SOME MEASUREMENTS OF TIME AND SPACE
CORRELATION IN WIND TUNNEL

By A. Favre, J. Gaviglio, and R. Dumas

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SOME MEASUREMENTS OF TIME AND SPACE CORRELATION IN WIND TUNNEL*¹

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SUMMARY

These researches are made at the Laboratoire de Mécanique de l'Atmosphère de l'I.M.F.M., for the Office National d'Etudes et de Recherches Aéronautiques (O.N.E.R.A.), with the aid of the Ministère de l'Air, and of the Centre National de la Recherche Scientifique (C.N.R.S.).

We shall sum up the results which we have obtained by means of the apparatus for measurements of time and space correlation and of the spectral analyser in the study of the longitudinal components \( u_1 \) of the turbulent velocities in a wind tunnel, downstream of a grid of meshes \( M \) (refs. 1 to 13); and we shall give the first results in the case of a flat-plate boundary layer (ref. 10).

The correlation \( R(VT/M, X_1/M, X_3/M) \) is measured between two velocities \( u_1 \) considered with a relative time delay \( T \), at two points of space at a distance \( X_1 \) from each other parallel to the general velocity \( V \) of the flow, and at a distance \( X_3 \) orthogonally.

I. TIME CORRELATION, TURBULENCE SPECTRA DOWNSTREAM OF A GRID

For \( X_1 = X_3 = 0 \), only one hot-wire anemometer is used. Numerous time correlation, or autocorrelation curves, \( R(VT/M) \) have been drawn, the spectral curves being obtained on one hand by transforming them, and on the other hand by direct measurements with the analyser, for velocity \( u_1 \), behind two grids of meshes \( M = 3 \ 1/4 \) and \( M = 1 \) inch, successively, at distances of 40 \( M \) downstream of the grids, and at various speeds and Reynolds mesh numbers \( B_M \) (refs. 2 to 9, 11, and 12).


²A communication to the 8th International Congress on Theoretical and Applied Mechanics, Istanbul, Aug. 1952.

² Analogous to those of the NBS (ref. 15), with elements being respectively of 1.6 and 0.5 centimeters diameter.
As an example, figure 1 (ref. 12) represents an autocorrelation curve of the velocities $u_1$ at 40 M downstream of that grid of mesh $M = 1$ inch, the mean velocity being $V = 12.27$ meters per second, $R_M = 21500$.

As an example also, figures 2 and 3 (ref. 12) show the values of the spectral function $F(n)$ obtained by Fourier transform of the preceding autocorrelation curve or by direct measurements by means of the analyzer ($n$ frequencies).

The autocorrelation curves have a limited radius of curvature at the origin (refs. 2 and 3). The equivalent frequency $N$ of the only sine wave, the autocorrelation curve of which would have the same curvature at the origin, can thus be measured and can show the decay rate of energy (ref. 16).

On figure 2 we have drawn the spectral curve given by H. L. Dryden as a first approximation by transformation of the correlation curve represented in an approximate way by an exponential curve (ref. 15).

In the spectra, from 40 to 2000 cps, the experimental points are neighboring this empirical curve. For the lower frequencies, from 1 to 40 cps, the measured energy is smaller. The same seems to hold true for the spectra of NBS and of NPL (ref. 13).

On figure 3 are also reproduced the spectra measured by R. W. Stewart and A. A. Townsend (ref. 18) with slightly different grids. The same peculiarity appears for the lower frequencies in the case of low Reynolds numbers $R_M$; for higher Reynolds numbers it is not detected; the band-pass of the spectral analyzer used is limited to about 20 cps.

As G. K. Batchelor (ref. 17) has shown, the tendency to isotropy — very strong for most frequencies — is very slight for the large scale components of homogeneous turbulence. In particular the large scale components of the turbulence behind a grid in a uniform flow are anisotropic. They are also nonhomogeneous (ref. 18). They can be dependent on the grid shape.

$$N^2 = \frac{1}{4\pi^2} \left( \frac{du}{dt} \right)^2 \frac{u^2}{V^2} \sim \frac{\int_0^\infty n^2 F(n) \, dn \sim \frac{U^2}{4\pi^2 \lambda^2}}{4\pi^2 \lambda}$$

$N = 435$ cps

$\lambda = 0.45$ cm

H. W. Liepmann (ref. 19) performed numerous measurements of $\lambda$, under different test conditions, varying the rates of energy dissipation, the spectral curves, and the number of passages through zero to $u_1$ per unit time.

