DOE-ER40757-003

CPP-93-03

CAN EPR NON-LOCALITY BE GEOMETRICAL ?

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1. Introduction

The presence in Quantum Mechanics of *non-local correlations* is one of the two fundamentally *non-intuitive* features of that theory; the other revolves around the (so-called) *collapse of the state vector*, i.e. the need to assign *special non-Schrödinger dynamics* to the (generalized) *"measurement"* process. The non-local correlations themselves fall into two classes: *"EPR"*[1] and *"Geometrical"*. The latter include the Aharonov-Bohm[2] and Berry (or Geometrical)[3] phases plus a variety of *global* solutions to quantum gauge theories, namely Maxwell's (à la Weyl), Yang-Mills theories with or without spontaneous symmetry breakdown, and Quantum Gravity state-of-the-art (i.e. simplified) models. This category includes *magnetic monopoles* (in U(1) gauge theories[4] and in some spontaneously broken Yang-Mills theories[5]), instantons[6], merons, etc.

The non-local characteristics of the "geometrical" type are well-understood and are not suspected of possibly generating acausal features, such as faster-than-light propagation of information. This has especially become true since the emergence of a *geometrical* treatment for the

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relevant gauge theories, i.e. *Fibre Bundle* geometry, in which the quantum non-localities are seen to correspond to pure *homotopy* considerations. We review this aspect in section 2.

Contrary-wise, from its very conception, the EPR situation was felt to be "paradoxical". [a] It has been suggested[7] that *the non-local features of EPR might also derive from geometrical considerations, like all other non-local characteristics of QM.* In [7], one of us was able to point out several plausibility arguments for this thesis, emphasizing in particular similarities between the non-local correlations provided by any gauge field theory (such as parallel-transport and the existence of a *connection*) and those required by the preservation of the quantum numbers of the original (pre-disintegration) EPR state-vector, throughout its (post-disintegration) spatially-extended mode. The derivation was, however, somewhat incomplete, especially because of the apparent difference between, on the one hand, the *closed spatial loops* arising in the analysis of the geometrical non-localities, from Aharonov-Bohm and Berry phases to magnetic monopoles and instantons, and on the other hand, in the EPR case, the *open line* drawn by the positions of the two moving decay products of the disintegrating particle. In what follows, we endeavor to remove this obstacle and show that as in all other QM non-localities, *EPR is* somehow *related to closed loops,* almost *involving homotopy considerations.* We shall develop this view in section 3.

Before presenting our "resolution" of the EPR "paradox", we should state our reading of the actual answers provided by experiment. This is necessary since some schools in the Foundations of Quantum Mechanics have not yet accepted the finality of these answers (pointing at possible "outs" which would have to be checked before a final verdict); in addition, there is the alternative de Broglie - Bohm interpretation which chooses to preserve the deterministic features, at the expense of having actual action-at-a-distance. EPR were assuming (in the hope of preserving the intuitive view of locality, as inherited from classical physics) that the uncertainty relations of QM are due to statistical or information-wise considerations (e.g. involving *hidden variables*), whereas there does exist nevertheless an *underlying determinisitic reality* in which the two components of the disintegrating particle have already acquired their new quantum numbers at the moment of disintegration, even though these still remain hidden from us – until a measurement has been performed on one of them. Bell's inequalities[8] made it possible to test this thesis and our understanding is that the Aspect experiments[9] indeed falsified it – with the possible alternative we mentioned, namely the *non-locally-acting hidden variables* stressed by the late David Bohm. In our present context of conjuring away the non-local features, however, Bohm's theory appears somewhat purposeless.

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[a] One advantage of this situation is the fact that after sixty years, there is still need for further clarification – thus also providing an excellent opportunity for a public celebration of yet another anniversary of our good friend Nathan Rosen..

2. Global Effects from Homotopy in Fibre Bundle Geometries

A Principal Fibre Bundle $\mathcal{P}(\mathcal{M},\mathcal{G},\pi,\times)^{10}$ is given by a base-manifold $\mathcal{M}$, the structure-group $\mathcal{G}$, the 'vertical' projection π and the multiplication $\times$. The Fibre Bundle trivializes locally into a direct product; the projection π maps an entire fibre over one point in the base manifold; and the "$\times$" product has the group $\mathcal{G}$ mapping the bundle manifold onto itself, while preserving the group's associativity:

$$\begin{aligned}&\forall(p\in\mathcal{P},\ x\in\mathcal{M},\ a\in\mathcal{G}),\ p=x\otimes a,\ \pi(p\times a)=\pi(p),\\&\mathcal{P}\times\mathcal{G}\to\mathcal{P},\ (p\times a)\times a'=p\times aa'\end{aligned}\tag{2.1}$$

