RESEARCH MEMORANDUM

NOTES ON THE PREDICTION OF SHOCK-INDUCED
BOUNDARY-LAYER SEPARATION

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INTRODUCTORY REMARKS

One of the fundamental problems that appears in the investigation of supersonic flow over a surface is that of the phenomena associated with the interaction of shock waves and boundary layers. The problem of whether a given shock wave will cause boundary-layer separation is one which occurs in all cases where a pressure increase is to be obtained as a result of the retardation of the flow. Such problems occur, for example, in the flow in supersonic diffusers and air inlets and in the flow at the rear of airfoils and bodies. Shock-induced boundary-layer separation generally results in poor aerodynamic efficiency in the former case and in undesirable airfoil characteristics in the latter case and, therefore, this problem is of considerable practical significance. The purpose of this paper is to discuss the status of information relative to the prediction of shock-induced boundary-layer separation. In order to study the fundamental features of the problem, the discussion is concerned principally with data obtained on flat plates in two-dimensional flow.

Prandtl has discussed separation of the incompressible boundary layer under the influence of a positive pressure gradient (refs. 1 and 2). The approximate methods such as those of Von Kármán, Pohlhausen, and Buri for predicting separation were derived on the assumption that the boundary layer has time to adjust itself to a prescribed pressure distribution. The Von Kármán-Pohlhausen approximation for a laminar boundary layer is

\[
\frac{\delta}{q_1} \frac{dp}{dx} = K_1 R_{\delta}^{-1}
\]  

(1)

and Buri's approximation for a turbulent boundary layer is

\[
\frac{\delta}{q_1} \frac{dp}{dx} = K_2 R_{\delta}^{-1/4}
\]  

(2)
where

\[ \delta \quad \text{boundary-layer thickness} \]

\[ \frac{\Delta p}{\partial} \quad \text{streamwise pressure gradient} \]

\[ \frac{q_1}{dx} \quad \text{free-stream dynamic pressure} \]

\[ K_1, K_2 \quad \text{empirical constants} \]

\[ \text{Re}_n \quad \text{Reynolds number based on distance } \delta \]

Experience with the use of these approximations has shown that the occurrence of separation depends chiefly upon the pressure gradient \( \frac{\Delta p}{\partial} \), and that the turbulent boundary layer can withstand a much greater pressure increase before separation than can a laminar boundary layer. When the influence of a shock wave on a boundary layer is considered, it is evident that, if the infinite free-stream pressure gradient which the shock wave represents could extend all the way to the wall, then separation would certainly result; however, as shown in the sketch of figure 1, it is known that the pressure difference across the shock is spread out in the lower levels of the boundary layer. (See refs. 3 to 7.) The work of Liepmann and Ackeret has shown that the amount of spread of the pressure rise at the wall depends upon the state of the boundary layer, that is, whether the boundary layer is laminar or turbulent (refs. 3 and 6). Thus, the pressure gradient appearing at the wall boundary is fixed by the physical properties of the boundary layer and by the strength of the shock wave. It seems logical to assume, then, that the occurrence of separation in this case depends principally upon the pressure rise \( p_2 - p_1 \) through the shock wave. It was further anticipated that as the pressure rise across the shock was decreased there would be one shock strength below which no separation of the boundary layer would occur. This concept was advanced by Beastall and Eggink (ref. 8) and, later, a simplified dimensional analysis presented in reference 9 indicated that the critical pressure rise across the shock \( \Delta p/q_1 \) which just causes separation of the boundary layer should be proportional to the local skin-friction coefficient, \( c_f \). These approximations are extended to the case for flat plates in terms of the Reynolds number based on \( x \). Thus, for a laminar boundary layer,

\[ \frac{\Delta p}{q_1} \propto c_f \propto \text{Re}_n^{-1} \propto \text{Re}_x^{-1/2} \quad (3) \]
and for a turbulent boundary layer

\[ \frac{\Delta p}{q_1} \propto c_f \propto R_{\theta}^{-1/4} \propto R_x^{-1/5} \]  \hspace{1cm} (4)