In the case of that grid the rods of which are of circular section.
II. COMPARISON OF TIME CORRELATION WITH LONGITUDINAL SPACE CORRELATION: G. I. TAYLOR'S HYPOTHESIS

According to Taylor's hypothesis (ref. 14) the time correlation curves \( R(VT/M) \) may be assimilated to the longitudinal space correlation curves \( R(X_1/M) \) or \( f \), if the relative intensity of turbulence is very low, with the condition \( VT/M = X_1/M \) and the general movement being rectilinear and uniform.

We have measured, in the same experimental conditions, the time correlation and the longitudinal space correlation.

It must be noted that the wake of the upstream wire disturbs the measurements of \( R(X_1/M) \) but not of \( R(VT/M) \); in order to lessen this effect, a small lag \( X_3 \) is used.

The results agree with an approximation similar to that of the measurements, as shown, for instance, in figure 4 (refs. 9 and 12).

Thus Taylor's hypothesis is directly verified in the case of the above-mentioned experiments made behind a grid in a wind tunnel by measurements of time correlation and longitudinal space correlation.

III. TIME AND SPACE CORRELATION, LONGITUDINALLY

The time and space correlation, longitudinally \( R(VT/M, X_1/M) \) behind a grid of mesh \( M = 1 \) inch, has been measured in the course of two series of experiments (refs. 9 and 12).

The first series of curves in figure 5 (ref. 9) relates to longitudinal distances \( X_1/M \) of

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<thead>
<tr>
<th></th>
<th>0.000</th>
<th>0.241</th>
<th>0.483</th>
<th>0.720</th>
<th>1.20</th>
<th>1.93</th>
<th>3.14</th>
<th>4.56</th>
<th>6.64</th>
<th>8.72</th>
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<tr>
<td></td>
<td>0.000</td>
<td>0.236</td>
<td>0.473</td>
<td>0.946</td>
<td>1.892</td>
<td>3.78</td>
<td>7.57</td>
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the mean velocity being \( V = 12.25 \) mps, the Reynolds number \( R_M = 21500 \).

The second series of curves in figure 5 (ref. 12) deals with longitudinal distances \( X_1/M \) of

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<td>1.20</td>
<td>1.93</td>
<td>3.14</td>
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</tbody>
</table>

the mean velocity being \( V = 12.27 \) mps, \( R_M = 21500 \).
The latter measurements have been made after several improvements of the experimental apparatus\(5\) (ref. 13).

Figure 6 gives the isocorrelation curves corresponding to the second series of the above-mentioned measurements, which are in first approximation ellipses whose axes have an inclination of about 45° and whose diameter ratios comprised between 0.027 and 0.055 are, on the average, of the order of 0.04, namely,

\[
\begin{array}{ccccccc}
R & 0.90 & 0.80 & 0.70 & 0.60 & 0.50 & 0.40 \\
\frac{a}{b} & 0.048 & 0.054 & 0.055 & 0.035 & 0.028 & 0.027 \\
\end{array}
\]

One finds that the time and space correlation longitudinally reaches a maximum for each distance \(X_1/M\) when the delay is close to the time necessary to cover this distance at the general velocity:

\[
VT/M \sim X_1/M
\]

The time and space correlation longitudinally downstream of a grid in a wind tunnel, with a delay compensative of the general movement \(R(VT/M = X_1/M, X_1/M)\), retains high values even for distances which are great in comparison with those that would practically suffice to make null the longitudinal space correlation \(R(X_1/M)\).

IV. TIME AND SPACE CORRELATION, TRANSVERSELY

The time and space correlation transversely \(R(VT/M, X_3/M)\) behind a grid of mesh \(M = 1\) inch has been measured in the course of two series of experiments (refs. 4 to 7, 11, and 12).

Figure 7 (ref. 11) shows the first results obtained for transversal distances \(X_3/M\) of

\[
\begin{array}{ccccccc}
0.000 & 0.039 & 0.078 & 0.157 & 0.315 & 0.630 & 1.26 \\
\end{array}
\]

the mean velocity being \(V = 12.20\) mps, \(RM = 21500\).

