RELAXATION TIME MEASUREMENTS FOR COLLISION PROCESSES
IN THE SURFACE LAYERS OF CONDUCTORS
AND SEMICONDUCTORS NEAR 10 GHZ

THESIS

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MASTER OF SCIENCE

By

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This thesis represents one phase of a joint effort of research on the properties of liquids and solids. This work is concerned primarily with the microwave properties of solids. Reference to additional phases of the total work can be found in the reference list of the thesis.
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CHAPTER I

INTRODUCTION

In recent years a number of papers which deal with the microwave properties of material media have appeared. Of particular interest is the property of materials which can have their current carrier density varied. In these type samples the carrier densities can be varied and plasma oscillations can be produced within the "free electron" gas which moves through the samples. These carrier concentrations are best observed in semiconductors.

In this investigation we are primarily concerned with the properties exhibited by conductor and semiconductor materials when they are subjected to electromagnetic radiation of microwave frequency. The method utilized in this experiment is the perturbation of a resonant cavity produced by introduction of a cylindrically shaped sample into it.

It is well known that a resonant cavity can be perturbed in resonant frequency and Q-value by introduction of material into it. The frequency shifts respond to the number of free ions in the cavity and the Q-value changes to the collision rates of these ions.

In this investigation the cavity is cylindrical in shape and the sample placed within the cavity is of cylindrical
geometry wherever practicable. The Slater Perturbation Technique will be employed extensively.¹

Extensive work with loaded cavities has been done since the early work of Slater in 1946.¹ The applications extend from cavities loaded with solids to gases, with microscopic properties of the loading media being inferred from measurement of the resonant frequency shifts and Q-change for the cavity.

In the following paragraphs a summary of some of the research already done by the cavity technique is given to bring the reader abreast of the field of cavity probing of materials.

Although some work has been done by Hall² on microwave properties of materials placed in a resonant cavity, the method employed by him can be improved by the cavity approach proposed herein. His work consisted of an investigation of the electromagnetic properties of plasmas through the use of microwave fields. The experimental method used is that of perturbation of a resonant cavity. The theoretical development centers around cavity perturbation by a spherical sample. However, Appendix B of the paper extends this development to samples of cylindrical shape.

Glover made a study of GaAs using a tapered section of waveguide for a resonant cavity.³ The sample is placed in the waveguide such that it receives maximum microwave power.
The microwave permittivity was found to change as a function of the strength of the microwave field.

Patton and Kohane developed a technique for highly sensitive measurements of microwave susceptibility on samples of ferrite materials. The technique employs a feedback system which locks the klystron signal on the half-power points of a high-Q cavity resulting in a high frequency stability.

Although Kohane explored this idea in conjunction with dielectric loss measurements, its most useful application is in the determination of effective linewidth parameters at magnetic fields far from resonance. However, our primary concern will be in its relation to the complex components of the microwave susceptibility of the sample. And since this technique requires only a small sample, resulting in slight effects on the cavity mode, perturbation theory is applicable.

Sen, Basu, and Ghoshal discussed two methods for determining the resonant behavior of a cylindrical cavity containing a tubular dielectric sample. The first method, exact solution, consists of a rigorous theoretical treatment of the situation. The second method, perturbation theory, with which we are primarily concerned, has for its basis the Slater method. Some of the experimental arrangement given in this paper is similar to that used in this study.
Sen, Basu, Ghoshal applied both perturbation and absolute calculation methods in the determination of the value of the dielectric constant of pyrex glass. A comparison of the two results revealed less than a 2 per cent variance of the perturbation method from the exact method. Therefore the approximate method, perturbation, appears feasible for our investigation so long as the cavity perturbation by the sample is small.

Tompkins and Spencer have investigated the effect of sample size on the frequency shift of a resonant cavity with special emphasis placed on the internal fields of the sample. Their treatment considers the case of cavity magnetic fields only. They observed that sample size has negligible effects on frequency shift as long as the sample is very small in comparison with the wavelength in the material. This allows the assumption to be made that static fields prevail in the perturbation approach to the frequency shift. We will consider sample size as related to the number of carriers available.

Silverman presents a discussion of the temperature dependence of the loss tangent of SrTiO₃ at high temperatures. He observed that the loss tangent increases linearly with temperature at temperatures above liquid air temperature. This property of SrTiO₃ is attributed to the temperature relations of the polarization mode, anharmonic damping,
and imperfection damping. His experimental results agree with the proposed theory.

Green and Kohane have reviewed the important properties of ferrite materials used in the production of microwave devices. They have also described methods and procedures used in the determination of these properties. A discussion is also given as to the effects on ferrite materials at high power levels.

However, our primary concern is with the discussions on perturbation of resonant cavities and determination of complex dielectric constant. Specifically we will try their suggestion of a direct current magnetic bias along the axis of the cylindrical sample which is supposed to reduce to zero the magnetic loss due to the sample. This should increase the accuracy of the $\frac{1}{Q}$ measurement used to determine the imaginary part of the complex dielectric constant.

