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CALCULATING THRUST DISTRIBUTION AND EFFICIENCY OF AIR PROPELLERS

By Theodor Bienen

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CALCULATING THRUST DISTRIBUTION AND EFFICIENCY OF AIR PROPELLERS.*

By Theodor Bienen.

Supplementary to various articles published by Professor Von Karman and myself on the propeller theory,** I am now proposing a method for the preliminary approximate calculation of the thrust distribution and efficiency of air propellers under any operating conditions.

The calculation is based on the following manner of reasoning. The propeller is imagined to be replaced by a suitable mechanism, with whose aid a certain mass of air is driven opposite to the direction of flight (and in the peripheral direction). Such a mechanism would, for example, be represented by a series of blade elements which are warped with reference to one another in the radial direction from element to element, corresponding to the pitch and which, in the peripheral direction, fill the propeller circle with infinite closeness. Every blade element generates a certain thrust, the magnitude and di-

*"Die rechnerische Ermittlung der Schubverteilung und des Wirkungsgrades für ausgefuhrte Luftschrauben bei beliebigen Betriebszustanden." From "Zeitschrift für Flugtechnik und Motorluftschiffahrt," Nov. 27, 1926, pp. 485-487.

**Bienen and Karman, "Zur Theorie der Luftschrauben," Z.d.V.D.I. ("Zeitschrift des Vereines deutscher Ingenieure"), Nov. 29, and Dec. 20, 1924. Th. Bienen, "Die gunstigste Schubverteilung für die Luftschraube usw.," Z.F.M., May 28, and June 13, 1925.

rection of which are known (provided the performance coefficients of the elements are known) and whose axial component represents the contribution of that element to the propeller thrust.

Hence our task becomes the following, namely, to determine the speed relations and the forces developed on a section with a given direction and velocity of the air current, a problem which can be approximately solved with the aid of the momentum theory.

There still remains to be considered the effect of the finite number of blades. This is determined in the following manner, as suggested by Professor Prandtl.

Effect of the Number of Blades

In accordance with our assumption, the thrust at any definite distance from the center of the propeller circle is uniformly distributed throughout the circumference, while the thrust distribution along the radius can be assumed to be distributed as may be desired.

Figure 1 shows the speed relations of the propeller blade at the distance r from the propeller axis.

w = flight speed,

ω = angular velocity of propeller,

β_r = $\arc\tan \frac{w}{r\,\omega}$, the effective angle of pitch,

β_r' = $\arc\tan \frac{w + \frac{w'}{2}}{r\left(\omega - \frac{\omega'}{2}\right)}$, the induced angle of pitch.

c' = the added (or induced) velocity. Its components in the axial and peripheral direction are respectively w' and $r\omega'$. (The induced velocity first reaches half its value at the position of the propeller blade itself.)

With a finite number of blades, there is a decrease in the thrust at the blade tips, which is a natural result of the release of an especially large number of vortices at those points. The velocity distribution in the periphery is no longer uniform. If the periphery is developed and the induced velocity is plotted over it, it is then found that it attains its maximum value at the propeller blade, but falls to a certain value before and behind the blade.

The mean induced velocity was approximately calculated by Prandtl* as $w_{mean} = \kappa w'$, in which w' denotes the maximum effective velocity at the propeller blade and κ equals

$$\frac{2}{\pi} \text{ arc cos } e^{-\frac{R - r}{R} \frac{z}{\sin \beta'}};$$

in which z denotes the number of blades and β' the induced angle of attack at the blade tip. The propeller with a finite number of blades can now be replaced by a dynamically equivalent propeller with an infinite number of blades. Prandtl computes the reduced radius of this propeller by the formula

$$R_{red.} = R\left(1 - \frac{1.39 \sin \beta'}{z}\right)$$

*See A. Betz, "Schraubenpropeller mit geringstem Energieverlust" with Appendix by L. Prandtl, "Nachr. d. Ges. d. Wissensch. zu Göttingen," "Math.-physik. Kl." 1919, p.193 ff. The quantity κ is given in the form of a curve in the previously mentioned paper by the writer.

A knowledge of the maximum velocity at the propeller blade, as also of the mean velocity, is very important for determining the shape of the propeller. Prandtl's correction factors seem to approximate the actual relations closely enough. At least the agreement was excellent between them and the actual velocity and thrust distribution obtained in the wind-tunnel experiments at the Aachen Aerodynamic Institute.

Calculation of the True Angle of Attack

As shown in Fig. 1, the task is only to determine the difference between the geometrical and the induced angle of attack. The induced velocity c', with its axial and peripheral components, can be calculated directly from its geometrical relations. With the induced velocity we then know the thrust distribution and the efficiency.

If we apply the momentum theory to an annular area, of width dr, in the propeller circle at the distance r from the propeller axis, the thrust developed by the propeller elements, perpendicular to the direction of attack, can be written as follows:

$$m\, c'\, \kappa = A$$

$$\frac{\gamma}{g}\left(w + \frac{w'}{2}\right) 2\, r\, \pi\, dr\, c'\, \kappa = \frac{\gamma}{g}\, \frac{v'^2}{2}\, c_a\, z\, t\, dr \qquad (1)$$

in which $c'/2$ is the maximum value of the induced velocity; κ, the Prandtl correction; c_a, the lift coefficient of the blade section; and zt, the sum of the blade chords of all the sections comprised in the radius under consideration. The remaining quantities, in so far as they are not given in Figure 1, are designated in the usual way.

