RESEARCH MEMORANDUM

THE EFFECTS OF A MODIFIED ROLL-COMMAND SYSTEM ON THE FLIGHT-PATH STABILITY AND TRACKING ACCURACY OF AN AUTOMATIC INTERCEPTOR

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SUMMARY

In order to improve the tracking characteristics of an automatic interceptor, a revised roll-command computer was tested both on an analog computer and in flight. This report presents flight-test results which indicate a significant improvement in flight-path stability and tracking accuracy. The modified roll computer was designed on the basis of previous analytical studies.

INTRODUCTION

The NACA has been conducting a general investigation of automatic interceptor systems with particular emphasis on problems affecting flight-path stability and tracking accuracy during the final attack phase. This program has involved both flight tests and analog simulation studies of a typical interceptor system. Flight tests have proved extremely useful not only for verification of analytic work but also in uncovering problem areas where the analog facilities can be put to best use. In this manner it is possible to isolate basic problems and to study various means for alleviating system deficiencies.

References 1 and 2 give the results of the initial phases of this program and are concerned primarily with the dynamic response of the radar antenna and its effect on over-all system performance.

The present report describes a series of flight tests in which a modified roll-command system was used. This system was developed on the basis of analytical studies reported in reference 3. The present report compares system responses obtained with the modified and with the normal roll computer and also shows how the problems associated with antenna dynamics are influenced by the roll-control system.
NOTATION

\( A_a \) antenna angle in azimuth, deg

\( A_j, A_k \) acceleration commands in azimuth and elevation, respectively, g

\( A_{D,D} \) desired normal acceleration, g

\( A_R \) resultant acceleration, \( \sqrt{A_j^2 + A_k^2} \), g

\( A_z \) normal acceleration (positive downward), g

\( E_a \) antenna acceleration (positive downward), g

\( E_a \) antenna angle in elevation, deg

\( p \) interceptor rolling velocity, radians/sec

\( R \) target range, ft

\( S_j, S_k \) steering signals in azimuth and elevation, respectively, yd/sec

\( s \) operator, \( \frac{d}{dt} \)

\( T \) time-to-go until impact, sec

\( t \) time, sec

\( \varphi \) interceptor bank angle, deg

\( \varphi_e \) bank-angle error, radians (except as noted)

\( \delta_H \) horizontal stabilizer angle, deg

\( \delta_a \) aileron angle, deg

\( \eta \) angle that defines the direction of \( A_R \) from the vertical

TEST EQUIPMENT

The interceptor used in the tests is an F-86D airplane (fig. 1) with an E-4 fire-control system and a Hughes developed automatic attack coupler (CSTT). The complete system is described in reference 1; however, pertinent details of the attack coupler are repeated in the following paragraphs.
A block diagram of the attack coupler is shown in figure 2. The steering signals $S_j$ and $S_k$ are converted to acceleration commands by the proportionality factor $K_1$. The desired normal acceleration is then expressed as

$$A_{D} = QA_k + \cos \varphi$$

where $\cos \varphi$ is the gravity component in g units and the roll command to the aileron servo as

$$\varphi_c = \frac{QA_j \cdot \sin \varphi - KPD}{|QA_k| + 1}$$

When $A_{D}$ exceeds a preset maximum allowable value, the gain $Q$ of a pair of variable gain amplifiers is automatically reduced until $A_{D}$ is within the desired limits. Thus when $A_{D}$ is less than its limit value, $Q$ is 1.0; but when $A_{D}$ exceeds the limit, $Q$ is effectively equal to

$$A_{D_{max}} \cdot \cos \varphi$$

$$A_k$$

The same gain reduction takes place in the azimuth channel to preserve the coordination between bank angle and normal acceleration.

For the present investigation the dividing network was modified so that

$$\varphi_c = \frac{QA_j \cdot \sin \varphi - KPD}{|QA_k| + |QA_j| + 1}$$

The flight instrumentation was the same as described in reference 1.

**TEST PROCEDURES**

The flight tests consisted of $90^\circ$ lead-collision beam attacks against an F-84F target equipped with radar reflectors to make its reflection
characteristics more typical of a bomber-type airplane. Attacks were initiated with various initial steering errors in azimuth. All flights were made at an altitude of 30,000 feet with the target and interceptor initially at a Mach number of 0.8.

ANALYSIS

Because the relation between azimuth steering error and desired bank angle is not linear, the command to the aileron servo must be modified in some fashion to prevent excessive roll commands (and even instability) in the presence of large azimuth steering errors and still retain rapid response to small errors. One of the most direct approaches, as discussed in reference 3, is to compute the instantaneous bank-angle error for use as a roll-rate command. The analysis of reference 3 is repeated briefly in the following paragraphs with application to the particular interceptor system in question.