Srivastava, Mathur, Krishna, and Mehra proposed an empirical approach for improvement of an extrapolation method of determining the dielectric constant of material from the dielectric constant of its crystalline powder. They suggest that Lorentz's expression for the internal E-field of the dielectric material is an overestimation and needs correction terms. They proposed a modification consisting of dividing the Lorentz E-field expression by $(\varepsilon^*)^{1/2}$ where $\varepsilon^*$ is the complex dielectric constant of the material. This revised
expression appears to hold for applications to which they extended the calculations. We will verify the need of a packing factor and look at its relation to carrier density.

Robbins and Ancker-Johnson have investigated the lifetimes of injected electron-hole plasmas in p-type InSb, and observed these lifetimes to be dependent upon the injected plasma density. It was found that the lifetime was quite short for low densities but generally increased as the density increased. However, beyond a certain density the lifetime began decreasing.

The paper presents a detailed explanation of the above observations, which involves the theories of carrier trapping in the bulk, surface recombination, and radiative recombination. The radiative recombination was dismissed after experimentation due to its negligible effect.

A paper by Ernst Bonek deals with the effect that an applied direct or alternating current electric field has on the relaxation time of a semiconductor. The semiconductors in his study were InSb, GaSb, InAs, and GaAs. The relaxation effects were observed using a perturbation technique in which the sample was placed over a slit cut into a rectangular cavity. Both alternating current and direct current fields were applied with similar results. Application of the field parallel to the microwave field produced the greatest conductivity. This was due to the lack of additional carrier heating by the microwave field.
It was found that relaxation effects are both temperature- and frequency-dependent. These properties are attributed to the change in the energy and momentum distributions of the carriers. However, the energy relaxation time remained constant while the momentum relaxation time decreased with increasing field at higher frequencies.

Eldumiati and Haddad investigated the properties of n-type InSb when subjected to microwave and direct current magnetic fields. This approach eliminated the hot-electron effect for temperatures above 77 degrees Kelvin.\(^1\)\(^2\)

The conductivity change of the sample was found to be dependent on magnetic field strength and temperature. Magnetic field orientation effects with respect to the sample were also observed.

It was also found that the dielectric constant is temperature-independent between 77 degrees Kelvin and 290 degrees Kelvin with no applied magnetic field. However, at 4.2 degrees Kelvin the dielectric constant seems to decrease with increasing magnetic field strength.

Loschner's and Kranzer's investigation considers the discrepancy of Drude's theory and experiment, while proposing a possible solution.\(^1\)\(^3\) The article claims that Drude's error lay in neglecting statistical effect. These are taken into account in the proposed theory, which is confirmed by experiment over a range of 83 degrees Kelvin to 170 degrees
Kelvin. An n-type InSb sample was investigated by Loschner and Kranzer.

It is with the above in mind that we will determine the effect on carrier density due to a change in carrier energy. The use of a direct current electric field to vary the carrier energy is not practical for our investigation. However, we can accomplish this end through changing the temperature of the sample by passing nitrogen vapor through the cavity and out the iris around the sample.

The Micro-Now paper advertises a microwave spectrometer for the measurement of the photoconductivity of various materials. Operation is based on the Q-change of the cavity due to the presence of free-electrons caused by photoconductivity. The spectrometer uses a Gunn Diode Oscillator for its source. This type of source improves the frequency stability of the klystron because of its low heat dissipation.

From the foregoing discussion the reader can see that application of the resonant cavity in the laboratory can lead to useful and worthwhile results. Specifically, by measuring the resonant frequency shifts the number of charge carriers can be determined, and by measuring the Q-changes the energy transfer mechanisms can be ascertained. Therefore alterations in carrier density and energy can be utilized in determining the characteristics of various materials with various conditions. We will use this property to display some of the resulting effects.
Further details of the experimental arrangements will be found in Chapter II.
CHAPTER II
EQUIPMENT AND DESIGN

In this chapter we will discuss the various components of the experimental equipment utilized, as well as give a brief summary of the theory behind their operation. Further information concerning these devices can be obtained from various textbooks on microwave experimentation. A complete block diagram of the experimental arrangement is given in Figure 1

Reflex Klystron

For our source of microwave radiation we employed a reflex klystron which operates on a variation of the klystron principle of electron velocity and amplitude modulation. The reflex klystron employs the use of a resonant cavity and reflector to provide a feedback necessary for oscillation. A V-58 type reflex klystron was used for this investigation.

Frequency Multiplier

A frequency multiplier was used for producing a frequency comparable to that produced by the klystron in order that a mixing process could be established to produce a frequency difference which could be received by the radio receiver. The frequency multiplier used in this experiment
FIGURE 1. Block Diagram of the Experimental Arrangement of the Apparatus used to make this study.
is a model 900 B Sweep Signal Generator made by Jerrold Electronics.

Frequency Marker Detection

The above mentioned process of heterodyning the two frequencies was carried out for the purpose of producing two reference frequency markers to be used in the measurement of resonant frequency shift and cavity Q-change. This process involved beating the klystron frequency against a multiplier frequency in order to obtain the markers from the mixing involved. A radio receiver was employed to detect these frequency markers and thereby provide an accurate determination of their frequency. The receiver then provides a modulating signal for the oscilloscope in order to display these markers which are used in an investigation of the resonant absorption curve.\(^{15-17}\)

The frequency of separation of the above mentioned markers can be determined by using

\[ f = f_r + n f_s \]

where \( f_r \) is the radio receiver frequency, \( f_s \) is the modulated klystron frequency, \( f_m \) is the frequency multiplier, and \( n \) is the order of the harmonic.

