REPORT No. 686

STABILITY OF CASTERING WHEELS FOR AIRCRAFT LANDING GEARS

By Arthur Kantrowitz

SUMMARY

A theoretical study has been made of the shimmy of castering wheels. The theory is based on the discovery of a phenomenon called kinematic shimmy. Experimental checks, use being made of a model having low-pressure tires, are reported and the applicability of the results to full scale is discussed. Theoretical methods of estimating the spindle viscous damping and the spindle solid friction necessary to avoid shimmy are given. A new method of avoiding shimmy—lateral freedom—is introduced.

INTRODUCTION

In many installations of castering rubber-tired wheels, there is a tendency for the wheel to oscillate violently about the spindle axis. This phenomenon, popularly termed "shimmy," has occurred in some airplane tail wheels and has been corrected in two ways: First, by the application of friction in the spindles of the tail wheels; and, second, by locking the wheels while taxying at high speeds. Shimmy is common with the large wheels used as nose wheels in tricycle landing gears and, because it is impossible to lock the wheels, friction in the nose-wheel spindle has been the sole means of correction. Because the nose wheel is larger than the conventional tail wheel and usually carries a greater load, the larger amounts of spindle friction necessary to prevent shimmy are objectionable.

Several experimental investigations undertaken to find methods of avoiding shimmy showed that shimmy could be prevented in small tail wheels by friction in the spindle. The most thorough work on the general subject of stability of castering wheels is that of Becker, Fromm, and Maruhn (reference 1) on shimmy in automobiles. Because they do not include the effect of lateral tire deflection, it is believed that their results would, in general, not be applicable to airplanes.

The present paper presents a theoretical and an experimental study of the problem of the stability of castering wheels for airplane landing gears. On the basis of simplified assumptions induced from experimental observations, a theory of the phenomenon is presented. The theory is then compared quantitatively with the results of model experiments.

THEORY OF THE STABILITY OF CASTERING WHEELS

KINEMATIC (OR STATIC) SHIMMY

Some preliminary experimental results on shimmy were obtained by the N. A. C. A. with the aid of the belt-machine apparatus shown in figure 1. This machine consists of a continuous fabric belt mounted on two rotating drums and driven by a variable-speed electric motor. Provision is made for rolling a castering wheel up to about 6 inches in diameter on the belt in such a way that it is free to move vertically but not horizontally.

On this belt machine, the following phenomenon (see fig. 1) was discovered while pushing the belt very slowly by hand. With the wheel set at an angle with the belt as in (a) and the belt pushed slowly, the bottom of the tire would deflect laterally as is shown in (b). When the belt was pushed farther, the wheel straightened out gradually as is shown in (c). The bottom of the tire would then still be deflected, however, and the wheel would continue to turn as in (d). The wheel would thus finally overshoot, as shown in (e) and (f). The process would then be repeated in the opposite direction.

Figure 2 is a photostatic record of the track left by the bottom of the tire on a piece of smoked metal. Two things will be noticed: First, that the bottom of the tire did not skid; and, second, that the places where the wheel angle is zero (indicated by zeros on the track) correspond roughly to the places where the lateral deflection of the tire is a maximum. Thus the wheel angle lags the tire deflection by one-quarter cycle.

It was noticed that the oscillation could be interrupted at any point in the cycle by interrupting the motion of the belt without appreciably altering the phenomenon. From this observation it was deduced that dynamic forces play no appreciable part in this oscillation.

The distance along the belt required for one cycle was also found not to vary much with caster angle or caster length. (See fig. 3.) Caster length was therefore considered not to be of fundamental importance in this type of oscillation.

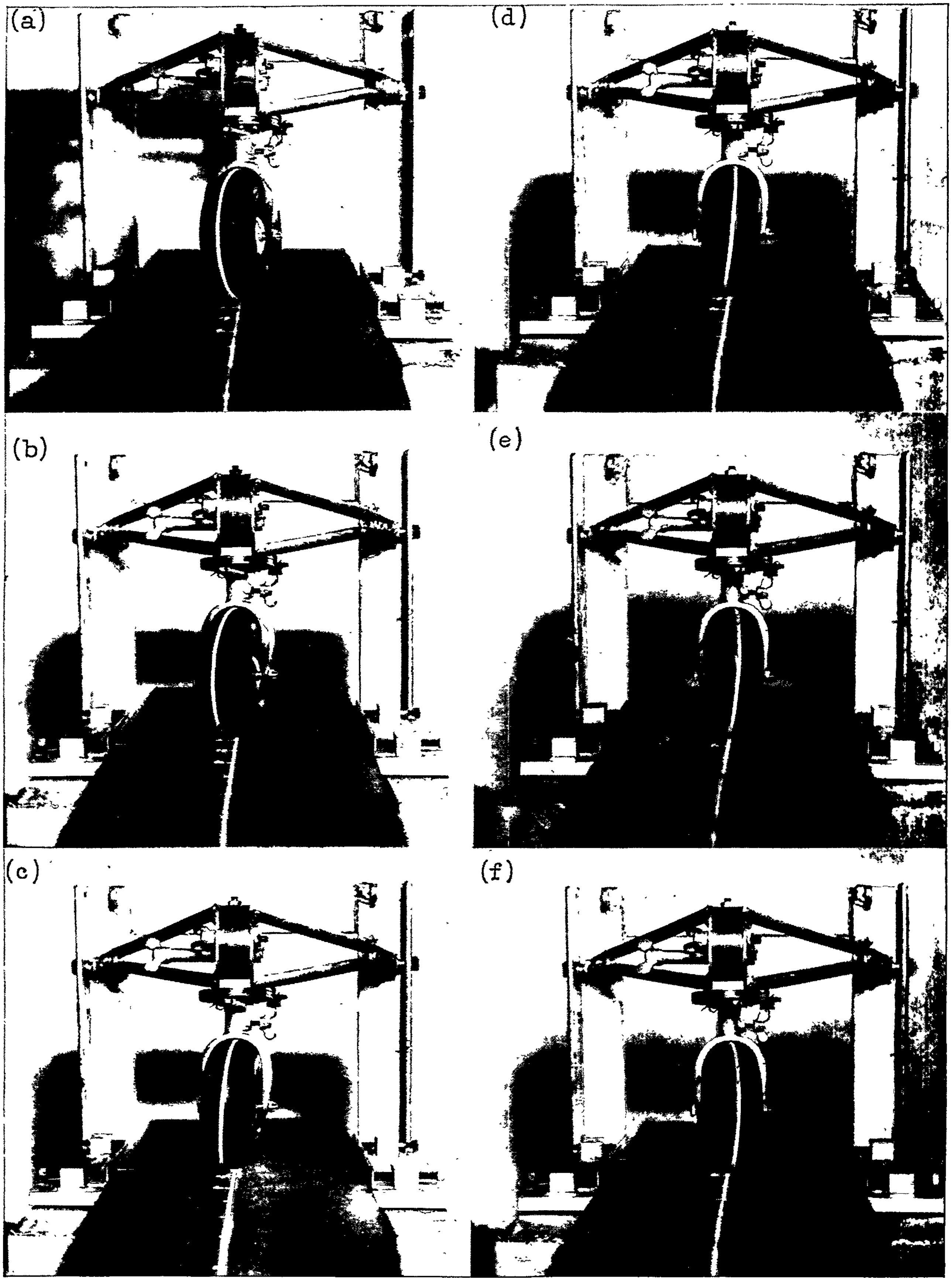


FIGURE 1.—Kinematic shimmy.

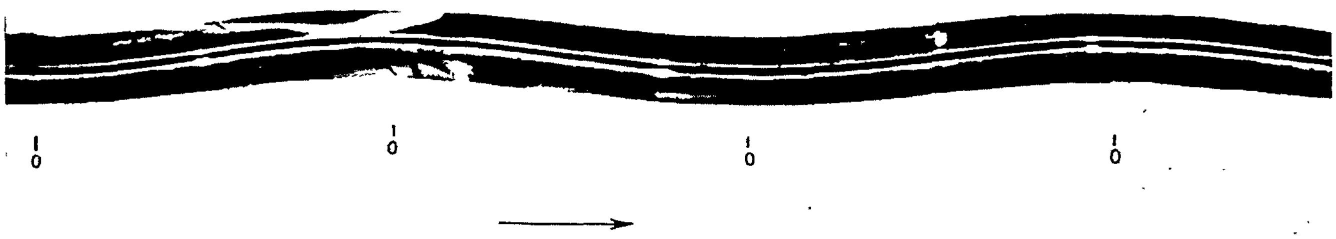


FIGURE 2.—Record of the track of a wheel executing a kinematic shimmy.

It should be pointed out that, in order to observe the kinematic shimmy, lateral restraint of the spindle is necessary to prevent the spindle from moving laterally when the bottom of the tire is deflected and thus neutralizing the tire deflection. This restraint is supplied by the dynamic reaction of the airplane when the airplane is moving forward rapidly but is not ordinarily present when the airplane is moving forward slowly. It has been observed, however, on airplanes towed slowly with two towropes so arranged as to provide some lateral restraint.

