REPORT No. 171

ENGINE PERFORMANCE AND THE DETERMINATION OF ABSOLUTE CEILING

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SUMMARY.

This report was prepared at the request of the National Advisory Committee for Aeronautics and contains a brief study of the variation of engine power with temperature and pressure. It is shown that for the conventional engines

$$BHP \propto \left(\frac{p}{p_o}\right)^{1.15}$$

when temperature and R. P. M. are held constant, and that

$$BHP \propto \left(\frac{T}{T_o}\right)^{-0.50}$$

when pressure and R. P. M. are held constant. Combining these in the standard atmosphere (N. A. C. A. Report No. 147 and Technical Note No. 99) gives

$$BHP \propto \left(\frac{p}{p_o}\right)^{1.055}$$

for constant R. P. M.

The variation of R. P. M. with altitude is then found from the flight tests reports of the U. S. Army Air Service to be

$$N \propto \left(\frac{p}{p_o}\right)^{0.10}$$

for the usual case, or constant in certain special cases where the engine is provided with adequate throttle control. These relations are sufficient to determine the variation of BHP in standard atmosphere.

The variation of propeller efficiency in standard atmosphere is obtained from the general efficiency curve which is developed in N. A. C. A. Report No. 168. The variation of both power available and power required are then determined and curves plotted, so that the absolute ceiling may be read directly for any known sea-level value of the ratio of power available to power required.

INTRODUCTION.

Standard nomenclature will be used in this report whenever practicable, but in order to avoid confusion the symbol HP will be used for power. The subscripts "a" and "r" will be used to denote power available, HP_a, and power required, HP_r. A second subscript "o" will be used to denote sea-level conditions, thus, HP_{ao} and HP_{ro}. These symbols are cumbersome, but they prevent ambiguity.

Obviously, the rate of climb of an airplane depends upon the excess power; that is, the difference between HP_a and HP_r. Consequently the absolute ceiling, or the altitude at which HP_a is equal to HP_r for only one speed, depends on the factors HP_{ao}, HP_{ro}, and their variation with altitude y. HP_{ao} and HP_{ro} may be obtained from a single performance calculation. The variation of HP_r with altitude is known from the relation between ρ and y, since the velocity for any given altitude in horizontal flight is proportional to $\sqrt{\frac{\rho_0}{\rho}}$. The drag is proportional to ρ and therefore HP_r is proportional to the velocity, or to $\sqrt{\frac{\rho_0}{\rho}}$.

There remains to be determined only the variation of HP_a with y. This factor must be subdivided into the variations of BHP and propeller efficiency η with y. It has frequently been assumed that BHP varied as $\left(\frac{\rho}{\rho_0}\right)$ or as $\left(\frac{p}{p_0}\right)$. There is considerable theoretical justification for each of these assumptions, although neither is entirely satisfactory in practice. The assumptions that BHP varies either as $\left(\frac{\rho}{\rho_0}\right)^{1.10}$ or as $\left(\frac{p}{p_0}\right)^{1.04}$ have also been used extensively. These assumptions are based on test data either from the altitude chamber or from flights at various altitudes and therefore represent a fair approximation to the true conditions. It will be shown, however, that both temperature and pressure must be considered in order to obtain accurate results. That is, strictly speaking, the BHP of an engine does not depend on the density of the air supply. This has been explained in Br. A. C. A., R. & M. No. 462, and elsewhere as a result of the temperature rise which takes place between the time the charge passes through the carbureter and the time of closing of the inlet valve. This time is small but finite, and owing to the high temperature of the valves, passages, and cylinder walls a considerable heat transfer must occur. The density of the charge therefore depends more upon the pressure than upon the temperature of the air supply.

The variation of propeller efficiency with altitude is not simple. The common assumption of constant efficiency is not justified by available performance data. In general, the air speed increases and the R. P. M. decreases with altitude in a climb. The effect is to increase $\frac{V}{ND}$ and the efficiency. The magnitude of this increase may be calculated by the aid of the general efficiency curve developed in N. A. C. A. Report No. 168.

VARIATION OF *BHP* WITH *p*.

The variation of BHP when the air temperature is held constant and the pressure varied is not well known. Occasional reference will be found to the relation

$$BHP \propto \left(\frac{p}{p_0}\right)^{1.04} \tag{1}$$

based on altitude chamber, or free flight tests, in which the air temperature is varied also. The true relation must be found from accurate test data which, fortunately, are available in ample quantity to give a definite conclusion.

