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AN ANALYSIS OF THE FACTORS THAT DETERMINE THE PERIODIC TWIST OF AN AUTOGIRO ROTOR BLADE, WITH A COMPARISON OF PREDICTED AND MEASURED RESULTS

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SUMMARY

An analysis is presented of the factors that determine the periodic twist of a rotor blade under the action of the air forces on it. The results of the analysis show that the Fourier coefficients of the twist are linear expressions involving only the tip-speed ratio, the pitch setting, the inflow coefficient, the pitching-moment coefficient of the blade airfoil section, and the physical characteristics of the rotor blade and machine. The validity of the analysis was examined by using it to predict the twist of a rotor whose twist characteristics had previously been measured in flight. The agreement between the calculated and experimental results was satisfactory. An examination of the assumption used in the analysis—that the twist is a linear function of the radius—disclosed that the approximation introduced no appreciable error. From this examination, a formula for the torsional rigidity of the rotor blade was derived.

INTRODUCTION

The development of the wingless direct-control autogiro has been hampered by a number of secondary difficulties. Probably the most troublesome are the avoidance of excessive or unstable center-of-pressure travel in the rotor and the elimination of rotor and control-stick vibrations. The production of a few designs that are satisfactory in these respects has demonstrated that the difficulties are the designer's problem and are not inherent in the direct-control type of rotor; however, the large number of unsuccessful machines is evidence that the basic factors controlling the behavior of the rotor are as yet not clearly understood.

A general survey of the problem indicated that both center-of-pressure travel and rotor vibrations are markedly affected by the periodic twist of the rotor blade arising from the interaction of air forces, elastic forces, and inertia forces during the flapping oscillation. It was accordingly decided that the factors controlling this twist must be understood before any real attack on the initial problems would be fruitful. This paper presents an analysis of periodic blade twist in which the factors controlling the twist are studied. The analysis is supported by a comparison of predicted and measured twist on a direct-control type of autogiro.

ANALYSIS

The motion of an autogiro rotor blade consists chiefly of rotation about the rotor axis, oscillation about the flapping hinge, and oscillation in twist about the blade-span axis. Additional components of the motion are oscillation as a pendulum in the plane of the rotor disk about a second hinge and oscillation in bending in a plane containing the blade span and the rotor axis. Experimental evidence has shown that these additional components have only a second-order influence on the air forces acting on the rotor blade, and they will consequently be neglected in the subsequent discussion.

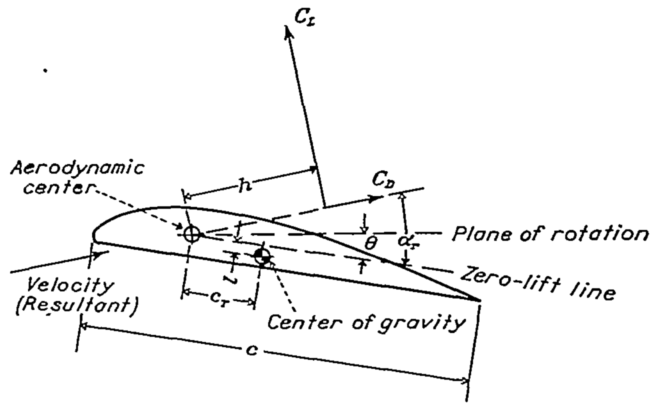

FIGURE 1.—Force diagram of autogiro rotor-blade element.

The coefficients of the air-twisting forces on an element of the autogiro rotor blade are diagrammed in figure 1. In general, the air forces on an airfoil will not pass through the aerodynamic center but will assume such a position that the moment of the air forces expressed in coefficient form is constant about the aerodynamic center. The component of the centrifugal force normal to the blade and the inertia forces of the blade pass through the center of gravity of the blade. Then the moment of the air forces about the center of gravity is the twisting moment on the blade. Let C_M be the moment coefficient of the air forces about the center of gravity; then from figure 1

$$C_M=-C_L\left(\frac{h}{c}-\frac{c_T}{c}\cos\alpha_r+\frac{l}{c}\sin\alpha_r\right)+C_D\left(\frac{c_T}{c}\sin\alpha_r+\frac{l}{c}\cos\alpha_r\right) \quad (1)$$

where the symbols are defined by figure 1; c_T is positive when the center of gravity is farther from the leading edge than the aerodynamic center, and l is positive when the center of gravity is below the aerodynamic center.

From the definition of moment about the aerodynamic center, if h is positive from the aerodynamic center toward the trailing edge,

$$h=\frac{-C_m c}{C_L} \tag{2}$$

where C_m is the coefficient of the moment about the aerodynamic center. Then

$$C_M=C_m+\frac{c_T}{c}(C_L \cos \alpha_r+C_D \sin \alpha_r)-\frac{l}{c}(C_L \sin \alpha_r-C_D \cos \alpha_r) \tag{3}$$

It will be assumed that α_r is sufficiently small that the cosine differs negligibly from unity and the sine can be equated to the angle; also, C_L will be assumed a linear function of the angle of attack. These assumptions do not accurately represent the conditions in the rotor when the resultant velocity is small and the angle of attack high, but the error introduced by them has been found to be small (reference 1). Then

$$C_M=C_m+\alpha_r\frac{c_T}{c}(a+C_D)-\frac{l}{c}(a\alpha_r^2-C_D) \tag{4}$$

where a is the lift-curve slope (in radian measure).

The lift-curve slope a at infinite aspect ratio lies between 5.8 and 5.9 for most airfoil sections. Inasmuch as C_D will have actual values varying from 0.009 to 0.03 below the stall, with a weighted average of about 0.015, it is thought unnecessary to use the C_D term in the expression for C_M. In addition, l/c will normally be less than 0.02 and $(a\alpha_r^2-C_D)$ will be of the same order of magnitude; the l/c term will consequently be dropped.

From reference 1, if xR be substituted for r, the nondimensional velocity components at the blade element of an autogiro rotor traveling at a speed V equal to $\mu\Omega R/\cos \alpha$ are:

$$u_T=\frac{U_T}{\Omega R}=x+\mu \sin \psi \tag{5}$$

$$\left.\begin{aligned}u_P=\frac{U_P}{\Omega R}=\lambda+\frac{1}{2}\mu a_1+\left(-\mu a_0+xb_1+\frac{1}{2}\mu a_2\right)\cos \psi+\left(-xa_1+\frac{1}{2}\mu b_2\right)\sin \psi+\left(\frac{1}{2}\mu a_1+2xb_2\right)\cos 2\psi \\ +\left(\frac{1}{2}\mu b_1-2xa_2\right)\sin 2\psi+\frac{1}{2}\mu a_2 \cos 3\psi+\frac{1}{2}\mu b_2 \sin 3\psi\end{aligned}\right\} \tag{6}$$

where u_T is the component of the resultant velocity perpendicular to the blade-span axis and to the rotor axis, u_P is the component of the resultant velocity perpendicular to the blade-span axis and to u_T, and ψ is the azimuth angle of the blade from its down-wind position. Also

$$u_T^2=x^2+\frac{1}{2}\mu^2+2\mu x \sin \psi-\frac{1}{2}\mu^2 \cos 2\psi \tag{7}$$

$$\left.\begin{aligned}u_Tu_P=x\lambda+\frac{1}{4}\mu^2b_2+\left(-\mu xa_0+b_1\left[x^2+\frac{1}{4}\mu^2\right]-\frac{1}{2}\mu xa_2\right)\cos \psi+\left(\mu\lambda-a_1\left[x^2-\frac{1}{4}\mu^2\right]-\frac{1}{2}\mu xb_2\right)\sin \psi \\ +\left(\mu xa_1+2x^2b_2\right)\cos 2\psi+\left(-\frac{1}{2}\mu^2a_0+\mu xb_1-2x^2a_2\right)\sin 2\psi+\left(-\frac{1}{4}\mu^2b_1+\frac{3}{2}\mu xa_2\right)\cos 3\psi \\ +\left(\frac{1}{4}\mu^2a_1+\frac{3}{2}\mu xb_2\right)\sin 3\psi-\frac{1}{4}\mu^2b_2 \cos 4\psi+\frac{1}{4}\mu^2a_2 \sin 4\psi\end{aligned}\right\} \tag{8}$$

The acute angle β between the blade and the plane perpendicular to the rotor axis is described by the expression

$$\beta=a_0-a_1 \cos \psi-b_1 \sin \psi-a_2 \cos 2\psi-b_2 \sin 2\psi-\ldots \tag{9}$$

The notation of reference 1 will be used throughout this analysis; a list of symbols employed and of their definitions is given at the end of this section.