The radio receiver used in this detection was a Model HQ-180-A made by Hammarland. For our purpose the frequency region ranging from zero to 36MHz was employed.
Detector Response

Silicon crystal diodes were used in this experiment for microwave power detection and for signal mixing purposes involved in modulation. The detectors were coupled with the circuit through directional couplers.

Tuned Amplifiers

The output signal at the detector was not sufficiently strong for accurate measurements at low power levels; therefore, a tuned amplifier was included in the circuit to increase the strength of the signal and to reduce the amount of noise. The amplifier was a high gain high-Q unit designed to reduce noise components not near the modulation frequency of the signal driving the reflex klystron.

Waveguide System

The following discussion gives a description of the components used to transport, control, and measure the microwave radiation produced by the reflex klystron. Care has been taken in the choice of these components in order to reduce any unwanted losses and waveguide leakages.

Directional Couplers

Two directional couplers were utilized in this experiment. The first, a cross coupler, was employed to serve as a mixer and multiplier in the creation of the frequency markers, while the second, a reflectometer, aided in a comparison of the initial and reflected signal.
Resonant Cavity

The resonant cavity chosen for our investigation is cylindrical in geometry and is silver coated on the inner surface in order to reduce losses. The TM_{010} mode was used for our study. The sample was advanced along the cylinder axis where the electric field is a maximum.

The Cavity Fields

Our cavity was operated in the TM_{010} mode. This mode was chosen for its simplicity in structure as well as for its applicability to our particular requirements. The mode provided an electric field which varied from a maximum at the center of the cavity to zero at the cavity walls.\textsuperscript{18} This characteristic is ideal for perturbation experiments in which the perturbing sample is introduced into the center of the cavity. A sketch of the fields is given in Figure 2.

Signal Shape

The shape of the signal produced by the reflex klystron is that of a power spectrum representing the frequency components transmitted. See Figure 3(a). This shape is then broadened through the application of a horizontal sawtooth sweep voltage provided by the oscilloscope. See Figure 3(b). The addition of a resonant cavity to the waveguide system causes a power absorption at its resonant frequency. See Figure 3(c). This depression represents cavity resonance absorption caused by reflection of the signal from
Electric Field

Magnetic Field

FIGURE 2. Diagram of Fields for TM_{0,10} Mode.
FIGURE 3. Nature of the Microwave Signals as each change is made in preparing the Apparatus for data taking.
the added cavity. This absorption signal is of primary importance in our investigation since its distortion is related to the type of material used in loading the cavity.

The final shape of the signal is one which has been differentiated through modulation using a high frequency sine wave voltage. See Figure 3(d). The purposes being convenience in measurement and an increase in accuracy of obtaining half-power points for the resonance signal.

A discussion of measurements and their relation to theory will be found in Chapter III.
CHAPTER III

THEORY AND MEASUREMENTS

In this chapter a discussion is given of the procedure for obtaining measurements, as well as development of the theory involved in the interpretation of these measurements. Some properties of the samples under consideration are also related to parameters in the Slater Perturbation Equation for resonant cavities.

We will begin our discussion with a development of the equations employed in the "Slater Perturbation Technique."

From Maxwell's equations for a hollow cavity with sample we can obtain the general expression for the wave equation

\[
\varepsilon_0 \mu_0 \frac{d^2}{dt^2} \int E \cdot E_a dV + k^2 \int E \cdot E_a dV = -\mu_0 \frac{d}{dt} \left( \int J \cdot E_a do - \int (n \times H) \cdot E dS' \right) - k \int (n \times E) \cdot H dS
\]

(1)

where \( \varepsilon_0 \) represents the dielectric constant in a vacuum.

\( \mu_0 \) is the permeability in a vacuum. \( \dot{E} \) represents the electric field applied to the cavity, \( \dot{E}_a \) is the electric field of the unperturbed cavity, and \( k_a \) is the wavenumber. \( \dot{J} \) is the current density of the sample. \( \tau \) is the sample volume, \( V \) the cavity volume, and \( S, S' \) the surfaces of cavity.
Now, our cavity presents a situation analogous to an LRC series circuit,

\[ L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{q}{c} = 0 \]  

(2)

From which we can obtain

\[ \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} - j \frac{1}{Q} = 0 \]  

(3)

where \( \omega_0 = \frac{1}{\sqrt{LC}} \) represents the angular frequency and \( \frac{1}{Q} = \frac{R}{Q} \frac{1}{L\omega_0} \) is the Q-value of the circuit. This will be discussed in further detail in this chapter.