Table I contains actual test data selected at random from the indicated references. The values of BHP from this table are plotted logarithmically against pressures in Fig. 1. The constant slope of the lines of this figure, each of which represents a test at constant R. P. M. and air temperature, shows that

$$BHP \propto \left(\frac{p}{p_0}\right)^{1.15} \tag{2}$$

over the range of pressures used in service. This relation is very important since it apparently holds true for any reasonable air temperature and R. P. M.

VARIATION OF *BHP* WITH *T*.

The variation of BHP with the absolute temperature of the air supply when the R. P. M. and air pressure are constant is not generally known except to those who specialize in aircraft engine research. Assuming the BHP to vary as $\left(\frac{\rho}{\rho_0}\right)$ would be equivalent to assuming BHP to vary as $\left(\frac{T_0}{T}\right)$, i. e., inversely as the absolute temperature. The HP of an internal combustion engine depends directly on the weight of the charge in the cylinders, but this weight is not proportional to the air density as has been shown before. There is a continuous transfer of heat from the manifold and cylinder walls to the charge so that the temperature of the charge in the cylinder at the time of closing the intake valve tends toward constancy. While the effect of this factor can not be calculated it may be obtained from test data.

Representative test data selected at random from sources as indicated, are given in Table II. The values of BHP from this table are plotted logarithmically against absolute temperature in Fig. 2. Each line in this figure represents a series of tests at constant R. P. M. and air pressure The uniform slope shows that

$$BHP \propto \left(\frac{T}{T_o}\right)^{-0.50} \qquad (3)$$

over the range of temperatures likely to be encountered in service.

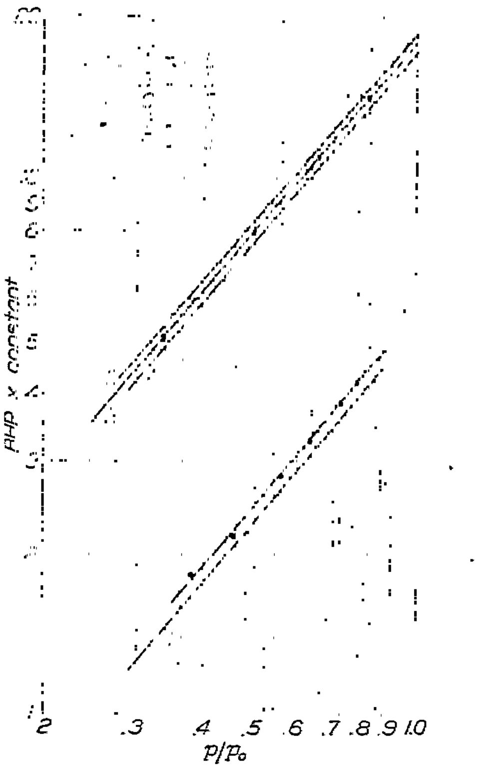

FIG. 1. Variation of BHP with air pressure (N and T constant).

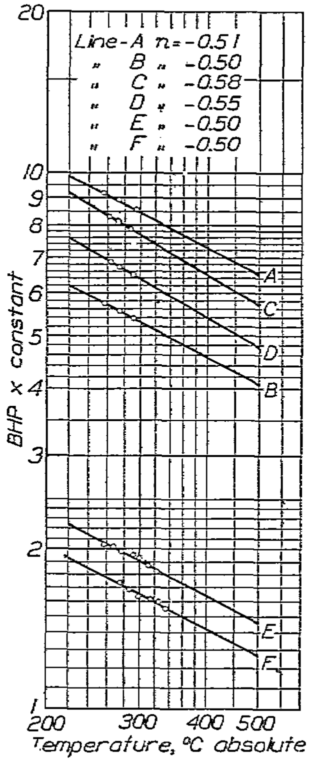

FIG. 2. Variation of BHP with air temperature (N and T constant).

It is found that there is a slight variation in the exponent in equation (3) for different engines. This variation is small and ordinarily the exponent is between -0.48 and -0.55. Part of the variation is undoubtedly due to experimental error and the small number of points used in defining the slope as in the case of line C, Fig. 2, which is included to show the extreme case so far noted in this study. Some variation with manifold design is to be expected, but this factor appears to be negligible in practice.

VARIATION OF *BHP* WITH ALTITUDE *y*.

