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MEMORANDUM REPORT

AN APPROXIMATE METHOD OF CALCULATION OF RELATIVE HUMIDITY REQUIRED TO PREVENT FROSTING ON INSIDE OF AIRCRAFT PRESSURE CABIN WINDOWS

By Alun R. Jones

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INTRODUCTION

This report has been prepared in response to a request received from an aircraft company for information. A typical example was selected for the presentation of an approximate method of calculation of the relative humidity required to prevent frosting on the inside of a plastic window in a pressure-type cabin on a high-speed airplane. The conditions assumed for this example are as follows:

Window ................ Single pane of transparent plastic 5/8 inches thick
(k for plastic = 0.1827 kg. cal./m °C hr.)

Outside temperature ... -40°F
Cabin temperature ...... 40°F
Air speed .............. 400 m.p.h.
Altitude ............... 25,000 ft.
Cabin pressure .......... 10.9 lb./sq. in.

The solution for this particular set of conditions is 15 per cent relative humidity. The method used to obtain this solution follows.
METHOD

It is assumed that the velocity of the air across the window is equal to the air speed, 400 m.p.h. The temperatures involved are shown in figure 1.

Since all temperatures involved will be low, the factor of radiation may be neglected and the heat transmission process will be free convection, $t_1$ to $t_2$, conduction through the plastic, $t_2$ to $t_3$, and forced convection into the boundary layer, $t_3$ to $t_4$.

The equations of the heat flow at steady state (from reference 1) give:

$$h_1 A (t_1 - t_2) = \frac{k A (t_2 - t_3)}{l} = h_2 A (t_3 - t_4) \quad (1)$$

where

- $h_1 =$ transfer coefficient, cabin air to window
- $A =$ window area
- $t_1$, $t_2$, $t_3$, $t_4$, $t_5 =$ as shown in figure 1
- $k =$ conductivity of plastic
- $l =$ window thickness
- $h_2 =$ transfer coefficient, window to outside air
The unknowns are $h_1$, $t_2$, $t_3$, $t_4$, and $h_2$. Since only two equations are available we must evaluate three of these in some manner.

An approximation for $h_1$ is obtained from equation (29), page 244, reference 1. Noting the remark on page 245 that $h$ is further affected by pressure, we write:

$$h_1 = 0.3(t_1 - t_2)^0.25 \sqrt{\frac{p}{14.7}}$$

The value of $t_4$ (temperature of air in boundary layer) can be calculated from equation (7), reference 2.

$$t_5 - t_4 = -0.45 V^2 \times 10^{-3}$$

This equation is for $V$ in m/sec and $t_5 - t_4$ in degrees Centigrade. Applying $V = 400$ m.p.h., we obtain $t_5 - t_4 = 25^\circ$ Fahrenheit, hence $t_4 = -15^\circ$ Fahrenheit.

The value of $h_2$ is difficult to state since very few data are available at such high speed. However, application of equations from reference 1 and experience with transfer coefficients at lower velocities, lead to the conclusion $h_2$ must be approximately equal to 50 and would certainly be no less.

The problem now resolves to a trial and error method, but certain reasoning will lead to a quick result. An inspection of equation (2) indicates that $h_1$ is not much affected by changes in $t_1 - t_2$, and for values of $t_2$ which seem within reason $h_1$ must be approximately 1.0.
Therefore, assume \( h_1 = 0.8 \), \( h_2 = 50 \) and equation (1) becomes

\[
0.8 \left( 40 - t_2 \right) = \frac{0.123 (t_2 - t_3) 8 \times 12}{5} = 50(t_3 + 15)
\]

which gives \( t_2 = -0.6^\circ \) Fahrenheit and \( t_3 = -14.4^\circ \) Fahrenheit.

Substituting this value of \( t_2 \) in equation (2) results in \( h_1 = 0.7 \) which is close enough to the assumed value of 0.8. Notice that the effect of assuming higher values of \( h_2 \) will merely result in reducing \( t_3 - t_4 \), and for all practical purposes \( t_3 \) could have been assumed equal to \( t_4 \).

Condensation of moisture upon the inside of the window will occur when the water vapor present in the cabin air has a saturation temperature equal to or greater than the temperature of the inside surface of the window. Assume the water vapor has an actual vapor pressure of \( P_v \) and is at cabin temperature of \( 40^\circ \) Fahrenheit. As this vapor is cooled it will maintain its same pressure as long as it is a vapor. In order for no condensation to occur until \(-0.6^\circ \) Fahrenheit is reached, the value of \( P_v \) must be the saturation pressure corresponding to \(-0.6^\circ \) Fahrenheit.

Thus, relative humidity = \[
\frac{\text{saturation pressure at } -0.6^\circ \text{ F.}}{\text{saturation pressure at } 40^\circ \text{ F.}} \times 100
\]

\[
= \frac{0.92 \text{ mm Hg}}{6.37 \text{ mm Hg}} \times 100 = 15 \text{ per cent}
\]

Air in cabin at \( 40^\circ \) Fahrenheit and pressure of 10.9 pounds per one square inch must have relative humidity less than 15 per cent in order to prevent frosting of the windows.
From the problem above it can be seen that frost can be prevented by (1) reducing the humidity or (2) heating the window. As regards (1), a humidity of 15 per cent is considerably below the comfort limit of 30 per cent. If the cabin humidity is raised, extremely dry air would have to be introduced into the cabin air near the windows to prevent condensation.

Method (2) appears to be the best solution, and a simple calculation indicates that a window inner-surface temperature of 15°F Fahrenheit would prevent frosting when the cabin air was at 40°F Fahrenheit and relative humidity of 30 per cent. Since this temperature is not much above the -0.6°F Fahrenheit from the problem just solved, probably an inner secondary window about 1/16-inch thick and separated from the outer window by a 1/16-inch air gap would be above 15°F Fahrenheit.

If this should not be sufficient, the hot air method described in reference 3 was extremely successful in tests, and the cabin heating air could be admitted between double panes. This is the best method concerning which we have information, although any means that will raise the glass temperature to the required value will be equally successful.

The value of the heat-transfer coefficient depends upon a number of complex variables such as velocity of fluid flow, nature and shape of contact surfaces, and physical properties of the fluid. The coefficient is greatly influenced by type of flow, whether laminar or turbulent. An increase in velocity usually tends to decrease the boundary layer thus increasing the transfer coefficient.

The value of this heat-transfer coefficient has been established for a great many specific conditions and interested parties are directed to references 1 and 2, and also the volume of the Durand series concerned with heat flow, reference 4.
REFERENCES.


