Coupling of Optical Energy From One Optical Fiber to Another of Like Kind, and to a Tapered Fiber, With and Without Small Sphere Lenses

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P. E. Klingsporn

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Topical Report

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Abstract

When light leaves the end of an optical fiber, the exit beam is divergent with an included angle approximately 25° for typical multimode, step-index fiber. In some applications, it is desired to have the exit beam pass through an optical interface and be returned to a second fiber. Experiments were conducted to measure the coupling efficiency using a single sphere lens, and also pairs of sphere lenses, to capture the divergent beam, alter it, and pass it back into a fiber of the same diameter. Three sizes of sphere lenses were used in the studies. The measured coupling efficiencies were analyzed in terms of expected results based on ray tracing and reflection losses. Coupling of optical energy into a tapered fiber was also investigated. An analysis was made of the paths of light rays propagated in the tapered fiber and the conditions that determine the maximum angular cone of light that will be accepted and transmitted by a tapered fiber of given taper angle and taper length. Measurements of coupling efficiency were made for the passage of light from a straight optical fiber into a tapered fiber with and without the placement of a sphere lens between the sending fiber and the receiving face of the tapered fiber. The experimental results were in agreement with the analysis of the propagation characteristics of a tapered optical fiber.

Summary

When optical energy propagated in a multimode step-index optical fiber is allowed to exit the end of the fiber, the energy distribution undergoes an angular spread because of the very nature of the way in which the energy is propagated within the fiber itself. For typical optical fiber, the total included angle of the exit beam is approximately 25° . So, at a distance of only 2.22 fiber diameters beyond the fiber end, the exit beam fills a diameter twice that of the fiber. This is why two fiber ends must be very nearly aligned and nearly in contact for maximum coupling of energy in a fiber-to-fiber connection. In many applications, it is desirable to allow optical energy to exit a fiber, pass through an optical interface of interest, and then enter a second optical fiber. It is therefore necessary to convert the diverging exit beam into a convergent beam in order to achieve maximum coupling into the second optical fiber.

Experiments were conducted to measure the efficiency of coupling optical energy from one optical fiber into a second fiber using

a pair of simple sphere lenses and, also, using a single sphere lens. Simple sphere lenses were used in the study because they are smaller, easier to align, and less expensive than GRIN (graded refractive index) rod lenses. The study was conducted at a wavelength of 850 nm, using 100- μ m-diameter multimode, step-index optical fiber. For studies with a pair of lenses, the lens medium was the high refractive index (1.83) material, *LaSF9*, and the sphere diameters investigated were 2 mm, 3 mm, and 5 mm. The maximum measured transmission from one fiber to a second, with the aid of two sphere lenses of the same diameter, was in the range 56.7 percent to 59.3 percent. A substantial portion of the loss was shown to result from Fresnel reflection at each of the six glass/air interfaces. Calculations showed that the optical transmission would be 87 percent to 91 percent if (AR) anti-reflection coatings were used on the fiber end faces and on the sphere lenses. The remaining losses in the range of 9 percent to 13 percent were shown to result from the presence of lens aberrations for the particular pair of sphere-lenses used.

Transmission coupling efficiency was also measured using a single-sphere lens to capture the emergent optical energy from one fiber and transfer it to a second fiber. The measurements showed a coupling efficiency from 60.9 percent to 65.1 percent among the sphere lens sizes 2 mm, 3 mm, and 5 mm diameter. Taking into account Fresnel reflection losses, the calculated transmission is expected to be in the range 78.1 percent to 83.5 percent if fiber end faces and the sphere lens were AR-coated.

The measurements showed that use of a single-sphere lens is less efficient in coupling energy from one fiber to another than a pair of sphere lenses. On the basis ray tracing, it was shown that the losses from aberration are greater for a single sphere lens than for a pair of sphere lenses, with the optical fiber diameter investigated.

All the foregoing measurements were done with optical fiber of constant cross-sectional diameter. Studies were also done to couple into tapered optical fibers having input faces three to six times the diameter of the exit face diameter. An analysis was made of the path of light rays propagated along a tapered fiber, as a function of the angle of incidence of a ray on the large entrance face and the angle of the taper. From this, equations were derived giving the maximum length of the tapered region for which rays within an incident angle will be propagated through the taper, for a given ratio of entrance and exit fiber face diameters. Results of the analysis were verified through experiments with tapered fibers having 300- μ m/100- μ m entrance/exit face diameters and 600- μ m/100- μ m diameters. For example, coupling directly from a 100- μ m diameter fiber into the 300- μ m face of a 300- μ m/100- μ m fiber gave a measured transmission of only 23.4 percent. According to the analysis, the low coupling efficiency results from the fact that the 300- μ m/100- μ m tapered fiber cannot accept the full angular spread of energy delivered from the 100- μ m fiber. However, use of a single sphere lens placed appropriately between the 100- μ m exit face of the delivery fiber and the 300- μ m receiving face of the tapered fiber gave a transmission of 63 percent. The lens placement reduced the maximum included angle to a level that could be transmitted by the tapered fiber. The practical utility of a tapered optical fiber is that it provides a large entrance face diameter for receiving light distributions that may otherwise be too large to enter a smaller fiber diameter, but this advantage is offset by the fact that the numerical aperture of the tapered fiber is smaller than that of a straight fiber.

Discussion

Scope and Purpose

Studies were conducted to measure the efficiency of coupling optical energy from one optical fiber to another with the aid of a single sphere lens and also with a combination of two sphere lenses. Additionally, measurements were made of coupling efficiency from a fiber of the foregoing type to a tapered fiber with and without the use of a lens. The results were analyzed in terms of ray tracing and Fresnel reflection losses. The work was undertaken to gain an understanding of fiber coupling losses in general, with application to fiber-optic interfaces in advanced optical firing system designs.