In the Standard Atmosphere the relations between p, T, p and y are fixed. The variation of BHP in standard atmosphere may therefore be obtained from equations 2 and 3, just derived. Referring to N. A. C. A. Technical Note No. 99,

$$\left(\frac{T}{T_o}\right)=\left(\frac{p}{p_o}\right)^{0.19} \qquad (4)$$

Therefore

$$\left(\frac{T}{T_o}\right)^{-0.50}=\left(\frac{p}{p_o}\right)^{-0.095} \quad \text{(5)}$$

substituting (5) in (3) and combining with (2) gives

$$BHP\propto\left(\frac{p}{p_o}\right)^{1.055} \quad \text{(6)}$$

If desired BHP may be obtained in terms of y by the use of

$$\left(\frac{p}{p_o}\right)=(1.0-0.000006878y)^{5.255} \quad \text{(7)}$$

Equation (6) gives the decrease in BHP with altitudes (as determined by $\left(\frac{p}{p_o}\right)$. This is for constant R. P. M. Unless the engine be equipped with altitude throttle control there will be a gradual decrease in N with increase in y. This decrease shows a remarkable uniformity yet it appears to have been overlooked by previous investigators. That is, the loss in power has been lumped into a single function with no attempt to separate the component factors.

Table III contains observed values of N in climbs at various altitudes for a number of representative airplanes and engines. These data are taken from the United States Army Air Service information circulars as indicated. The engines in the airplanes listed in columns (A) to (G) inclusive have either no altitude control or else only a manual control on the throttle. Values of N from these columns are plotted logarithmically against $\left(\frac{p}{p_o}\right)$ in Fig. 3. It is found that the slope of the lines is substantially constant with a slope of about 1 in 10, thus giving

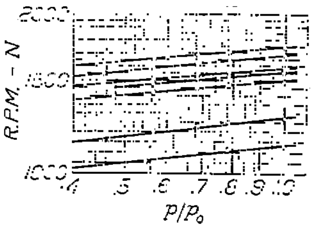

FIG. 3.—Variation of N with air pressure.

$$N\propto\left(\frac{p}{p_o}\right)^{0.10} \quad \text{(8)}$$

Since the engine is operating under a "propeller load" the BHP will vary as N^3. Consequently there will be a loss in power due to drop in N, given by

$$BHP\propto\left(\frac{p}{p_o}\right)^{.30} \quad \text{(9)}$$

This is in addition to the loss in power given by (6) so that the total loss in power is given by

$$BHP\propto\left(\frac{p}{p_o}\right)^{1.355} \quad \text{(10)}$$

When adequate altitude throttle control is provided, the value of N does not decrease appreciably at high altitudes. This is shown conclusively by the data in columns (H) and (I). For this case the total loss in power is given by (6).

VARIATION OF PROPELLER EFFICIENCY WITH ALTITUDE y.

The variation of propeller efficiency with altitude is complex but capable of a certain generalization. An approximation often used is that given in Br. A. C. A., R. and M. No. 324, which assumes that η may be expressed in terms of the air density. The method there employed is open to considerable error, however, and frequently gives results which are wholly unreliable. An original method based on a reasonable and proved variation in $\frac{V}{ND}$ will be used in this study. It has the disadvantage of complexity but the results obtained are well worth the effort. In order that the method may be made clear, the derivation will be given in full.

In the first place, η is a function of $\left(\frac{V}{ND}\right)$. The nature of this function is the same for all conventional propellers. In N. A. C. A. Report No. 168, it is shown that there is a general efficiency curve applying to all propellers. In this curve, $\frac{\eta}{\eta_m}$ is plotted against $\left(\frac{V}{ND}\right)/\left(\frac{V}{ND}\right)_m$, the subscript m referring to the maximum efficiency and its corresponding $\left(\frac{V}{ND}\right)$. Any variation in $\left(\frac{V}{ND}\right)$ must therefore produce a definite proportional variation in η. D is fixed so that we are concerned only with variations in V and N. The variation of N has been shown to be

$$N \propto \left(\frac{p}{p_o}\right)^{0.10} \qquad (8)$$

only the variation of V is yet to be determined.

In most cases it will be found that there is a decrease in *indicated* air speed as the altitude increases during a climb. This is due to the relation between HP_a and HP_r changing so that the maximum excess horsepower occurs at a larger angle of attack as the density decreases. However, at the ceiling the airplane must always fly at that angle of attack at which the ratio HP_{ao}/HP_{ro} is greatest, and the air speed at this angle of attack will vary as $\sqrt{\frac{\rho_o}{\rho}}$. That is

$$V = V_o\sqrt{\frac{\rho_o}{\rho}} \qquad (11)$$

where V_o and V are the true air speeds at sea level and altitude y, respectively. The variation of $\left(\frac{V}{ND}\right)$ with altitude is fully determined by equations (8) and (11) for the usual case, or by equation (11) alone when N is constant.