The calculation of the air-twisting moment M_Q at the hub end of the blade will be made on the assumption that the air forces lie in a plane perpendicular to the blade span and depend only upon the resultant velocity in that plane. The angle φ between the resultant velocity and the plane perpendicular to the rotor axis will be assumed equal to its sine and tangent, and to have a cosine of unity. On this basis, the angle of attack of a blade element is

$$\alpha_r=\varphi+\theta=\frac{u_P}{u_T}+\theta \tag{10}$$

where θ is the pitch angle of the blade, measured as the acute angle between the plane perpendicular to the rotor axis and the zero-lift line of the blade airfoil section.

The pitch of the rotor blade will be the sum of the pitch setting and the instantaneous value of the angle of twist to which the blade is deflected by the twisting moment. If the pitch setting is given as $\theta_0+x\theta_1$ and it is assumed that the twist is a linear function of the radius, the pitch angle θ may be expressed in the form

$$\theta=\theta_0+x\theta_1+x\epsilon_0+x\epsilon_1 \cos \psi+x\eta_1 \sin \psi+x\epsilon_2 \cos 2\psi+x\eta_2 \sin 2\psi+\ldots \tag{11}$$

The use of a Fourier series in ψ for the angle of twist is justified, as it was for the flapping angle β, by the fact that the twist angle must be a repeated function of ψ. Some question concerning the assumption that the twist is linear along the radius naturally arises; the problem will subequently be examined in more detail.

The air-twisting moment at the blade hub can now be expressed in integral form as

$$M_Q=\int_0^B \frac{1}{2}\rho\Omega^2R^3u_T^2C_Mc^2dx \tag{12}$$

The integration is performed from B to 0 (where B is arbitrarily assigned the value $1-c/2R$) to allow for tip losses. Substituting for C_M from (4)

$$M_Q=\int_0^B \frac{1}{2}\rho\Omega^2R^3c^2u_T^2\left(C_m+a\alpha_r\frac{c_T}{c}\right)dx \tag{13}$$

Expression (10) for α_r can now be used

$$M_Q=\int_0^B \frac{1}{2}\rho\Omega^2R^3c^2u_T^2\left(C_m+a\frac{c_T}{c}\left[\frac{u_P}{u_T}+\theta\right]\right)dx \tag{14}$$

Substitute for θ, $u_T{}^2$ and $u_T\ u_P$; then integrate and collect; and

$$\begin{aligned}
M_Q=\frac{1}{2}\rho\Omega^2R^3cac_T\Bigg\{&\bigg(\frac{1}{2}\lambda B^2+\theta_0\left[\frac{1}{3}B^3+\frac{1}{2}\mu^2B\right]+\theta_1\left[\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2\right]+\frac{1}{4}\mu^2b_2B+\epsilon_0\left[\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2\right]+\frac{1}{3}\mu\eta_1B^3-\frac{1}{8}\mu^2\epsilon_2B^2\\
&+\frac{C_mc}{ac_T}\left[\frac{1}{3}B^3+\frac{1}{2}\mu^2B\right]\bigg)+\bigg(-\frac{1}{2}\mu a_0B^2+b_1\left[\frac{1}{3}B^3+\frac{1}{4}\mu^2B\right]-\frac{1}{4}\mu a_2B^2+\epsilon_1\left[\frac{1}{4}B^4+\frac{1}{8}\mu^2B^2\right]+\frac{1}{3}\mu\eta_2B^3-\frac{1}{8}\mu^2\epsilon_3B^2\bigg)\cos\psi\\
&+\bigg(\mu\lambda B+\mu\theta_0B^2+\frac{2}{3}\mu\theta_1B^3-a_1\left[\frac{1}{3}B^3-\frac{1}{4}\mu^2B\right]-\frac{1}{4}\mu b_2B^2+\frac{2}{3}\mu\epsilon_0B^3+\eta_1\left[\frac{1}{4}B^4+\frac{3}{8}\mu^2B^2\right]\\
&-\frac{1}{3}\mu\epsilon_2B^3-\frac{1}{8}\mu^2\eta_3B^2+\mu\frac{C_mc}{ac_T}B^2\bigg)\sin\psi+\bigg(-\frac{1}{2}\mu^2\theta_0B-\frac{1}{4}\mu^2\theta_1B^2+\frac{1}{2}\mu a_1B^2+\frac{2}{3}b_2B^3\\
&-\frac{1}{4}\mu^2\epsilon_0B^2-\frac{1}{3}\mu\eta_1B^3+\epsilon_2\left[\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2\right]+\frac{1}{3}\mu\eta_3B^3-\frac{1}{2}\mu^2\frac{C_mc}{ac_T}B\bigg)\cos 2\psi\\
&+\bigg(-\frac{1}{2}\mu^2a_0B+\frac{1}{2}\mu b_1B^2-\frac{2}{3}a_2B^3+\frac{1}{3}\mu\epsilon_1B^3+\eta_2\left[\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2\right]-\frac{1}{3}\mu\epsilon_3B^3\bigg)\sin 2\psi\\
&+\bigg(-\frac{1}{4}\mu^2b_1B+\frac{3}{4}\mu a_2B^2-\frac{1}{8}\mu^2\epsilon_1B^2-\frac{1}{3}\mu\eta_2B^3+\epsilon_3\left[\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2\right]\bigg)\cos 3\psi\\
&+\bigg(\frac{1}{4}\mu^2a_1B+\frac{3}{4}\mu b_2B^2-\frac{1}{8}\mu^2\eta_1B^2+\frac{1}{3}\mu\epsilon_2B^3+\eta_3\left[\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2\right]\bigg)\sin 3\psi\Bigg\}
\end{aligned} \tag{15}$$

In order to examine the variation of blade twist with radius, it is necessary first to establish an expression for the total air-twisting moment M_{Q_x} outboard of any station x. This moment can be expressed simply as

$$M_{Q_x}=\int_x^B \frac{1}{2}\rho\Omega^2R^3c^2u_T^2\left(C_m+a\frac{c_T}{c}\left[\frac{u_P}{u_T}+\theta\right]\right)dx \tag{16}$$