Activity

Introduction

Consider the cross section of a step-index optical fiber shown schematically in Figure 1. The core, circular in cross section, has a diameter d and is characterized by refractive index n_1 . Surrounding the core in intimate contact with it is the cladding, which has a

refractive index n_2 , slightly lower in value than n_1 . Based on geometric optics, a ray incident on the flat face at angle θ , as shown, will be refracted toward the perpendicular to the surface as it enters the core. Provided θ is sufficiently small, the ray will undergo total internal reflection when it arrives at the core/cladding boundary, and it will continue propagating along the core. The condition for total internal reflection can be satisfied even when the fiber is bent into a configuration characterized by moderate bend radii. If the angle θ is too large, the entering ray will not be reflected when it encounters the boundary but, instead, will enter the cladding material and be "lost" from propagation along the core. The well-known relation giving the maximum angle of incidence in terms of the core and cladding refractive indexes is¹

$$\sin \theta_{\max} = \sqrt{n_1^2 - n_2^2} \quad (1)$$

FIG.1

Figure 1. Schematic Cross Section of a Multimode Step-Index Optical Fiber of Constant Core Diameter

In general, not all those incident rays characterized by $0 < \theta < \theta_{\max}$ will be propagated along the core. This is true because of the electromagnetic nature of light. As such, successive total internal reflections of the light at the core/cladding boundary, as it travels along the core, must satisfy phase conditions associated with Maxwell's electromagnetic field equations that describe the propagation of the light in the core. The phase conditions will be satisfied only for discrete values of incident angle θ within the interval $0 < \theta < \theta_{\max}$. The propagation associated with each discrete value of θ is referred to as a "mode" and, if λ is the wavelength of the light, it can be shown that the approximate number of possible modes is given by²

$$M = 2 \frac{\pi^2 d^2}{\lambda^2} (n_1^2 - n_2^2) \quad (2)$$

If $d \gg \lambda$, then the number of modes is quite large. For example, for a fiber with core diameter $d = 100 \mu m$ passing light in the near infrared at wavelength $\lambda = 850 nm$, the calculated number of possible propagation modes is $M \cong 13,450$, where typical values of $n_1 = 1.457$ for silica and

$n_2 = 1.440$ for fluorine-doped silica cladding were used. A case such as this, where $d \gg \lambda$ is referred to as "multimode" step index. The number of possible modes is quite large, and the refractive index of the core is constant over its diameter. For given values of wavelength and refractive indexes, the core diameter corresponding to a single mode ($M = 1$) is much smaller than for multimode. For example, setting $M = 1$ and solving for d with $\lambda = 850 \text{ nm}$, and the above values of n_1 and n_2 gives $d \cong 0.9 \mu m$.

For the multimode case, the number of discrete values of θ for which Maxwell's equations are satisfied is very large and, hence, the possible values of angle θ in the interval $0 < \theta < \theta_{\max}$ represent essentially a continuum. For this case, it is adequate to use simple geometric optics to represent the paths of light rays as they propagate along the optical fiber core.

Many applications arise in which it is desirable to inject light into one end of an optical fiber, let it pass out the far end of the fiber and intersect with any of a number of optical devices, and then be brought back into another fiber of the same diameter and type. This is indicated schematically in Figure 2. If light injected into the fiber fills the entire acceptable angular interval, then the light passing out the other end will fill the same angular range and, hence, the light will diverge rapidly with distance beyond the exit end of the first fiber in Figure 2. Only if the receiving fiber face is placed very nearly in contact with the exit face can any appreciable amount of light be passed into the second fiber. However, this would leave very little space for desired optical devices of practical size to be placed between the exit end of Fiber 1 and the receiving end of Fiber 2.

FIG.2

Figure 2. Illustration of Applications in Which It Is Desired to Pass the Divergent Exit Beam From a Fiber Through an Optical Interface and Then Return as Much Energy as Possible Into Another Fiber

Coupling From One Optical Fiber to Another With Two Identical Sphere Lenses

This situation can be remedied with the aid of two lenses placed symmetrically in relation to the exit face of Fiber 1 and the receiving face of Fiber 2, as shown schematically in Figure 3. The first lens is placed essentially at one focal distance from the exit face of Fiber 1, so that the light is collimated after passing through this lens, but the diameter of the collimated beam is much larger than the fiber core diameter because the focal length of the lens is many times the core diameter. The second lens, of identical type and focal length to that of the first, is placed so that the light passing through it is focused on the receiving face of the second fiber. Use of the lenses in this way will aid in capturing a significant amount of the light exiting the first fiber and passing it to the second fiber. However, the coupling is by no means perfect for several reasons. First, the light passes through two air/lens boundaries and two lens/air boundaries in its path from one fiber to the next, in addition to the fiber/air and air/fiber boundaries at the exit of

Fiber 1 and entry into Fiber 2. At each boundary, a loss of a few percent occurs through reflection unless the lenses are coated with anti-reflection coatings to minimize the reflection loss. Secondly, the "sending" fiber face is not a mathematical point source, but has a finite size and, hence, represents an extended source. Finally, even if the fiber diameter d were very small, lens aberrations for significant angular spreads of θ in the range $0 < \theta < \theta_{\max}$ make it impossible to capture all of the light from Fiber 1 and pass it into Fiber 2 for simple lenses.

FIG.3

Figure 3. Coupling of Optical Energy From One Optical Fiber to Another With the Aid of Two Symmetrically Located Sphere Lenses

One of the simplest and most economical lenses for the coupling depicted in Figure 3 is a lens in the form of a solid sphere. If n is the refractive index of a lens of this type, then, according to thick-lens formulas³, the focal length of the sphere, measured relative to the sphere center, is $f = nR/2(n-1)$, where R is the radius of the sphere. Experiments were conducted with pairs of solid-sphere lenses of diameter 2 mm, 3 mm, and 5 mm to determine the percentage of light from Fiber 1 that could be coupled into Fiber 2. The source of light introduced into Fiber 1 was provided by a very stable LED operating at wavelength of 850 nm, and the energy exiting Fiber 2 was measured with a silicon photodetector. The stabilized source and detector system are integral parts of the HP8153A Lightwave Multimeter referenced in Appendix A. First, two 2-mm-diameter *LaSF9* spheres were positioned with a separation distance of $2f = 2.2048$ mm and fixed in place. Next, the 100- μ m-core optical fibers were positioned so that the axes lay on the axis defined by the line passing through the centers of the sphere lenses. The fiber ends were mounted on precision slides so they could be moved toward or away from the lenses. The input Fiber 1 was positioned with its face at distance $x_1 = 0.05$ mm from the sphere lens, on axis, and the detector was used to measure the light energy received by Fiber 2 as a function of the distance x_2 of its face from the exit face of the second sphere lens. The data for 2-mm-diameter *LaSF9* sphere lenses, with refractive index 1.83, are given in Figure 4. A plot of the energy coupled from Fiber 1 through the lenses into Fiber 2 is shown by the curve with solid dots in Figure 4. The maximum coupled energy of 57.7 percent occurs for distance $x_2 = 0.125$ mm. Next, the input Fiber 1 was positioned at distance $x_1 = 0.10$ mm from the first lens, and the