The next step is to determine the initial value of $\left(\frac{V}{ND}\right)$. This may be obtained from free flight tests. Table IV contains data taken from the U. S. Army Air Service Information Circulars as indicated. It is found that for all practical purposes the initial $\left(\frac{V}{ND}\right)$ in climb is 66 per cent of the $\left(\frac{V}{ND}\right)$ at high speed. It has been explained that the initial air speed in climb is somewhat higher than that corresponding to the angle of attack which obtains at the absolute ceiling. The average change in both V and N has the effect of reducing the figure just given in the order of 10 per cent so that the initial $\left(\frac{V}{ND}\right)$ at the angle of attack which obtains at the absolute ceiling may be written as

$$\left(\frac{V}{ND}\right)_o = .60\left(\frac{V}{ND}\right)_m \qquad (12)$$

Assuming that the propeller efficiency is a maximum at high speed, the probable value of $\frac{\eta}{\eta_o}$ for a series of altitudes have been calculated in Table V for the case where $N \propto \left(\frac{p}{p_o}\right)^{0.10}$, and in Table VI for the case where N is constant. The procedure is straight forward and partially explained by the column headings. Obviously $\left(\frac{V}{V_o}\right)\left(\frac{N_o}{N}\right)$ is the ratio $\left(\frac{V}{ND}\right)/\left(\frac{V}{ND}\right)_o$. $\frac{\eta}{\eta_m}$ is the efficiency ratio corresponding to the ratio $\left(\frac{V}{ND}\right)/\left(\frac{V}{ND}\right)_m$ from the efficiency curve given in N. A. C. A. Report No. 168.

THE CALCULATION OF ABSOLUTE CEILING.

CASE I.—$N \propto \left(\frac{p}{p_o}\right)^{0.10}$

The calculation of absolute ceiling is obviously a determination of the altitude (pressure or density) at which HP_a is equal to HP_r at only one speed. This condition must occur at the angle of attack at which the ratio HP_{ao}/HP_{ro} is greatest. The maximum value of this ratio is therefore a measure of the absolute ceiling.

Since HP_r at any given angle of attack is directly proportional to the true velocity

$$\frac{HP_r}{HP_{ro}}=\sqrt{\frac{\rho_o}{\rho}} \qquad 13)$$

The increase in propeller efficiency partially counteracts the decrease in HP_a given by equation (10) so that the net HP_a is given by

$$\frac{HP_a}{HP_o}=\frac{\eta}{\eta_o}\left(\frac{p}{p_o}\right)^{1.355} \qquad (14)$$

where $\frac{\eta}{\eta_o}$ is to be taken from Table V for each value of $\left(\frac{p}{p_o}\right)$.

Dividing (14) into (13) gives

$$\left(\frac{HP_{ao}}{HP_{ro}}\right)\left(\frac{HP_r}{HP_a}\right)=f(y) \qquad (15)$$

since $HP_a = HP_r$ at the absolute ceiling; the value of $\frac{HP_{ao}}{HP_{ro}}$ corresponding to any value of the absolute ceiling may be determined by solving for $f(y)$ in equation (15). This has been done in Table VII, where the headings to the columns should be self explanatory. The values of HP_{ao}/HP_{ro} so obtained are plotted against y in Fig.4. The absolute ceiling of any airplane equipped with the conventional engines and carburettors may be read from the curve when HP_{ao}/HP_{ro} is known.

CASE II.—WHEN N IS CONSTANT.

In this case the procedure is similar to that just outlined except in calculating HP_a, which is now obtained from equation (6) together with the values of $\frac{\eta}{\eta_o}$ from Table VI. That is

$$\frac{HP_a}{HP_{ao}}=\left(\frac{p}{p_o}\right)^{1.055}\frac{\eta}{\eta_o} \qquad (16)$$

Calculations for the values of HP_{ao}/HP_{ro} corresponding to the usual values of y are made in Table VIII. These values are plotted in Fig. 5, which is to be used in place of Fig. 4, when the engine is equipped with adequate altitude throttle control. Whether or not the control is adequate must be determined by the criterion of constancy in N.

CONCLUSION.

There is only one doubtful factor in the calculation of absolute ceiling, the variation of N with altitude. In a surprisingly large number of cases, equation (8) holds true; a few cases have been noted where N was substantially constant from sea level to the highest altitude attained, and it is to be expected that in some cases the variation will lie between these limits. In the absence of accurate data on the performance of a particular engine Case I corresponding to equation (8), should be used.