Substituting as in (14) and integrating

$$\begin{aligned}
M_{Q_x}=&\frac{1}{2}\rho\Omega^2R^3cac_T\Bigg\{\Bigg(\mu^2\left[\frac{1}{2}\theta_0+\frac{1}{4}b_2+\frac{1}{2}\frac{C_mc}{ac_T}\right][B-x]+\left[\frac{1}{2}\lambda+\frac{1}{4}\mu^2\theta_1+\frac{1}{4}\mu^2\epsilon_0-\frac{1}{8}\mu^2\epsilon_2\right][B^2-x^2]+\left[\frac{1}{3}\theta_0+\frac{1}{3}\frac{C_mc}{ac_T}+\frac{1}{3}\mu\eta_1\right]\\
&[B^3-x^3]+\left[\frac{1}{4}\theta_1+\frac{1}{4}\epsilon_0\right][B^4-x^4]\Bigg)+\Bigg(\frac{1}{4}\mu^2b_1[B-x]+\left[-\frac{1}{2}\mu a_0-\frac{1}{4}\mu a_2+\frac{1}{8}\mu^2\epsilon_1-\frac{1}{8}\mu^2\epsilon_3\right][B^2-x^2]\\
&+\left[\frac{1}{3}b_1+\frac{1}{3}\mu\eta_2\right][B^3-x^3]+\frac{1}{4}\epsilon_1[B^4-x^4]\Bigg)\cos\psi+\Bigg(\left[\mu\lambda+\frac{1}{4}\mu^2a_1\right][B-x]+\mu\left[\theta_0-\frac{1}{4}b_2+\frac{C_mc}{ac_T}+\frac{3}{8}\mu\eta_1-\frac{1}{8}\mu^2\eta_3\right]\\
&[B^2-x^2]+\left[-\frac{1}{3}a_1+\frac{2}{3}\mu\theta_1+\frac{2}{3}\mu\epsilon_0-\frac{1}{3}\mu\epsilon_2\right][B^3-x^3]+\frac{1}{4}\eta_1[B^4-x^4]\Bigg)\sin\psi+\Bigg(\mu^2\left[-\frac{1}{2}\theta_0-\frac{1}{2}\frac{C_mc}{ac_T}\right][B-x]\\
&+\left[-\frac{1}{4}\mu^2\theta_1-\frac{1}{4}\mu^2\epsilon_0+\frac{1}{2}\mu a_1+\frac{1}{4}\mu^2\epsilon_2\right][B^2-x^2]+\left[\frac{2}{3}b_2-\frac{1}{3}\mu\eta_1+\frac{1}{3}\mu\eta_3\right][B^3-x^3]+\frac{1}{4}\epsilon_2[B^4-x^4]\Bigg)\cos 2\psi\\
&+\Bigg(-\frac{1}{2}\mu^2a_0[B-x]+\left[\frac{1}{2}\mu b_1+\frac{1}{4}\mu^2\eta_2\right][B^2-x^2]+\left[-\frac{2}{3}a_2+\frac{1}{3}\mu\epsilon_1-\frac{1}{3}\mu\epsilon_3\right][B^3-x^3]+\frac{1}{4}\eta_2[B^4-x^4]\Bigg)\sin 2\psi\\
&+\Bigg(-\frac{1}{4}\mu^2b_1[B-x]+\left[\frac{3}{4}\mu a_2-\frac{1}{8}\mu^2\epsilon_1+\frac{1}{4}\mu^2\epsilon_3\right][B^2-x^2]-\frac{1}{3}\mu\eta_2[B^3-x^3]+\frac{1}{4}\epsilon_3[B^4-x^4]\Bigg)\cos 3\psi\\
&+\Bigg(\frac{1}{4}\mu^2a_1[B-x]+\left[\frac{3}{4}\mu b_2-\frac{1}{8}\mu^2\eta_1+\frac{1}{4}\mu^2\eta_3\right][B^2-x^2]+\frac{1}{3}\mu\epsilon_2[B^3-x^3]+\frac{1}{4}\eta_3[B^4-x^4]\Bigg)\sin 3\psi\Bigg\}
\end{aligned} \tag{17}$$

The torsional deflection of the rotor blade can now be determined. Let G designate a twisting moment that is distributed along the radius in the same manner as M_{Q_x} and is of such magnitude that the blade tip is deflected through 1 radian; then denoting the instantaneous torsional deflection at the tip of the blade by v

$$q\frac{d^2v}{dt^2}+Gv=M_Q \tag{18}$$

where q is the equivalent moment of inertia of the blade about the elastic axis and

$$v=\epsilon_0+\epsilon_1\cos\psi+\eta_1\sin\psi+\epsilon_2\cos 2\psi+\eta_2\sin 2\psi+\ldots \tag{19}$$

The value of q may be arrived at by considering the problem to be that of the torisonal deflection of a bar connecting two masses having moments of inertia of infinity and zero. The system then has but one degree of freedom and only principal modes of vibration will be assumed existent. Rayleigh's approximation to the exact solution of the problem (reference 2, p. 59), which is sufficiently accurate for use here, is

$$q=\frac{1}{3}il \tag{20}$$

where i is the moment of inertia of the bar per unit length, and l is the length of the bar.

For a particular case, $i=0.0050$, $l=20$, $G=1{,}700$, and $\Omega=21$; then $q=0.0333$ and $\Omega^2q=14.7$. It is seen that 4 Ω^2q and even 9 Ω^2q are quite small in comparison with G; consequently inertia effects on the torsional vibration will be neglected. Then

$$\begin{aligned}
&G\{\epsilon_0+\epsilon_1\cos\psi+\eta_1\sin\psi+\epsilon_2\cos 2\psi+\eta_2\sin 2\psi+\epsilon_3\cos 3\psi+\eta_3\sin 3\psi\}\\
&=\frac{1}{2}\rho\Omega^2R^3cac_T\Bigg\{\Bigg(\frac{1}{2}\lambda B^2+\theta_0\left[\frac{1}{3}B^3+\frac{1}{2}\mu^2B\right]+\theta_1\left[\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2\right]+\frac{1}{4}\mu^2b_2B+\epsilon_0\left[\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2\right]+\frac{1}{3}\mu\eta_1B^3-\frac{1}{8}\mu^2\epsilon_2B^2\\
&+\frac{C_mc}{ac_T}\left[\frac{1}{3}B^3+\frac{1}{2}\mu^2B\right]\Bigg)+\Bigg(-\frac{1}{2}\mu a_0B^2+b_1\left[\frac{1}{3}B^3+\frac{1}{4}\mu^2B\right]-\frac{1}{4}\mu a_2B^2+\epsilon_1\left[\frac{1}{4}B^4+\frac{1}{8}\mu^2B^2\right]+\frac{1}{3}\mu\eta_2B^3-\frac{1}{8}\mu^2\epsilon_3B^2\Bigg]\cos\psi\\
&+\Bigg(\mu\lambda B+\mu\theta_0B^2+\frac{2}{3}\mu\theta_1B^3-a_1\left[\frac{1}{3}B^3-\frac{1}{4}\mu^2B\right]-\frac{1}{4}\mu b_2B^2+\frac{2}{3}\mu\epsilon_0B^3+\eta_1\left[\frac{1}{4}B^4+\frac{3}{8}\mu^2B^2\right]-\frac{1}{3}\mu\epsilon_2B^3-\frac{1}{8}\mu^2\eta_3B^2\\
&+\mu\frac{C_mc}{ac_T}B^2\Bigg)\sin\psi+\Bigg(-\frac{1}{2}\mu^2\theta_0B-\frac{1}{4}\mu^2\theta_1B^2+\frac{1}{2}\mu a_1B^2+\frac{2}{3}b_2B^3-\frac{1}{4}\mu^2\epsilon_0B^2-\frac{1}{3}\mu\eta_1B^3+\epsilon_2\left[\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2\right]\\
&+\frac{1}{3}\mu\eta_3B^3-\frac{1}{2}\mu^2\frac{C_mc}{ac_T}B\Bigg)\cos 2\psi+\Bigg(-\frac{1}{2}\mu^2a_0B+\frac{1}{2}\mu b_1B^2-\frac{2}{3}a_2B^3+\frac{1}{3}\mu\epsilon_1B^3+\eta_2\left[\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2\right]-\frac{1}{3}\mu\epsilon_3B^3\Bigg)\sin 2\psi\\
&+\Bigg(-\frac{1}{4}\mu^2b_1B+\frac{3}{4}\mu a_2B^2-\frac{1}{8}\mu^2\epsilon_1B^2-\frac{1}{3}\mu\eta_2B^3+\epsilon_3\left[\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2\right]\Bigg)\cos 3\psi\\
&+\Bigg(\frac{1}{4}\mu^2a_1B+\frac{3}{4}\mu b_2B^2-\frac{1}{8}\mu^2\eta_1B^2+\frac{1}{3}\mu\epsilon_2B^3+\eta_3\left[\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2\right]\Bigg)\sin 3\psi\Bigg\}
\end{aligned} \tag{21}$$