On the basis of the geometrical relations in Figure 1, we obtain

$$
\begin{aligned}
c' &= 2\ v \sin\delta = \frac{2\ w}{\sin\beta}\sin\delta \\
v' &= v\cos\delta = \frac{w}{\sin\beta}\cos\delta \\
w' &= c'\cos\beta' = \frac{2\ w}{\sin\beta}\sin\delta\ \cos(\beta+\delta) \\
r\ \omega' &= c'\sin\beta' = \frac{2\ w}{\sin\beta}\sin\delta\ \sin(\beta+\delta)
\end{aligned}
\qquad (2)
$$

If i denotes the angle between the line of zero thrust ($c_a = 0$) of the section and the "effective" direction of attack (i.e., of the apparent direction given by the operating conditions), we can write, in the usual manner,

$$c_a = c_a'\ (i - \delta) \qquad (3)$$

in which c_a' is derived from c_a, according to the angle of attack. In what follows, we will treat c_a' as a constant. For ordinary blade sections and the usual angle-of-attack region, this approximation answers very well. We can then put $c_a' = 5.25$ (the angle in circular measure). In general, c_a' has smaller values only at very great angles of attack. This should be remembered when the following conclusions are applied to the operation of propellers under conditions in which the above assumption is not (or is only partially) fulfilled. This case will occur especially with the propeller on the test stand and there more often with propellers having a large V/NO ratio (i.e., large ratio of flight speed to peripheral velocity, hence in low-speed propellers). Since, however, this disadvantage is chiefly confined to the elements near the hub, the resulting error is small, for these blade elements furnish only a small fraction of the total thrust.

From equations (1), (2), and (3), we now obtain

$$\left\{1 + \frac{\sin \delta}{\sin \beta_r} \cos(\beta_r + \delta)\right\} 4 \pi r \sin \delta \, \kappa = \frac{zt}{2t} \frac{\cos^2 \delta \, c_a{}' (i - \delta)}{\sin \beta_r}$$

Since δ is always very small, we expend $\sin \delta$ and $\cos \delta$. By neglecting everything after the first term, we obtain the difference between the induced and effective angle of pitch (angle of slip).

$$\delta = \frac{i}{1 + \kappa \frac{8 \pi r}{c_a{}' \, zt} \sin \beta_r} \tag{4}$$

Taking into account the quadratic terms of δ we obtain

$$\delta = -\frac{1}{2} \frac{\kappa \frac{8\pi r \sin\beta_r}{z\,t\,c_a{}'} + 1}{\kappa \frac{8\pi r \cos\beta_r}{z\,t\,c_a{}'} + i} \left(1 - \sqrt{1 + 4\,i \frac{\frac{\kappa 8\pi r \cos\beta_r}{z\,t\,c_a{}'} + i}{\left(\frac{\kappa 8\pi r \sin\beta_r}{z\,t\,c_a{}'} + 1\right)^2}}\right) \tag{5}$$

Even this formula, with a suitable tabular arrangement, can be evaluated much simpler and quicker than perhaps appears possible at first glance. Any one desiring greater accuracy can make a second calculation with the more exact value of c_a'. He will then obtain accurate values even for points at any desired distance from normal operating conditions.

Calculation of Thrust Distribution and Efficiency

The induced velocities in the axial and peripheral directions can be calculated with the aid of equation (2). From the momentum and blade-area theories, in the usual manner for the annular element with the radius r and the width dr, we then obtain the contribution to the thrust

$$\frac{dS}{dr} = \frac{\gamma}{g}\left(w + \frac{w'}{2}\right) w' (1 - \epsilon \operatorname{tg} \beta_r') \, 2 \, r \, \pi$$

and the contribution to the torque (6)

$$\frac{dM}{dr} = \frac{\gamma}{g}\left(w + \frac{w'}{2}\right) r^2 \omega' \, (1 + \epsilon \operatorname{ctg} \beta_r') \, 2 \, r \, \pi$$

in which $\epsilon = \frac{c_w}{c_a}$, the D/L ratio of the element (for an infinite aspect ratio). Since the angle of attack is given by δ, both c_a and c_w are known for the sections tested as models. The D/L ratio can be approximately determined for elements for which no results of model tests are available. We will recur to this point later.

The thrust and torque distribution along the radius is given by equation (6). The total thrust and torque are obtained by integration.

The efficiency, at the distance r from the propeller axis, is

$$\eta_r = \frac{d\,S\,w}{d\,M\,\omega} \tag{7}$$

We finally obtain for the total efficiency

$$\eta = \frac{\int \eta_r \frac{d\,M}{dr}\,dr}{\int \frac{d\,M}{dr}\,dr} \tag{8}$$

This integration can be quite easily made, either numerically or graphically.