Now, for a small perturbation the angular frequency may be expressed as

\[ \omega_0 = \omega + \Delta \omega \]  

(4)

where \( \omega \) is one of the resonant frequencies. Therefore, equation (3) becomes to a close approximation

\[ j \left( \frac{\omega}{\omega} - \frac{\omega_0}{\omega} \right) + \Delta \frac{1}{Q} - 2j \frac{\Delta \omega}{\omega} = 0. \]  

(5)
Now, assuming a time-dependent solution varying as $j t$ for equation (1) and combining equations (1) and (5) we obtain

\[
\begin{align*}
\Delta - & \frac{-2j}{Q} \frac{\Delta \omega}{\omega_a} = - \frac{j}{\omega_a (\varepsilon_0 \mu_0)^{\frac{1}{2}}} \int \frac{(\mathbf{n} \cdot \mathbf{E}) \cdot \mathbf{H}_a}{\mathbf{E} \cdot \mathbf{E}_a} dS \\
& - \frac{1}{\varepsilon_0} \frac{1}{\omega_a} \int \frac{\mathbf{J} \cdot \mathbf{E}_a}{\mathbf{E} \cdot \mathbf{E}_a} d\tau.
\end{align*}
\]

(6)

For our particular situation in which the perturbation is one of currents produced within the cavity by the introduction of a sample, we assume perfectly conducting cavity walls, thereby eliminating the surface integrals from equation (6). The resulting equation is

\[
\begin{align*}
\Delta - & \frac{-2j}{Q} \frac{\Delta \omega}{\omega} = \frac{1}{\varepsilon_0} \frac{1}{\omega} \int \frac{\mathbf{J} \cdot \mathbf{E}_a}{\mathbf{E} \cdot \mathbf{E}_a} d\tau.
\end{align*}
\]

(7)

The currents from the sample are related to its polarization $P$

\[
\dot{P} = (\varepsilon - \varepsilon_0) \hat{E}_s = \varepsilon_0 (\varepsilon' - j\varepsilon'' - 1) \hat{E}_s
\]

(8)

where $\hat{E}_s$ is the local field of the sample and $\varepsilon' - j\varepsilon''$ is the complex dielectric constant of the sample. This will be discussed in further detail later in this chapter.
Now from Maxwell's Equations

\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial}{\partial t} \left[ \varepsilon_0 \mathbf{E} + \mathbf{P} \right] = \mathbf{J} + j\omega \left[ \varepsilon_0 \mathbf{E} + \mathbf{P} \right]. \quad (9) \]

Therefore from equations (7), (8), and (9)

\[ \frac{1}{Q} - \frac{2j}{\omega_a} \frac{\Delta \omega}{a} = j \left[ (\varepsilon' - 1) - j\varepsilon'' \right] \frac{\int \mathbf{E}_s \cdot \mathbf{E} dV}{\int \mathbf{E} \cdot \mathbf{E} dV}. \quad (10) \]

Equating real and imaginary parts of equation (10) we obtain Slater's Perturbation equations in a form suitable to our particular investigations.

\[ \frac{\Delta f}{a} = -\frac{\varepsilon' - 1}{2} \frac{\int \mathbf{E}_s \cdot \mathbf{E} d\tau}{\int \mathbf{E} \cdot \mathbf{E} dV}. \quad (11) \]

\[ \frac{\Delta}{Q} = \varepsilon'' \frac{\int \mathbf{E}_s \cdot \mathbf{E} d\tau}{\int \mathbf{E} \cdot \mathbf{E} dV}. \]

where \( f \) represents the resonant frequency of the cavity and the other parameters have been defined earlier.

The introduction of a sample into the resonant cavity causes a shift in the resonant frequency and changes the "Q-value of the cavity", as exemplified by equation (5).

The shift in resonant frequency is dependent upon the location
of the perturbing sample. In this case the perturbing sample is advanced along the symmetry axis of the cavity. The resultant change in the resonant frequency is monitored as the depth of penetration is varied. The changes of the "Q-value of the cavity" are due to energy exchanges between the cavity and the perturbing sample. These two parameters, resonant frequency shift and Q-change, are related to the real and imaginary parts of the permittivity of the perturbing sample.

The permittivity or dielectric constant of the sample is a complex quantity arising from the "current" in the sample. This property arises as follows from Maxwell's equations

\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} = \sigma \mathbf{E} + j \omega \varepsilon \varepsilon' \mathbf{E} = j \omega \varepsilon_0 (\varepsilon' - j \varepsilon'') \mathbf{E} \]

where \( \varepsilon = \varepsilon_0 (\varepsilon' - j \varepsilon'') = \varepsilon_r \varepsilon_c \). \( \mathbf{H} \) represents the magnetic field, \( \mathbf{D} \) is the electric field, and \( \mathbf{J} \) is the current density. \( \varepsilon_r \) is the relative dielectric constant and \( \varepsilon_c \) represents the complex dielectric constant of the sample. \( \sigma \) is the conductivity of the sample.
In the complex permittivity term \((\varepsilon' - j\varepsilon'')\), \(\varepsilon'\) is related to the energy storing ability of the dielectric material whereas \(\varepsilon''\) is related to the dissipation of energy within the material.\(^\text{16}\)

A discussion of the measurements required in our investigation to analyze the parameters \(\varepsilon'\) and \(\varepsilon''\) is given in the following two paragraphs.