FIG 4

Figure 4. Measured Coupling of Optical Energy From One Fiber to Another With the Aid of Two Sphere Lenses, as a Function of the Placement of the Lenses Relative to the Fiber Faces

coupled energy into Fiber 2 was again measured as a function of distance x_2 . The data are given by the curve with open circles in Figure 4, from which it is seen that the maximum coupling of 59.3 percent occurs for $x_2 = 0.075 \text{ mm}$. Finally, the input Fiber 1 was set at distance $x_1 = 0.2 \text{ mm}$ from the first sphere, and the coupling was again measured as a function of distance x_2 . The results are given by the curve with crosses. From Figure 4, it is seen that the most efficient coupling occurs for input Fiber 1 distance $x_1 = 0.10 \text{ mm}$ and Fiber 2 distance $x_2 = 0.075 \text{ mm}$. For this case, the coupling is within 98 percent of the maximum for the receiving Fiber 2 distance in the range $x_2 = 0.075 \text{ mm} \pm 0.037 \text{ mm}$.

The primary reason for the relatively low coupling efficiency of 59.3 percent is that there are six fiber/air and lens/air boundaries that the light traverses in its path from Fiber 1 to Fiber 2. At each boundary, a reflection loss occurs. The combined effect of these losses can be calculated by taking into account the refractive indexes of the fiber core material and the lens material. For angles of incidence near normal, the optical power transmitted from Fiber 1 to Fiber 2 in Figure 3 can be calculated from the relation⁴

$$P_T = \left[1 - \left(\frac{n_f - 1}{n_f + 1} \right)^2 \right]^2 \left[1 - \left(\frac{n_L - 1}{n_L + 1} \right)^2 \right]^4, \quad (3)$$

where the first term on the right takes into account the losses as the light leaves Fiber 1 and as it enters Fiber 2, and the second term gives the combined loss at the four air/lens boundaries. Substituting $n_f = 1.4527$ for the silica fiber core and $n_L = 1.83$ for the LaSF9 lens material, at $\lambda = 850 \text{ nm}$, gives

$$P_T = 65.1\% , \quad (4)$$

which assumes perfect alignment and perfect lens action. This means that the combined reflection losses amount to approximately 34.9 percent. Essentially all of this loss can be avoided with today's technology of multilayer dielectric "anti-reflection" coatings which can reduce the reflection loss at each surface to approximately 0.1 percent.

The actual maximum measured coupling of 59.3 percent is 5.8 percent lower than the calculated value of 65.1 percent for uncoated fiber faces and uncoated lenses. This difference can be explained by the fact that the simple combination of two sphere lenses cannot perfectly image all of the light from Fiber 1 into Fiber 2 of the same diameter. To further investigate this, a single-mode optical fiber with central modal field diameter approximately $5 \mu\text{m}$ was used to measure the size of the focused spot produced by the second lens in the plane of the receiving Fiber 2 face. This was done by removing Fiber 2 and placing the $5\text{-}\mu\text{m}$ -diameter probe at the same distance from the second lens and then measuring the energy as a function of transverse distance through the center of the focused spot. The energy E as a function of transverse distance, expressed as a ratio to the maximum energy E_o at the center of the focused spot, is given in Figure 5. The dashed vertical lines at $+0.05 \text{ mm}$ and -0.05 mm denote the $100\text{-}\mu\text{m}$ -diameter of the Fiber 2 receiving face. As can be seen, some of the energy falls outside the $\pm 50\text{-}\mu\text{m}$ radius for $E/E_o < 0.4$. The fraction of total energy falling outside the $100\text{-}\mu\text{m}$ -diameter Fiber 2 face can be estimated as follows. If the energy profile is approximated by two straight-line regions, one for the region $0.7 < E/E_o \leq 1$ and the other for the region $0 \leq E/E_o \leq 0.7$, then the total energy represented by the profile is

$$E_T = \int_{\varphi=0}^{2\pi} d\varphi \left[\int_{r=0}^{r_1} \left(1 - \frac{0.3}{r_1} \right) r dr + \int_{r=r_1}^{r_2} \left(0.7 - \frac{0.7(r-r_1)}{r_2-r_1} \right) dr \right] . \quad (5)$$

fig 5

Figure 5. Measured Ratio of Energy E at Distance r From Fiber Center to Energy E_0 at Fiber Center in the Plane of Fiber 2 Face in Figure 4

Taking $r_1 = 46 \mu m$ and $r_2 = 58 \mu m$ from the profile gives

$$E_T = 267.6 \text{ units} . \quad (6)$$

The energy falling outside the Fiber 2 radius of $50 \mu m$ can be estimated from

$$E_{>0.5\text{ mm}} = \int_{\varphi=0}^{2\pi} d\varphi \int_{r=50\mu\text{m}}^{r_2} \left(0.45 - \frac{0.45(r - 50\mu\text{m})}{(r_2 - 50\mu\text{m})} \right) dr, \quad (7)$$

which gives $E_{>0.5} = 9.9$ units. Thus, the energy falling outside the 100- μ m Fiber 2 face is 3.7 percent of the total energy. So, the foregoing calculated estimate accounts for all but 2.1 percent of the difference between the calculated coupling of 65.1 percent in Equation 4 and the measured coupling of 59.3 percent. Without more sophisticated nonlinear functions to represent the profile in Figure 5, the calculated fraction of 3.7 percent can be considered in reasonable agreement with the difference of 5.8 percent.