The rate of climb of an airplane and its variation with altitude should be made the object of a separate study, but it is to be noted at this time that the assumption of a uniform decrease in climb from a maximum at sea level to zero at the absolute ceiling implies a uniform decrease in excess power. This assumption, while not necessarily true, according to the values of HP_a and HP_r from Tables VII and VIII, appears to be justified by the results of free flight tests. An explanation may be found in the change of angle of attack, previously mentioned. That is, the excess power used in climb is not the difference between the HP_a and the HP_r used in calculating the absolute ceiling, but in general it is somewhat greater. This follows from the fact that the relation between the L/D of the airplane and its speed is

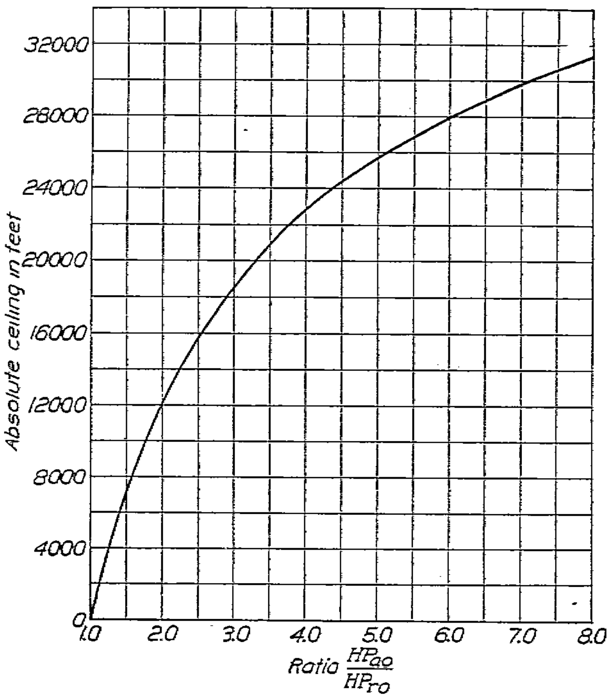

Fig. 4.—Absolute ceiling as determined by $\frac{HP_{ao}}{HP_{ro}}$.
Case I: $N \propto \left(\frac{\rho}{\rho_0}\right)^{0.16}$.

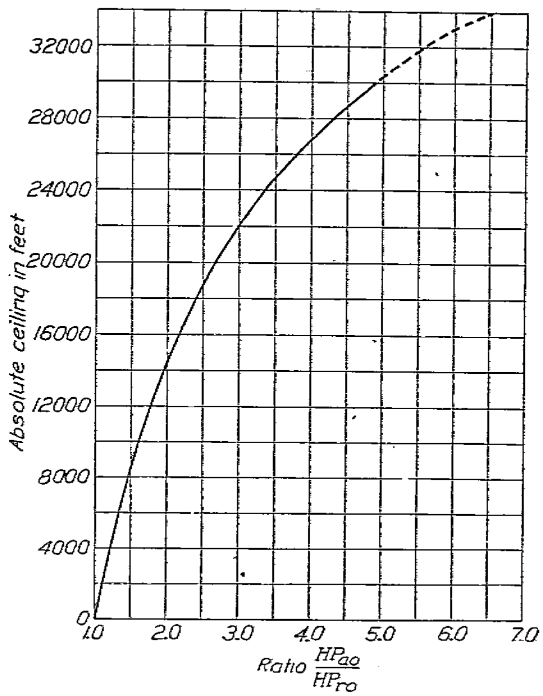

Fig. 5.—Absolute ceiling as determined by $\frac{HP_{ao}}{HP_{ro}}$.
Case II: N constant.

such that HP_r is increasing slowly over a considerable range of speed while the HP_a is increasing rapidly—in comparison. The maximum value of HP_{ao}/HP_{ro} will occur near the minimum value of HP_{ro} but the maximum excess power will occur at some higher speed.

It should be noted that equations (2), (3), (8), etc., may be used in reducing observed performance to standard atmosphere conditions. The air forces on the airplane may be assumed to vary directly as the air density and proper corrections made for the true power delivered by the engine.

REFERENCES.

1. National Advisory Committee for Aeronautics Reports Nos. 45, 147, and 168.
2. National Advisory Committee for Aeronautics Technical Note No. 99.
3. United States Army Air Service Information Circulars Nos. 37, 40, 50, 51, 71, 109, 132, 155, 173, 280, 286, 287, 288, 306, and 310.
4. British Advisory Committee for Aeronautics—R. and M. Nos. 324, 462, 474, and 534.
5. Internal Combustion Engine Subcommittee Reports Nos. 19, 35, 39, and 44.