The following sketch is a projection of the aiming point in a plane normal to the flight path of the interceptor. Superimposed on the sketch is an acceleration diagram. The bank-angle error \( \phi_e \) can be expressed in terms of the bank angle \( \phi \) and the acceleration commands \( A_k \) and \( A_j \).

In order for the resultant acceleration vector of the interceptor to be directed toward the aiming point, the interceptor should roll through an additional angle \( \phi_e \) and develop a normal acceleration equal to \( A_{LD} \). From the sketch it can be seen that
\[
\phi_e = \tan^{-1} \frac{A_j - \sin \varphi}{A_k + \cos \varphi} = \sin^{-1} \frac{A_j - \sin \varphi}{A_{LD}}
\]  

(1)

where

\[
A_{LD} = \sqrt{(A_j - \sin \varphi)^2 + (A_k + \cos \varphi)^2}
\]

As expressed in equation (1), \( \phi_e \) is a true indication at any instant of the bank-angle error, and \( \phi_e \) is zero whenever \( A_j = \sin \varphi \).

In the basic system the roll-rate command is essentially a small angle approximation to the arc tangent function in which \( \cos \varphi \) was replaced by 1. Furthermore, to prevent the denominator from passing through zero, the absolute magnitude of \( A_k \) was used. Thus \( \phi_e \) was mechanized as

\[
\phi_e = \frac{A_j - \sin \varphi}{|A_k| + 1}
\]

(2)

This expression may be interpreted also as an approximation to the arc sine function if \( A_j \) and \( \varphi \) are both small. In any event, the term \(|A_k| + 1\) acts as a variable gain to reduce the sensitivity of the roll channel as \( A_k \) is increased.

Although this type of roll control appeared to be adequate under most conditions, previous analog-computer studies indicated that a more efficient system could be obtained by making the gain a function of \( A_j \) as well as \( A_k \). This can be accomplished directly by using the arc sine function of equation (1) as a modifying network. However, it was found that an approximation for \( A_{LD} \) of the form \(|A_j| + |A_k| + 1\) would produce the desired results and would be much easier to mechanize. Thus, the modifying network as proposed in reference 3 and used in the present tests is the following small angle approximation to the arc sine function:

\[
\phi_e = \frac{A_j - \sin \varphi}{|A_k| + |A_j| + 1}
\]

(3)

Figure 3, in which \( \phi_e \) computed by equations (2) and (3) is plotted against the true bank-angle error as defined by the previous sketch, demonstrates the relative merit of the two types of roll computers. Here the interceptor bank angle is zero so that \( A_k = A_R \cos \eta \) and \( A_j = A_R \sin \eta \). The curves are drawn for various values of \( A_R \) from 1g to infinity. With
the normal system for a large value of \( A_j \) (a true bank-angle error of about 90°), the computed \( \varphi_e \), and hence the commanded roll rate, can become extremely large. This means that to prevent an oscillatory response with excessive roll rates the gain through the aileron channel must be reduced to the point where the system response to small errors becomes sluggish. With the modified system, however, the computed \( \varphi_e \) will not exceed 1 radian and is relatively insensitive to the magnitude of the azimuth error. In this case gain can be adjusted to give a desirable maximum roll rate without compromising the response to small error signals.

**TEST RESULTS**

The analysis presented in the previous section is greatly idealized and can be used only as a guide in judging the limitations of a particular configuration. In order to show how the roll response is influenced by the dynamics of the system and also to select proper gain levels, the complete system was examined on the analog computer. The simulation procedures were the same as described in reference 1, and the results showed that with the modified roll command a significant improvement in flight-path stability could be obtained without sacrificing speed of response.

Because of the limited capacity of the analog equipment certain simplifying assumptions had to be made. Furthermore it was not feasible to include radar noise or certain nonlinearities such as saturation, hysteresis, and backlash that occur at various places in the system. Thus it was felt that actual flight tests were required to give a fair appraisal of the modified roll computer and also to assess the importance of the factors that had been neglected in the simulation.

The results of these flight tests are presented in the following paragraphs and are compared with data previously obtained with the normal system. In general, the flight tests indicated the same sort of improvement in system response that was noted on the analog computer; however, because of the difficulties in establishing precise initial conditions in flight, only qualitative comparisons could be made.

The effectiveness of the modified roll system is most pronounced under conditions of short lock-on range and large azimuth steering errors. Figure 4(a) is a flight record of the normal system under such conditions in which the target was engaged at about 15,000 feet. The time history shows the characteristic oscillations in roll and normal acceleration that generally accompany a negative antenna elevation angle (ref. 1).