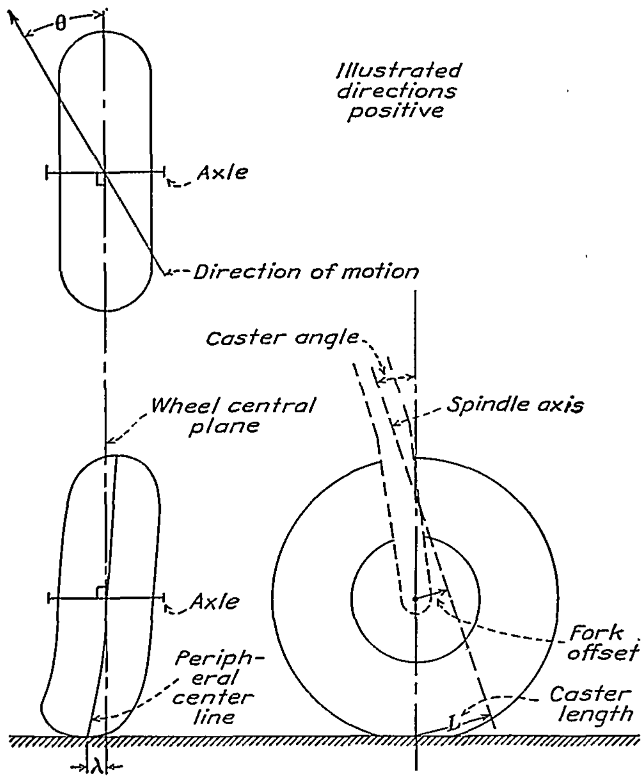


FIGURE 3.—Definition of terms.

Figure 1 (a) shows that, when the center line of the wheel is at an angle θ (see fig. 3) with the direction of motion, the bottom of the tire deflects. This situation is represented schematically in figure 4. It is seen that a typical point on the peripheral center line must have a component of motion perpendicular to the wheel center line if the tire is not to skid. Thus

$$d\lambda = -\sin \theta ds$$

(The minus sign follows from the conventions used as shown in fig. 3.) Since only small oscillations are to be considered, the approximation

$$\frac{d\lambda}{ds} = -\theta \tag{1}$$

may be substituted.

The effect of tire deflection on θ will now be considered. For the purposes of rough calculation, it will be assumed that, as illustrated in figure 5, the projection of the peripheral center line on the ground is a circular arc intersecting the wheel central plane at the extremities of the projection of the tire diameter. (See fig. 1 (d).) Thus, in figure 5, r is the tire radius. (It will be assumed for the time being that the caster length

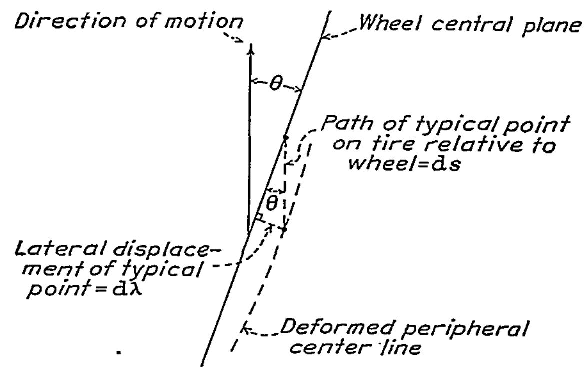


FIGURE 4.—Geometrical relationships involved in kinematic shimmy, illustrating that $d\lambda = -\theta ds$.

and the caster angle are zero.) Now if the tire is deflected in the form of a circular arc, then the condition that the torque about the spindle axis be zero is that the strain be symmetrical about the projection of the wheel axle on the ground. Clearly, if the wheel is displaced, it will be turned about the spindle axis by the asymmetrical elastic forces until, if it is allowed time to reach equilibrium, the symmetrical strain condition is reached. Thus, if the tire is deflected an amount λ as in figure 5 (a) and if the wheel rolls forward a distance ds to the condition shown in figure 5 (b), in order for the strain to remain symmetrical the wheel must turn about the spindle axis an amount $d\theta$. From figure 5, $Rd\theta = ds$. The value of R may be readily obtained from geometry in terms of r and λ. Thus $R^2 = r^2 + (R-\lambda)^2$, from which, if $\lambda^2 << r^2$, it is seen that $R = r^2/2\lambda$.

Then substituting for R,

$$\frac{d\theta}{ds}=\frac{2}{r^2}\lambda \tag{2}$$

If the caster length is finite, the strain will not be symmetrical about the axle, as was assumed here, but will be symmetrical with respect to some line parallel to the axle but a certain fixed distance ahead. Hence, the essential elements of the geometry are unchanged and all the reasoning that led to equation (2) is still valid for this case.

Since the phenomena represented by equations (1) and (2) occur simultaneously, they must be combined to get the total effect. Thus

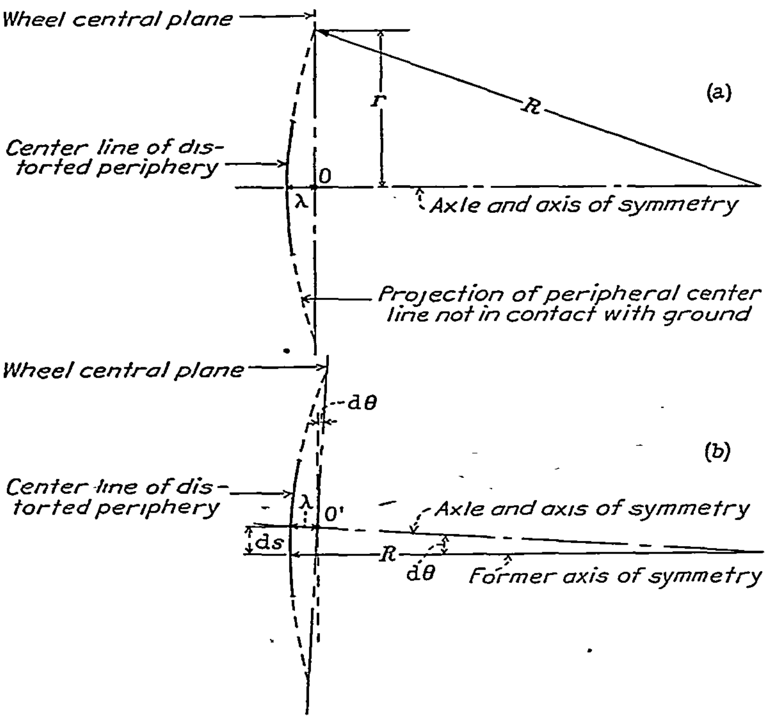


FIGURE 5.—Geometrical relationships involved in kinematic shimmy, illustrating that $d\theta=2\lambda/r^2=K_1\lambda$.

$$\frac{d^2\theta}{ds^2}=-\frac{2}{r^2}\theta \tag{3}$$

This differential equation corresponds to a free simple harmonic oscillation occurring every time the wheel moves a distance $S=2\pi/\sqrt{2/r^2}=\pi r\sqrt{2}$.

Measurements of the space interval S of kinematic shimmy have been made for three similar tires of the type illustrated in figure 1. These tires all had radii of approximately 2 inches so that the theoretical interval was about 0.74 foot. Their experimental intervals were 0.65 foot, 0.74 foot, and 0.79 foot. This agreement is closer than might have been anticipated in view of the roughness of the assumption. It will be seen from equation (1) that λ is one-fourth cycle out of phase with θ, which is in agreement with the information obtained from figure 2.

In order to take account of tires for which the assumption made concerning the projection of the peripheral center line is not quantitatively valid, an empirical constant K_1 will be used in place of $2/r^2$ in equations (2) and (3), thus obtaining

$$\frac{d\theta}{ds}=K_1\lambda \tag{4}$$

and

$$\frac{d^2\theta}{ds^2}=-K_1\theta \tag{5}$$

The constant K_1 can be measured by observing the space interval of kinematic shimmy. Where experimental values of K_1 are available, they will be used rather than the rough theoretical value $2/r^2$.