TABLE I.

Variation in brake horsepower with air pressure.

[R. P. M. and air temperature constant for each test.]

A.			B.			C.			D.			E.		
Press. p	$\frac{p}{p_r}$	BHP	Press. p	$\frac{p}{p_o}$	BHP	Press. p	$\frac{p}{p_o}$	BHP	Press. p	$\frac{p}{p_o}$	BHP	Press. p	$\frac{p}{p_o}$	BHP
61.1	0.804	133.3	62.1	0.817	140.4	60.6	0.797	142.0	24.70	0.826	41.20	23.08	0.772	42.00
48.2	.634	103.3	49.8	.655	110.8	49.7	.654	115.2	22.78	.761	38.15	21.64	.723	38.05
35.5	.467	71.0	37.6	.495	80.4	37.6	.495	84.8	19.31	.645	31.22	19.03	.635	32.60
27.7	.364	52.4	25.6	.337	50.6	25.7	.338	53.3	16.96	.567	27.36	16.78	.560	28.07
									14.56	.486	22.66	13.60	.454	21.92
									11.56	.386	16.84	11.38	.380	18.38

Sources: Data in columns A, B, and C are taken from Tables I-III, N. A. C. A. Report No. 45, Part I. Data in columns D and E are from Tables I and II, Br. A. C. A. Internal Combustion Engine Subcommittee Report No. 44.

TABLE II.

Variation in brake horsepower with air temperature.

[R. P. M. and air pressure constant for each test.]

A.		B.		C.		D.		E.		F.	
T °C	BHP	T °C	BHP	T °C	BHP	T °C	BHP	T °C	BHP	T °C	BHP
255.0	91.3	257.2	57.6	261.4	83.3	263.3	68.4	257.5	202.4	275.2	17.3
270.4	89.3	273.0	55.8	274.0	80.8	273.6	66.3	268.5	201.2	287.0	16.7
293.8	85.1	289.2	54.1	287.7	78.5	289.6	64.4	279.1	197.4	299.9	16.3
								290.3	194.4	313.0	16.1
								299.0	190.8	323.0	15.9
								311.6	186.0	333.0	15.5
								325.4	181.5		
H=19,200 feet.		H=29,750 feet.		H=19,250 feet.		H=23,170 feet.		H=1,950 feet.		H=500 feet.	

Sources follows: (A) N. A. C. A. Report No. 45, Part III, Table II. (B) N. A. C. A. Report No. 45, Part III, Table II. (C) N. A. C. A. 45, Part III, Table III. (D) N. A. C. A. Report No. 45, Part III, Table III (E) N. A. C. A. Report No. 45, Part III, Table VI. C. A I. C. E. S. C., Report No. 19, Table I.
(A) to (E) inclusive are for Hispano-Suiza 8-cylinder engine.
(F) is for RAF 4TD engine (single cylinder).

TABLE III.

Variation of R. P. M. with altitude in climb.

H. Feet.	$\frac{p}{p_o}$	A. Fokker D VII.	B. Thomas Morse S-6.	C. Roland D VI B.	D. DH4	E. DH4	F. Fokker D VIII.	G. Fokker D VII.	H. Junker JL-6.	I. Spad 13.
0	1.000	1,555	1,130	1,490	1,730	1,575	1,262	1,680	1,365	2,040
6,500	.786	1,540	1,125	1,475	1,705	1,560	1,250	1,645	1,365	2,040
10,000	.688	1,520	1,110	1,460	1,690	1,540	1,238	1,625	1,365	2,040
15,000	.564	1,470	1,055	1,430	1,655	1,500	1,210	1,595	1,350	2,030
20,000	.459	....	....	(1,395)	....	....	1,160	1,555	....	2,010
		Engine.								
		Liberty "6."	Le Rhone.	Benz.	Hispano-Suiza.	Liberty.	Oberursel.	Packard "1237."	BMW.	Wright "220."
BHP		215	80	200	300	400	110	350	242	220
n		0.09	0.104	0.095	0.09	0.09	0.104	0.10	....	....
Reference: A. S. I. C. No.		71	109	132	155	287	288	310	173	286

TABLE IV.