Remarks on the Method of Calculation

The calculation is not difficult when test results of the individual propeller elements are available, which is rarely the case. For calculating δ from equations (4) and (5), we must know the $c_a = 0$ line (or the angle i) and the c_a and c_w values in terms of the angle of attack $(i - \delta)$. For the approximate determination of the $c_a = 0$ line of a known element, the following method is more suitable.*

Connect the trailing edge of the elements (Fig. 2) with the leading edge (contact points of a circumscribed circle) by the straight line a. Next draw the line b, so as to bisect the angle formed by the tangents to the sides of the trailing edge. The straight line bisecting the angle between a and b

*See Karman and Trefftz, "Potentialströmung um gegebene Tragflachenquerschnitte," Z.F.M. 1918, p.111 ff.

is the $c_a = 0$ line (or line of zero thrust). This is exactly true only for elements having a trailing-edge angle of 0, i.e., for so-called "Joukowsky profiles." For finite trailing-edge angles, it would be necessary to add a correcting term dependent on the magnitude of this angle. This term is, however, so small as to be practically negligible.

If the same basic profile is employed for the different cross sections by geometrically distorting it, then a single determination of the $c_a = 0$ line is sufficient. For any other thickness ratio d/t, the angle between the $c_a = 0$ line and the chord is

$$\alpha = \frac{\alpha_o}{\left(\frac{d}{t}\right)_o} \frac{d}{t}$$

in which α_o and $\left(\frac{d}{t}\right)_o$ are the corresponding values, either measured or otherwise determined. For any desired angle of attack c_a is then found according to equation (3).

In determining the c_w values, care must be exercised that they represent only the coefficient of the pure profile drag (i.e., without induced drag). The profile drag may, however, with sufficient accuracy, be regarded as constant in the customary angle-of-attack region. It can be estimated by comparison with known profiles.

Summary

A method is presented whereby the thrust and torque distribution, the efficiency, and therewith the requisite engine power, can be determined for any propeller under any working conditions. With the aid of the momentum and blade-area theories, on the one hand, and the results of the elementary airfoil theory, on the other hand, the only unknown quantity, namely, the actual aerodynamic angle of attack of each propeller-blade element is determined. Herewith all other quantities can be calculated in a simple manner.

Translation by Dwight M. Miner,
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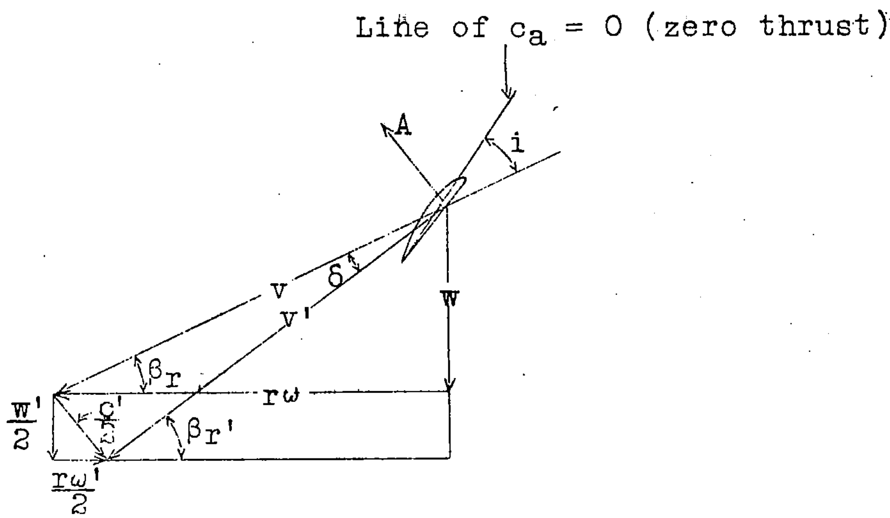

Fig.1

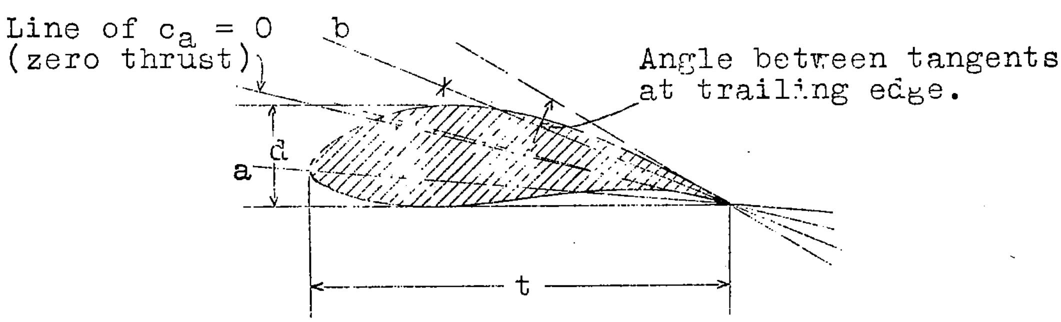

Fig.2 Graphic determination of line of zero thrust, (c_a = 0).