The expression in Equation 3 giving the calculated optical power transmission, taking into account Fresnel reflection losses, assumes that the incident and refracted rays are all very nearly perpendicular to the surfaces as the light passes the various glass→air and air→glass boundaries. Although this is nearly true, the light leaving Fiber 1 can exit the face at angles as large as $\theta_{\text{max}} = 12.7^\circ$ for typical optical fiber, which in turn results in angles of incidence and refraction of several degrees at the various surface boundaries in the overall propagation through the two lenses and into Fiber 2. Taking into account the non-perpendicular directions of the rays for Fresnel reflection losses leads to the following relation, in place of Equation 3⁵.

$$P_s = \left[1 - \frac{\sin^2(\theta - r)}{\sin^2(\theta + r)} \right] \left[1 - \frac{\sin^2(\theta_1 - r_1)}{\sin^2(\theta_1 + r_1)} \right] \left[1 - \frac{\sin^2(\theta_2 - r_2)}{\sin^2(\theta_2 + r_2)} \right] \quad (8)$$

$$X \left[1 - \frac{\sin^2(\theta_3 - r_3)}{\sin^2(\theta_3 + r_3)} \right] \left[1 - \frac{\sin^2(\theta_4 - r_4)}{\sin^2(\theta_4 + r_4)} \right] \left[1 - \frac{\sin^2(\theta_5 - r_5)}{\sin^2(\theta_5 + r_5)} \right]$$

The subscript s indicates that the expression holds for light polarized perpendicular to the plane of incidence. For light polarized parallel to the plane of incidence, the expression for P_p is obtained by replacing the trigonometric sine functions by tangent functions in Equation 8.

The first term on the right in Equation 8 takes into account the Fresnel loss associated with angles of incidence and refraction, θ and r , as the light leaves Fiber 1 face. The second and third terms on the right take into account the losses as the light enters and leaves the first sphere lens, with incident/refracted angles (θ_1, r_1) and (θ_2, r_2) , respectively, at the entrance and exit sphere lens surfaces. The fourth and fifth terms take into account the losses at the entrance, (θ_3, r_3) and exit (θ_4, r_4) surfaces, respectively, of the second sphere lens. Finally, the last term on the right takes into account Fresnel losses of the light entering the Fiber 2 face. To calculate the transmitted optical power from Equation 8 for non-perpendicular paths, all the angles of incidence and refraction at all the interfacial boundaries must be known. These angles can be determined through ray tracing. In general, a ray leaving Fiber 1 face at a point at distance h from the fiber center and making angle θ with the perpendicular to the fiber face will enter the first lens at a different height and angle, leave the first lens at still a different height, pass through the second lens, and eventually arrive at Fiber 2 face at angle of incidence ϕ^1 , at distance y^1 from the Fiber 2 center. This is indicated in Figure 6. The equations associated with the ray tracing are derived in Appendix B. From these, all the angles of incidence and refraction at the various boundaries in the arrangement of Figure 3 were calculated so that the transmitted optical power expressed in Equation 8 could be determined. The results are given in Figure 7 as a function of the angle θ of departure from Fiber 1 face, for each of three different heights of departure from the center of Fiber 1. The calculated transmitted optical power depends on whether the light is polarized perpendicular to the plane of incidence, given by the dashed curves labeled P_s , or whether the light is polarized parallel to the plane of incidence, the latter given by the solid curves labeled P_p in Figure 7. For increasing angle of departure θ , the

transmission increases for parallel polarization, whereas the transmission decreases with θ for perpendicular polarization. It can be seen that the increases in transmission, P_P , relative to $\theta = 0^\circ$, very nearly compensate for the decreases in transmission, P_S , as a function of angle θ . Therefore, for randomly polarized light, with equal amounts of "p" and "s" polarization, the total optical transmission is expected to be essentially independent of angle θ over the angular interval $0 < \theta \leq 12.7^\circ$, characteristic of typical optical fiber with numerical aperture of approximately 0.22.

FIG 6

Figure 6. Schematic Representation of Path of a Ray From Fiber 1 Face Through Both Lenses and to Fiber 2 Face in Figure 4

Figure 7. Optical Transmission for Pair of 2-mm Diameter *LaSF9* Coupling Sphere Lenses, as a Function of Angle θ in Figure 6, Taking Into Account Fresnel Reflection Losses Calculated From Equation 8 for Perpendicular Polarization. (Curves for parallel polarization were calculated from Equation 8 with the sine functions replaced by tangent functions.)

Measurements of the type given in Figure 4 were also made for a separation of the sphere lenses by a distance of $4f$ and for a distance of $8f$. As the distance between the sphere lenses increases, the effects of aberration become more pronounced not only by the fact that more energy falls outside the Fiber 2 receiving face but, also, based on ray tracing, some of the energy arriving on the fiber face is incident at angles that exceed the angle for which the energy will propagate in the core of Fiber 2. Hence, the maximum energy coupled from Fiber 1 to Fiber 2 will decrease significantly for sufficiently large lens separations. This is represented by Figure 8, which shows the maximum measured coupled energy for the three lens separations investigated. The value at $2f$ -separation is that referred to above.

FIG 8

Figure 8. Measured Percent Coupling of Optical Energy From Fiber 1 to Fiber 2 in the Arrangement of Figure 6, Versus Distance Between the Sphere Lenses

Measurements of the foregoing type were also made with a pair of 3-*mm* diameter *LaSF9* sphere lenses, and with a pair of 5-*mm* diameter lenses. The results are summarized in Table 1, along with the results discussed above for the pair of 2-*mm*-diameter sphere lenses. The measurements with the 3-*mm* spheres show that the coupling efficiency is essentially independent of sphere separation over the interval $2f$ to $8f$, unlike the case for the 2-*mm* spheres where a separation of $8f$ gave a markedly lower coupling efficiency. For the 5-*mm* lenses, the coupling is nearly the same as with the 3-*mm* lenses, although the values for $8f$ separation are a little lower.