The measurements for resonant frequency shift consisted of placing the sample at a given depth and determining the resonant frequency. The resonant frequency shift versus penetration depth is plotted to make a comparison of initial resonant frequency and the resonant frequency due to perturbation.

The measurement of the Q-change of the cavity consisted of recording the sample depth versus the frequency separation of the two half-power points of the resonance absorption curve. The difference of the two frequencies is the "linewidth" of the resonance absorption curve. The Q-change of the cavity is then given by a comparison of the initial Q-value and the Q-value due to a certain depth of penetration of the perturbing sample. Plots of these two parameters \(\Delta f\) versus depth and \(\Delta \frac{1}{Q}\) versus depth are given in Figures 4 through 19.

We will now consider the "local fields" in the sample and their effect on the "Slater Perturbation Equations." According to Lorentz's Model, the total electric field acting at a point within the sample is composed of electric field
components due to the external electric field, the polarization of the sample, and the interaction of atoms and molecules within the sample.\textsuperscript{20} The last of these can be neglected provided the crystal structure of the sample is of simple cubic, body-centered cubic, or face-centered cubic symmetry.\textsuperscript{21} Lorentz's Model applies to spheres, whereas Kittel related a generalization for the class of ellipsoidal shapes.\textsuperscript{21} From this we find for our particular situation (cylindrical samples) that there is no electric field contribution, due to the polarization of the sample. Therefore, the "local field" in the sample is merely the external electric field applied to the sample.

Next we consider the polarizability of the sample and its effect on the Slater Perturbation Equations.\textsuperscript{19} We define

\[
p = \alpha E_{\text{local}} \tag{13}
\]

where \( p \) represents the dipole moment and \( \alpha \) represents the total polarizability. The total polarizability consists of the sum of the polarizabilities due to electronic, ionic, orientational, and interfacial polarization. These types of polarization are frequency dependent at higher frequencies because of the inertias of the dipoles responsible for their effects.\textsuperscript{21}
The total polarization of the sample is then given by

\[ \mathbf{P} = N\mathbf{a}\mathbf{E} \]  

(14)

where \( N \) is the number of molecules per unit volume. From equations (6) and (11)

\[ \mathbf{P} = (\varepsilon - \varepsilon_0) \mathbf{E} = N\mathbf{a}\mathbf{E} \]

\[ \varepsilon_0 \mathbf{r} = N\mathbf{a} \]

(15)

\[ \varepsilon_r = \frac{N\mathbf{a}}{\varepsilon_0} + 1 \]

Now using the simple harmonic oscillator equation to describe the motion of the dipoles of a dielectric in an alternating field, we obtain a polarizability expression which is frequency-dependent as well as complex.

\[ \hat{\alpha} = C \left[ \frac{\omega_p^2 - \omega^2}{(\omega_p^2 - \omega^2)^2 + b\omega^2/m^2} - j \frac{b\omega/m}{(\omega_p^2 - \omega^2)^2 + b^2\omega^2/m^2} \right] \]

(16)

where \( C \) is a constant dependent upon the dipoles in question. \( \omega_p \) represents the natural frequency of oscillation of the dielectric, \( \omega \) is the applied frequency of oscillation and \( m \) is the mass in question. \( b \) represents a damping constant.
Therefore,
\[ \hat{\alpha} = \alpha' - j\alpha'' \]  \hspace{1cm} (17)

Substituting equation (17) into equation (15),
\[ \varepsilon_r = 1 + \frac{N (\alpha' - j\alpha'')}{\varepsilon_0} \]
\[ = (1 + \frac{Na'}{\varepsilon_0}) - j \frac{jNa''}{\varepsilon_0} \]  \hspace{1cm} (18)
\[ \varepsilon_r = \varepsilon' - j\varepsilon'' \]

We can see from a comparison of equation (11), (16), and (18) that the dielectric constant is dependent upon frequency. This brings us to our next topic of consideration: the natural frequency of materials and how it can be changed.

The natural frequency of oscillation of a material is the angular frequency at which the atoms of the material resonate. For our particular situation, in which we are dealing with a contained free electron gas, the natural frequency is referred to as the plasma frequency. This frequency is dependent upon the number of carriers, as seen from equation (15) and (16). In semiconductors, with which we are concerned, this carrier density can be varied by the temperature of the material or by the amount of impurity.
We intend also to observe the effects of applying an external static magnetic field to the sample. Computations for this case require further consideration of the Slater approach. In particular, it is necessary to include the impressed magnetic field given in equation (9). The magnetic field due to the microwave field is assumed to be very nearly zero over the sample, as its profile for the $\text{TM}_{0,1,0}$ mode is expressed by $J_1 (kr)$. Impressing the external magnetic field requires that both magnetic and electric fields within the sample be considered. It is not one of the objectives of this experiment to determine precisely how collision times depend upon the impressed magnetic field, but to show that they do. Additional work should be done in which the magnetic field is varied in a systematic way and these results are used to deduce a model for the behavior of the "collision times" as a function of applied magnetic field.
CHAPTER IV

CONCLUSION

Some of the conclusions which can be reached from the results of this investigation are given below.