DYNAMIC SHIMMY

In the foregoing derivation for the oscillation called kinematic shimmy, it was assumed that the strain of the tire was always symmetrical, that is, the wheel was moving so slowly that any torque arising from dynamic effects involved in the oscillation would be negligible. For this case, from equation (4)

$$\left(\lambda-\frac{1}{K_1}\frac{d\theta}{ds}\right)=0$$

If, now, the wheel is assumed to be moving at a velocity such that the effect of the moment of inertia about the spindle axis is significant, then the strain can no longer be symmetrical and, for small asymmetries, the torque exerted by the tire on the spindle will be proportional to the amount of the asymmetry. Thus, the value in parentheses will no longer be zero but it can be assumed that it will be proportional to the dynamic torque; hence

$$\frac{d^2\theta}{dt^2}=C_1\left(\lambda-\frac{1}{K_1}\frac{d\theta}{ds}\right)$$

where C_1 is an appropriate constant of proportionality and includes the moment of inertia about the spindle axis I_w. If the forward velocity V of the wheel is constant

$$\frac{d^2\theta}{dt^2}=V^2\frac{d^2\theta}{ds^2}$$

and

$$V^2\frac{d^2\theta}{ds^2}=C_1\left(\lambda-\frac{1}{K_1}\frac{d\theta}{ds}\right) \tag{6}$$

The constant C_1 may be measured by deflecting the bottom of the tire a known amount, moving the wheel forward, and balancing the torque T exerted by the tire on the spindle so that θ stays constant. The method of deflecting the bottom of the tire a known amount will be described later. In this case (6) becomes

$$\frac{T}{I_u}=C_1\lambda$$

from which C_1 may be found. It has been found that C_1 increases with increasing caster angle. Thus, for a tire like the one in figure 1, C_1 was 71,000 radians per second^2 per foot for a caster length L of 0.17 inch (caster angle, 5°; no fork offset) and was 104,000 radians per second^2 per foot for caster length 0.68 inch (caster angle, 20°).

In the study of kinematic shimmy, it was also seen that the only change in λ was due to the fact that a component of the forward motion was perpendicular to the central plane of the wheel. This circumstance is expressed by equation (1). It is, however, found that, if the spindle is clamped at $\theta=0$ and the bottom of the tire is deflected, the deflection will gradually neutralize itself; that is, the bottom of the tire will roll under the wheel. Thus the asymmetrical ($d\theta/ds=0$)

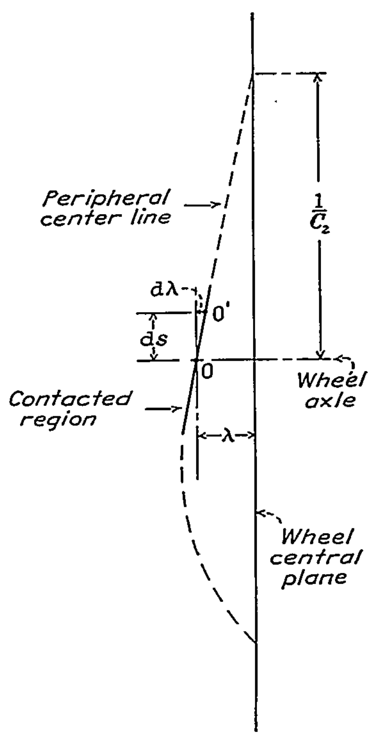


FIGURE 6.—Illustrating the contribution of asymmetrical strain to $d\lambda/ds$.

strain that exists in this case contributes to $d\lambda/ds$. The case of θ and $d\theta/ds$ zero is illustrated in figure 6. If the effect is again supposed to be proportional to the cause, there is obtained

$$\frac{d\lambda}{ds}=-C_2\lambda \tag{7}$$

In this equation, the constant C_2 is a geometrical constant of the tire that can be obtained from static measurements. The order of magnitude of C_2 can be obtained by assuming that the periphery of the tire intersects the extremity of the extended central plane of the wheel. In that case $C_2=1/r$.

If θ is not zero, there will be a component of the forward motion contributing to $d\lambda/ds$. As in equation (1), this component will be $-\theta$. Adding this component to the part of $d\lambda/ds$ due to asymmetry, then ($d\theta/ds$ still assumed zero)

$$\frac{d\lambda}{ds}=-\theta-C_2\lambda \tag{8}$$

This equation expresses that, for $\frac{d\theta}{ds}=0$, the contribution of the asymmetrical strain to $d\lambda/ds$ was $-C_2\lambda$. Also for the symmetrical strain, in which case $\lambda-\frac{d\theta/ds}{K_1}=0$ (kinematic shimmy), there was, of course, no contribution due to asymmetry. Assume now a linear interpolation between these two limiting cases. Thus, finally,

$$\frac{d\lambda}{ds}=-\theta-C_2\left(\lambda-\frac{1}{K_1}\frac{d\theta}{ds}\right) \tag{9}$$

A method of measuring C_2 is provided by equation (7) which, when integrated, gives

$$\log_e\frac{\lambda}{\lambda_0}=-C_2(s-s_0)$$

If, with the spindle clamped at $\theta=0$, the tire is deflected a known amount λ_0 and rolled ahead a known distance and the new λ measured, C_2 may be computed. It was seen earlier than C_2 was of the order of magnitude of $1/r$; that is, it would be of the order of 6 for a 2-inch-radius tire. The constant C_2 was measured on two model tires under different loads and found to be 6.2 and 3.4. Considerable variations of this constant with tire pressure and load have been found.

A method of obtaining the constant known λ necessary for the measurement of C_1 is provided through equation (8). Here it is seen that, if the wheel is pushed along at a constant angle θ, λ will increase (negatively) until $\theta=-C_2\lambda$, in which case $\frac{d\lambda}{ds}=0$ and equilibrium is reached.

When the wheel is moving ahead at a finite constant velocity, the phenomena represented by equations (6) and (9) occur simultaneously. Therefore, to get the total effect, combine the two equations, thus obtaining

$$\frac{V^2}{C_1}\frac{d^3\theta}{ds^3}+\left(\frac{1}{K_1}+\frac{C_2}{C_1}V^2\right)\frac{d^2\theta}{ds^2}+\theta=0 \tag{10}$$

The solution for the natural modes of motion represented by equation (10) is

$$\theta=Ae^{\alpha_1 s}+Be^{\alpha_2 s}+Ce^{\alpha_3 s} \tag{11}$$

where the α's are the three solutions of the so-called auxiliary equation

$$\frac{V^2}{C_1}\alpha^3+\left(\frac{1}{K_1}+\frac{C^2}{C_1}V^2\right)\alpha^2+1=0 \tag{12}$$

One of these α's is real and negative and corresponds to a nonoscillatory convergence. The other roots are conjugate complex numbers and correspond to the shimmy under consideration. The roots will be of the form $a \pm \omega i$. If the divergence a is positive, the oscillation will steadily increase in amplitude (while its amplitude is not large enough for skidding to occur); and, if a is negative, the oscillation will steadily decrease in

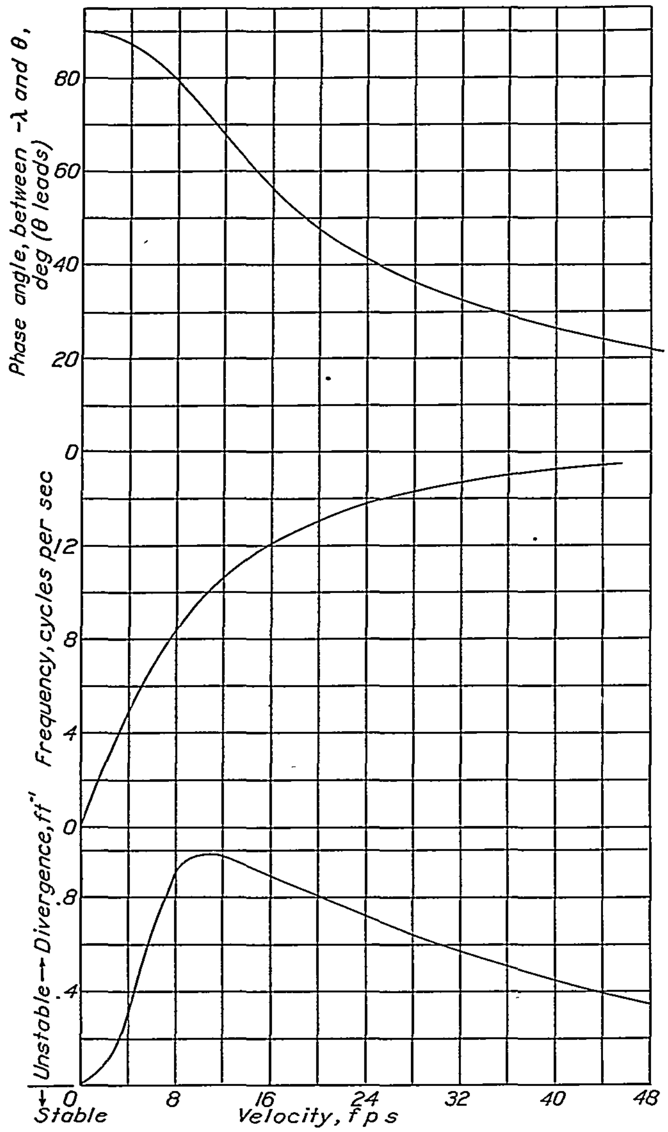


FIGURE 7.—Theoretical values of divergence, frequency, and phase angle against velocity.

amplitude and eventually disappear. The meaning of the quantity "divergence" may be illustrated by saying that it is approximately equal to the natural logarithm of the ratio of successive maximum amplitudes to the distance between them. The quantity ω is equal to 2π times the number of oscillations per foot. The frequency therefore is $\omega V/2\pi$. The phase angle is obtained by substituting for θ in equation (6) the value obtained from the foregoing procedure and solving for λ.