The expressions for the thrust and the flapping-motion coefficients have been established in reference 3 for the rotor with varying twist; they are:

$$\left.\begin{aligned}T=\frac{1}{2}bc\rho a\Omega^2R^3\Big\{&\frac{1}{2}\lambda\left(B^2+\frac{1}{2}\mu^2\right)+\theta_0\left(\frac{1}{3}B^3+\frac{1}{2}\mu^2B-\frac{4}{9\pi}\mu^3\right)+\theta_1\left(\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2-\frac{1}{32}\mu^4\right)+\frac{1}{8}\mu^3a_1\\&+\frac{1}{4}\mu^2b_2B+\epsilon_0\left(\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2-\frac{1}{32}\mu^4\right)+\frac{1}{3}\mu\eta_1B^3-\frac{1}{8}\mu^2\epsilon_2B^2\Big\}\end{aligned}\right\} \tag{22}$$

$$\left.\begin{aligned}a_0=\frac{1}{2}\gamma\Big\{&\lambda\left(\frac{1}{3}B^3+0.080\mu^3\right)+\theta_0\left(\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2-\frac{1}{32}\mu^4\right)+\theta_1\left(\frac{1}{5}B^5+\frac{1}{6}\mu^2B^3\right)+\frac{1}{8}\mu^2b_2B^2\\&+\epsilon_0\left(\frac{1}{5}B^5+\frac{1}{6}\mu^2B^3\right)+\frac{1}{4}\mu\eta_1B^4-\frac{1}{12}\mu^2\epsilon_2B^3\Big\}-\frac{M_W}{I_1\Omega^2}\end{aligned}\right\} \tag{23}$$

$$a_1=\frac{2\mu}{B^4-\frac{1}{2}\mu^2B^2}\left\{\lambda\left(B^2-\frac{1}{4}\mu^2\right)+\theta_0\left(\frac{4}{3}B^3+0.106\mu^3\right)+\theta_1B^4-\frac{1}{3}b_2B^3+\epsilon_0B^4+\frac{\eta_1}{\mu}\left(\frac{2}{5}B^5+\frac{1}{2}\mu^2B^3\right)-\frac{1}{2}\epsilon_2B^4\right\} \tag{24}$$

$$b_1=\frac{4\mu}{B^4+\frac{1}{2}\mu^2B^2}\left\{a_0\left(\frac{1}{3}B^3+0.035\mu^3\right)+\frac{1}{6}a_2B^3-\frac{\epsilon_1}{\mu}\left(\frac{1}{5}B^5+\frac{1}{12}\mu^2B^3\right)-\frac{1}{4}\eta_2B^4\right\} \tag{25}$$

$$\left.\begin{aligned}a_2=\frac{\mu^2\gamma}{\gamma^2B^8+144}\Big\{&\lambda B\left(16+\frac{7\gamma^2B^8}{108}\right)+\theta_0B^2\left(\frac{46}{3}+\frac{7\gamma^2B^8}{144}\right)+\theta_1B^3\left(12+\frac{7\gamma^2B^8}{180}\right)+\epsilon_0B^3\left(12+\frac{7\gamma^2B^8}{180}\right)\\&-\frac{1}{30}\frac{\epsilon_1}{\mu}\gamma B^8+\frac{2}{5}\frac{\eta_1}{\mu}B^4+\frac{24}{5}\frac{\epsilon_2}{\mu^2}B^5+\frac{2}{5}\frac{\eta_2}{\mu^2}\gamma B^9\Big\}\end{aligned}\right\} \tag{26}$$

$$b_2=\frac{-\mu^2\gamma^2}{\gamma^2B^8+144}\left\{\frac{5}{9}\lambda B^5+\frac{25}{36}\theta_0B^6+\frac{8}{15}\theta_1B^7+\frac{8}{15}\epsilon_0B^7+\frac{2}{5\gamma}\frac{\epsilon_1}{\mu}B^4+\frac{1}{30}\frac{\eta_1}{\mu}B^8+\frac{2}{5}\frac{\epsilon_2}{\mu^2}B^9-\frac{24}{5\gamma}\frac{\eta_2}{\mu^2}B^5\right\} \tag{27}$$

It will be expedient to substitute average values for B and γ in a_2 and b_2; in actuality, B will be but little different from 0.970 for solidities near 0.05, and γ will be between 10 and 18 for present rotor blades. The assumption that B is 0.970 and that γ is 15.0 will accordingly introduce little error. The substitution will be made in such a way that the resultant expressions will be linear, and the coefficients of λ, θ, ϵ, and η will have the same form and exponents as the similar factors already present in the expressions for the twisting moment and the flapping motion. Examination of these equations discloses that the consequent forms for a_2 and b_2 are:

$$\left.\begin{aligned}a_2=&\mu^2\{0.0854\gamma\lambda B+0.0746\gamma\theta_0B^2+0.0588\gamma(\theta_1+\epsilon_0)B^3\}\\&+\mu\{-0.0184\epsilon_1+0.0012\gamma\eta_1B^4\}+0.0150\gamma\epsilon_2B^5+0.2200\eta_2B\end{aligned}\right\} \tag{28}$$

$$\left.\begin{aligned}b_2=&-\mu^2\{0.306\lambda B^{-3}+0.382\theta_0B^{-2}+0.294(\theta_1+\epsilon_0)B^{-1}\}\\&-\mu\left\{\frac{0.2200}{\gamma}\epsilon_1B^{-4}+0.0184\eta_1\right\}-0.2200\epsilon_2B+\frac{2.650}{\gamma}\eta_2B^{-3}\end{aligned}\right\} \tag{29}$$

Inspection of (26), (27), (28), and (29) shows that substitution for γ^2B^8 has been made in the denominator of (26) and (27), and in the numerator of (26) whenever γ has been raised higher than the first power. The quantity in braces of (27) has been divided by B^8 and the resultant γ^2B^8 outside the braces has been evaluated.

The solution of equation (21) for ϵ and η now will follow, after substituting for the a and b coefficients, by equating the coefficients of identical trigonometric terms in (21). Before this operation is performed, the work may be simplified to some extent by considering the order of accuracy required in the substitution.

It has already been shown (reference 1) that the expressions for the thrust and torque are evaluated to a sufficiently high order of μ (the fourth) if a_0, a_2, and b_2 are expressed to the order μ^2 and a_1 and b_1 to the order μ^3. Reference 3 shows that the same order of accuracy for the thrust and torque will be obtained if ϵ_0 is evaluated to μ^4, ϵ_1 and η_1 to μ^3, and ϵ_2 and η_2 to μ^2. The coefficients ϵ_3 and η_3 do not, to the order of μ^4, influence the thrust and torque. Reference to equation (21) establishes that ϵ_n and η_n are of the order μ^n, a fact that has already been implied by the form of the expressions for a_2 and b_2.

