TECHNICAL MEMORANDUMS
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 781

REDUCTION OF LIFT OF A WING DUE TO ITS DRAG

By J. Stüper

Zeitschrift für Flugtechnik und Motorluftschifffahrt
Vol. 24, No. 16, August 28, 1933
Verlag von R. Oldenbourg, München und Berlin

Washington
November 1935
In his expositions, Professor A. Betz had already established the methods available now for the calculation of wing characteristics. The work by Kutta and others, particularly the introduction of the circulation concept, made it possible to compute the lift of a wing. Inasmuch as the potential theorems resorted to, disregarded the friction, it precluded the inclusion of the wing properties induced by friction. This resulted in a discrepancy between theory and measurement. The first experiments in this direction were those by Professor Betz in 1915 with a Joukowski airfoil mounted between two parallel walls (reference 1). His recorded lift obtained to only 70 percent of the theoretical figure.

A further step in the explanation of the effect of drag on the lift was that by C. Wieselsberger (reference 2) with his investigations of the effect of varying friction due to changes in roughness on the lift. It being impossible at that time to make any quantitative predictions regarding friction layers, no theoretical experiments with a view to the effect of drag on the lift could be undertaken. This palpable gap was closed by E. Gruschwitz in his calculation of the turbulent friction layer with pressure rise and pressure drop (reference 3). 1931 then saw the publications by Miss J. Lotz and Professor Betz, regarding the reduction of the lift of a wing by its drag (reference 4). In connection with this work the calculations were made which form the subject of this report. The analysis is, briefly, as follows:

Compute for a predetermined airfoil and given Reynolds Number the course of the "displacement thickness," i.e., the course of the layer by which the streamlines of the potential flow are pushed away from the wing through the frictional layer. The result is, to a certain extent, a new wing contour. This contour is conformably transformed

in the plane in which the original wing contour is a circle. The new contour may be derived from the old one by assuming adequately chosen sources and sinks on the wing contour or circle. Owing to the unsymmetrical position of the sources and sinks on the circular contour, the rear stagnation point is displaced, and to return it to its old position the circulation, and through it the lift, must be reduced.

The calculation proceeded on the basis of a Joukowski airfoil, which rendered the distribution of the velocity or pressure quite simple. The following angles of attack were chosen: $-4^\circ, -2^\circ, 0^\circ, 2^\circ, 4^\circ,$ and $6^\circ$.

The calculation of the frictional layers is in two parts: the determination of the laminar entrance zone and determination of the turbulent friction layer.

The first may be effected by Pohlhausen's approximation method (reference 5). He approached the velocity profiles of the laminar friction layer through a four-term polynome, and defined the coefficients by means of the boundary conditions and the momentum theorem. In this manner he obtained the distribution of the friction layer thickness in form of a differential equation, which may be used to calculate a directional field from the predetermined velocity distribution. Figure 1 is an illustrative example. The solution curve starts at a singular point which may be computed from the velocity gradient in the stagnation point. The determination of the point of transition from laminar to turbulent is still very uncertain. The only available reference is found in Gruschmitz's report; transition takes place when the Reynolds Number

$$U(x) \cdot \frac{\delta(x)}{\nu}$$

($U(x) =$ velocity of potential flow, $\delta(x) =$ momentum growth of frictional layer) reaches the value 650. From $U(x)$ and $\delta(x)$ the location of the transition point can be determined for different Reynolds Numbers $R = \frac{U \infty \cdot t}{\nu}$ ($U \infty =$ undisturbed air stream, $t =$ wing chord) (figs. 2 and 3).

The turbulent part was established at $R = 7 \times 10^6$, while the laminar friction layers could be computed for any Reynolds Number. The calculation of the turbulent
friction layer, according to Gruschwitz is, in contrast to that of the laminar part, quite simple. Foregoing the details of his report, we attempt to prove the essential features on an example (fig. 4). The predetermined pressure distribution affords a "polar" (the points denoted by + in fig. 4) for each abscissa x. The solution curve must be so plotted that the tangent at each point x passes through the corresponding polar. The effect of profile curvature also enters into the determination of the polars, according to an unpublished report by Schmidbauer. From the course of the friction layers the distribution of the displacement thickness along the profile can be obtained by extrapolation (fig. 5). The new contour is conformably transformed in the plane in which the original contour is a circle (fig. 6); the distance between circle and the new contour is considerably exaggerated. Then we plot sources on the circular contour, one sink with one half the total yield of the sources in the center of the circle, another sink at infinity.