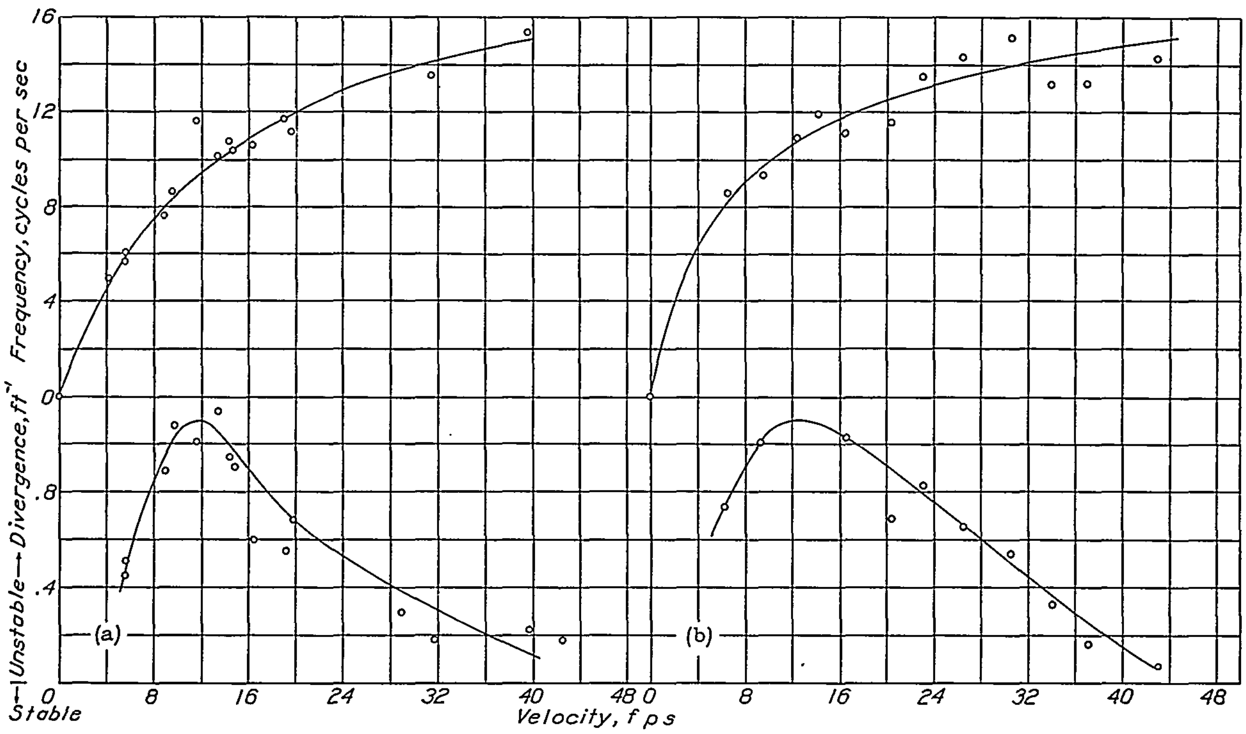
The divergence, the frequency, and the phase relations thus derived for typical model tire constants are plotted in figure 7. For small velocities (0 to 6 feet per second) the frequency corresponds to kinematic shimmy; it is proportional to velocity. The divergence increases rapidly, however, because the spindle angle lags on account of the moment of inertia about the spindle axis, thus allowing more lateral deflection than would occur in a kinematic shimmy. On the next half cycle, a larger spindle angle is reached and the process repeats. As the velocity is further increased, the lag, and hence the asymmetry of the strain, further increase until the strain becomes almost entirely asymmetrical. For this condition, $\frac{d\theta}{ds}\frac{1}{K_1} << \lambda$. Then the restoring torque on the spindle is approximately proportional to λ. (See equation (6).) The tire deflection λ (measured negatively) will, however, still lag somewhat behind θ because, after the wheel is turned through a given angle, a certain forward distance is required for equilibrium tire deflection to be reached. Thus, the restoring force will again lag the displacement. As the velocity increases in the high-velocity range, the frequency stays nearly constant (see fig. 7) and the distance corresponding to a single oscillation increases. Hence this constant lag becomes a smaller part of the cycle and the divergence decreases at high velocities.

It will be appreciated that the foregoing theory considers only the fundamental phenomena taking place in shimmy. Other phenomena occurring simultaneously have been neglected. Some of the more important of the neglected phenomena are:

1. Miscellaneous strains (other than lateral tire deflection) occurring in the tire. A rubber tire being an elastic body will distort in many complicated ways while shimmying. In particular, there will be a twist in the tire due to the transmission of torque from the ground to the wheel.

2. Two effects will cause the stiffness constants of the tire to change with speed. First, centrifugal force on the rubber will make the tire effectively stiffer at high speeds. Second, much of the energy used to deflect the tire will go into compressing the air. The compressibility of the air will change with the speed of compression owing to the different amounts of heat being transferred from it.

3. There will be a gyrostatic torque about the spindle axis caused by the interaction of the rotation of the wheel on the axle and the effective rotation of part of the tire about a longitudinal axis on account of the lateral tire deflection. This torque will later be shown to have a noticeable effect on the results.



(a) Caster angle, 5°. (b) Caster angle, 20°.

FIGURE 8.—Experimental values of divergence and frequency of shimmy against velocity as determined on belt machine.

The inclusion of items 1 and 2 in the theory would obviously be very difficult. It is therefore necessary to resort to experiment to determine whether the present theory gives an adequate description of the phenomena. If so, the omission of these and any other items will be justified.

An experimental check on the theory was obtained by measuring the divergence and the frequency of the shimmy on the belt machine at two caster angles and at a series of velocities. These measurements were made by placing a lighted flashlight bulb on a 6-inch sting ahead of a model castering wheel with a ball-bearing spindle and then taking high-speed moving pictures of the flashlight bulb with the wheel free. The photographs were made with time recordings on the film, and the belt carried an object that interrupted light from a fixed flashlight bulb and thus recorded the belt speed on the film.

The divergence and the frequency of the shimmy were obtained by measuring the displacements and the times corresponding to successive maximum amplitudes (while the amplitude was still small enough to make all the assumptions valid). The results are plotted in figure 8.

In order to compare these results with the theory, the constants C_1, C_2, and K_1 were measured on the same tire at the two caster angles by the previously described methods. It was found that $C_2=6.2$ feet $^{-1}$; that $K_1=62.5$ feet $^{-2}$; and that, for 5° caster angle, $C_1=71{,}100$ feet $^{-1}$-second $^{-2}$ and, for 20° caster angle, $C_1=104{,}000$ feet $^{-1}$-second $^{-2}$. The roots of equation (12) were then found and the divergence and the frequency of the shimmy were computed for a series of velocities and at caster angles of 5° and 20°. These results are plotted in figure 9 and, for purposes of comparison, the experimental curves are also reproduced.

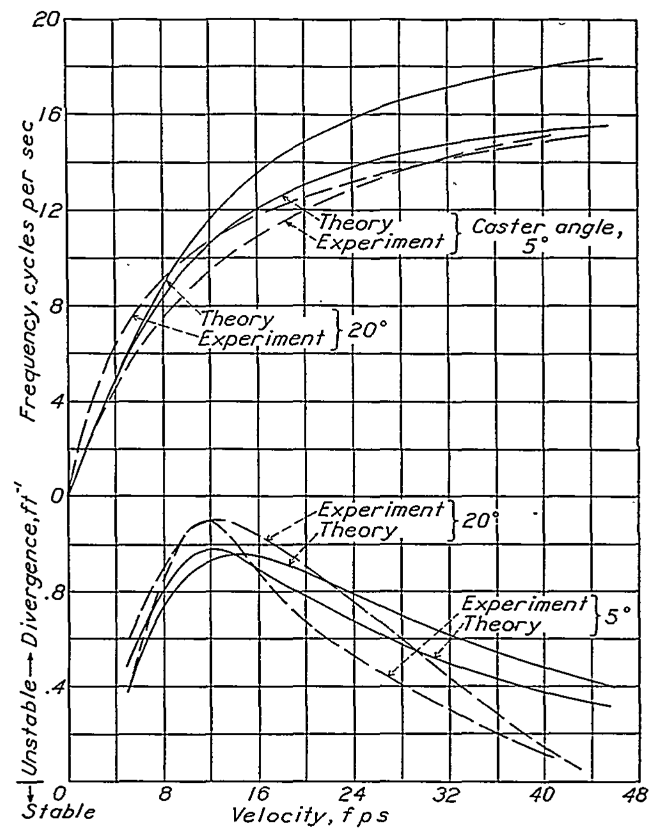


FIGURE 9.—Divergence and frequency of shimmy on belt machine compared with theory.