It is seen now that a_1 and b_1 may be expanded in a linear form which is developed only to the μ^3 order and that all terms in a_0 above the order μ^2 may be dropped. Then, substituting for a_2 and b_2,

$$a_0=\frac{1}{6}\gamma\lambda B^3+\gamma\theta_0\left(\frac{1}{8}B^4+\frac{1}{8}\mu^2B^2\right)+\gamma(\theta_1+\epsilon_0)\left(\frac{1}{10}B^5+\frac{1}{12}\mu^2B^3\right)+\frac{1}{8}\mu\gamma\eta_1B^4 \tag{30}$$

$$\left.\begin{aligned}a_1=&\mu\lambda\left(\frac{2}{B^2}+\frac{0.704\mu^2}{B^4}\right)+\mu\theta_0\left(\frac{8}{3B}+\frac{1.588\mu^2}{B^3}\right)+\mu(\theta_1+\epsilon_0)\left(2+\frac{1.196\mu^2}{B^2}\right)+\frac{0.147}{\gamma B^5}\mu^2\epsilon_1+\eta_1\left(\frac{4}{5}B+\frac{1.412\mu^2}{B}\right)\\&-0.853\mu\epsilon_2-\frac{1.768}{\gamma B^4}\mu\eta_2\end{aligned}\right\}\tag{31}$$

$$\left.\begin{aligned}b_1=&\mu\gamma\lambda\left(\frac{2}{9}B^2-0.0542\mu^2\right)+\mu\gamma\theta_0\left(\frac{1}{6}B^3+0.1330\mu^2B\right)+\mu\gamma(\theta_1+\epsilon_0)\left(\frac{2}{15}B^4+0.0836\mu^2B^2\right)-\epsilon_1\left(\frac{4}{5}B-0.055\frac{\mu^2}{B}\right)\\&+0.1676\mu^2\gamma\eta_1B^3+0.0100\mu\gamma\epsilon_2B^4-0.853\mu\eta_2\end{aligned}\right|\tag{32}$$

The term $M_W/I_1\Omega^2$ in a_0 has been neglected, since it amounts to 2 percent or less of a_0.

Substitute for a_0, a_1, b_1, a_2, and b_2 in (21); there results a set of linear equations in ϵ and η that can be solved in succession by starting with the highest order. Thus, from the coefficient of cos 3ψ,

$$\left.\begin{aligned}G\epsilon_3=&\frac{1}{2}\rho\Omega^2R^3cac_T\{0.0085\mu^3\gamma\lambda B^3+0.0143\mu^3\gamma\theta_0B^4+0.0108\mu^3\gamma(\theta_1+\epsilon_0)B^5+0.0621\mu^2\epsilon_1B^2+0.0009\mu^2\gamma\eta_1B^6\\&+0.0112\mu\gamma\epsilon_2B^7-0.1683\mu\eta_2B^3\}\end{aligned}\right\}\tag{33}$$

From sin 3ψ

$$G\eta_3=\frac{1}{2}\rho\Omega^2R^3cac_T\left\{0.271\frac{\mu^3}{B}\lambda+0.381\mu^3\theta_0+0.280\mu^3(\theta_1+\epsilon_0)B-\frac{0.1650}{\gamma B^2}\mu^2\epsilon_1+0.0612\mu^2\eta_1B^2+0.1683\mu\epsilon_2B^3+\frac{1.988}{\gamma B}\mu\eta_2\right\}\tag{34}$$

From cos 2ψ, retaining only terms in μ^2 or lower,

$$\left.\begin{aligned}G\epsilon_2=&\frac{1}{2}\rho\Omega^2R^3cac_T\left\{0.796\mu^2\lambda+0.578\mu^2\theta_0B+0.554\mu^2(\theta_1+\epsilon_0)B^2-\frac{0.147}{\gamma B}\mu\epsilon_1+0.055\mu\eta_1B^3+0.1033\epsilon_2B^4+\frac{1.768}{\gamma}\eta_2\right.\\&\left.-\frac{1}{2}\mu^2\frac{C_m}{ac_T}B\right\}\end{aligned}\right\}\tag{35}$$

From sin 2ψ,

$$\left.\begin{aligned}G\eta_2=&\frac{1}{2}\rho\Omega^2R^3cac_T\{-0.0291\mu^2\gamma\lambda B^4-0.0290\mu^2\gamma\theta_0B^5-0.0225\mu^2\gamma(\theta_1+\epsilon_0)B^6-0.0544\mu\epsilon_1B^3-0.0008\mu\gamma\eta_1B^7\\&-0.0100\gamma\epsilon_2B^8+0.1033\eta_2B^4\}\end{aligned}\right\}\tag{36}$$

From cos ψ, retaining all terms up to μ^4,

$$\left.\begin{aligned}G\epsilon_1=&\frac{1}{2}\rho\Omega^2R^3cac_T\left\{-\mu\gamma\lambda\left(\frac{1}{108}B^5-0.0161\mu^2B^3\right)-\mu\gamma\theta_0\left(\frac{1}{144}B^6-0.0049\mu^2B^4\right)-\mu\gamma(\theta_1+\epsilon_0)\left(\frac{1}{180}B^7-0.0048\mu^2B^5\right)\right.\\&\left.-\epsilon_1\left(\frac{1}{60}B^4-0.073\mu^2B^2\right)-0.0069\mu^2\gamma\eta_1B^6-0.0005\mu\gamma\epsilon_2B^7-0.0057\mu\eta_2B^3\right\}\end{aligned}\right\}\tag{37}$$

From sin ψ,

$$\left.\begin{aligned}G\eta_1=&\frac{1}{2}\rho\Omega^2R^3cac_T\left\{\mu\lambda\left(\frac{1}{3}B+0.341\frac{\mu^2}{B}\right)+\mu\theta_0\left(\frac{1}{9}B^2+0.233\mu^2\right)+0.175\mu^3(\theta_1+\epsilon_0)B+\frac{0.006}{\gamma B_2}\mu^2\epsilon_1+\eta_1\left(\frac{1}{60}B^4-0.008\mu^2B^2\right)\right.\\&\left.+0.006\ \mu\epsilon_2B^3-\frac{0.073}{\gamma B}\mu\eta_2+\mu\frac{C_mc}{ac_T}B^2\right\}\end{aligned}\right\}\tag{38}$$

From the constant term

$$\left.\begin{aligned}G\epsilon_0=&\frac{1}{2}\rho\Omega^2R^3cac_T\left\{\frac{1}{2}\lambda B^2+\theta_0\left(\frac{1}{3}B^3+\frac{1}{2}\mu^2B\right)+\theta_1\left(\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2\right)+\frac{1}{4}\mu^2b_2B+\epsilon_0\left(\frac{1}{4}B^4+\frac{1}{4}\mu^2B^2\right)+\frac{1}{3}\mu\eta_1B^3-\frac{1}{8}\mu^2\epsilon_2B^2\right\}\\&+\frac{1}{2}\rho\Omega^2R^3c^2C_m\left(\frac{1}{3}B^3+\frac{1}{2}\mu^2B\right)\end{aligned}\right\}\tag{39}$$

Comparison with (22) shows that the first part of (39) differs only by insignificant terms from the thrust multiplied by $cc_T/\sigma\pi R$, or, since σ is $bc/\pi R$,

$$G\epsilon_0=\frac{Tc_T}{b}+\frac{1}{2}\rho\Omega^2R^3c^2C_m\left(\frac{1}{3}B^3+\frac{1}{2}\mu^2B\right) \tag{40}$$