$\frac{V}{ND}$ *in climb—Initial value.*

Airplane.	Prop. diameter (feet). D	High speed. V	High speed. RPM. N	High speed. $\frac{V}{ND}$ J	Climb. V	Climb. RPM. N	Climb. $\frac{V}{ND}$ J_1	$\frac{J_1}{J}$	Reference: A. S. I. C. No.
USXBIA	9.17	124.0	1,730	0.687	72.0	1,520	0.456	0.663	37
Thomas Morse MB-3	8.13	152.0	1,835	.895	81.0	1,595	.550	.613	40
Loening monoplane	8.67	143.5	1,900	.766	83.0	1,630	.518	.675	50
Ordnance D	8.5	147.0	1,885	.807	72.0	1,585	.470	.582	51
Fokker D-VII	8.5	120.0	1,750	.710	71.0	1,555	.473	.665	71
Thomas Morse S-6	8.2	97.0	1,260	.825	58.0	1,130	.551	.668	109
Roland D VI B	9.39	114.0	1,610	.662	72.0	1,490	.453	.683	132
Junkers JL-6	9.51	111.2	1,445	.712	66.0	1,365	.449	.630	173
Sperry Messenger	6.5	96.7	1,880	.697	60.0	1,640	.496	.712	280
Spad 13	8.18	131.5	2,300	.615	78.0	2,040	.412	.670	286
Orenco D	8.5	139.5	1,810	.797	80.0	1,520	.546	.685	306
Fokker D-VII	8.67	151.0	1,975	.775	80.0	1,680	.484	.624	310
Average								.656	

TABLE V.

Variation of propeller efficiency with altitude.

CASE I.—$\left(N \propto \left(\frac{p}{p_o}\right)^{0.10}\right)$.

Altitude (feet).	Standard atmosphere. $\frac{p}{p_o}$	Standard atmosphere. $\frac{\rho}{\rho_o}$	True velocity ratio $\frac{V}{V_o}$ $\left(\sqrt{\frac{\rho_o}{\rho}}\right)$	$\left(\frac{N_o}{N}\right)$ $\left(\frac{p_o}{p}\right)^{0.10}$	$\frac{\left(\frac{V}{V_o}\right)}{\left(\frac{N_o}{N}\right)}$	$\frac{\left(\frac{V}{ND}\right)}{\left(\frac{V}{ND}\right)_m}$	$\frac{\eta}{\eta_m}$	$\frac{\eta}{\eta_o}$
0	1.0000	1.0000	1.0000	1.0000	1.0000	0.600	0.825	1.000
2,000	.9298	.9428	1.0299	1.007	1.0371	.622	.843	1.022
4,000	.8637	.8881	1.0611	1.015	1.0770	.646	.861	1.044
6,000	.8013	.8358	1.0938	1.022	1.1179	.671	.879	1.065
8,000	.7428	.7860	1.1279	1.030	1.1617	.697	.897	1.087
10,000	.6877	.7384	1.1640	1.038	1.2082	.725	.914	1.108
12,000	.6359	.6931	1.2011	1.046	1.2564	.754	.930	1.127
14,000	.5874	.6500	1.2404	1.055	1.3086	.785	.945	1.145
16,000	.5409	.6089	1.2815	1.063	1.3622	.817	.960	1.164
18,000	.4993	.5699	1.3247	1.072	1.4200	.852	.973	1.179
20,000	.4595	.5328	1.3701	1.081	1.4811	.889	.985	1.194
22,000	.4222	.4975	1.4177	1.090	1.5453	.927	.993	1.204
24,000	.3875	.4641	1.4679	1.099	1.6132	.968	.998	1.210
26,000	.3551	.4324	1.5207	1.109	1.6865	1.012	1.000	1.212
28,000	.3249	.4024	1.5763	1.119	1.7639	1.058	.994	1.205
30,000	.2969	.3741	1.6348	1.129	1.8457	1.107	.978	1.185

TABLE VI.

Variation of propeller efficiency with altitude.

CASE II.—N CONSTANT.

Altitude (feet).	$\frac{V}{V_o}$	$\frac{\left(\frac{V}{ND}\right)}{\left(\frac{V}{ND}\right)_m}$	$\frac{\eta}{\eta_m}$	$\frac{\eta}{\eta_o}$
0	1.0000	0.600	0.825	1.000
2,000	1.0299	.618	.840	1.018
4,000	1.0611	.637	.855	1.036
6,000	1.0938	.656	.869	1.055
8,000	1.1279	.677	.884	1.072
1,0000	1.1640	.698	.898	1.088
1,2000	1.2011	.721	.911	1.104
1,4000	1.2404	.744	.925	1.121
1,6000	1.2815	.769	.938	1.137
1,8000	1.3247	.795	.950	1.152
20,000	1.3701	.822	.962	1.166
22,000	1.4177	.851	.973	1.179
24,000	1.4679	.881	.982	1.190
26,000	1.5207	.912	.990	1.200
28,000	1.5763	.946	.996	1.207
30,000	1.6348	.981	.999	1.211

TABLE VII.