It should be emphasized that the relationships given in equations (1) to (4) are only approximations. For incompressible flow more refined methods have been developed (refs. 10 to 13); however, the applications of these methods for predicting separation have met with only limited success. A collection of the available data for supersonic flow (ref. 9) appeared to bear out the predictions shown by equations (3) and (4) at the time they were first derived; however, since that time, more experimental data have come to light, especially for the turbulent boundary layer, which show that the problem must be reexamined. The discussion of these data forms the subject of this paper which now follows for both laminar and turbulent boundary layers.

**LAMINAR BOUNDARY LAYER**

The available data for shock-induced separation for laminar boundary layers on flat plates are given in figure 2, where the critical pressure rise \( \Delta p \) across the shock divided by the free-stream dynamic pressure \( q_1 \) is plotted against Reynolds number on logarithmic scales. The Reynolds number is based on the distance from the leading edge of the plate to the point of intersection of the shock wave and the boundary layer. The sources of these data are given at the top of the figure. (See refs. 3, 6, 8, 14, 15, and 16.) The data at Mach numbers of 1.4, 1.44, 2.00, and 2.05 (refs. 3, 16, and 15) were obtained from tests in which shock waves of varying strength were made to impinge upon the boundary layer on a flat plate. The data at Mach numbers of 1.93 and 2.48 (refs. 14 and 8, respectively) were obtained in the separated region ahead of a forward-facing step. It can be seen that the available data are rather limited in scope and, therefore, are not conclusive; however, there are some trends in the data which should be mentioned. For example, at free-stream Mach numbers \( M_1 \) of 1.93, 2.00, 2.05, and 2.48 the Reynolds number effect on the critical pressure coefficient appears to follow the inverse square root of the Reynolds number as denoted by the dashed lines in figure 2. Except for the data at Mach numbers of 1.40 and 1.44, the critical pressure coefficient also decreases with increasing Mach number. These trends of Reynolds number and Mach number agree with the predictions of equation (3); however, the magnitude of the Mach number effect shown, especially between Mach numbers of 1 and 2, is much greater than that which would be predicted by reference 9. Recent data obtained at the
Ames Laboratory in the separated region ahead of a forward-facing step show an increase in $\Delta p/q_1$ with increase in Reynolds number which is opposite to that obtained at Mach numbers of 1.93 and 2.48. The forward-facing-step data shown in figure 2 should be ignored, therefore, until more systematic data are available. Stewartson (ref. 17) has made a detailed analysis of the interaction process which leads to the inference that the dimensionless pressure rise required to produce laminar separation would be proportional to $R_x^{-2/5}$. Also shown in figure 2 is a curve which traces the criterion of separation advanced by Pabst (ref. 18) in a recent Argentine paper; however, this criterion cannot account for the Mach number effect and does not correlate with any of the experimental data shown.

**TURBULENT BOUNDARY LAYER**

Investigations of shock—boundary-layer interaction for the turbulent boundary layer have shown that a given shock wave may or may not separate the boundary layer. Data are now available from a number of sources in which turbulent boundary-layer separation has been investigated by three methods: (1) the forward-facing-step technique, (2) the wedge technique, and (3) the incident-shock technique.

In order to remove all doubt as to whether the turbulent boundary layer has been separated, several investigators have forced separation by means of a forward-facing step mounted on a flat plate (see refs. 8, 9, 19, and 20). Typical data for this type of configuration are given in figure 3 which shows the pressure distribution along the surface and (to the same scale) a sketch of the flow field in the interaction region as determined from shadowgraphs. These data were obtained in a blowdown jet of the Langley gas dynamics laboratory at a Mach number of 3.03. The flow diagram at the top of the figure shows that a wedge-shaped separation region is formed ahead of the step and is bounded on its upstream edge by the shock wave. The direction of the circulatory flow within the separated region is shown by the arrows.