Let $\rho ca\Omega^2R^3c_T/2G$ be put equal to A; then after substituting from (35), (36), (37), and (38) for ϵ_2, η_2, ϵ_1, and η_1 and neglecting insignificant terms, the final expressions for the twist coefficients are obtained:

$$\epsilon_0=\frac{Tc_T}{bG}+A\frac{C_mc}{ac_T}\left(\frac{1}{3}B^3+\frac{1}{2}\mu^2B\right) \tag{41}$$

$$\left.\begin{aligned}\epsilon_1=&-\mu\gamma A\left\{\lambda\left(\frac{1}{108}B^5-0.0161\mu^2B^3\right)+\theta_0\left(\frac{1}{144}B^6-0.0049\mu^2B^4\right)+\left(\theta_1+\epsilon_0\right)\left(\frac{1}{180}B^7-0.0048\mu^2B^5\right)\right\}\\&+\mu^3\gamma A^2\left\{0.0021\lambda B^7+0.0007\theta_0B^8-0.0002(\theta_1+\epsilon_0)B^9+0.0071\frac{C_mc}{ac_T}B^8\right\}\end{aligned}\right\} \tag{42}$$

$$\left.\begin{aligned}\eta_1=&\mu A\left\{\lambda\left(\frac{1}{3}B+0.341\frac{\mu^2}{B}\right)+\theta_0\left(\frac{1}{9}B^2+0.233\mu^2\right)+0.175\mu^2(\theta_1+\epsilon_0)B+\frac{C_mc}{ac_T}B^2\right\}\\&+\mu^3A^2\left\{0.007\lambda B^3+0.006\theta_0B^4+0.005(\theta_1+\epsilon_0)B^5-0.003\frac{C_mc}{ac_T}B^4\right\}\end{aligned}\right\} \tag{43}$$

$$\left.\begin{aligned}\epsilon_2=&\mu^2A\left\{0.796\lambda+0.578\theta_0B+0.554(\theta_1+\epsilon_0)B^2-\frac{1}{2}\frac{C_mc}{ac_T}B\right\}\\&-\mu^2A^2\left\{0.032\lambda B^4+0.044\theta_0B^5+0.039(\theta_1+\epsilon_0)B^6-0.055\frac{C_mc}{ac_T}B^5\right\}\end{aligned}\right\} \tag{44}$$

$$\left.\begin{aligned}\eta_2=&-\mu^2\gamma A\{0.0291\lambda B^4+0.0290\theta_0B^5+0.0225(\theta_1+\epsilon_0)B^6\}\\&-\mu^2\gamma A^2\left\{0.0078\lambda B^8+0.0057\theta_0B^9+0.0052(\theta_1+\epsilon_0)B^{10}-0.0050\frac{C_mc}{ac_T}B^9\right\}\end{aligned}\right\} \tag{45}$$

$$\left.\begin{aligned}\epsilon_3=&\mu^3\gamma A\left\{0.0085\lambda B^3+0.0143\theta_0B^4+0.0108(\theta_1+\epsilon_0)B^5\right\}\\&+\mu^3\gamma A^2\left\{0.0135\lambda B^7+0.0111\theta_0B^8+0.0097(\theta_1+\epsilon_0)B^9-0.0047\frac{C_mc}{ac_T}B^8\right\}\end{aligned}\right\} \tag{46}$$

$$\left.\begin{aligned}\eta_3=&\mu^3A\left\{\frac{0.271}{B}\lambda+0.381\theta_0+0.280(\theta_1+\epsilon_0)B\right\}\\&+\mu^3A^2\left\{0.098\lambda B^3+0.048\theta_0B^4+0.049(\theta_1+\epsilon_0)B^5-0.023\frac{C_mc}{ac_T}B^4\right\}\end{aligned}\right\} \tag{47}$$

The expressions for the twist coefficients disclose that only ϵ_0, η_1, and ϵ_2 involve the factor $A\frac{C_mc}{ac_T}$, which reduces to $\frac{1}{2G}\rho c^2\Omega^2R^3C_m$ and is independent of c_T. The rest of the coefficients—ϵ_1, η_2, ϵ_3, and η_3, and parts of ϵ_0, η_1, and ϵ_2—are proportional to A and consequently to c_T. Thus if c_T is zero, only ϵ_0, η_1, and ϵ_2 differ from zero, and then only if C_m is not zero. Exclusive of the moment arising from C_m, the factor A represents in nondimensional form the ratio of the moment of the air forces to the torsional rigidity of the rotor blade.

The probable magnitude and range of values of A can be estimated. The rigidity G will be proportional to the polar moment of inertia of the blade spar and inversely proportional to the length of the blade. The moment of inertia of the spar will be proportional to the fourth power of the blade thickness. Then

$$G\propto\frac{c^4t^4}{R} \tag{48}$$

where t is the blade thickness divided by the chord.

The numerator of A can be examined by the following considerations. In the design of a rotor the pitch setting chosen is almost invariably the one that results in the highest efficiency. The rotor speed is then adapted to varying maximum speeds by adjusting the solidity. The rotor disk loading is fixed between fairly narrow boundaries by the requirement of good low-speed performance. For a given pitch setting, C_T/σ (ratio of thrust coefficient to solidity) is constant and $C_T\rho\Omega^2R^2$ is equal to the disk loading. Thus, $\sigma\rho\Omega^2R^2$ will be almost constant. Now, from (48) and this discussion,

$$A \propto \frac{R^3 c_T}{b c^4 t^4} \tag{49}$$

Or, from $c=\frac{\sigma\pi R}{b}$,

$$A \propto \frac{b^2 c_T}{R \sigma^4 t^4} \tag{50}$$

It has been found that a value of A of 0.600 is associated with a rotor that has a radius of 20 feet, a chord of 1.00 foot, a blade thickness ratio of 0.175, and c_T equal to 0.038 foot. It seems unlikely that R/ct will increase by more than 25 percent above the value given here of 114; assuming this increase, and an increase in c_T/c from 0.038 to an upper limit of 0.06, A would become approximately 1.90. The lower limit is obviously zero, since c_T/c may become zero. It will be found that normal designs will result in a value of A of less than unity and that the given value of 0.600 is larger than the average.

LIST OF SYMBOLS

R, blade radius.
b, number of blades.
c, blade chord.
c_T, distance between aerodynamic center and center of gravity of rotor-blade element.
r, radius of blade element.
x, r/R.
θ_0, blade pitch angle at hub, radians.
θ_1, difference between hub and tip pitch angles, radians.
ϵ_n, coefficient of cos $n\psi$ in expression for θ, radians.
η_n, coefficient of sin $n\psi$ in expression for θ, radians.
θ, instantaneous pitch angle, radians.
ψ, blade azimuth angle measured from down wind in direction of rotation, radians.
Ω, rotor angular velocity, $d\psi/dt$, radians per second.
$\lambda\Omega R$, speed of axial flow through rotor.
$\mu\Omega R$, component of forward speed in plane of disk, equal to $V\cos\alpha$, where V is forward speed, feet per second.
β, blade flapping angle, radians.
a_n, coefficient of cos $n\psi$ in expression for β, radians.
b_n, coefficient of sin $n\psi$ in expression for β, radians.
I_1, mass moment of inertia of rotor blade about rotor hinge.
α, rotor angle of attack, radians.
C_m, pitching-moment coefficient of rotor-blade airfoil section.
M_T, thrust moment about horizontal hinge.
M_W, weight moment of blade about horizontal hinge.
$u_T\Omega R$, velocity component at blade element perpendicular to blade span and parallel to rotor disk.
$u_P\Omega R$, velocity component at blade element perpendicular to blade span and to $u_T\Omega R$.
T, rotor thrust.
$C_T=T/\rho\Omega^2\pi R^4$
a, slope of curve of lift coefficient against angle of attack of blade airfoil section, in radian measure.

$\varphi=\tan^{-1}\frac{u_P}{u_T}$

α_r, blade-element angle of attack, radians.

$\gamma=\frac{c\rho a R^4}{I_1}$, mass constant of rotor blade.

$B=1-\frac{c}{2R}$, factor allowing for tip losses.

G, torsional rigidity of rotor blade, ft.-lb. per radian.