\(5\) New amplifiers of hot-wire anemometers, improvement of the compensation by means of the square waves method, use of wire of 5\(\mu\) in diameter, extension from 2.5 to 1 cps of the band-pass of the apparatus for measurements of time correlation, reduction of the intensity of the wind-tunnel turbulence from 0.00045 to 0.00038.
Figure 8 (ref. 12) relates to the new measurements, made after the above-mentioned improvements, for the same values of $X_3/M$, the mean velocity being $V = 12.27$ mps, $R_M = 21500$.

Figure 9 gives the isocorrelation curves corresponding to the new measurements (ref. 12), which are in first approximation ellipses whose diameter-ratios comprised between 0.36 and 0.49 are, on the average, of the order of 0.44, namely,

\[
\begin{array}{cccccccc}
R & 0.90 & 0.80 & 0.70 & 0.60 & 0.50 & 0.40 & 0.30 & 0.20 & 0.10 & 0.00 \\
a/b & 0.36 & 0.39 & 0.43 & 0.49 & 0.49 & 0.49 & 0.47 & 0.43 & 0.41 & 0.48
\end{array}
\]

V. TIME AND SPACE CORRELATION LONGITUDINALLY AND TRANSVERSELY,
WITH COMPENSATORY DELAY OF THE GENERAL MOVEMENT

Figure 10 represents time and space correlation longitudinally and transversely $R(VT/M, X_1/M, X_3/M)$ for zero delay $VT/M = 0$ and also for a delay compensative of the general movement $VT/M \sim X_1/M$ behind a grid of mesh $M = 1$ inch for distances $X_1/M$ of

\[
0.000 \quad 0.236 \quad 0.473 \quad 0.946 \quad 1.89 \quad 3.78 \quad 7.57
\]

the mean velocity being $V = 12.27$ mps, $R_M = 21500$.

Figure 11 gives the corresponding isocorrelation curves. They are in first approximation ellipses whose diameter ratios decrease with $R$: \[\begin{array}{cccccccc}
R & 0.90 & 0.80 & 0.70 & 0.60 & 0.50 \\
a/b & 0.31 & 0.25 & 0.20 & 0.15 & 0.12
\end{array}\]

One finds that the influence on the correlation between the components $u_1$ of the velocities at two points behind a grid of the distance between these points in the direction of the general movement is partly compensated, even for relatively great distances, by delays equal to the time necessary to cover this distance at the speed of the general movement.
VI. TIME CORRELATION, TURBULENCE SPECTRA IN THE BOUNDARY LAYER OF A FLAT PLATE

The measurements are made at 0.91 m from the leading edge of a flat plate; the mean velocity is \( V = 12.20 \) mps, the Reynolds number \( R_x = 766000 \) (ref. 10).

The grid being taken off, the preturbulence was of 0.00045 with four screens, and the boundary layer was laminar; a grid of \( M = 1 \) inch being set in, the preturbulence is of 0.01480, and the boundary layer is turbulent (thickness \( \delta = 24 \) mm).

The autocorrelation has been measured for the component \( u_1 \) of the turbulent velocity, at various distances \( X_3/8 \) from the plate (fig. 12):

<table>
<thead>
<tr>
<th>Distance (m)</th>
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<td>0.06</td>
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An important evolution of these curves as a function of the distance to the plate takes place in the boundary layer.

The smallest delay \( T \) for which the correlation is zero assumes the respective values:

\[
\begin{align*}
7.5 & \quad 10 & \quad 12 & \quad 24 & \quad 8.3 & \quad 5.5 & \quad 5 \\
\text{ms} & & & & & & \\
\end{align*}
\]

On the contrary, the equivalent frequency \( N \) and the rate of energy decay change only slightly.

Figure 13 gives the spectra corresponding to the above-mentioned experiments, obtained either by transformation of the autocorrelation curves or directly.

They differ but little from those of the turbulence behind a grid for frequencies from 40 to 2000 cps; for frequencies lower than 40 cps, they show a marked evolution as a function of the distance from the wall in the boundary layer.

Translated by A. Favre
REFERENCES


4. Favre, A.: Mesures de corrélation dans le temps et l'espace en aval d'une grille de turbulence, pour la composante longitudinale de la vitesse. 15/7/49.

5. Favre, A.: Nouvelles mesures de corrélation dans l'espace et le temps en aval de grille de turbulence avec appareillage modifié. 31/12/49.


correlation dans le temps et l'espace, longitudinalement, trans-
versalement, longitudinalement et transversalement en aval d'une
grille de turbulence. 10/7/52.

correlation dans le temps et l'espace. VIII\textdegree Cong. Intern. M\=ecan.
Th\=eor. et Appl., Istanbul, 1952. Recherche A\=eronautique No. 31,
1953. J.A.S.