The "$\times$" action is achieved through the group $\mathcal{G}$'s generating Lie algebra γ, ($a(g)=exp(ig^i\gamma_i)$), as represented by vector-fields (of the mathematical terminology) on the bundle's tangent manifold $\gamma\subset\mathcal{P}_*$, acting on $\mathcal{P}$; the Lie bracket is realized through the usual vector-field construction (i.e. the differential operators acting mutually as derivatives on each other's coefficient functions). The "$\times$" action can thus be reinterpreted as a mapping from the *abstract* Lie algebra Γ_{abs} onto a submanifold of $\mathcal{P}_*$,

$$(\times):\ \Gamma_{abs}\to\mathcal{P}_*,\ \gamma\to\tilde{\gamma}\in\mathcal{P}_*,\ \forall\gamma\in\Gamma_{abs}.\tag{2.2}$$

The dimensionalities obey $dim(\mathcal{P}_*)=dim(\Gamma_{abs})+dim(\mathcal{M}_*)$. The "inverse" mapping[11], with kernel $\mathcal{M}_*$, is performed by the *connection* ω:

$$\omega:\ \mathcal{P}_*\to\Gamma_{abs},\ \forall\gamma\in\Gamma_{abs},\ \omega(\tilde{\gamma})=\gamma.\tag{2.3}$$

with the "abstract" (or matrix commutator) Lie bracket relating to the vector-field realization through

$$\widetilde{[\gamma,\gamma']}_{matrixcom.}=[\tilde{\gamma},\tilde{\gamma}']_{vectorfield}\tag{2.4}$$

The connection is a one-form, whose action in the above map is realized through an *inner product* or contraction with the vector-fields of the Lie algebra's realization in $\mathcal{P}_*$,

$$\omega^i(\tilde{\gamma}_j)=\delta^i_j\tag{2.5}$$

where the coefficient functions of dp^{Ξ} in the ω one-form and of ∂_{Φ} in the vector-field $\tilde{\gamma}$ are quantum-fields with arguments p^{Ξ}, p'^{Φ}, respectively, and where the right-hand side of (2.5) is supplemented, within the inner product, by the appropriate Dirac delta function $\delta(p-p')$ in the corresponding integration.

The Fibre Bundle (FB) is trivial, if a cross-section can be drawn on it globally. Taking the example of the Möbius strip, what makes it non-trivial is the *fold* or *twist*. Should we select some point p and draw a cross-section through it, the twist can always be pushed further away, as long as we do not close a loop. As a result, *(I) any open line (such as is drawn in the EPR particle's disintegration) will follow the geometrical constraints of the direct product $\mathcal{M} \times \mathcal{G}$. If $\mathcal{M}$ is Minkowski spacetime, Poincaré invariance is thus guaranteed.* To the extent that we shall show that EPR occurs within a FB manifold, this is then why it will not involve faster-than-light communication.

The corolary is also relevant to our physical issue: *(II) Closed loops – and only closed loops – probe the connectedness characteristics of a FB Manifold.* Thus, *only homotopy can reflect global topological features.* This is why all the global quantum effects we cited (Aharonov-Bohm[2] and Berry[3] phases, monopole[4],[5] and instanton[6] YM solutions, etc) do involve closed loops, when they display their non-local characteristics.

The emergence of these effects in the formalism of plain quantum mechanics follows. The *wave-functions*, given as sections on the bundle, are represented over the fibre bundle Projective Hilbert Space representations. The connectedness features revealed by homotopy are induced from the bundle over these representations. It is not possible to use one coordinate system without encountering singularities. On the bundle, instead, one utilizes *coordinate patches*, clean of singularities, each in its own sector; continuity is ensured by requiring a smooth transition beteen different patches in the overlap region[12,13]. Such a requirement, coupled with the constraint of obtaining single-valued wave functions there, defines the *gauge transformations*. The reader, if unfamiliar with this description, is encouraged to read the example treated in ref. [14], where $\mathcal{M}$ is the 2-sphere with a "hole" at the center (containing a magnetic monopole) $[S^2 - (0)]$; at least two patches are required (one for the "northern" and one for the "southern" hemispheres), with the overlap region covering a broad equatorial belt. $\mathcal{G}$ emerges as a $U(1)$ gauge group, as a result of the single-valuedness requirement in the overlap transition function. This is the *passive* approach to gauge transformations and to fibre bundles.

3. Fibre Bundle Embedding of the EPR Processes

In the usual treatment of the EPR processes, non-relativistic quantum mechanics serves as the arena. This in itself makes one wonder why would the EPR authors or anybody else expect the portrayed experiments to display a relativistic behavior, i.e. exclude faster-than-light transmission of information. From a non-relativistic treatment, one might have expected Galilean rather than Lorentz invariance. In the FB approach, $\mathcal{M}$ obeys special relativity because of the local triviality of the FB (reducing to a direct product, with the Poincaré group as the local isometry of $\mathcal{M}$, one of the factors in the direct product). From I-II we also know that this also automatically relegates non-local effects to closed loops.