The agreement between theory and experiment is considered satisfactory as regards qualitative results. It will be noticed, however, that the theoretical values of the divergence are decidedly too large at high velocities, say 25 feet per second. A velocity of 25 feet per second for the model would, however, correspond to 100 feet per second for even a small airplane, such as the Weick W-1A (reference 2). It was thought that the discrepancy might be attributed to a gyroscopic torque (mentioned earlier) that was due to the interaction of the rotation of the wheel on its axle with lateral motion of the bottom of the tire. This lateral motion may be considered to be equivalent to a rotation of *part* of the tire about an axis perpendicular to the spindle and the

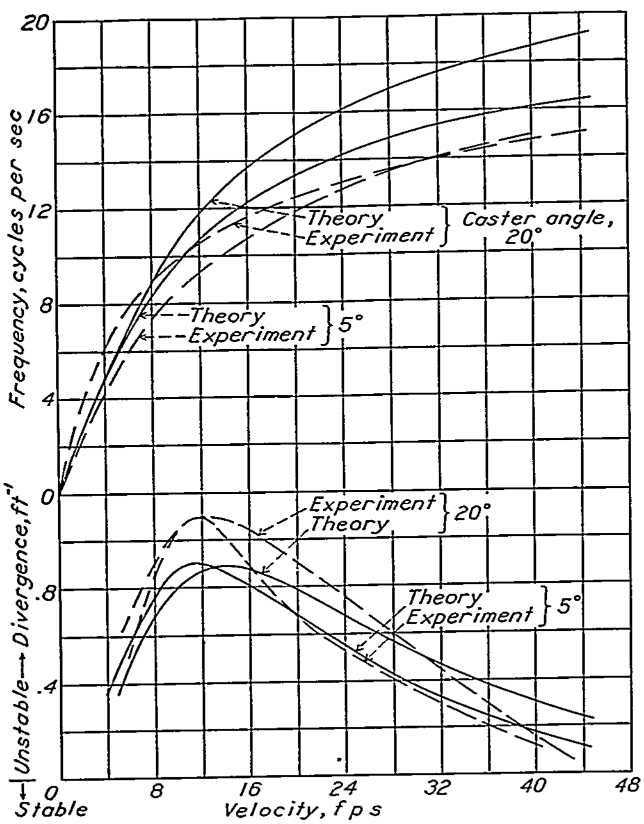


FIGURE 10.—Divergence and frequency of shimmy on belt machine compared with theory including gyrostatic torque.

wheel axle. This interaction gives rise to a spindle torque, which must be included in equation (6). The numerical value of this torque will be $I_a\dot{\psi}\dot{\varphi}$, where I_a is the moment of inertia about the wheel axle of the part of the tire that engages in the lateral motion (say, possibly one-fourth of the total moment of inertia of the tire); ψ is the angular rotation about the longitudinal axis ($\dot{\psi}=\dot{\lambda}/r$); and φ is the angular rotation about the axle ($\dot{\varphi}=V/r$). Thus, the value of the gyrostatic torque is $I_a\dot{\lambda}V/r^2$; but $\dot{\lambda}=V\frac{d\lambda}{ds}$ so, finally, the torque is $I_a\frac{V^2}{r^2}\frac{d\lambda}{ds}$. The angular acceleration due to this torque is $\frac{I_a}{I_w}\frac{V^2}{r^2}\frac{d\lambda}{ds}$. Thus (6) becomes

$$V^2\frac{d^2\theta}{ds^2}-\frac{I_a}{I_w}\frac{V^2}{r^2}\frac{d\lambda}{ds}=C_1\left(\lambda-\frac{d\theta/ds}{K_1}\right)$$

Combining this equation with (9)

$$\frac{V^2}{C_1}\frac{d^3\theta}{ds^3}+\left(\frac{C_2}{C_1}V^2-\frac{C_2V^2I_a}{C_1K_1r^2I_w}+\frac{1}{K_1}\right)\frac{d^2\theta}{ds^2}+\frac{I_a}{I_wr^2}\frac{V^2}{C_1}\frac{d\theta}{ds}+\theta=0$$

Here the constants C_1, C_2, and K_1 are the same as before. For the assembly on which the measurements plotted in figure 8 were made, I_w was 1.06×10^{-4} slug-feet², I_a was 1.09×10^{-5} slug-feet², and r was 0.165 foot. These values were used to compute new theoretical divergences and frequencies, which are compared with the experiments in figure 10. It will be seen that, when the gyrostatic torques are included, the agreement of the theoretical divergences with experimental values is considerably improved at high velocities.

The gyrostatic torque is proportional to $d\lambda/ds$. Hence, at high velocities, it is nearly proportional to $d\theta/ds$ since $-\lambda$ and θ are nearly in phase. Thus, this torque is approximately in the right phase to damp the oscillation.

EFFECT OF MOTION OF AIRPLANE

When a shimmying wheel is attached to an airplane, the airplane will respond to a certain extent to the forces supplied by the shimmy. This lateral motion of the nose of the airplane will in turn affect the shimmy and does, in fact, reduce the divergence at low velocities.

In order to calculate an approximation to this effect, the airplane will be assumed to be a rigid body. By this assumption, any deflection occurring in the structure or at the rear tires is neglected. The gyrostatic torque due to lateral tire deflection is also omitted.

The following symbols are used in the calculations:

- I_z moment of inertia about vertical axis through center of gravity of the airplane.
- m mass of airplane.
- E_λ lateral force exerted by nose-wheel tire per foot deflection.
- θ angle between central plane of wheel and average direction of motion.
- ψ angle between longitudinal axis of airplane and average direction of motion. Measured positive in the same direction as θ.
- y lateral distance between center of gravity of airplane and path of average motion. Positive in same direction as λ.
- Y lateral force exerted by the rear wheels on the airplane.
- l_2, l_1 horizontal distance from nose wheel and rear wheels, respectively, to center of gravity.

$l=l_1+l_2$.

The side force developed by the shimmy is $E_\lambda\lambda$. The lateral acceleration is

$$\frac{E_\lambda\lambda+Y}{m}=\ddot{y}=V^2\frac{d^2y}{ds^2} \tag{13}$$

Similarly, the angular acceleration of the airplane is given by

$$\frac{-l_2E_\lambda\lambda+l_1Y}{I_z}=\ddot{\psi}=V^2\frac{d^2\psi}{ds^2} \tag{14}$$

There is, in addition, the geometrical relationship

$$\frac{dy}{ds}=-\psi-\frac{d\lambda}{ds}\frac{l_1}{l}-(\theta-\psi)\frac{l_1}{l} \tag{15}$$

The lateral motion of the nose per unit forward motion, assuming that the rear wheels do not skid, is

$$\psi+l\,\frac{d\psi}{ds}$$

This motion will naturally affect the rate of deflection of the nose wheel $d\lambda/ds$; equation (9) becomes

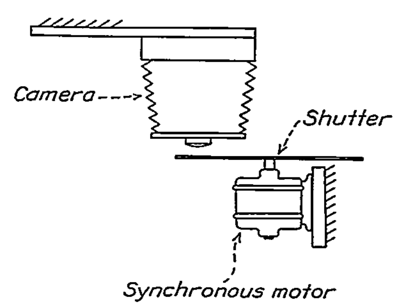
$$\frac{d\lambda}{ds}=-\theta-C_2\left(\lambda-\frac{1}{K_1}\frac{d\theta}{ds}\right)+\psi+l\frac{d\psi}{ds} \tag{16}$$

Equation (6) is unaffected. Combining these equations

$$V^4(I_z+ml_1^2)+V^2\left[l_1m\left(V^2+l_1\frac{C_1}{K_1}\right)+I_z\left(\frac{C_1}{K_1}+V^2C_2\right)\right]\frac{d^3\theta}{ds^3}$$
$$+V^2\left[l_1m\left(\frac{C_1}{K_1}+\frac{l_2}{l}V^2C_2\right)+l^2E_\lambda\right]\frac{d^2\theta}{ds^2}$$
$$+\left(C_1l_1^2mV^2+I_zV^2C_1+lE_\lambda V^2+l^2E_\lambda\frac{C_1}{K_1}\right)\frac{d\theta}{ds}$$
$$+C_1\left(l_1mV^2+\frac{lE_\lambda}{K_1}\right)\theta=0 \tag{17}$$

This differential equation has two real roots, which are of little interest to the problem at hand, and two complex roots, which give the divergence and the frequency of the shimmy under consideration.

An experimental check on the theory was obtained by running on a flat surface (see fig. 11) two models similar to the one shown in figure 12. The model was launched by a compressed-air catapult, which gave it a very smooth start. The nose wheel was locked during the



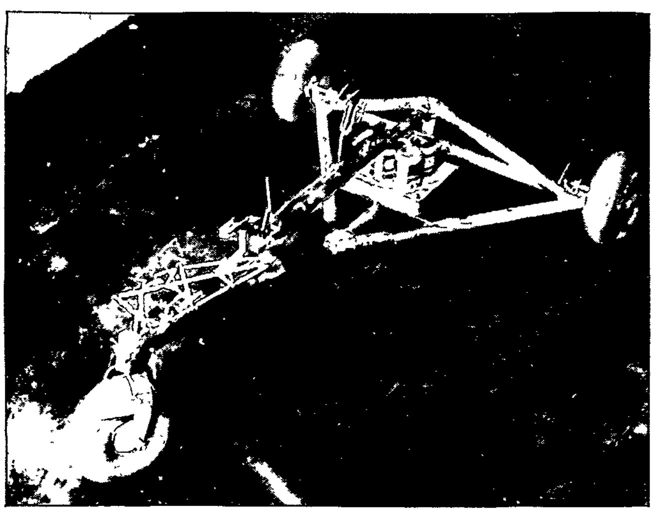


FIGURE 12.—Model used for free-model test.

