Kinetic Theory and Boundary Conditions for Flows of Highly Inelastic Spheres.

Quarterly Progress Report

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Introduction

In the last quarter, we focused on steady, fully developed, gravity-driven flows of identical, smooth spheres down bumpy inclines, with flow depths greater than one particle diameter, solid fraction profiles everywhere less than .65. For prescribed boundary bumpiness (r and Δ), restitution coefficients of the boundary (e_w) and the flow particles (e), and angle of inclination (ϕ), we determined whether *at least one* such flow could be maintained. We did so by determining whether there was at least one value of dimensionless granular temperature T_{top} within the range $0 < T_{\text{top}} \leq 10$ for which the appropriate boundary value problem (described in our 7/1 - 9/30/93 Quarterly Report) could be solved.

In this quarter, we choose values of r , Δ , e , e_w , and ϕ for which we know that at least one solution of the type described above may be maintained, and determine the *complete* range of T_{top} (within $0 < T_{\text{top}} \leq 20$) for which such flows can be maintained. For each value of T_{top} in this range, we can calculate the corresponding values of mass hold-up (m_t), mass flow rate ($\dot{m}$), depth β , total fluctuation energy per unit area (E), slip velocity (v) at the base, velocity ($u(y=\beta)$) at the top, and the depth-averaged values of solid fraction, velocity, and granular temperature. In this manner, we thoroughly characterize *all* the steady, fully developed, gravity driven flows that are possible for prescribed sets of r , Δ , e , e_w , and ϕ .

Overview

In Figure 1, we show as a darkened area in the ϕ - Δ plane the values of ϕ and Δ for which at least one value of T_{top} within the range $0 \leq T_{\text{top}} \leq 10$ yielded a solution to the boundary value problem for steady, fully developed, inclined flows (within the bounds $\beta \geq 1$ and $v \leq .65$) for $0 < \Delta < .732$, when $r=1$ and $e=e_w=.5$. This figure was included in our last quarterly report, but is included here for completeness. It demonstrates that when $\Delta=0$, for example, steady, full developed flows are possible when ϕ is approximately between 14° and 26° . In what follows, we consider three intermediate angles ϕ for this value of Δ , and for each angle determine the *full range* of T_{top} that yields solutions to the boundary value problem.

In addition, we characterize the solutions by calculating the corresponding values of mass hold-up (m_t), mass flow rate ($\dot{m}$), depth β , total fluctuation energy per unit area (E), slip velocity (v) at the base, the velocity ($u(y=\beta)$) at the top, and the depth-averaged values of solid fraction $\bar{v}$, velocity $\bar{u}$, and granular temperature $\bar{T}$. Here, the dimensionless depth β and distance y from the base are nondimensionalized by particle diameter σ , the dimensionless velocities u , v , and $u(y=\beta)$ are nondimensionalized by $(\sigma g)^{1/2}$,

the dimensionless granular temperature T is nondimensionalized by σg , the depth-average of any quantity $q(y)$ is defined by,

$$\bar{q} = \frac{1}{\beta} \int_0^{\beta} q(y) dy \quad , \quad (1)$$

and the depth-totaled quantities m_t , $\dot{m}$, and E are equal to $\beta \bar{v}$, $\beta \bar{v} u$, and $\beta \bar{v} T$, respectively.

Results and Discussion

Here we consider the case in which $r=1$, $e=e_w=.5$, $\Delta=0$, and choose three angles $\phi=21^\circ$, 22.5° , and 24° in the range $14^\circ < \phi < 26^\circ$. In Figure 2, we show the variations of β , m_t , $\dot{m}$, $\bar{T}$, $\bar{v}$, E , $\bar{u}$, v , and $u(y=\beta)$ with $T_{\text{top}} \equiv T(y=\beta)$. At the maximum value of $T(y=\beta)=20$, the flows are quite deep and quite dilute. The minimum values of $T(y=\beta)$ for $\phi=21^\circ$, 22.5° , and 24° are 0.013613, 0.014549, and 0.015889, respectively. In each case, as $T(y=\beta)$ decreases to its minimum value, the flows become more shallow, more massive, and therefore more dense. In fact, the theory predicts that for values of $T(y=\beta)$ below the minima, the solid fraction somewhere within the flows exceeds .65. For each inclination, when $T(y=\beta)$ is exactly equal to its minimum value, the flow rate assumes a corresponding *finite maximum* value.

To make the variations with $T(y=\beta)$ near its minimum value more clear, in Figure 3 we replot the variations of $\dot{m}$, m_t , and $\bar{v}$ with $T(y=\beta)$ on log-log scales. The values of $\dot{m}$, m_t , $\bar{v}$ and β corresponding to the minimum value of $T(y=\beta)$ for each angle shown in Figures 2 and 3 are summarized in the table below:

ϕ	$T(y=\beta)$ (minimum)	$\dot{m}$	m_t	$\bar{v}$	β
21°	.013613	32.236	2.672	.494	5.357
22.5°	.014549	73.566	3.142	.416	7.478
24°	.015889	168.411	3.727	.318	11.621

The variations of $\dot{m}$ with $T(y=\beta)$ shown in both Figures 2 and 3 also indicate that corresponding to each inclination is a *nonzero minimum* value of $\dot{m}$. Indeed, we have found that corresponding to every angle in the range

$14^\circ < \phi < 26^\circ$ is a nonzero minimum value and a finite maximum value of $\dot{m}$ between which steady, fully developed flows can be maintained.