The need for the FB embedding here can best be understood in terms of the need for *active* gauge transformations and for a connection. What is at issue is *parallel transport*: How else could we know that the symmetry group, used to describe the pre-disintegration compound in terms of its components (e.g. spin angular momentum, in the singlet state $S = 0$), is unbroken and that the eigenvalues of its Lie algebra retain their value, scale and meaning all along the paths of the decay products? In a FB, this is guaranteed by the existence of the connection. Any measurement performed at the disintegration point $x_0 \in \mathcal{M}$ can be parallel-transported to any points along this section of the FB (the decay path), using a covariant derivative $D_\mu \Psi^m(x) = (\delta^m_n \partial_\mu - \omega^i_\mu(x)(\gamma_i)^m_n)\Psi \;\; (x)$.. The connection regenerates the Lie algebra at any point along the path, following (2.3).

What about the non-local features? They have to relate to closed paths and holonomy. We do have a closed path, since the punchline in EPR – namely checking that when particle A is measured to have its spin 'up', particle B indeed turns out to have its spin 'down' – requires closure of the path with 'messages' from A and B back to the origin I, or, more precisely, to F, where we have meanwhile arrived from I. In a space-time diagram, in which space is reduced to one dimension, we have a diamond shaped path (fig. 3.1)

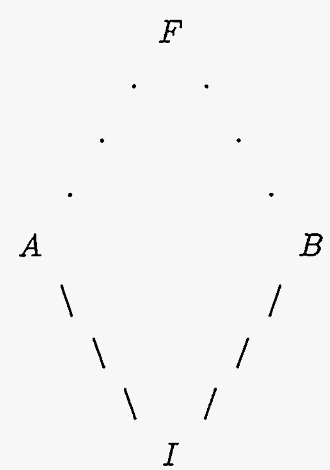

Fig. 3.1: Closing the Loop in EPR

The "paradox" in EPR consists in the existence of non-local correlations between A and B, i.e. in the survival of the original amplitude as long as the measurement has not been performed, either at A or at B. This is a natural geometrical result in the FB.

Note the complementarity between the two types of non-local quantum effects: the explicitly homotopical situations require the loop to be uninterrupted, i.e. having no localized interaction *along the paths between I and F, whereas EPR by definition requires a "measurement" at A or B.* It might be possible to invent an EPR experiment in which the carriers of the information from A or B to F would consist of the same type of particle as those resulting from the disintegration at I (and thus moving along IA and IB). However, this would still not lead to a "combined" effect, because the interaction at A or B would automatically exclude the explicit global result.

4. Discussion

This FB realization of the EPR situation posits that (non-relativistic) QM is indeed "incomplete", but only in the following sense: (1) there would be no justification, anyhow, to expect a non-relativistic theory to display "Einstein locality" or "separability" (modern "local causality" in axiomatic relativistic quantum field theory); (2) the marriage of QM with Special Relativity has been long (since 1948) known to *require* Quantum Field Theory, but we now also know (from our acquaintance with the Standard Model of Particle Physics) that these have to be *Gauge Field Theories.* Such theories are geometrical in their nature and are realized by Fibre Bundle geometries. All EPR situations can therefore be embedded in some bundle geometry deriving from an appropriate combination of the basic geometries of the Standard Model gauges and/or Gravity. (3) Thus, QM is a truncation of the $\mathcal{R}^3$ spatial submanifold of a FB's Minkowski base manifold. As such, it carries a structure allowing it to fit properly into the complete relativistic FB and excluding faster-than-light effects. QM has no formal 'knowledge' of relativity; however, having emerged from studies of electromagnetism, it is endowed with several such "pre-coded" interfaces with Special Relativity. Example: $E = h\nu$ and $p = h/\lambda$ will yield $E^2 - (cp)^2 = 0$. Similarly, the non-local structure does not represent a violation of SR because it is a part of the FB 'heritage', its active aspect being limited to global homotopy. This is the structure behind that situation which has been termed "peaceful coexistence" by Shimony[14].

In an indirect way, this picture is related to the fact that we have a complex Hilbert space, involving phases, the "basic element" of all gauge groups $\mathcal{G}$. The complex Hilbert space itself derives directly from the Heisenberg algebra $[x, p] = ih$, once we require x and p to have real eigenvalues, i.e. to be hermitean. Intuitively, one might have indeed expected quantum non-locality to reflect the blurring of spacetime, due to the uncertainty relations. Something of the sort is happening in the FB approach, but only in a rather loose sense.

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