The pressure coefficients on the plate first reach a maximum value, noted herein as the first peak, at a point about halfway between the location of the shock wave and the location of the step. This distance is roughly the equivalent of 8 boundary-layer thicknesses or 133 momentum thicknesses, on the assumption of a $1/7$-power velocity distribution in the boundary layer just ahead of the shock. The pressures then dip slightly behind the first peak and subsequently rise sharply, showing the large influence of the circulatory flow. Also pertinent to the discussion of the flow in the separated region are the pressure coefficients measured along the front vertical face of the step given in figure 4.
The three isolated points at a Reynolds number of $4 \times 10^6$ were obtained at $M_1 = 1.86$ (ref. 21), and the data for Reynolds numbers ranging from $12 \times 10^6$ to $32 \times 10^6$ were obtained at $M_1 = 3.03$. The pressure orifices were located at the base of the step and at two other vertical locations above the surface of the plate as denoted by $z/h$. The data at $M_1 = 3.03$ show no significant Reynolds number effect on the pressure coefficients. The results show that there is one stagnation point at the foot of the step and one near the top of the step, and calculations based on the data at $M_1 = 3.03$ and utilizing the incompressible Bernoulli equation show that the velocity downward along the vertical face is about one-fifth the free-stream velocity; whereas the velocity along the plate in a direction opposite to the main flow is about one-third the free-stream velocity. Thus, the separated region cannot be treated as a dead-air space as is commonly assumed. The results at both Mach numbers also show that a considerable error would result if the pressures on the front face of the step were assumed to be the same as that obtained on the plate surface ahead of the step in the separated region. The first peak pressure coefficients obtained ahead of the step are shown by the dashed lines at both Mach numbers for comparative purposes in this case. It is clear then, from the results given in figures 3 and 4, that the first peak pressure coefficient is obtained as a result of the mutual effects of the shock on the boundary layer and of the circulatory flow in the separated region and should not be interpreted as the value of the pressure rise across the minimum strength of shock wave which just causes separation of the boundary layer. These results have been obtained for cases where the step height is about 3 times the local boundary-layer thickness and may be changed somewhat for cases where the step height is very large compared with 8.

A summary of the available data obtained from the use of the step technique for forcing boundary-layer separation is given in figure 5 which shows $\Delta p/q_1$ taken at the first peak plotted against Reynolds number on a logarithmic scale. The Reynolds number is based on the distance from the leading edge of the plate to the point of intersection of the shock wave with the boundary layer. All the data were obtained from pressure distributions (see refs. 8, 14, 20, 22, and 23), and the sources are given at the top of the figure. The Mach number range of the data is from 1.55 shown by the long string of points at the top of the data to 3.65 shown by the lowest data points. The pressure-distribution data at $M_1 = 3.03$ given by the circles are new data which have not been published. The data given in reference 9 (NACA TN 2770) for $M_1 = 3.03$ represented by the dashed line which varies as $R_x^{-1/5}$ were obtained by measuring shock angles close to the point of intersection of the shock wave and the boundary layer, where, as shown previously, the pressures on the plate are changing rapidly; therefore this method for obtaining pressure coefficients is too crude and the data should be
ignored. It is apparent from the mass of data that, except for the data at Mach numbers of 1.86 and 2.48, the Reynolds number effect on the value of $(\Delta p/q_1)_{1st \ peak}$ is very slight. On the basis that there is no Reynolds number effect, figure 6 has been prepared to show the decrease in $(\Delta p/q_1)_{1st \ peak}$ with increase in free-stream Mach number for Mach numbers between 1.55 and 3.65. All the data from the previous figure have been included in this plot, and the vertical lines connecting some of the symbols show the extent of the Reynolds number effect obtained. Included on this plot is the empirical relationship derived by Beastall and Eggink from a curve which best fit their data for both forward-facing steps and backward-facing steps (refs. 8 and 24). This approximation is independent of both Reynolds number and Mach number and, therefore, does not correlate well with the available experimental data for forward-facing steps.

