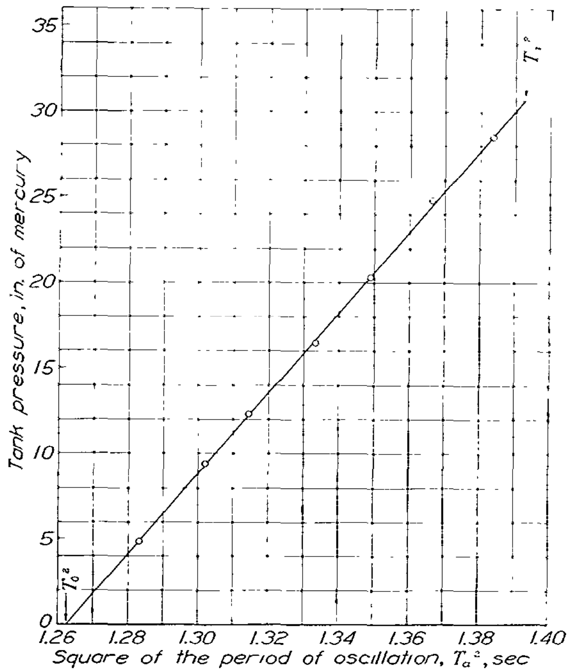

FIGURE 11.—Variation of T_a^2 with tank pressure for plate of aspect ratio 8. (1940 NACA tests.)

method permits the determination of the moment of inertia of the plate in air and in vacuum without changing the make-up of the plate. For this reason, and because the air density may be measured with good precision, this method is believed to yield more accurate measurements of the additional-mass effect than either method 1 or method 2.

Suggested explanations to account for the discrepancies shown in figures 8 and 9 include: scale effect, sharpness of the edges of the plates, and the fact that some of the tests were performed in air below atmospheric pressure.

The effect of scale, it is believed, can be eliminated, inasmuch as the present NACA results (obtained by testing small plates) fall between the 1937 German and the 1933 NACA curves (both resulting from tests of relatively large plates).

The effect of the edge shape was investigated by the NACA in 1933. The results, which were not published, showed a negligible variation of k' for plates with round, square, and elliptical edges. These results, however, are at variance with the theoretical data of reference 2 in which it is shown that the additional mass of rectangular cross sections is appreciably higher than for flat plates. The increase depends on the t/c (thickness/chord) ratio and is about 14 percent for $t/c=0.1$. The investigation of edge shape just described was conducted on three plates with dimensions, in inches, ¼ by 12 by 48, for which the theoretical increase is about 4 percent. Inasmuch as the precision of the tests was of this order, the conclusion of a negligible effect of edge shape is accounted for. It should be noted that, for a series of plates of constant thickness, the t/c ratio and consequently the increase in the additional mass varies directly with span-chord ratio. The effect of plate thickness therefore offers a possible explanation for the values of k and k' above 1.0 at the higher aspect ratios, even when the cross sections are not exactly rectangular. Theoretically, of course, plates with circular or elliptical edges (in which case the cross section approximates that of an elongated ellipse) should yield the same additional-mass data as thin plates.

Finally, tests conducted at air densities other than atmospheric should not be invalidated because: (1) The present tests showed linear variations of the square of the period of oscillation with air density, or pressure, the system being isothermal (fig. 11); and (2) the moments of inertia of the plates determined from these variations checked the computed values with reasonably good precision.

A review of the results of the various tests discloses the fact that the data obtained in each investigation fall with good precision along well-defined curves. Although several possible sources of error have been pointed out, it is difficult, because of a lack of certain details of the foreign tests and because of the complexity of the factors involved, to assign to each test the errors pertinent to that investigation. The difficulty of properly evaluating the data of the various tests is therefore obvious. In view of the good precision of each test and in the absence of any definite sources of error, the only conclusion that can be drawn at this time is that the discrepancies in the various results are apparently due to consistent errors, which are probably embodied in differences in the experimental methods and the apparatus (types of plate, means of suspension, etc.). One piece of evidence in support of the assumption of consistent errors may be found in the present NACA tests in which the experimental values of I_0 were found to be consistently higher than the computed values, a fact showing that the experimental curves would have been displaced upward had k and k' been determined on a basis of the computed values for I_0.