According to Figure 2, as $T(y=\beta)$ decreases from 20 over most of its range, the flows become less thermalized (as might be expected) and slower. However as $T(y=\beta)$ continues to decrease near its minimum value, these trends are reversed. Furthermore, the quantities m_t , $\dot{m}$, $\bar{v}$, and to a lesser extent E are extremely sensitive to changes in $T(y=\beta)$ near its minimum value but relatively insensitive to changes in $T(y=\beta)$ away from its minimum value. These observations indicate that there is no simple relationship between $T(y=\beta)$, which can not be controlled experimentally, and such parameters as $\dot{m}$ and E , which may be controllable. For this reason, when presenting the results, it is probably better to parameterize the solutions by either $\dot{m}$ or E .

In Figure 4, for example, we eliminate $T(y=\beta)$ and plot the variations of E and m_t with $\dot{m}$ when $r=1$, $e=e_w=.5$, and $\Delta=0$, for $\phi=21^\circ$, 22.5° , and 24° . At each inclination, there are two flows for each flow rate near the minimum. One is less thermalized and more massive than its counterpart. At higher flow rates, only the less thermalized and more massive flow is possible; as the flow rate increases to its maximum, this flow becomes more thermalized and more massive.

In a typical experiment, the thermal energy at the inlet of an incline is influenced by the height from which the particles are dropped and/or by vibrating the inlet. The left-hand panel of Figure 4 gives some indication of how much thermal energy should be imparted to the particles at the inlet to ensure that their initial states are near to the steady, fully developed states predicted by the theory. Conversely, the left-hand panel of Figure 4 indicates that the steady, fully developed flows predicted by the theory may not be established if, for a fixed flow rate, the magnitude of difference between the thermal energy at the inlet and the predicted value of E is too great. We anticipate that the maximum mismatch between the thermal energy at the inlet and the predicted value of E (beyond which a steady, fully developed flow can not be established) increases as the length of the chute increases.

In Figure 5, we plot the variations of β , $\bar{v}$, and $\bar{u}$ with $\dot{m}$ corresponding to those of E and m_t shown in Figure 4. For a given angle, Figure 5 demonstrates that of the two flows that are possible at flow rates near the minimum, the less thermalized and more massive flow is also more shallow, more dense, and slower than its counterpart. At higher flow rates, only the more shallow, more dense, slower flow is possible; as the flow rate increases to its maximum the depth of this flow changes only slightly as it becomes more dense and faster. For completeness, we have also included in Figure 5 the corresponding variations of β , $\bar{v}$, and $\bar{u}$ with E .

Figure Captions

- Figure 1: The area in the ϕ - Δ plane in which steady, fully developed, gravity driven flows are possible when $r=1$, $e=e_w=.5$, and $0<\Delta<.732$.
- Figure 2: The variations of β , m_t , $\bar{m}$, $\bar{T}$, $\bar{v}$, E , $\bar{u}$, v , and $u(y=\beta)$ with $T(y=\beta)$ for $\phi=21^\circ$, 22.5° , and 24° when $r=1$, $e=e_w=.5$, and $\Delta=0$.
- Figure 3: The variations of $\bar{m}$, m_t , and $\bar{v}$ with $T(y=\beta)$ for $\phi=21^\circ$, 22.5° , and 24° when $r=1$, $e=e_w=.5$, and $\Delta=0$.
- Figure 4: The variations of E and m_t with $\bar{m}$ for $\phi=21^\circ$, 22.5° , and 24° when $r=1$, $e=e_w=.5$, and $\Delta=0$.
- Figure 5: The variations of β , $\bar{v}$, and $\bar{u}$ with $\bar{m}$ and E for $\phi=21^\circ$, 22.5° , and 24° when $r=1$, $e=e_w=.5$, and $\Delta=0$.

Figure 1: The area in the ϕ - Δ plane in which steady, fully developed, gravity driven flows are possible when $r=1$, $e=e_w=.5$, and $0 < \Delta < .732$.

Figure 2: The variations of β , m_t , $\tilde{m}$, $\bar{T}$, $\bar{\nu}$, E , $\bar{u}$, v , and $u(y=\beta)$ with $T(y=\beta)$ for $\phi=21^\circ, 22.5^\circ$, and 24° when $r=1$, $e=e_w=.5$, and $\Delta=0$.

Figure 3: The variations of $\bar{m}$, m_t , and $\bar{\nu}$ with $T(y=\beta)$ for $\phi=21^\circ$, 22.5° , and 24° when $r=1$, $e=e_w=.5$, and $\Delta=0$.

Figure 4: The variations of E and m_t with m for $\phi=21^\circ$, 22.5° , and 24° when $r=1$, $e=e_w=.5$, and $\Delta=0$.

Figure 5: The variations of β , $\bar{\nu}$, and $\bar{u}$ with m and E for $\phi=21^\circ$, 22.5° , and 24° when $r=1$, $e=e_w=0.5$, and $\Delta=0$.