Table 1. Comparison of Measurements for Coupling From One Fiber to Another With Pairs of Sphere Lenses

Sphere Size	$2f$	$4f$	$8f$
2 mm	59	54	14
3 mm	59	54	54
5 mm	59	54	50

2 mm	Maximum measured % coupling	59.3%	54.6%	12.8%
	Calculated coupling for AR-coated	91%	83.8%	19.7%
	Alignment tolerance for 98% coupling	$\pm 37 \mu m$	$\pm 70 \mu m$	$\pm 40 \mu m$
3 mm	Maximum measured % coupling	56.7%	57.8%	57.6%
	Calculated coupling for AR-coated	87.1%	88.8%	88.5%
	Alignment tolerance for 98% coupling	$\pm 33 \mu m$	$\pm 60 \mu m$	$\pm 50 \mu m$
5 mm	Maximum measured % coupling	59.3%	58.4%	52.9%
	Calculated coupling for AR-coated	91%	89.7%	81.2%
	Alignment tolerance for 98% coupling	$\pm 20 \mu m$	$\pm 50 \mu m$	$\pm 80 \mu m$

Use of a Single Sphere Lens to Couple From One Fiber to Another

The pairs of sphere lenses used earlier in the arrangements for coupling energy from Fiber 1 to Fiber 2 represent a symmetric configuration in that each fiber face is located at nearly the same distance (i.e., the focal distance from its corresponding sphere). The angles of divergence and convergence at Fiber 1 and Fiber 2, respectively, are nearly equal in magnitude, and the light rays are nearly parallel in the region between the spheres.

A single sphere lens can also be used in a symmetric position between the fiber faces to couple energy from Fiber 1 to Fiber 2. This arrangement, in general, is shown schematically in Figure 9a. Based on simple geometric optics, the Fiber 1 face can be treated as an object point and Fiber 2 face as an image point. If x_o and x_i are the conjugate object/image distances, then they obey the relation $1/x_o + 1/x_i = 1/f$, where f is the focal distance of the lens. If $x_o \equiv x_i$ for symmetry, then $x_o = x_i = 2f$. For this relation, the angular divergence of the light leaving Fiber 1 is essentially the same in magnitude as the angular convergence of the light

entering Fiber 2, as indicated in Figure 9b.

FIG.9

Figure 9. Use of a Single Sphere Lens to Couple From One Optical Fiber to Another. [In (a), the magnitudes of angular divergence and convergence at the fiber faces are different, while in (b), the magnitudes are the same as a result of the symmetric location of the sphere.]

The input Fiber 1 and receiving Fiber 2 were aligned on separate precision slides with their axes collinear, and a single 2-*mm*-diameter *LaSF9* sphere was positioned between the fiber tips with the sphere center on the axis defined by the fibers. The Fiber 1 was moved to an appropriate distance from the 2-*mm* sphere, and then the Fiber 2 was moved until the energy coupled from Fiber 1 through the lens into Fiber 2 was a maximum. If the fiber faces are treated as points, then the two positions for maximum

coupling should represent conjugate object/image distances obeying the relation $1/x_o + 1/x_i = 1/f$, where $f = nR/2(n-1)$, with $R = 1.0$ mm and $n = 1.83$. The Fiber 1 was positioned at various other distances from the sphere lens, and the corresponding position of Fiber 2 was determined to achieve maximum coupling for each Fiber 1 position. The Fiber 2 location, x_2 , is plotted versus Fiber 1 location, x_1 , in Figure 10. The solid curve in Figure 10 is the calculated relation for x_2 as a function of x_1 , based on the conjugate object/image relationship.

FIG 10

Figure 10. Distance of Receiving Fiber Face From Sphere Lens to Achieve Maximum Coupling Versus Distance of Sending Fiber Face From Sphere

The percentage coupling of the energy from Fiber 1 into Fiber 2 is plotted in Figure 11 as a function of the distance x_1 from the Fiber 1 face to the 2-*mm*-diameter lens surface. Note from Figures 10 and 11 that the maximum measured coupling percentage of 63.6 percent is achieved for x_1 and x_2 nearly equal to 1.2 *mm*. That is, the conjugate object/image distances are nearly equal. This leads to symmetry in that the angular convergence of the light arriving on the Fiber 2 face is very nearly equal in magnitude to the angular divergence of the light leaving the Fiber 1.

FIG 11

Figure 11. Measured Percentage Coupling of Energy From Fiber 1 Into Fiber 2 as a Function of Distance From Sphere Lens to Fiber 1 Face

Taking into account the Fresnel reflection losses that exist as the light leaves Fiber 1, passes through the lens, and finally enters Fiber 2, the calculated coupling efficiency for uncoated surfaces, for assumed small angles of incidence and refraction, is

$$P_T = \left[1 - \left(\frac{n_f - 1}{n_f + 1} \right)^2 \right]^2 \left[1 - \left(\frac{n_2 - 1}{n_2 + 1} \right)^2 \right]^2 . \quad (9)$$

Using the values $n_f = 1.4527$ for the silica fiber and $n_2 = 1.83$ for *LaSF9* at the wavelength 850 nm gives

$$P_T = 77.9\% . \quad (10)$$

The maximum measured coupling was 63.6 percent, which occurred for $x_1 = x_2$. This value is lower than the value in Equation 10 by 14.3 percent. Moreover, the latter shows that a single 2-mm -diameter sphere lens, even though placed symmetrically, is less efficient for coupling from Fiber 1 to Fiber 2 than the combination of two 2-mm -diameter sphere lenses. The primary reason for this is that the effects of aberration are more severe for the single sphere lens than for the pair. Equations for ray tracing for the single sphere lens case in Figure 9 are derived in Appendix C. A comparison of ray tracing results for the two-sphere lens case of Figure 3, with those for the single-sphere lens of Figure 9, calculated from the relations in Appendices B and C, are given in Figure 12. The results are for a sphere lens diameter of 2.0 mm , with refractive index $n = 1.83$ corresponding to lens material *LaSF9* at 850-nm wavelength. The fiber diameter is $100 \mu\text{m}$ for both Fiber 1 and Fiber 2. In Figure 12a, a comparison plot is given for the maximum angle θ at which light can leave Fiber 1 face, as a function of distance h from Fiber 1 center, and still arrive within the radial boundary of the $100\text{-}\mu\text{m}$ -diameter Fiber 2 face. For example, rays leaving Fiber 1 face at radial distance $30 \mu\text{m}$ from center cannot exceed $\theta = 7^\circ$ if they are to arrive on the face of Fiber 2, for the single-sphere lens case. However, light leaving the Fiber 1 surface at angles up to $\theta = 12^\circ$ will arrive within the core diameter of Fiber 2 for the two-sphere lens case.