FLIGHT TESTS AND CALCULATIONS

Data for investigating the validity of the analysis were obtained in flight tests of a Kellett KD-1 direct-control wingless autogiro. The physical characteristics of the machine and its rotor are given in table I. Measurements were made in a steady glide of the air speed, the rotor speed, and the blade motion. The air speed was obtained with a trailing pitot-static head and an N. A. C. A. air-speed recorder; the rotor speed was ob served with a calibrated rotoscope; and the blade motion was photographed with a motion-picture camera mounted on and turning with the rotor hub. The photographs obtained with the motion-picture camera established the blade flapping motion and, in addition, the instantaneous twist at the rotor radius of the markers on the blade.

TABLE I.—PHYSICAL CHARACTERISTICS OF KD-1 AUTOGIRO

Gross weight, W	2,100 pounds.
Rotor radius, R	20.0 feet.
Blade weight, w_b	61.5 pounds.
Blade-weight moment, M_W	482 pound-feet.
Blade moment of inertia, I_1	175 slug-feet^2.
Blade chord, c	1.00 foot.
Chordwise location of blade center of gravity from leading edge	0.280 foot.
Chordwise location of aerodynamic center from leading edge	0.242 foot.
Number of blades, b	3.
Rotor solidity, σ	0.0478.
Blade airfoil section	Göttingen 606.
Blade pitch setting (constant), θ_0	0.0960 radian.
Airfoil section moment coefficient, C_m (about aerodynamic center)	−0.056.
Blade torsional rigidity constant, G	1,700 pound-feet.

In order to investigate the validity of the analysis, the flight-test data were used in two ways. The analysis was checked directly by predicting the blade twist at 0.75 R from the physical constants of the rotor and the value of the inflow coefficient λ. The factor λ was calculated from the experimental thrust coefficient by the substitution of known values in the expression for the thrust coefficient given in the analysis. A further examination of the analysis was made by substituting the experimental values of the inflow factor, the blade-motion coefficients, and the twist coefficients at one tip-speed ratio in the equation expressing the twisting moment as a function of the radius. The resultant twist deflection for a blade of constant rigidity followed directly and could be qualitatively compared with the basic assumption of a linear variation of twist with radius.

RESULTS AND DISCUSSION

Measured values of rotor speed and thrust coefficient are shown in figure 2, and derived values of the inflow factor λ are given in table II. The experimental blade-motion coefficients are presented in figure 3.

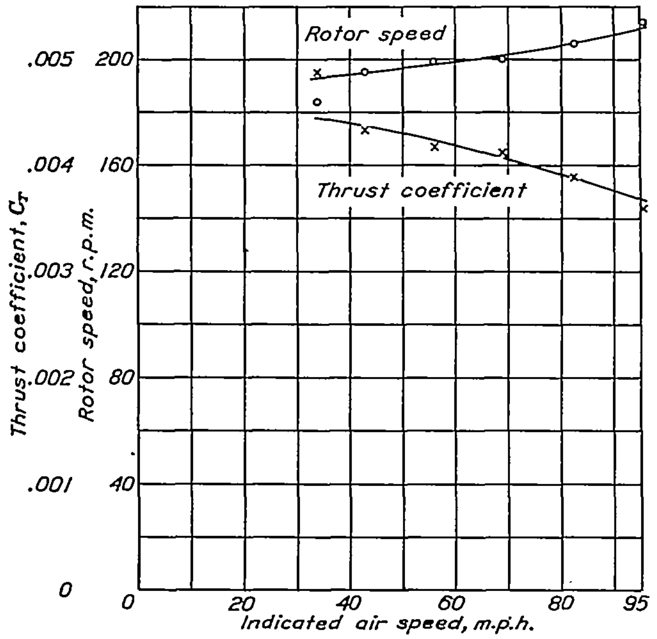

FIGURE 2.—Rotor speed and thrust coefficient of KD-1 autogiro rotor as measured in flight; ρ=0.00231 slug/cu. ft.

TABLE II.—DERIVED VALUES OF INFLOW FACTOR λ

μ	λ	μ	λ	μ	λ
0.125	0.0214	0.200	0.0209	0.275	0.0191
.150	.0213	.225	.0204	.300	.0185
.175	.0212	.250	.0197	.325	.0179

The twist coefficients shown in figure 4 represent a comparison of the experimental points with the calculated values for the radius at which the measurements were made. The agreement of theory with experiment is satisfactory, and strikingly so for η_1, the largest coefficient. The calculated ϵ_0 is consistently smaller than the measured value, but a reasonable explanation of the disagreement will be given later. Unfortunately, the experimental results for ϵ_1 and ϵ_2 are badly dispersed; the mean of the points is not appreciably different, however, from the predicted values.

The variation of twisting moment and twist angle with radius is illustrated in figures 5 and 6. The

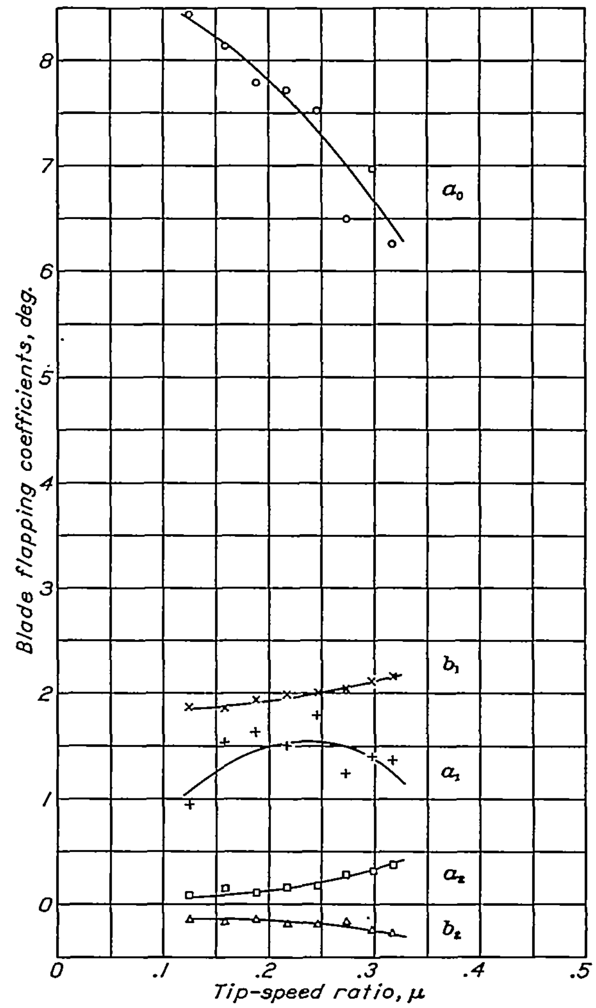

FIGURE 3.—Flapping motion coefficients of KD-1 autogiro rotor as measured in flight.

twisting moment at the hub for each component of the twist has been considered unity; the blade rigidity from x=0 to x=0.05 has been considered infinite as an approximation to the high rigidity inboard of the vertical pin; and the rigidity outboard of x=0.05 was chosen to make ϵ_0 unity at x=1.00. Examination of the figures discloses that ϵ_0 attains a larger value at x=0.75 for the same twisting moment at the hub than any of the remaining components of twist. This result can be considered a partial explanation of the underestimation of ϵ_0 in figure 4. The curves in figures 5 and 6 further indicate that the assumption of linear twist is a reasonably accurate approximation to the actual variation.