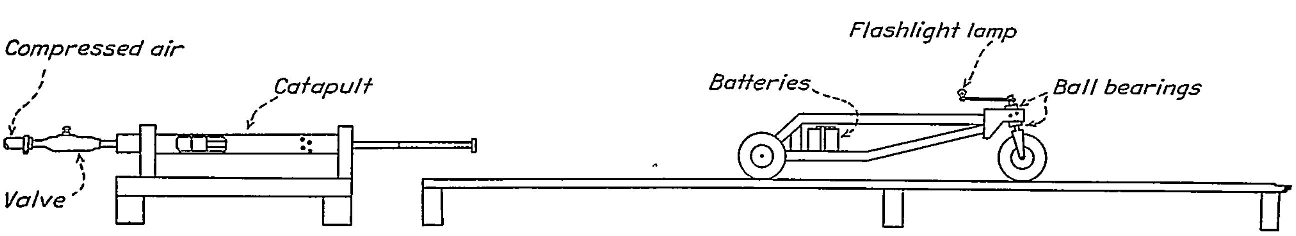


FIGURE 11.—Apparatus for conducting free-model experiments (elevation view.)

acceleration so that the shimmy would not start to build up before constant velocity was reached. The wheel was freed by a lever tripped by a wire. It then came into the field of view of an elevated camera and the flashlight bulb made a track of light on the camera film. The light entering the camera was interrupted 60 times a second by a shutter driven by a synchronous motor. A typical trace is shown in figure 13. The curve was read only in the region between the two arrows. Several such traces were made at various velocities and the divergence and the frequency of the shimmy were measured and plotted against velocity in figure 14. Two models with different mass characteristics and different caster angles and tires were used to get two checks on the theory. The theoretical divergences and frequencies in these models are also shown in figure 14.

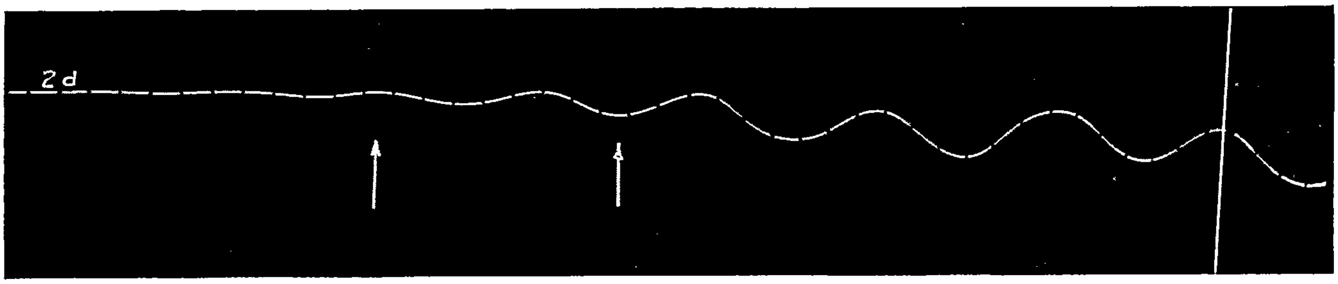
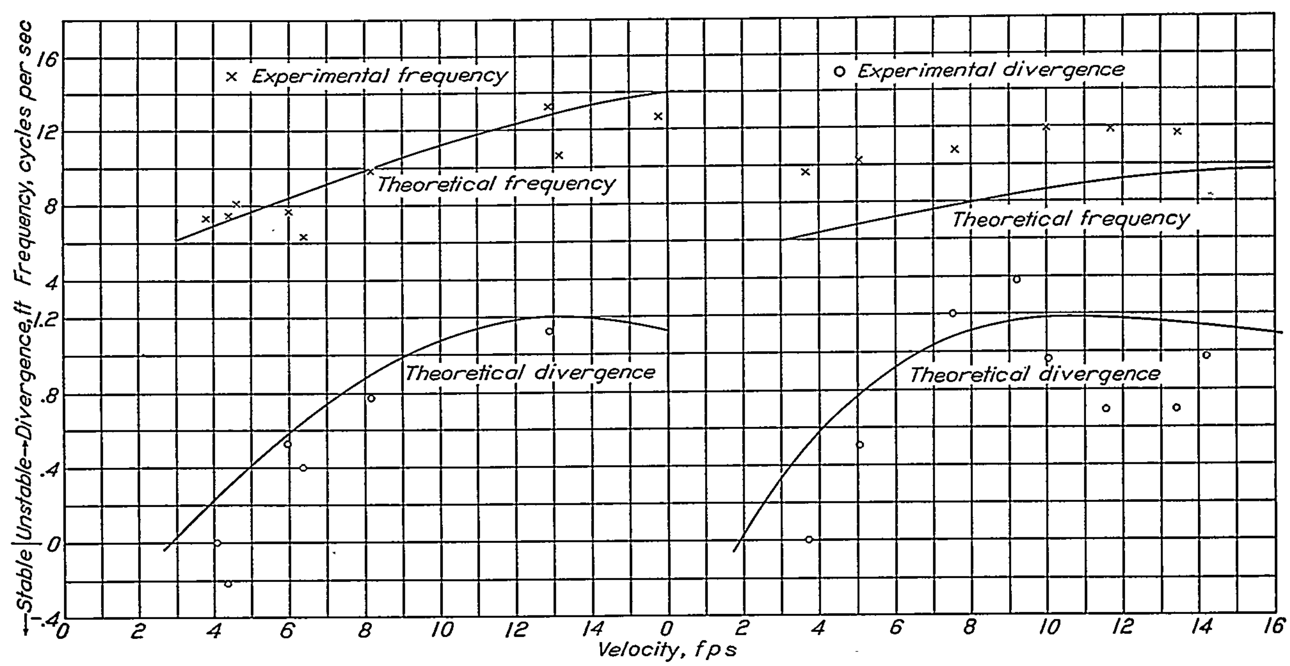


FIGURE 13.—Record from free-model apparatus.

It will be noticed that both the theoretical and the experimental divergences are somewhat smaller at low velocities than in the case of the belt-machine shimmy. In general, however, the differences between the two cases are small. The theory therefore seems adequate to account for the shimmy of freely rolling models when corrections are made for the response of the model to the shimmy.



(a) Model 1. (b) Model 2.

FIGURE 14.—Divergence and frequency of shimmy of nose-wheel models compared with theory.

Full-scale experiments similar to the ones with the model would be valuable in further checking the theory. One very rough check was made on the W-1A airplane (reference 2). The friction in the spindle of this airplane was considerable but, when the friction was reduced, the wheel shimmied. Motion pictures were taken of the shimmying wheel and some of the enlarged frames are shown as figure 15. The shimmy diverged and finally reached an equilibrium amplitude. It will be seen that deflection of the bottom of the tire plays a prominent part in the shimmy. The frequency of the shimmy of this airplane was measured at a velocity of 47 feet per second and found to be 9 cycles per second. The frequency was then computed for this velocity (on the basis of constants found by the method to follow) and a frequency of 10.5 cycles per second was obtained. These results are taken to be indications that the theory will be at least approximately valid for full scale.

In the application of the theory, it is necessary to know the constants of the tire to be used. In the absence of such measurements for full-scale tires, it will be necessary to use the theoretical values of the parameters $K_1=2/r^2$ and $C_2=1/r$. For I_wC_1 (the spindle torque per unit lateral tire deflection for a completely asymmetrical strain), the expression nrE_λ will be used, where E_λ is the lateral force per unit deflection, r is the radius of the tire, and n is an empirical nondimensional factor to be obtained from experiments. The factor n will, of course, increase with increasing caster length; n for the model tires was 0.69 at a caster angle of 5° and 1.00 at a caster angle of 20°.

These expressions for C_1, C_2, and K_1 will be used for numerical calculations in the rest of the report.

FIGURE 15.—Shimmy in W-1A airplane, 64 frames per second.

METHODS OF AVOIDING SHIMMY

SPINDLE DAMPING

In the past the only method that has been used to avoid shimmy in single swiveling wheels has consisted in the application of friction or hydraulic damping to the spindle. It will therefore be desirable to set up a criterion for the amount of spindle damping necessary to avoid shimmy.

In this calculation and also in later calculations of other methods of avoiding shimmy, the damping effect of gyrostatic torque due to lateral tire deflections will be neglected as well as that due to airplane motions; these omissions result in small errors and usually give a conservative result.