APPLICATIONS TO AIRPLANES

BIPLANE EFFECT

The effect on the additional moment of inertia due to the mutual interference of two plates was studied by the German investigators in 1937 and by the NACA investigators in 1933. The results are shown in figure 12 to be in good agreement. The Russian tests also included one determination of this biplane effect. It was concluded from these tests that, for normal gap-chord ratios, the two wings of a biplane may be considered as separate plates.

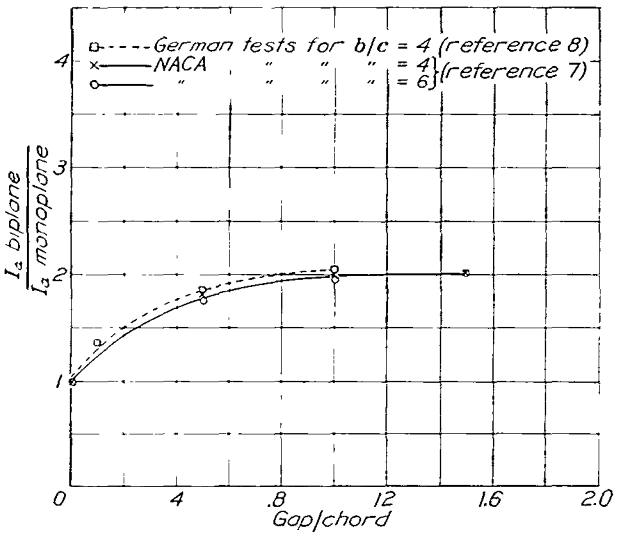

FIGURE 12. Variation of additional moment of inertia with gap-chord ratio for orthogonal biplanes.

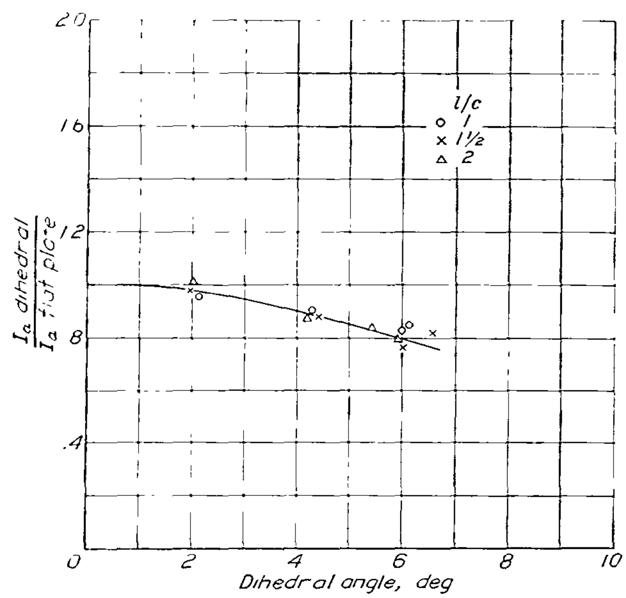

FIGURE 13. Variation of the additional moment of inertia of a single plate with dihedral angle; b/c, 4. (NACA tests of 1933; reference 7.)

DIHEDRAL ANGLE

The effect of dihedral angle on the additional moment of inertia was investigated during the 1933 NACA tests; the results are shown in figure 13. The ratio of the additional moment of inertia of a plate with dihedral to that of a flat plate was found to decrease with the dihedral angle, the decrease being of the order of 20 percent for 6° dihedral. Although the additional moment of inertia might be expected to decrease with dihedral, a decrease of this magnitude is questionable. The British conducted tests with plates having positive and negative dihedral angles of 3½°. Inasmuch as these results were inconsistent, the British authors made no attempt to analyze them, and the results are therefore omitted.