The curves in figures 5 and 6 suggest that it would be erroneous to calculate the torsional rigidity from

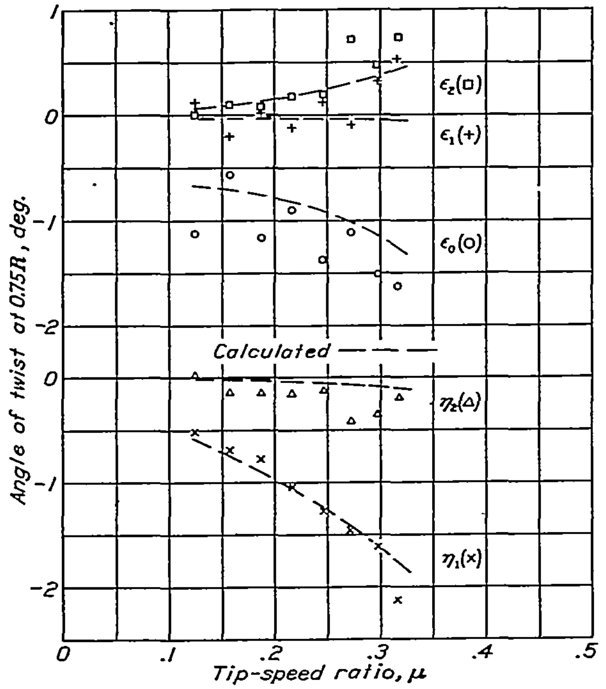

FIGURE 4.—Measured and calculated twist coefficients of KD-1 autogiro rotor.

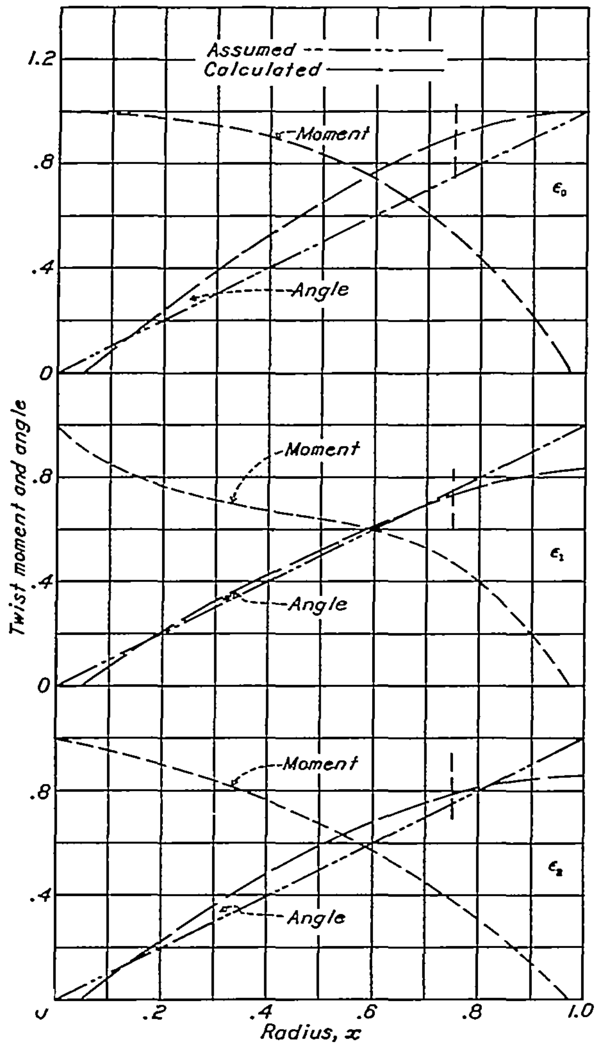

FIGURE 5.—Radial distribution of twisting moment and twist angle of KD-1 autogiro rotor; cos $n\psi$ components; $\mu=0.325$.

the twist produced by a constant moment along the blade. Instead, it is recommended that a moment be assumed which has a value G at the blade root and varies with the radius according to the expression (B^2-x^2). The definition of G suggested is that it be the moment at the hub, distributed as indicated, which will produce a twist of 0.80 radian at 0.75 R. Thus, if I_p is the polar moment of inertia of the blade cross

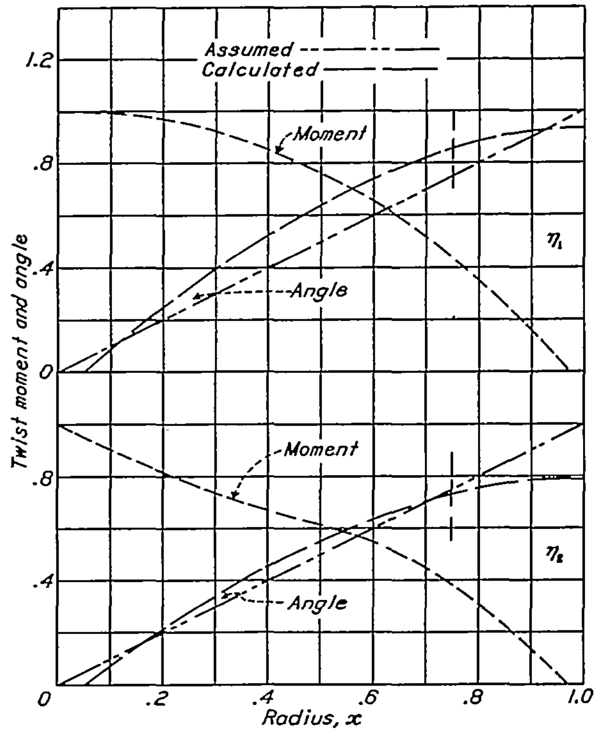

FIGURE 6.—Radial distribution of twisting moment and twist angle of KD-1 autogiro rotor; sin $n\psi$ components; $\mu=0.325$.

section, R the radius (blade length), and E_s the modulus of elasticity in shear,

$$G=\frac{64}{585}\frac{E_sI_p}{R}$$

where G is in pound-feet.
E_s is in pounds per square inch.
I_p is in inches^4.
R is in inches.

The value of 0.80 rather than 0.75 radian for the determination of G appears to result in a curve that is better approximated by a straight-line distribution to 1 radian at the tip, as evidenced in figures 5 and 6.

The merit of the analysis made in this paper depends upon the accuracy and facility with which it may be used to predict the twist of a rotor blade before the rotor itself has passed the drawing-board stage of design. The use of the analysis in this manner is not obvious since at first it appears that the rotor speed is required in order to calculate the twist; whereas the calculation of the rotor speeds can be made only after the twist, and consequently the thrust coefficient, is known. In one sense this objection is valid but in another the difficulty mentioned is not insurmountable. Assume a rotor design in which the known factors are the blade moment coefficient, the distance between the aerodynamic center and the center of gravity, the radius, the chord, the torsional rigidity, and the design maximum speed. Because of the variation of rotor efficiency with tip-speed ratio, it is mandatory that at maximum speed the tip-speed ratio shall be between 0.40 and 0.45. When the tip-speed ratio for design maximum speed is chosen, the tip speed is fixed. The twist coefficients ϵ_0, η_1, and ϵ_2, which depend principally upon C_m, can now be found with satisfactory accuracy by using the values of pitch setting and inflow coefficient λ that would be assigned to the rotor if the twist were zero. For a given airfoil section there is a mean lift coefficient which results in maximum efficiency and which fixes the ratio C_T/σ; the values of λ and the pitch setting corresponding to this mean lift coefficient should be used. The coefficients ϵ_0, η_1, and ϵ_2 now are known and fixed, and their effect upon C_T/σ can be evaluated. The desired value of C_T/σ is now attained by adjusting the pitch setting to offset the effect of twist, and all design requirements have been met. A final check of the twist coefficients using the final values of λ and the pitch setting can be made but, since C_m dominates the only coefficients that influence C_T/σ, it is found that only in exceptional and peculiar designs will the twist coefficients be affected.

CONCLUSIONS

1. The assumption that the twist of a rotor blade varies linearly with the radius is a satisfactory approximation to actual conditions.

2. The analysis of blade twist predicted without important error the twist of a rotor used as an example.

3. The torsional rigidity of the rotor blade should be calculated on the basis that the twisting moment varies with radius as (B^2-x^2).

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., *April 14, 1937.*

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