In Figure 12b, a plot is given for the maximum angle θ of departure of light from Fiber 1 that will arrive at incident angle $\leq 12.7^\circ$ on Fiber 2 face, as a function of radial distance of departure on Fiber 1 face. For the two-sphere lens case, it is seen that all light departing at angle $\theta < 12.0^\circ$ will arrive on Fiber 2 face incident at angle $\leq 12.7^\circ$, regardless of location on Fiber 1 face. However, for the single-sphere lens case, a departure angle of $\theta = 9^\circ$ is the maximum that will arrive on Fiber 2 face at incident angle less than 12.7° . So, for a given lens size and material, the two-sphere case is more efficient than the single sphere case, for a given fiber size.

Note, for example, from Figure 11 that the percent coupling is relatively low in the range 42-45 percent for $x_1 = 0.8 \text{ mm}$, and for $x_1 = 2.0 \text{ mm}$. A $5\text{-}\mu\text{m}$ -diameter fiber probe was used to measure the focused light distributions in the plane of Fiber 2 face for the Fiber 1 face distances $x_1 = 0.8 \text{ mm}$ and $x_1 = 2.0 \text{ mm}$. These distributions, plotted as a ratio to the peak value E_o , versus radial distance, are given in Figure 13. Also given in Figure 13 is a plot of the light energy distribution for the case of maximum coupling efficiency, at $x_1 = 1.28 \text{ mm}$. Consider first the distribution for $x_1 = 2 \text{ mm}$, corresponding to Fiber 2 face at $x_2 = 0.7 \text{ mm}$. For this case, the rays exiting the lens make relatively large angles with the axis in order to converge at distance $x_2 < x_1$. Ray tracing based on the relationship derived in Appendix C shows that many of the rays exceed the value of 12.7° , which means that they will not be propagated by the Fiber 2 core. On the other hand, for $x_1 = 0.8 \text{ mm}$, the converging rays are characterized by smaller angles such that they do not violate the condition of numerical aperture, but many of the rays arrive in the plane of Fiber 2 at distances beyond the radius of the fiber core and hence cannot enter the core.

FIG 12

Figure 12. Results of Ray Tracing Showing (a) the Maximum Angle θ of Departure Versus Distance h From Center of Fiber 1 Face for Which Rays Will Fall Within the Core Diameter of Fiber 2,

and (b) the Maximum Angle of Departure θ Versus Distance h on Fiber 1 for Which Ray Will Be Incident on Fiber 2 Face Within the Numerical Aperture, i.e., With Angle $< 12.7^\circ$

FIG 13

Figure 13. Measured Focused-Light Profiles in the Plane of Fiber 2 for Various Distance x_l Between Fiber 1 and the Sphere Lens in Figure 9

Finally, the ray tracing results derived in Appendix C for the single-sphere lens case were used to calculate the angles of incidence and refraction at each of the four glass-air boundaries the light passes as it leaves Fiber 1 face, passes through the lens, and enters Fiber 2 in the arrangement of Figure 9. From these results, the Fresnel reflection losses were calculated from a relationship similar to that of Equation 8 for the two-sphere lens case. Figure 14 gives the results of the single-sphere lens case, where the percent transmitted optical power is plotted as a function of the angle of departure from Fiber 1 face. Note that for θ near zero, the transmission is approximately 78 percent, the same, of course, as that given in Equation 10, calculated from Equation 9 for the special case of small incident and refracted angles. As the angle θ increases, for any distance h from the Fiber 1 center, the transmission decreases from the value 78 percent, for light polarized perpendicular to the plane of incidence. However, for light polarized parallel to the plane of incidence, the transmission increases slightly with increasing angle θ . Moreover, the slight increase is approximately equal in magnitude to the slight decrease for perpendicular polarized light. Therefore, for random polarization, with essentially equal amounts of parallel and perpendicular polarization, the transmitted optical power would be expected to be nearly independent of angle θ over the interval $0 < \theta < 12.7^\circ$ for typical optical fiber numerical aperture.

Figure 14. Optical Transmission for a Single Coupling Sphere Lens of 2-mm Diameter, as a Function of Angle θ in Figure 9, Taking Into Account Fresnel Reflection Losses, Calculated From Equation 8 for Polarization Perpendicular to the Plane of Incidence. (The curves for parallel polarization were calculated from Equation 8 with the trigonometric sine functions replaced by tangent functions.)

Similar measurements of the type described above for the single 2-mm diameter coupling sphere were also made with a single 3-mm diameter sphere and with a single 5-mm diameter sphere. The results are summarized in Table 2, along with the foregoing for the 2-mm diameter sphere.

Table 2. Comparison of Coupling Measurements Using a Single Sphere Lens

Sphere Size

	2 mm	3 mm	5 mm
Maximum measured % coupling	63.6%	65.1%	60.9%
Calculated coupling for AR coated	81.6%	83.5%	78.1%
Alignment tolerance for 98% of maximum coupling:			
Fiber 1	$\pm 100 \mu m$	$\pm 125 \mu m$	$\pm 310 \mu m$
Fiber 2	$\pm 50 \mu m$	$\pm 105 \mu m$	$\pm 250 \mu m$

Analysis of Light Paths in a Tapered Optical Fiber

Consider the tapered optical fiber shown in the schematic cross section in Figure 15. The large end of the taper has a diameter of $2R_o$ and, over a distance of L , the fiber diameter decreases linearly to a diameter of $2R_i$, beyond which the diameter remains constant. The taper is characterized by the half-angle α given by

$$\alpha = \text{arc tan } (R_o - R_i)/L . \quad (11)$$

Figure 15. Schematic Representation of a Tapered Optical Fiber Having Large Face Diameter $2R_o$ and Decreasing in Diameter to $2R_i$ Over the Tapered Length L . (A ray of light is incident at angle θ on the large face and, provided θ is sufficiently small, the ray will undergo successive total internal reflections and pass into the fiber portion of constant diameter, $2R_i$.)