The surface carrier density of a material varies according to the macroscopic and microscopic properties of the material. These properties involve such items as mass density, purity, surface area, skin depth, and environment. We have investigated the reactions of several materials submitted to a change in one or more of the above properties.

The effect of mass density on the number of carriers in the sample "skin" is exemplified by the necessity of a packing factor when dealing with powders, as was shown in the case of Cu$_2$O. This "packing effect" was noted in the case of carbon which was compressed by different degrees. It seems that density change causes the skin depth to become an important factor in the conductivity of the powder samples.

Sample surface area also affected the number of carriers interacting with the microwave field, as was observed in the case of carbon rods of different diameters. Changes would have been more significant with a larger variation in cross-section of the samples, but the perturbation condition would have been violated for samples which are too large.
Next, the effect of etching a semiconductor and two conductors was examined. It is interesting to note that the results of etching display an opposite effect on the conductivity of the two different types of samples. The microwave conductivity appeared to increase in the conductor while it decreased in the semiconductor. It seems that the etching process removed carriers from the semiconductor while decreasing the "resistance" of the conductor surface. Further etching of the In Sample produced little or no effect.

Finally we considered the situation in which the sample is enveloped in an external magnetic field. The field was applied both parallel and perpendicular to the axis of the sample, with the latter resulting in the greatest conductivity change. The motion of the carriers when the field is perpendicular to the sample seems to decrease the depth of penetration for the microwave into the sample. The magnetic field was approximately 500 Gauss.

Table I is a list of laboratory relaxation times calculated from a ratio of resonant width change to resonant frequency shift. The relaxation time relation can be obtained from a comparison of equation (11) and the Kramers-Kronig dispersion relations.

We attempted the temperature study of the carriers but were unable to obtain satisfactory results due to unexpected complications. The geometry of the cavity was altered sufficiently by small temperature changes to cause inconsistent
TABLE I

EXPERIMENTAL COLLISION TIMES FOR MOMENTUM TRANSFER AMONG CURRENT CARRIERS IN SOLIDS AT 300° KELVIN

<table>
<thead>
<tr>
<th>Sample</th>
<th>Collision Time ((10^{-12}) \text{ second})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pb</td>
<td>1.41</td>
</tr>
<tr>
<td>Cu</td>
<td>0.42</td>
</tr>
<tr>
<td>Al</td>
<td>0.75</td>
</tr>
<tr>
<td>In</td>
<td>1.49</td>
</tr>
<tr>
<td>CuO</td>
<td>3.74</td>
</tr>
<tr>
<td>Cu\textsubscript{2}O</td>
<td>0.14</td>
</tr>
<tr>
<td>C</td>
<td>1.81</td>
</tr>
<tr>
<td>n-InSb</td>
<td>3.70\textsuperscript{a}</td>
</tr>
<tr>
<td></td>
<td>4.30\textsuperscript{b}</td>
</tr>
<tr>
<td></td>
<td>5.65\textsuperscript{c}</td>
</tr>
<tr>
<td></td>
<td>7.25\textsuperscript{d}</td>
</tr>
</tbody>
</table>

\textsuperscript{a} - sample was un-etched
\textsuperscript{b} - sample etched
\textsuperscript{c} - magnetic field of 500 Gauss applied such that \(B\parallel\) microwave E-field
\textsuperscript{d} - magnetic field of 500 Gauss applied such that \(B\perp\) microwave E-field
frequency shifts. A possible solution to this problem would be to attach the sample to a heat sink, excluding the cavity in the cooling process.

Studies of this type are necessary in determining the qualities of usable materials for microwave applications as well as their behavior under the stress to which they are to be submitted.

Future consideration in this field could involve varying the carrier density by altering the energy of the carriers through temperature change, direct current electric field application, infrared irradiation, and carrier impact.\(^2\) The amount of impurity doping appears to be significant, and the effect of impurity doping could be explored in the future. A theoretical investigation into these properties would also be beneficial in understanding the microscopic effects involved in producing the results appearing in this macroscopic behavior.
APPENDIX

Plots of resonant frequency shift versus penetration length and resonant width versus penetration length of samples considered. The last plot is resonant width versus resonant frequency shift.
FIGURE 4

Resonant frequency shift versus penetration length for Cu$_2$O. Bold lines represent a computer least square fit to the polynomial $\Delta F = A + Bx^N$.

For:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loosely packed</td>
<td>1.98 x 10$^4$</td>
<td>1.50 x 10$^4$</td>
<td>1.62</td>
</tr>
<tr>
<td>Firmly packed</td>
<td>1.82 x 10$^4$</td>
<td>2.11 x 10$^4$</td>
<td>1.63</td>
</tr>
</tbody>
</table>
Loosely Packed
Firmly Packed

FIGURE 4. Cu$_2$O
Resonant width change versus penetration length for Cu$_2$O. Bold lines represent a computer least square fit to the polynomial $\Delta W = A + Bx^N$.