First, the effect of idealized viscous damping applied to the spindle will be calculated, that is, the effect of a torque proportional to the angular velocity of the spindle, say $K\dot{\theta}=KV\frac{d\theta}{ds}$. The angular acceleration resulting from this torque $KV\frac{d\theta}{ds}/I_w$ must be added to equation (6).

$$V^2\frac{d^2\theta}{ds^2}=C_1\left(\lambda-\frac{1}{K_1}\frac{d\theta}{ds}\right)-\frac{KV}{I_w}\frac{d\theta}{ds} \tag{18}$$

Combining this equation with equation (9), which is unaffected by the addition of spindle friction,

$$\frac{V^2}{C_1}\frac{d^3\theta}{ds^3}+\left(\frac{V}{C_1}\frac{K}{I_w}+\frac{1}{K_1}+\frac{C_2}{C_1}V^2\right)\frac{d^2\theta}{ds^2}+\frac{C_2}{C_1}V\frac{K}{I_w}\frac{d\theta}{ds}+\theta=0 \tag{19}$$

In the case of a self-excited oscillation represented by a third-order differential equation, Routh's discriminant provides a simple method of deciding whether the oscillation will or will not diverge. If all the coefficients are positive (as they are here), then the oscillation will diverge if and only if the product of the coefficients of the third- and zero-order terms is greater than that of the first- and second-order terms. Hence for the shimmy to have zero divergence

$$\frac{V^2}{C_1}=\left(\frac{V}{C_1}\frac{K}{I_w}+\frac{1}{K_1}+\frac{C_2}{C_1}V^2\right)\frac{C_2}{C_1}V\frac{K}{I_w}$$

or, rearranging and solving for K/I_w,

$$2\frac{K}{I_w}=-\left(\frac{C_1}{VK_1}+C_2V\right)\pm\sqrt{\left(\frac{C_1^2}{V^2K_1^2}+2\frac{C_1C_2}{K_1}+C_2^2V^2\right)+4\frac{C_1}{C_2}} \tag{20}$$

for the condition of zero divergence.

Only the positive value of K is of interest because the application of negative damping to the spindle would make the wheel unstable. From equation (20) a maximum positive value of K is found at the velocity $\sqrt{C_1/C_2K_1}$, which for the model wheels at 5° caster is 13.6 feet per second. Thus, the amount of spindle damping necessary to avoid shimmy is seen to be a maximum near the velocity at which maximum divergence occurs with an undamped wheel. (See fig. 9.) This velocity will be denoted by V_0. The damping required at this velocity, i. e., the maximum damping required, is

$$K_{max}=I_w\sqrt{\frac{C_1C_2}{K_1}}\left(\sqrt{1+\frac{K_1}{C_2^2}}-1\right)\frac{\text{lb-ft}}{\text{radians per sec}} \tag{21}$$

Equation (21) shows that it is desirable to make the caster length small for in this way C_1, and hence K_{max}, may be made small. A rough approximation to K_{max} may be obtained from equation (21) by the use of the theoretical formulas for the tire parameters given earlier. Using these formulas and the value of n obtained for 5° caster angle,

$$K_{max}=0.43r\sqrt{I_wE_\lambda}\,\frac{\text{lb-ft}}{\text{radians per sec}}$$

Using these formulas, also

$$V_0=0.59r^2\sqrt{E_\lambda/I_w}\text{ fps}$$

for the velocity at which this maximum damping is required. For the W-1A airplane, using $r=0.67$ ft, $I_w=0.11$ slug-ft^2 (estimated), and $E_\lambda=2{,}400$ lb/ft (from rough measurements).

$K_{max}=4.6$ lb-ft per radian per sec, and
$V_0=39$ fps

For the Hammond Model Y airplane, which has had considerable difficulty with shimmy, $r=0.83$ ft, I_w (estimated)$=0.33$ slug-ft^2, E_λ (estimated)$=4{,}130$ lb/ft, which gave

$K_{max}=13.2$ lb-ft per radian per sec
$V_0=46$ fps

(If full-scale tire-constant measurements as yet unpublished are used, equation (21) becomes

$$K_{max}=\text{constant}\times r\sqrt{I_w}\frac{\text{lb-ft}}{\text{radians per sec}}$$

where the constant may vary between 16 and 32 depending on the type of tire.)

The amount of damping required increases with the fourth power of the size of the airplane, as would be expected. It is necessary in the application of a practical viscous damper to insure that adequate damping is obtained at all amplitudes; that is, it would be necessary to allow for the nonlinearity of the damper.

Most practical shimmy dampers have used solid rather than viscous friction. Solid friction, however, is difficult to deal with analytically in a straightforward manner. Therefore, resort will be had to the following subterfuge. It will be assumed that, if solid friction takes an amount of energy out of an oscillation equal to the energy taken out by a given amount of viscous damping, it is equivalent to viscous damping for purposes of damping the oscillation. The energy taken out by viscous damping in one-quarter cycle is

$$\int_0^{\theta_{max}}K\dot{\theta}\,d\theta=\int_0^{\theta_{max}}KV\frac{d\theta}{ds}d\theta=\frac{\pi}{4}KV\omega\theta_{max}^2$$

where $\theta=\theta_{max}\sin\omega s$. The energy taken out by a solid friction torque T is $T\,\theta_{max}$. Therefore, to take out the same amount of energy

$$T=\frac{\pi}{4}KV\omega\,\theta_{max} \tag{22}$$

(Velocity enters because the independent variable is distance.)

Equation (22) gives an interesting explanation of the fact that shimmy (and other self-excited vibrations) will usually not start in the presence of solid friction until an initial displacement is supplied. From the theory of small oscillations the viscous friction required to damp a self-excited vibration is independent of the amplitude. (See equation (21).) It will be seen from (22) that the solid friction required to damp a self-excited vibration will be directly proportional to the initial amplitude of the oscillation. If solid friction is used to damp shimmy, it will be desirable to insure that the wheel is centered when it first makes contact with the ground. This precaution has been found useful on airplanes.

The value of ω^2 for the case of equilibrium viscous damping (zero divergence) is equal to the ratio of the coefficient of the $d\theta/ds$ term to the $d^3\theta/ds^3$ term in equation (19). This fact may be readily verified by substituting a simple harmonic solution for θ and equating the sum of the imaginary terms to zero. Thus

$$\omega=\sqrt{\frac{C_2K}{VI_w}} \tag{23}$$

It is difficult to obtain a good estimate of the maximum angle that should be used in computing the amount of solid friction necessary to damp shimmy on an actual airplane because the value depends on how much energy will be supplied to the shimmy after the tire starts to skid. If it be supposed that the energy supplied to the shimmy at angles greater than that at which skidding starts is entirely taken out by the solid friction, then the value of θ for which skidding begins can be put into (22) and a value of the friction will be obtained such that shimmy with all initial angles will be damped out. This supposition is probably true for small caster lengths. The tire will start to skid when the lateral force is large enough to overcome the friction, that is, when $E_\lambda\lambda=\mu W$, where μ is the coefficient of friction and W is the load on the swiveling wheel. Now, if the wheel moves in a direction at an angle θ with the central plane of the wheel, the equilibrium deflection of the tire will be given by

$$C_2\lambda=-\theta$$

from equation (8). In this way a rough estimate can be obtained of the angle θ_0 at which skidding starts. Thus

$$|\theta_0|=\frac{C_2\mu W}{E_\lambda} \tag{24}$$

Substituting in (22) from (24) for θ_{max}, the value from (23) for ω, V_0 or $\sqrt{C_1/C_2K_1}$ for V, and the value of K_{max} from (21) for K, then

$$T_{max}=\frac{\pi\mu WI_wC_1C_2^2}{4E_\lambda K_1}\left(\sqrt{1+\frac{K_1}{C_2^2}}-1\right)^{3/2} \tag{25}$$

is obtained as the amount of solid spindle friction required to damp shimmy at the velocity requiring most viscous spindle damping (which velocity will probably require the most solid spindle friction). The validity of formula (25) is restricted to small caster lengths; this restriction should not hamper its application because the caster length is preferably small. Using the theoretical formulas for the tire parameters, there is obtained the simple formula (again assuming small caster length),

$$T_{max}=0.17\mu rW\text{ lb-ft} \tag{26}$$

It should be remembered that the theoretical formulas for the tire parameters used in the derivation of (26) are approximately valid only for tires similar to the model tires used in the experiments. Thus (26) and all other formulas based on the theoretical tire parameters will be approximately valid only for air wheels or low-pressure tires. For high-pressure tires, it seems likely that the effective r would be smaller than the geometrical radius of the tire. It should be remembered that no conservative assumptions were made in the derivation of (26).

If now $\mu=0.55$, a value of 19 foot-pounds spindle friction is required for the W–1A airplane with $W=310$ pounds (including propeller thrust) and 44 foot-pounds for the Hammond Y airplane with $W=570$ pounds. The amount of spindle friction actually present in the spindle and steering mechanism of the W–1A was measured and found to be of the same order of magnitude as the computed value. This result explains why no shimmy was found with that airplane. As mentioned previously, when this solid friction was reduced, shimmy occurred in the castering front wheel of this airplane.