The second technique for producing turbulent boundary-layer separation is the use of wedges of different angles mounted on flat plates, and a limited amount of data is available. (See refs. 20 and 25.) This configuration is analogous to the deflection of a flap or a control surface. Typical data obtained at a Mach number of 3.03 are given in figure 7 which shows the pressure distribution along the plate and on the wedge and above it a sketch of the flow phenomena as determined by shadowgraphs. A double scale is given along the abscissa of the pressure distribution - one which gives $x$ in inches measured from the leading edge of the wedge and one which gives a measure of the boundary-layer thickness, $x/\delta$. As shown in the flow picture, the separation in the corner produced by this particular wedge angle results in a weak shock wave, which projects ahead of the main shock, and an inflection point is obtained in the pressure distribution on the surface. Downstream of this point the pressure coefficient continues to rise and levels off at a value somewhat less than that calculated from oblique-shock theory for this wedge angle in the absence of a boundary layer. In general, the limited available data at a given Mach number show that, for wedge angles greater than a certain value, the pressure distribution has an inflection point similar to that shown in figure 7; moreover, the value of $\Delta p/q_1$ measured at the inflection point remains almost constant with further increases in wedge angle. The data at $M_1 = 3.03$ also show that the value of $\Delta p/q_1$ obtained at the inflection point is essentially constant for Reynolds numbers ranging from $12 \times 10^6$ to $32 \times 10^6$. Results are available from tests utilizing the third technique in which shock waves of varying strength are made to impinge upon the boundary layer on a flat plate. (See refs. 16 and 26.) In these tests inflection points are obtained in the pressure distributions along the plate surface somewhat similar to those in the wedge tests, and these inflection points are also associated with local separation of the turbulent boundary layer. The tests of Gadd and Holder at a Mach number of 2 show no significant effect of Reynolds number on the value of $\Delta p/q_1$ obtained at the inflection point for Reynolds
numbers ranging from about $0.8 \times 10^6$ to $10 \times 10^6$. In figure 8 $\Delta \rho/q_1$ is plotted against Mach number, where the inflection-point pressure coefficients obtained in the wedge tests are given by the open symbols and the inflection-point pressure coefficients obtained by the incident-shock technique are given by the solid symbols. Also shown on this figure is the curve representing the data obtained by the forward-facing-step technique. The data given on this figure, therefore, constitute all information available at present on turbulent boundary-layer separation. The spread in $\Delta \rho/q_1$ obtained at $M_1 = 1.80$ in the wedge tests represents a Reynolds number effect, although, as mentioned previously, no such Reynolds number effect was obtained at $M_1 = 3.03$. The spread in $\Delta \rho/q_1$ at $M_1 = 2$ in the incident-shock tests represents the maximum scatter in the data. Although the available data are rather limited in scope, the results show that the inflection-point pressure coefficients obtained from both techniques generally have the same range of values with increasing Mach number and that on the average these values are about 20 percent lower than those obtained using the step technique. The application of these data for predicting separation should, therefore, be limited to these particular configurations, at least for the present. For example, the data from the incident-shock technique represent conditions of local separation of the flow and, because the experiments are performed on flat plates, the flow reattaches downstream of the separation point. This reattachment may be changed somewhat for conditions where a back pressure exists - for example, for conditions near the trailing edge of an airfoil. Also, flight data for a wing in transonic flow indicate that the $\Delta \rho/q_1$ for separation is predicted more accurately by the step data if extrapolated to the lower supersonic Mach numbers obtained in the flight tests (ref. 27). These data are useful, then, in providing a first approximation to the pressure coefficient for which separation is likely to be encountered.

CONCLUDING REMARKS

In conclusion, the present status of information relative to the prediction of shock-induced boundary-layer separation indicates that, although no universal value of pressure-rise coefficient which causes incipient separation of the boundary layer has been found, there is a fairly narrow band of pressure coefficients from which predictions of turbulent separation can be made with an accuracy probably sufficient for engineering purposes. On the basis of these results the following tentative conclusions are given:

1. The data obtained with forward-facing steps, wedges, and incident shock waves indicate that there is a dependency of the pressure
coefficient for separation on Reynolds number for the laminar boundary layer but little, if any, dependency on Reynolds number for the turbulent boundary layer. There is a dependency of this pressure coefficient on Mach number for both laminar and turbulent boundary layers.