TAPER RATIO

The decrease in the additional moment of inertia with taper ratio as determined by the present NACA tests is given in figure 14. The results are presented as the ratio of the additional moment of inertia of a tapered plate to that of an equivalent rectangular plate. By "equivalent rectangular plate" is meant a rectangle with the same span and area as a given tapered plate. The decrease in I_a is shown to be about 40 percent for a 5:1 taper. This correction is of particular importance for obtaining the true moment of inertia about the longitudinal axis of airplanes with tapered wings.

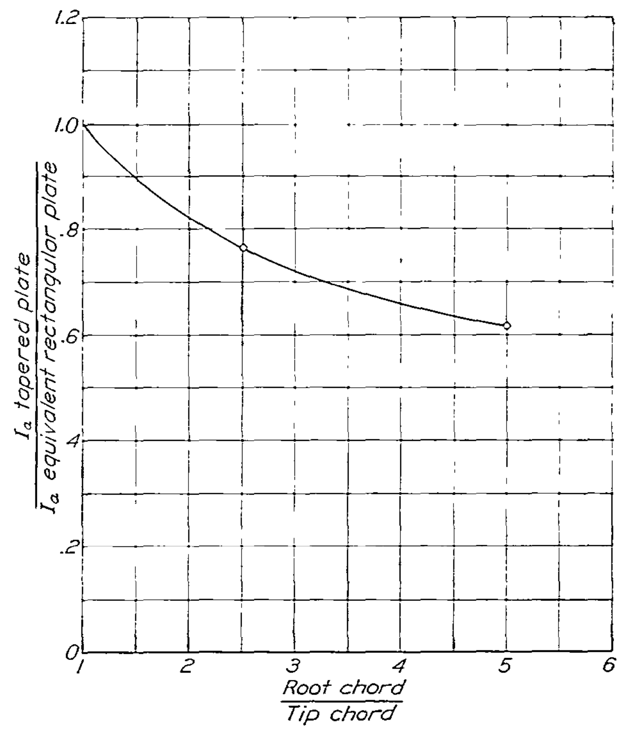

FIGURE 14.—Dependence of the additional moment of inertia on taper ratio.

CONCLUSIONS

1. The results of the present investigation of the additional-mass effect of rectangular plates fall a little above the data obtained by the NACA in 1933 and somewhat below values published in Germany in 1937. Sources of error indicated by previous tests having been avoided in the present investigation, the new results are believed to be the more accurate. The German

TABLE I.—EXPERIMENTAL RESULTS OF TESTS

ADDITIONAL MOMENTS OF INERTIA

Plan form	b/c	Number of sides covered	W_a (lb)	l (ft)	T_a^2 (sec)	T_0^2 (sec)	$\frac{W_a l T_a^2}{4\pi^2}$ (slug-ft²)	I_t (slug-ft²)	I_0 Experimental (slug-ft²)	I_0 Computed (slug-ft²)	I_0 Difference (percent)	I_a (slug-ft²)	k'
Rectangular	2.0	1	0.02130	0.444	0.927	0.8282	22.22×10^{-5}	9.15×10^{-5}	6.78×10^{-5}	6.66×10^{-5}	1.8	2.37×10^{-5}	0.61
Do	3.8	1	.03015	.441	1.622	1.315	55.52	36.86	26.36	25.56	.3	10.50	.76
Do	3.8	2	.03982	.445	1.550	1.291	69.51	44.27	33.51	32.41	3.3	10.73	.77
Do	6.0	1	.02291	.431	1.491	1.271	37.28	21.05	18.57	18.23	1.8	5.48	.97
Do	6.0	2	.02908	.430	1.441	1.270	45.64	28.68	23.50	22.71	3.4	5.18	.94
Do	8.0	1	.02051	.465	1.399	1.255	33.76	19.98	16.51	16.44	.4	3.47	1.07
Do	8.0	2	.02584	.432	1.393	1.263	39.36	24.02	20.70	20.28	2.0	3.32	1.02
Tapered 5 : 1	4.0	1	.02754	.442	1.283	1.085	39.59	22.83	16.72			6.11	.50
Tapered 2.5 : 1	1.0	1	.02789	.445	1.379	1.144	43.38	26.18	18.80			7.38	.61