The strength distribution of the sources may be effected in two ways: The stream function may be determined, whereby the source distribution is expressed in Fourier series with temporarily unknown coefficients, and the coefficients are so computed that the predetermined contour becomes streamlined. This method is quite cumbersome - a much more simple way is the following: Compute the velocity from the potential flow at points A and B (fig. 6); the mean value multiplied by the distance AB gives a measure for the sources which must be present between stagnation point S and point AB. The differentiation of the curves affords the desired source - sink distribution in first approximation (fig. 7), and from this source distribution the tangential velocity component in the rear stagnation point can be established. Next the original circulation is reduced until this velocity component is made to disappear. The result is a reduction of lift.

Having proceeded from the theoretical (i.e., abnormally great) pressure distribution, the lift reduction will be excessive. A second approximation can be made by repeating the process for the reduced circulation, and it may be repeated until the convergence is satisfactory. When diminishing the circulation of a wing with constant angle of attack, flow around the trailing edge must be permitted. Hereby the velocities resulting in the vicinity of the trailing edge become paradoxial because at the trailing edge itself the flow around it induces an infi-
nite velocity. It would become necessary to make plausible, albeit arbitrary, assumptions for this zone. On the other hand, the entire, very complicated calculation method must be repeated several times for approximations.

Now it is quite natural to effect the circulation decrease by lowering the angle of attack for a passably chosen amount; this removes the difficulty due to the flow around the trailing edge. The question remains as to how much the velocity and pressure distributions are at variance with these two methods. The full-drawn curve in figure 8 is the normal velocity distribution for \(0^\circ\) angle of attack, the dashed curve is the velocity distribution obtained when the same circulation strength is forced on the wing with a \(4^\circ\) angle of attack by flow around the trailing edge. Effecting the circulation decrease through angle-of-attack changes, the next approximations can be interpolated from different angle-of-attack values \((-4^\circ, -2^\circ, 0^\circ, 2^\circ, 4^\circ, 6^\circ)\) for the circulation decrease.

The result is illustrated in figure 9. The circles are experimental figures reduced from wind-tunnel tests; the \(X\) crosses denote the first, the \(+\) crosses the fifth approach. From the point of view of the Reynolds Numbers the calculated figures are slightly too low.

The present article is principally a preliminary report of an investigation which is still under way; the cited calculation method still needs improvement and polish. More exact knowledge about the location of the transition point is needed, as well as about any eventual effect of the Reynolds Number on the turbulent friction layer.

The uncertainty at present is probably the prime reason for the still not quite satisfactory accord between theory and test. The method for defining the laminar friction layer is also in need of substantial simplification (say, perhaps, according to Gruschwitz's method). A study of all these problems is under way.

The further extension of the method includes:

Determination of profile drag. — The source-sink distribution affords information about the wake; applying Betz's momentum method (reference 6), the determination of the profile drag will be attempted. This would supply the connection between drag and lift reduction;
Penetration in zones of the separating and separated flow, which renders the calculation of the total polars of a wing possible. Of special interest is the investigation of the effect of Reynolds Numbers, particularly the high numbers, on the polars (maximum lift) — a study which, experimentally, would involve great expense.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

REFERENCES


Figure 1. - Laminar friction layer; directional field, $x =$ development of profile, $\eta =$ criterion of friction layer thickness ($\eta = \frac{62}{42} \frac{1}{R}$). The figures denote the directions of $(d\eta/dx)$.

Figure 2. - Location of transition point from laminar to turbulent flow, suction side.
Figure 3.- Location of transition point from laminar to turbulent flow, pressure side.

Figure 4.- Graph of turbulent friction layer.
Enlarged 10x

Angle of attack 2°

Enlarged 10x

Enlarged 100x

Figure 5.- Airfoil with displacement thickness curve.

Figure 6.- Circle with new contour.

Figure 7.- Source-sink distribution.
Figure 8. - Velocity distribution.

Figure 9. - Theoretical versus experimental lift.