Calculation of absolute ceiling—$\frac{HP_{ao}}{HP_{ro}}$ *vs y*.

CASE I.—$N \propto \left(\frac{p}{p_o}\right)^{0.10}$

1 Altitude y feet.	2 $\left(\frac{p}{p_o}\right)$	3 $\left(\frac{BHP_a}{BHP_{ao}}\right)^1$ $\left(\frac{p}{p_o}\right)^{1.355}$	4 $\frac{\eta}{\eta_o}$ from Table V.	5 $\frac{HP_a}{HP_{ao}}$ (3)X(4)	6 $\frac{HP_{ao}}{HP_a}$	7 $\frac{HP_r}{HP_{ro}}$ $\sqrt{\frac{\rho_o}{\rho}}$	8 $\frac{HP_{ao}}{HP_{ro}}$ (6)X(7)
0	1.0000	1.0000	1.000	1.0000	1.0000	1.0000	1.0000
2,000	.9298	.9061	1.022	.9260	1.0799	1.0299	1.1122
4,000	.8637	.8199	1.044	.8560	1.1682	1.0611	1.2396
6,000	.8013	.7407	1.065	.7888	1.2677	1.0938	1.3866
8,000	.7428	.6684	1.087	.7266	1.3763	1.1279	1.5523
10,000	.6877	.6021	1,108	.6671	1.4990	1.1640	1.7448
12,000	.6359	.5415	1.127	.6103	1.6385	1.2011	1.9680
14,000	.5874	.4863	1.145	.5568	1.7960	1.2404	2.2278
16,000	.5409	.4349	1.164	.5062	1.9755	1.2815	2.5316
18,000	.4993	.3902	1.179	.4600	2.1739	1.3247	2.8798
20,000	.4595	.3487	1,194	.4163	2.4021	1.3701	3.2911
22,000	.4222	.3109	1.204	.3743	2.6717	1.4177	3.7877
24,000	.3875	.2768	1.210	.3349	2.9860	1.4679	4.3831
26,000	.3551	.2459	1.212	.2980	3.3557	1.5207	5.1030
28,000	.3249	.2180	1.205	.2627	3.8066	1.5763	6.0003
30,000	.2969	.1929	1,185	.2286	4.3745	1.6348	7.1514

TABLE VIII.

Calculation of absolute ceiling=$\frac{HP_{ao}}{HP_{ro}}$ *vs. y*.

CASE II.—N CONSTANT.

1 Altitude y feet.	2 $\left(\frac{p}{p_o}\right)$	3 $\left(\frac{BHP_a}{BHP_{ao}}\right)^1$ $\left(\frac{p}{p_o}\right)^{1.055}$	4 $\frac{\eta}{\eta_o}$ from Table V.	5 $\frac{HP_a}{HP_{ao}}$ (3)×(4)	6 $\frac{HP_{ao}}{HP_a}$	7 $\frac{HP_r}{HP_{ro}}$ $\sqrt{\frac{\rho_o}{\rho}}$	8 $\frac{HP_{ao}}{HP_{ro}}$ (6)×(7)
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2,000	.9298	.9261	1.018	.9428	1.0607	1.0299	1.0924
4,000	.8637	.8567	1.036	.8875	1.1268	1.0611	1.1956
6,000	.8013	.7916	1.055	.8351	1,1975	1.0938	1.3098
8,000	.7428	.7307	1.072	.7833	1.2766	1.1279	1.4399
10,000	.6877	.6737	1.088	.7330	1,3643	1.1640	1.5880
12,000	.6359	.6203	1.104	.6848	1.4603	1.2011	1.7540
14,000	.5874	.5705	1.121	.6395	1.5637	1.2404	1.9396
16,000	.5404	.5229	1,137	.5945	1.6821	1.2815	2.1556
18,000	.4993	.4806	1,152	.5537	1.8060	1.3247	2.3924
20,000	.4595	.4403	1.166	.5134	1.9478	1.3701	2.6687
22,000	.4222	.4026	1.179	.4747	2.1066	1.4177	2.9865
24,000	.3875	.3678	1.190	.4377	2.2847	1.4679	3.3537
26,000	.3551	.3354	1.200	.4025	2.4845	1.5207	3.7782
28,000	.3249	.3054	1.207	.3686	2.7130	1.5763	4.2765
30,000	.2969	.2777	1.211	.3363	2.9735	1.6348	4.8611