ADDITIONAL MASS

Plan form	b/c	Number of sides covered	W_a (lb)	l (ft)	T_a^2 (sec)	T_0^2 (sec)	$\frac{W_a l T_a^2}{4\pi^2}$ (slug-ft²)	I_t (slug-ft²)	I_0 Experimental (slug-ft²)	I_0 Computed (slug-ft²)	I_0 Difference (percent)	I_a (slug-ft²)	k
Rectangular	4.0	1	0.03078	0.425	1.982	1.320	65.68×19^{-5}	48.38×10^{-5}	26.44×10^{-5}	25.56×10^{-5}	3.4	21.94×10^{-5}	0.96
Do	6.0	2	.02893	.422	1.585	1.279	48.98	32.52	23.52	22.71	3.4	9.00	.91

values appear erroneously high on the basis of theoretical considerations.

2. The effect of taper ratio on the coefficient of additional moment of inertia was found to be considerable, being of the order of 40 percent for a 5:1 taper.

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National Advisory Committee for Aeronautics,
Langley Field, Va., *July 18, 1940.*

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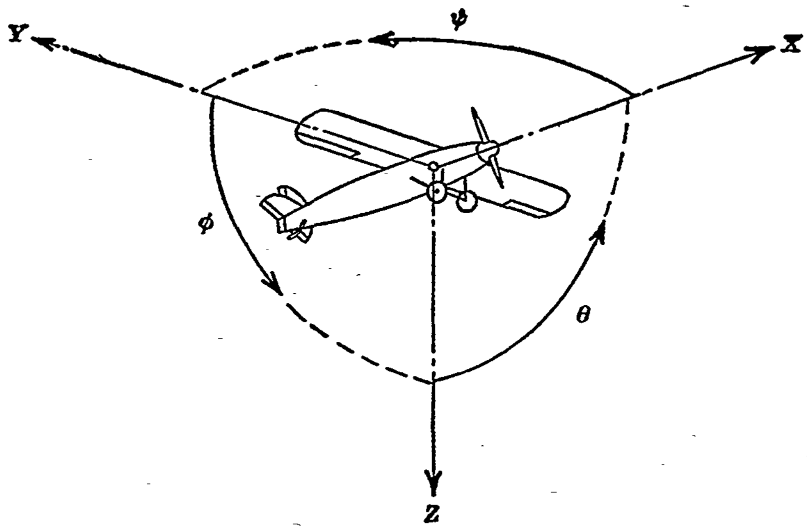

Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	Rolling	L	$Y \longrightarrow Z$	Roll	ϕ	u	p
Lateral	Y	Y	Pitching	M	$Z \longrightarrow X$	Pitch	θ	v	q
Normal	Z	Z	Yawing	N	$X \longrightarrow Y$	Yaw	ψ	w	r

Absolute coefficients of moment

$C_l=\frac{L}{qbS}$ (rolling) $C_m=\frac{M}{qcS}$ (pitching) $C_n=\frac{N}{qbS}$ (yawing)

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

- D Diameter
- p Geometric pitch
- p/D Pitch ratio
- V' Inflow velocity
- V_s Slipstream velocity
- T Thrust, absolute coefficient $C_T=\frac{T}{\rho n^2 D^4}$
- Q Torque, absolute coefficient $C_Q=\frac{Q}{\rho n^2 D^5}$
- P Power, absolute coefficient $C_P=\frac{P}{\rho n^3 D^5}$
- C_s Speed-power coefficient $=\sqrt[5]{\frac{\rho V^5}{P n^2}}$
- η Efficiency
- n Revolutions per second, rps
- Φ Effective helix angle $=\tan^{-1}\left(\frac{V}{2\pi rn}\right)$

5. NUMERICAL RELATIONS

1 hp=76.04 kg-m/s=550 ft-lb/sec
1 metric horsepower=0.9863 hp
1 mph=0.4470 mps
1 mps=2.2369 mph
1 lb=0.4536 kg
1 kg=2.2046 lb
1 mi=1,609.35 m=5,280 ft
1 m=3.2808 ft