The response of the modified system under similar flight conditions is illustrated in figure 4(b). It can be seen that the flight path is generally more stable with little tendency to oscillate. In general,
with the roll gain adjusted to give an adequately fast response, peak roll rates rarely exceeded 1.5 radians per second. This is in contrast to roll rates of 3 radians per second that were sometimes encountered in the normal system and which were objectionable to the pilot. For example, figure 5 illustrates a case of this type. Even though lock-on was at a fairly long range and the steering errors were not excessively large, the interceptor received a sharp roll command which resulted in a 180° roll at a maximum rate of almost 3 radians per second. The interceptor subsequently lost radar contact with the target. While this phenomenon was not common and may be related to system misalignment, it did happen on several occasions during the test program. The effective roll-rate limiting provided by the modified roll command successfully prevents a maneuver of this type.

One further example of the modified system is given in figure 6. Again lock-on was at short range (18,000 ft) with an azimuth error large enough to require the interceptor to make a maximum g turn. It should be noted that there is no tendency to oscillate and the steering errors are driven to zero at the time of firing. No comparable records for the normal system are available because attacks under such extreme conditions generally resulted in a loss of radar contact with the target.

The test results show that the modified roll-command system is also beneficial for the less severe attack situations which may be more typical of normal operation. Figures 7(a) and 7(b), for example, illustrate a pair of long-range beam attacks (lock-on at 8 or 10 miles) with small initial steering errors. In both cases the interceptor is essentially on course during the major portion of the attack and, hence, very little maneuvering is required. In a comparison of the two figures it is evident that with the modified system the steering signals are more stable. Furthermore, the response in roll and normal acceleration is less oscillatory, and peak roll rates are lower.

Reference 2 showed that the same sort of improvements could be attained by minimizing the interaction between antenna and interceptor motions, that is, improving the space stabilization of the radar antenna. The present investigation, however, indicates that a greater improvement in this regard can be obtained by modifying the roll command. When the radar modification was tested in conjunction with the revised roll computer, the results differed very little from those shown in figure 7(b).

Since the over-all stability and tracking efficiency of the system is reflected in the steering error signals, the following table has been prepared to give a quantitative comparison of the relative effectiveness of the various configurations tested. The table presents, for a number of attacks, average values of the standard deviations of the steering signals during phases II and III. In each case the attack was initiated at sufficiently long range so that initial steering errors were corrected before the start of phase II. The table gives relative values of $S_j$ and $S_k$. 


in phase II and $S_k$ in phase III. Because no significance should be attached to the actual values, these have been normalized with respect to the azimuth error $S_j$ for the normal system in phase II.

<table>
<thead>
<tr>
<th>Automatic</th>
<th>Number of runs</th>
<th>Phase II</th>
<th>Phase III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal CSTI</td>
<td>21</td>
<td>1.00</td>
<td>0.33</td>
</tr>
<tr>
<td>With modified radar antenna</td>
<td>13</td>
<td>0.79</td>
<td>0.55</td>
</tr>
<tr>
<td>With modified roll command</td>
<td>14</td>
<td>0.42</td>
<td>0.29</td>
</tr>
<tr>
<td>With both modifications</td>
<td>15</td>
<td>0.25</td>
<td>0.23</td>
</tr>
</tbody>
</table>

The first two rows of the table are taken from reference 2 and indicate the degree of improvement obtained by isolating the antenna from interceptor motions. This modification results in smoother steering signals and, hence, a more stable flight path. In general, this leads to smaller elevation errors at the start of phase III and greater terminal accuracy.

The third row of the table indicates, however, that a greater improvement was obtained with the modified roll computer. The restricted peak roll rates and reduced tendency to oscillate alleviated the major source of antenna interaction, and the net result was smoother steering signals during the entire attack.

Finally, the last row indicates the degree of improvement that was obtained by combining both modifications. It can be noted that there is no appreciable difference in phase III.

CONCLUDING REMARKS

Flight tests of an automatic interceptor system have been made to check the effectiveness of a roll-command system proposed on the basis of analog-computer studies. The tests indicated a marked improvement in stability and tracking accuracy. Furthermore, the modified system provides effective roll-rate limiting which tends to alleviate the effects of coupling between interceptor and antenna motions and also to eliminate the violent rolling maneuvers that were sometimes encountered with the basic system.

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REFERENCES


Figure 1.- Photograph of test airplane.
Figure 2.- Block diagram of automatic attack coupler.
Figure 3. - Computed versus true bank-angle error.
Figure 4.- Flight-time history of beam attack with short lock-on range.
(b) Modified roll computer.

Figure 4.- Concluded.
Figure 5.- Flight-time history of normal system.
Figure 6.- Flight-time history of modified system.
(a) Normal system.

Figure 7.- Time history of beam attack.
Figure 7. Concluded.

(b) Modified system.