2. For the particular case of the spoiler, the available data obtained by the forward-facing-step technique permit calculations of the loading on the surface ahead of the spoiler, the pressure on the front face of the spoiler, and the separation point ahead of the spoiler for a Mach number range of from 1.55 to 3.65 for the turbulent boundary layer.

3. For application to supersonic diffusers or scoop inlets, the available data from incident-shock-wave tests provide a first approximation to the minimum strength of shock which will separate the turbulent boundary layer for Mach numbers between 2 and 3.

4. From the data available from the wedge tests, a first approximation to the pressure coefficient for which separation becomes appreciable as a result of flap deflection can be made for a surface with a turbulent boundary layer for Mach numbers between 1.75 and 3.03.

5. Caution should be exercised in attempting to predict the separation or loading on configurations which differ considerably from those for which experimental data are available. For example, fair success has been obtained in predicting base pressure coefficients by the use of the forward-facing-step data, but reasons for this success are not at present fully understood.

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REFERENCES


SHOCK WAVE ENTERING A BOUNDARY LAYER

Figure 1

CRITICAL PRESSURE COEFFICIENTS FOR LAMINAR BOUNDARY LAYERS

Figure 2
SEPARATION OF TURBULENT BOUNDARY LAYER BY STEP

Figure 3

PRESSURE COEFFICIENTS ON VERTICAL FACE OF STEP

Figure 4
FIRST PEAK PRESSURE COEFFICIENTS AHEAD OF STEPS FOR TURBULENT BOUNDARY LAYERS

\[ M_I \]

- 3.03 LANGLEY GAS DYNAMICS
- 1.62 NPL
- 1.38 J. LANGLEY 9-IN.
- 2.41 SST
- 1.55 N.A.A.
- 2.48 TAYLOR MODEL BASIN
- 2.43 N.A.A.
- 2.80 AEROPHYSICS

\[ \frac{(\Delta \rho)}{q_I}_{1ST \ PEAK} \]

**Figure 5**

MACH NUMBER EFFECT ON PEAK PRESSURE COEFFICIENT TURBULENT BOUNDARY LAYER

\[ \frac{(\Delta \rho)}{q_I}_{ST \ PEAK} \]

- 0.357, BEASTALL AND EGGINK

**Figure 6**
SEPARATION OF TURBULENT BOUNDARY LAYER BY WEDGE

MAIN SHOCK
WEAK SHOCK

24.25° WEDGE
SEPARATION AND CIRCULATION

\[ \frac{\Delta p}{q_1} \]

\[ M_1 = 3.03 \]
\[ R_x = 16.8 \times 10^6 \]

Figure 7

SEPARATION PRESSURE COEFFICIENTS FOR TURBULENT BOUNDARY LAYERS

\[ \frac{\Delta p}{q_1} \] vs \[ M_1 \]

INCIDENT SHOCK DATA
PRINCETON
NPL
STEP DATA \[ \left( \frac{\Delta p}{q_1} \right)_{1st \text{ PEAK}} \]

Figure 8
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Subject

Flow, Supersonic
Flow, Laminar
Flow, Turbulent
Diffusers, Supersonic

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1.4.2.1.2

ABSTRACT

The present status of available information relative to the prediction of shock-induced boundary-layer separation is discussed. Experimental results showing the effects of Reynolds number and Mach number on the separation of both laminar and turbulent boundary layer are given and compared with available methods for predicting separation. The flow phenomena associated with separation caused by forward-facing steps, wedges, and incident shock waves are discussed. Applications of the flat-plate data to problems of separation on spoilers, diffusers, and scoop inlets are indicated for turbulent boundary layers.