Let a light ray lying in the plane of Figure 15 be incident on the large-diameter face of the taper at angle θ , at distance h from the axis. Upon entering the face of the taper, the ray will be refracted toward the perpendicular to the surface, and the angle r that the path of the ray makes upon entry is given by Snell's law,

$$r = \arcsin \left(\frac{\sin \theta}{n} \right), \quad (12)$$

where n is the refractive index of the fiber core material. The entry ray will travel a horizontal distance L_o before it arrives at the boundary between the core and the cladding, and L_o is related to the angle of refraction, r , through the relation

$$\tan r = (R_o - y_o - h)/L_o. \quad (13)$$

Noting that $\tan \alpha = y_o/L_o$, the result in Equation 13 can be used to obtain the following expression for L_o

$$L_o = \frac{R_o - h}{\tan \alpha + \tan r}. \quad (14)$$

The horizontal distance L_o at which the entry ray strikes the core/cladding boundary can be expressed explicitly in terms of the ray height, h , the angle of incidence, θ , and the taper geometry R_o , α by eliminating the angle r between Equations 12 and 14 to give

$$L_o = \frac{R_o - h}{\tan \alpha + \sin \theta (n^2 - \sin^2 \theta)^{-1/2}}. \quad (15)$$

Let i_1 be the angle that the entry ray makes with the perpendicular to the boundary between the core and cladding at the first point of reflection in the plane of Figure 15. Snell's law gives

$$n \sin i_1 = n^1 \sin i^1, \quad (16)$$

where i^1 is the angle that the ray makes with the perpendicular if it enters the cladding material of refractive index n^1 . Of course, the intent is that the ray not enter the cladding. To satisfy this condition, the angle i_1 must be sufficiently large so that total internal reflection occurs at the core/cladding boundary. The limiting or critical angle of incidence, i_{1c} , occurs when $i^1 = 90^\circ$, which, according to Equation 16, is

$$i_{1c} = \arcsin [n^1/n]. \quad (17)$$

From Figure 15, the entry ray makes an angle of incidence, i_1 , with the core/cladding boundary given by

$$i_1 = \frac{\pi}{2} - (\alpha + r), \quad (18)$$

and if $i_1 > i_{1c}$, then the entry ray will be totally reflected and remain in the core. The reflected ray will strike the core/cladding interface at point P_1 at a horizontal distance L_1 beyond the point where the entry ray first struck the core/cladding interface. If β_1 is the angle that the reflected ray path makes with the perpendicular to the fiber core axis, then it can be seen from Figure 15 that

$$\beta_1 = i_1 - \alpha \quad (19)$$

$$= \frac{\pi}{2} - (2\alpha + r),$$

and that

$$\tan \beta_1 = \frac{L_1}{2R_o - 2y_o - y_1} \quad (20)$$

Noting that $\tan \alpha = y_1/L_1$, the result in Equation 20 can be solved for L_1 and, by eliminating β_1 , expressed as follows.

$$L_1 = \frac{2(R_o - y_o)}{\tan \alpha + \tan(2\alpha + r)} \quad (21)$$

The angle of incidence i_2 of the ray with the core/cladding interface at point P_1 is

$$i_2 = \beta_1 - \alpha \quad (22)$$

$$= \frac{\pi}{2} - (3\alpha + r),$$

and, provided the angle i_2 is greater than the critical angle i_{1c} in Equation 17, the ray will be totally internally reflected at P_1 , as indicated. The angle β_2 that the reflected ray makes with the perpendicular to the fiber core axis is

$$\beta_2 = i_2 - \alpha \quad (23)$$

$$= \frac{\pi}{2} - (4\alpha + r).$$

The ray reflected from point P_1 will strike the core/cladding boundary at horizontal distance L_2 beyond point P_1 , and the relationship

$$\tan \beta_2 = \frac{L_2}{2R_o - 2y_o - 2y_1 - y_2} \quad (24)$$

holds, where $y_2 = L_2 \tan \alpha$. The latter can be solved for L_2 to yield

$$L_2 = \frac{2(R_o - y_o - L_1 \tan \alpha)}{\tan \alpha + \tan(4\alpha + r)} . \quad (25)$$

The angle of incidence of the ray at point P_2 is

$$i_3 = \beta_2 - \alpha \quad (26)$$

$$= \frac{\pi}{2} - (5\alpha + r) ,$$

and, provided $i_3 > i_{1c}$ in Equation 17, the ray will undergo total internal reflection at point P_2 as indicated.

Upon reflection from point P_2 , the ray will traverse the core and again strike the core/cladding interface at point P_3 , at horizontal distance L_3 beyond the point of incidence P_2 on the core/cladding interface. The angle that the line representing this ray makes with the perpendicular to the axis is

$$\beta_3 = (\beta_2 - \alpha) - \alpha \quad (27)$$

$$= \frac{\pi}{2} - (6\alpha + r) .$$

Therefore, the horizontal distance L_3 is related to β_3 through the relation

$$\tan \beta_3 = \frac{L_3}{2(R_o - y_o) - 2y_1 - 2y_2 - y_3} , \quad (28)$$

where

$$y_3 = L_3 \tan \alpha . \quad (29)$$

Combining Equations 27-29 and solving for the horizontal distance L_3 gives

$$L_3 = 2 \frac{R_o - y_o - (L_1 + L_2) \tan \alpha}{\tan \alpha + \tan(6\alpha + r)} . \quad (30)$$