<table>
<thead>
<tr>
<th>For:</th>
<th>A</th>
<th>B</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loosely packed</td>
<td>$5.18 \times 10^2$</td>
<td>$1.17 \times 10^1$</td>
<td>1.67</td>
</tr>
<tr>
<td>Firmly packed</td>
<td>$1.84 \times 10^2$</td>
<td>$2.81 \times 10^1$</td>
<td>2.64</td>
</tr>
</tbody>
</table>
Loosely Packed ●
Firmly Packed ○

FIGURE 5. Cu$_2$O
FIGURE 6

Resonant frequency shift versus penetration length for Carbon. Bold lines represent a computer least square fit to the polynomial \( \Delta F = A + Bx^N \).

For:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Soft</td>
<td>5.81 x 10^3</td>
<td>7.49 x 10^3</td>
<td>3.19</td>
</tr>
<tr>
<td>Medium</td>
<td>2.89 x 10^3</td>
<td>9.41 x 10^3</td>
<td>2.88</td>
</tr>
<tr>
<td>Hard</td>
<td>7.19 x 10^3</td>
<td>1.22 x 10^4</td>
<td>2.82</td>
</tr>
</tbody>
</table>
Figure 6. Carbon

Very Soft •
Medium ×
Hard ○

Resonant Frequency Shift, $\Delta f$ (MHz)

Sample Penetration Length (mm)
FIGURE 7

Resonant width change versus penetration length for Carbon. Bold lines represent a computer least square fit to the polynomial $\Delta W = A + Bx^N$. 

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Soft</td>
<td>$-3.1 \times 10^1$</td>
<td>$7.73 \times 10^2$</td>
<td>3.71</td>
</tr>
<tr>
<td>Medium</td>
<td>$5.58 \times 10^2$</td>
<td>$1.61 \times 10^3$</td>
<td>2.91</td>
</tr>
<tr>
<td>Hard</td>
<td>$9.74 \times 10^2$</td>
<td>$1.38 \times 10^3$</td>
<td>3.35</td>
</tr>
</tbody>
</table>
SAMPLE PENETRATION LENGTH (mm)

RESONANT WIDTH CHANGE, ΔW (MHz)

Very Soft •
Medium ×
Hard ○

FIGURE 7. Carbon
FIGURE 8

Resonant frequency shift versus penetration length for Carbon "Medium". Bold lines represent a computer least square fit to the polynomial $\Delta F = A + Bx^N$.

For:

<table>
<thead>
<tr>
<th>Diameter</th>
<th>$A$</th>
<th>$B$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.91 mm</td>
<td>$2.17 \times 10^3$</td>
<td>$1.22 \times 10^4$</td>
<td>2.58</td>
</tr>
<tr>
<td>1.20 mm</td>
<td>$2.89 \times 10^3$</td>
<td>$9.41 \times 10^3$</td>
<td>2.88</td>
</tr>
<tr>
<td>1.98 mm</td>
<td>$5.32 \times 10^3$</td>
<td>$4.63 \times 10^3$</td>
<td>3.35</td>
</tr>
</tbody>
</table>
0.91 mm diameter •
1.20 mm diameter ×
1.98 mm diameter ○

SAMPLE PENETRATION LENGTH (mm)

FIGURE 8. Carbon "Medium"
FIGURE 9

Resonant width change versus penetration length for Carbon "Medium". Bold lines represent a computer least square fit to the polynomial $\Delta W = A + Bx^N$.

For:

<table>
<thead>
<tr>
<th>Diameter</th>
<th>A</th>
<th>B</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.91 mm</td>
<td>5.35 x 10^2</td>
<td>1.05 x 10^3</td>
<td>3.58</td>
</tr>
<tr>
<td>1.20 mm</td>
<td>5.58 x 10^2</td>
<td>1.61 x 10^2</td>
<td>2.91</td>
</tr>
<tr>
<td>1.98 mm</td>
<td>-9.08 x 10^1</td>
<td>1.53 x 10^3</td>
<td>3.05</td>
</tr>
</tbody>
</table>
0.91 mm diameter •
1.20 mm diameter ×
1.98 mm diameter ○

SAMPLE PENETRATION LENGTH (mm)

FIGURE 9. Carbon "Medium"
FIGURE 10

Resonant Frequency Shift for a resonant cavity loaded with select lengths of fixed cross-section carbon samples. The horizontal curves represent the resultant frequency shift produced by the carbon volume alone. The other curves represent frequency shifts due to the teflon container plus carbon sample. The bold lines represent an average behavior for the cavity resonance.
SAMPLE PENETRATION LENGTH (mm)

FIGURE 10. Carbon "Medium" Diameter: 1.214 mm.
FIGURE 11

Resonant frequency shift versus penetration length for InSb. Bold lines represent a computer least square fit to the polynomial $\Delta F = A + Bx^N$.

For:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Un-etched</td>
<td>$5.52 \times 10^3$</td>
<td>$2.52 \times 10^2$</td>
<td>4.71</td>
</tr>
<tr>
<td>Etched</td>
<td>$3.03 \times 10^3$</td>
<td>$1.33 \times 10^3$</td>
<td>4.09</td>
</tr>
</tbody>
</table>
FIGURE 11. InSb

SAMPLE PENETRATION LENGTH (mm)

RESONANT FREQUENCY SHIFT, $\Delta F$ (MHz)

Un-etched •
Etched ○
FIGURE 12

Resonant width change versus penetration length for InSb. Bold lines represent a computer least square fit to the polynomial $\Delta W = A + Bx^N$.

For:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Un-etched</td>
<td>7.49 x $10^2$</td>
<td>2.52 x $10^2$</td>
<td>4.17</td>
</tr>
<tr>
<td>Etched</td>
<td>1.07 x $10^3$</td>
<td>8.71 x $10^2$</td>
<td>3.31</td>
</tr>
</tbody>
</table>
**FIGURE 12. InSb**

Resonant Width Change, $\Delta W$ (MHz) vs. Sample Penetration Length (mm) for Un-etched and Etched samples.
FIGURE 13

Resonant frequency shift versus penetration length for InSb. Bold lines represent a computer least square fit to the polynomial $\Delta F = A + Bx^N$.

For:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perpendicular Magnetic Field</td>
<td>$1.27 \times 10^3$</td>
<td>$6.89 \times 10^3$</td>
<td>3.27</td>
</tr>
<tr>
<td>Parallel Magnetic Field</td>
<td>$1.28 \times 10^3$</td>
<td>$1.64 \times 10^4$</td>
<td>2.69</td>
</tr>
</tbody>
</table>
Perpendicular Magnetic Field
Parallel Magnetic Field

FIGURE 13. InSb
FIGURE 14

Resonant width change versus penetration length for InSb. Bold lines represent a computer least square fit to the polynomial $\Delta W = A + Bx^N$.

For:

<table>
<thead>
<tr>
<th>Magnetic Field</th>
<th>A</th>
<th>B</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perpendicular</td>
<td>$7.13 \times 10^1$</td>
<td>$5.39 \times 10^3$</td>
<td>2.26</td>
</tr>
<tr>
<td>Parallel</td>
<td>$-1.13 \times 10^2$</td>
<td>$9.09 \times 10^3$</td>
<td>1.61</td>
</tr>
</tbody>
</table>
Perpendicular Magnetic Field ●
Parallel Magnetic Field ○

FIGURE 14. InSb

SAMPLE PENETRATION LENGTH (mm)

RESONANT WIDTH CHANGE, \( \Delta W \) (MHz)
FIGURE 15

Resonant frequency shift versus penetration length for In. Bold lines represent a computer least square fit to the polynomial $\Delta F = A + Bx^N$.

For:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Un-etched</td>
<td>$9.95 \times 10^2$</td>
<td>$7.06 \times 10^3$</td>
<td>2.88</td>
</tr>
<tr>
<td>Etched</td>
<td>$4.15 \times 10^2$</td>
<td>$1.05 \times 10^3$</td>
<td>4.25</td>
</tr>
<tr>
<td>Etched #2</td>
<td>$8.19 \times 10^2$</td>
<td>$9.84 \times 10^2$</td>
<td>4.16</td>
</tr>
</tbody>
</table>
Un-etched •
Etched •
Etched #2 •

FIGURE 15. In
Resonant width change versus penetration length for In. Bold lines represent a computer least square fit to the polynomial $\Delta W = A + Bx^N$.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Un-etched</td>
<td>$-2.08 \times 10^2$</td>
<td>$2.47 \times 10^3$</td>
<td>2.88</td>
</tr>
<tr>
<td>Etched</td>
<td>$-1.74 \times 10^2$</td>
<td>$1.18 \times 10^3$</td>
<td>1.86</td>
</tr>
<tr>
<td>Etched #2</td>
<td>$-3.06 \times 10^1$</td>
<td>$1.35 \times 10^3$</td>
<td>2.11</td>
</tr>
</tbody>
</table>
FIGURE 16. In
FIGURE 17

Resonant frequency shift versus penetration length for AL. Bold lines represent a computer least square fit to the polynomial $\Delta F = A + Bx^N$.

<table>
<thead>
<tr>
<th>For:</th>
<th>A</th>
<th>B</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Un-etched</td>
<td>$4.48 \times 10^2$</td>
<td>$8.61 \times 10^3$</td>
<td>2.70</td>
</tr>
<tr>
<td>Etched</td>
<td>$-2.59 \times 10^1$</td>
<td>$2.25 \times 10^3$</td>
<td>2.81</td>
</tr>
</tbody>
</table>
SAMPLE PENETRATION LENGTH (mm)

RESONANT FREQUENCY SHIFT, $\Delta f$ (MHz)

Un-etched •
Etched •

FIGURE 17. AL
FIGURE 18

Resonant width change versus penetration length for AL. Bold lines represent a computer least square fit to the polynomial $\Delta W = A + Bx^N$.

For:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Un-etched</td>
<td>$5.70 \times 10^2$</td>
<td>$3.53 \times 10^3$</td>
<td>3.07</td>
</tr>
<tr>
<td>Etched</td>
<td>$-1.35 \times 10^2$</td>
<td>$1.34 \times 10^3$</td>
<td>2.86</td>
</tr>
</tbody>
</table>
Un-etched •
Etched ○

SAMPLE PENETRATION LENGTH (mm)

FIGURE 18. AL
FIGURE 19. Slope
REFERENCES

1J. C. Slater, Rev. Mod. Phys. 18, 441 (1946).