(Later full-scale tire-constant measurements as yet unpublished indicate that a value $T_{max}=0.06\ \mu rW$ lb-ft would be more accurate.)

It will be seen that the amount of solid spindle friction necessary to damp shimmy in nose wheels is considerable even at small caster lengths. As is the case for the hydraulic damper, it increases with the fourth power of the size of the airplane.

LATERAL FREEDOM

The divergence of the shimmy has been seen to depend on the interaction of the lateral tire deflection with the angular rotation of the spindle. If the coupling between these two degrees of freedom could be sufficiently reduced, the shimmy would not diverge.

One method of reducing the coupling is to introduce a new degree of freedom into the oscillation. Thus, if the wheel is free to move in the direction of the axle, the tire deflections will depend on the inertia reactions of the wheel and of any other members moving laterally. The maximum mass that the wheel could have and still reduce the coupling sufficiently to avoid shimmy is calculated in the appendix. The masses of wheels used in practice are found to be small enough to prevent shimmy if lateral freedom were used. The amount of lateral freedom required is of the order of the distance that the wheel will deflect laterally without skidding. (See the appendix.)

This system was tried on the belt machine and on the W-1A airplane. Shimmy could be prevented if the spindle were prevented from turning through an angle large enough to make the tire skid. Experiments made here and elsewhere without this angular restriction have failed to prevent shimmy. The necessity of limiting the angular travel of the spindle indicates that another type of oscillation occurs at large spindle angles. It seems likely that this oscillation is due to the decrease of skidding friction with increase of relative skidding velocity. As is well known, this effect is apt to cause a chattering type of oscillation and, in the present tests, probably fed energy into the shimmy.

CONCLUDING REMARKS

The phenomenon called kinematic shimmy seems to be the fundamental phenomenon in the oscillation of model wheels. A dynamical theory based on this discovery gives an adequate description of the shimmy of small model wheels. It seems unlikely that shimmy of full-scale airplanes will be fundamentally different.

On the basis of this theory, a method has been provided of estimating numerically the spindle friction necessary to damp shimmy. An objection to this method of avoiding shimmy lies in its indicated interference with the steering of nose wheels.

A new method of avoiding shimmy by the use of lateral freedom for the wheel is introduced but requires further investigation before complete design data can be given.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., *August 11, 1937.*

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2. Weick, Fred E.: Everyman's Airplane—A Development toward Simpler Flying. S. A. E. Jour., vol. 38, no. 5, May 1936, pp. 176–187.

APPENDIX

LATERAL FREEDOM

If the lateral acceleration of the wheel is $\ddot{x}$ and the mass of the wheel and tire (and other members moving laterally) is m', then

$$E_\lambda \lambda = m'\ddot{x} = m'V^2\frac{d^2x}{ds^2}$$

or

$$\int \frac{E_\lambda \lambda}{m'V^2}ds = \frac{dx}{ds} \tag{27}$$

This lateral motion of the wheel must be added to equation (9),

$$\frac{d\lambda}{ds} = -\theta - C_2\left(\lambda - \frac{d\theta/ds}{K_1}\right) - \frac{dx}{ds}$$

or

$$\frac{d\lambda}{ds} = -\theta - C_2\left(\lambda - \frac{d\theta/ds}{K_1}\right) - \int \frac{E_\lambda \lambda}{m'V^2}ds \tag{28}$$

The effect on equation (6) will be small because only small lateral motions will be involved. Combining equation (6) with equation (28),

$$\frac{V^2}{C_1}\frac{d^3\theta}{ds^3} + \left(V^2\frac{C_2}{C_1} + \frac{1}{K_1}\right)\frac{d^2\theta}{ds^2} + \frac{E_\lambda}{C_1 m'}\frac{d\theta}{ds} + \left(1 + \frac{E_\lambda}{K_1 m' V^2}\right)\theta = 0 \tag{29}$$

It is seen that a damping term has been added to the final equation of motion. (Cf. equation (10).) If this term is sufficiently large, it will counteract the divergence of the oscillation. Applying Routh's discriminant to obtain the condition of zero divergence and simplifying,

$$m_0' = \frac{C_2 E_\lambda}{C_1} \text{ slugs}$$

It is noted that m_0' is independent of velocity. If m' is smaller than m_0', the shimmy will converge; if greater, it will diverge. Putting in the theoretical formulas for the constants

$$m_0' = 1.45\frac{I_w}{r^2} \text{ slugs} = 47\frac{I_w}{r^2} \text{ pounds}$$

For the W–1A airplane this value would be 11 pounds and for the Hammond Y, 22 pounds. Fortunately, these values are of the same order of magnitude as the weights of the wheels used on these airplanes.

The necessary amount of lateral freedom will now be estimated. The distance is of the order of magnitude of the amplitude of the oscillation in x, which corresponds to a violent enough oscillation to skid the tire. After skidding starts, lateral freedom would not be expected to have any beneficial effect. For skidding to start,

$$E_\lambda \lambda_{max} = \mu W$$

But

$$m'\ddot{x} = E_\lambda \lambda$$

Hence

$$m'\ddot{x}_{max} = \mu W$$

Suppose the value of m' used is that for equilibrium damping ($m_0' = C_2E_\lambda/C_1$), then the maximum possible $\ddot{x}$ is

$$\ddot{x}_{max} = \frac{\mu W C_1}{E_\lambda C_2}$$

But

$$\ddot{x}_{max} = V^2\omega^2 x_{max}$$

Hence

$$x_{max} = \frac{\mu W C_1}{V^2\omega^2 E_\lambda C_2}$$

As before, for equilibrium damping, ω^2 equals the ratio of the coefficient of the $d\theta/ds$ term to that of the $d^3\theta/ds^3$ term in equation (29), for $m' = C_2E_\lambda/C_1$,

$$\omega^2 = \frac{C_1}{C_2 V^2}$$

and

$$x_{max} = \frac{\mu W}{E_\lambda}$$

It will therefore be seen that the minimum amount of lateral freedom necessary on either side of the wheel is equal to the amount that the tire will deflect laterally before skidding when the weight of the wheel is the required value. This approximation is rough and no conservative assumptions were made other than the one implicit in the neglect of whatever spindle friction may be present. For the W–1A airplane, the calculated amount of lateral freedom required is 0.85 inch on either side of the wheel and, for the Hammond Y airplane, 0.91 inch.

In the application of lateral freedom to an airplane wheel, means for keeping the wheel centered in the lateral travel must be provided. It can be shown that the application of spring-restoring forces sufficiently large to accomplish this purpose would tend to make the wheel shimmy. It was found, however, in a lateral-freedom arrangement that, when the wheel was tilted, it would tend to ride on the lowest portion of the axle. From figure 16 the point A has a smaller rolling radius than the point B but the point A must make as many revolutions as point B. Hence, if no skidding occurs, point A will tend to lag behind point B. Then the resultant couple will turn the wheel about the spindle axis in such a direction that the forward motion will supply a side force causing the wheel to slide "downhill" on the axle. This effect is found to be more than

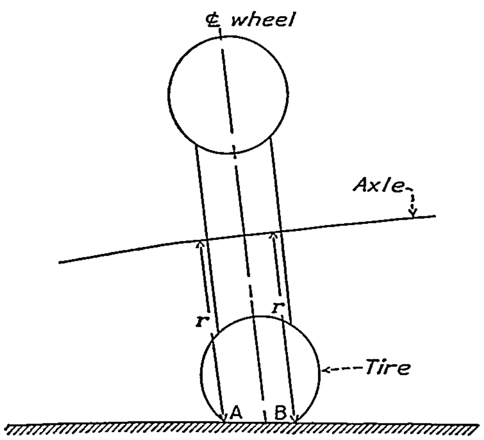

FIGURE 16.—Tilted axle.

large enough to overcome the component of the weight of the airplane that would tend to make the wheel slide "uphill." If an axle that is convex downward is used, the wheel will therefore always seek the lowest point, which in this case will be the center of the lateral travel. Both theoretically and experimentally, the lowest point has been found to be a point of stable equilibrium.

Two methods of applying lateral freedom to a castering wheel were tested on the belt machine. The first method was simply to allow the wheel to slide along the axle. The second method was to allow the wheel to rotate about two axes in a vertical plane perpendicular to the axle. Shimmying in models could be avoided by either method. The second method, however, did not involve making the fork any wider.

Additional full-scale tests of the first system were made of the W–1A airplane. An axle curved downward was used to keep the wheel centered. A radius of curvature equal to 6 times the tire radius was arbitrarily chosen. The wheel centered satisfactorily and, on a rough turf-covered flying field, shimmy was prevented by the use of one-half inch of lateral freedom on either side of the wheel. On concrete with a lateral freedom of three-fourths inch, the wheel could be made to shimmy by taxying over a severe bump at a moderate speed and at an angle that would introduce a large initial angular deflection of the wheel. In order to prevent shimmy under these conditions, the rotation of the spindle was limited by stops to $\pm 13°$. The pilot did not regard this restriction of spindle freedom as objectionable for the ground handling of the airplane.