Ser A, 164, 476, 1938.


17. Batchelor, G. K., and Stewart, R. W.: Anisotropy of the spectrum of

18. Stewart, R. W., and Townsend, A. A.: Similarity and self preserva-

19. Liepmann, H. W., Laufer, J., and Liepmann, Kate: On the spectrum of
isotropic turbulence. NACA TN 2473, 1951.
Figure 1.- Autocorrelation of turbulence behind a grid. \( V = 12.27 \) mps; 
\( M = 1 \) inch; distance = 40 M; \( R_M = 21500 \).
Figure 2.- Spectrum of turbulence downstream of a grid obtained by Fourier transform of autocorrelation, or directly measured. $V = 12.27 \text{ mps}; M = 1 \text{ inch}; \text{distance} = 40 \text{ M}; \text{Compensation square waves method, } R_M = 21500.$
\[ k_x = (\varepsilon/\lambda)^t \]

(a) Stewart-Townsend spectra.

(b) Spectra obtained by Favre, Gaviglio, and Dumas. \( V = 12.27 \text{ mps}; \) 
\( M = 1 \text{ inch}; \) distance = 40 M; \( E_M = 21500; \) N = 435 cps.

Figure 3.- Spectra of turbulence downstream of a grid by transform of autocorrelation or measured directly.
Figure 4. - Comparison of time correlation $R(VT/M)$ and longitudinal space correlation $f$ (i.e., $R(X_1/M)$). G. I. Taylor's hypothesis.
(a) First measurements (ref. 9). $V = 12.25$ mps; $M = 1$ inch; distance = 40 M; $R_M = 21500$.

(b) New measurements (ref. 12). $V = 12.27$ mps; $M = 1$ inch; distance = 40 M; $R_M = 21500$.

Figure 5.- Time and space correlation longitudinally $R(VT/M, X_1/M)$ for turbulence downstream of a grid.
Figure 6. - Time and space isocorrelation longitudinally $R(VT/M, X_1/M)$ for turbulence downstream of a grid. $V = 12.27$ mps; $M = 1$ inch; distance = 40 M; $R_M = 21500$. 


Figure 7.- First series of measurements. Time and space correlation transversely $R(VT/M, X_3/M)$ for turbulence downstream of a grid. $V = 12.20$ mps; $M = 1$ inch; distance $= 40$ M; $R_M = 21400$. 
Figure 8. - New measurements. Time and space correlation transversely $R(VT/M, X_3/M)$ for turbulence downstream of a grid. $V = 12.27$ mps; $M = 1$ inch; distance = 40 M; $R_M = 21500$. 
Figure 9. - Time and space isocorrelation transversely $R(V T / M, X_{3} / M)$ for turbulence downstream of a grid. New series of measurements. $V = 12.27$ mps; $M = 1$ inch; distance $= 40 M$; $R_{M} = 21500$. 
Figure 10. - Time and space correlation longitudinally and transversely \( R(VT/M \sim X_1/M, X_1/M, X_3/M) \) with compensatory delay of general movement for turbulence downstream of a grid. \( V = 12.27 \) mps; \( M = 1 \) inch; distance = 40 M; \( R_M = 21500 \).
Figure 11. - Time and space isocorrelation longitudinally and transversely, with compensatory delay of general movement $R(VT/M \sim X_1/M, X_1/M, X_3/M)$ for turbulence downstream of a grid. $V = 12.27$ mps; $M = 1$ inch; distance $= 40$ M; $R_M = 21500$; $T \sim X_1/V$. 
Figure 12. - Autocorrelation in turbulent boundary layer of a flat plate. 
$V = 12.20$ mps; $M = 1$ inch; distance = 40 M; $R_x = 766000$. 

\[ R(T) \]

\[ X/A = 0.05 \]

\[ X/A = 0.12 \]

\[ X/A = 0.25 \]

\[ X/A = 0.50 \]

\[ X/A = 0.75 \]

\[ X/A = 1.00 \]

\[ X/A = 5.62 \]
Figure 13. Turbulence spectra in flat-plate boundary layer as a function of the distance from the wall. $R_x = 766000$. 