From the form of Equations 18, 22, and 26, it can be seen that the angle of incidence, i_j , at the j^{th} point of incidence of the ray is given by

$$i_j = \frac{\pi}{2} - [(2j - 1)\alpha + r] \quad (31)$$

and from Equations 19, 23, and 27, the corresponding angle β_j is

$$\beta_j = \frac{\pi}{2} - (2j\alpha + r) \quad . \quad (32)$$

Noting the form of Equations 21, 25, and 30, the horizontal distance, L_j , between the j^{th} point of incidence and the $(j-1)^{\text{th}}$ point can be expressed as

$$L_j = 2 \left[\frac{R_o - y_o - \tan \alpha \sum_{p=1}^{j-1} L_p}{\tan \alpha + \tan(2j\alpha + r)} \right] \quad . \quad (33)$$

In Equation 33, the summation on L_p is defined only for $j \geq 2$. As the ray continues its path in the taper region, let P_N be the last point of incidence just before it leaves the taper portion and enters the region of fiber of constant diameter, $2R_i$. The first point of incidence in this portion is designated as P_{N+1} in Figure 15. The angle of incidence at point P_{N-1} in the taper section is

$$i_N = \frac{\pi}{2} - [(2N-1)\alpha + r] \quad , \quad (34)$$

and, therefore, the angle of incidence at the last point of incidence, P_N , within the taper region is $i_N - 2\alpha$. Let i_s be the angle of incidence of the ray at the first point where it intersects the core-cladding interface within the straight region of the fiber, at point P_{N+1} . At any given reflection within the taper, the angle of incidence is smaller than the angle at the immediately preceding

reflection, by the amount 2α , as can be seen from Equation 34 noting that $i_N - i_{N-1} = -2\alpha$. However, in going from the last reflection in the taper at point P_N to the first reflection in the straight section, the angle of incidence is smaller by only α and hence,

$$i_s = i_N - 3\alpha \quad (35)$$

$$= \frac{\pi}{2} - [2(N+1)\alpha + r] \quad .$$

Applying Snell's law at the first reflection at point P_{N+1} in the straight section of the fiber gives

$$n \sin i_s = n^1 \sin \omega \quad (36)$$

where, in general, ω is the angle of refraction of the ray if it enters the cladding. However, for propagation through the straight section, the ray must not enter the cladding, and the limiting value of i_s occurs when $\omega = 90^\circ$, so that Equation 36 becomes

$$\sin i_{sc} = n^1/n . \quad (37)$$

At each successive total internal reflection of a given ray in the tapered region, the angle of incidence decreases by twice the taper angle, or 2α . Therefore, there is a limiting value of the number of reflections for which $i_{sc} \leq \arcsin (n^1/n)$ in Equation 37 can be satisfied. The limiting value of N at which i_s satisfies Equation 37 can be found by substituting i_s from Equation 35 into Equation 37 and solving for the critical value of N , which gives

$$N_c = \frac{1}{2\alpha} \left[\arccos \left(\frac{n^1}{n} \right) - \arcsin \left(\frac{\sin \theta}{n} \right) \right]^{-1} , \quad (38)$$

where Equation 12 was used to express the result in terms of the angle of incidence, θ , of the ray entering the large diameter face of the taper. So, knowing the angle α characterizing the taper and knowing the refractive indices n^1 and n of the fiber cladding and core, respectively, the maximum number of reflections, N_c , which can be made along the taper so that a ray incident at angle θ at the large diameter end will just be propagated with total internal reflection as it enters the straight fiber section can be calculated from Equation 38. Of course, the maximum length of the tapered section will depend upon the large and small diameters, $2R_o$ and $2R_i$, as related to the taper angle α through Equation 11.

The maximum length of the taper is given by

$$L = L_o + \sum_{j=1}^{N_c} L_j , \quad (39)$$

where L_o is given by Equation 14, and L_j is given by Equation 33. The summation extends to the value N_c in Equation 38, which is the maximum value for which total internal reflection occurs; that is, the maximum value for which i_s in Equation 35 is $\leq \arcsin (n^1/n)$.

As a particular ray incident at angle θ on the entrance face of the taper undergoes total internal reflection at the core-cladding interface in the course of its propagation through the taper, the largest angle of reflection occurs at the entrance of the taper, and the smallest angle of reflection occurs at the exit of the taper region. Correspondingly, the horizontal distance between two successive internal reflections is greatest at the entrance end of the taper and smallest at the exit end. Let L_{avg} represent the average horizontal distance between two successive total internal reflections. Then, the total taper length L can be written as

$$L = N_c L_{avg} , \quad (40)$$

where N_c is the critical or maximum number of reflections that can occur (Equation 38) and still satisfy total internal reflection conditions for propagation into the straight fiber segment. Let ω_{avg} be the average angle that the ray paths make with the axis of the taper. At the midpoint of the taper length, the taper diameter is $R_o + R_i$, which is the average taper diameter, and hence L_{avg} in Equation 40 is related to ω_{avg} through the expression

$$\tan \omega_{avg} = \frac{R_o + R_i}{L_{avg}} . \quad (41)$$

Let N (identically equal to N_c) be the last reflection before the ray leaves the taper. According to Equation 32, the angle the ray

makes with the normal to the axis is $\beta_N = \frac{\pi}{2} - (2N\alpha + r)$. The angle ω that the ray makes with the horizontal axis is $\omega_N = \pi/2 - \beta_N$ or

$$\omega_N = 2N\alpha + r \quad (42)$$

The angle ω that the incoming ray makes with the horizontal axis as it enters the taper is merely $\omega \equiv r$, as seen in Figure 15. Therefore, the average angle is

$$\begin{aligned} \omega_{\text{avg}} &= \frac{1}{2}[r + \omega_N] \\ &= N\alpha + r, \quad (43) \end{aligned}$$

and combining this with Equations 40 and 41 gives an approximate expression for the total taper length L

$$L = \frac{N(R_i + R_o)}{\tan(N\alpha + r)} \quad (44)$$

So, for a given fiber taper characterized by angle α and core and cladding refractive indexes n and n^1 , the maximum number of reflections N that a ray incident at angle θ can make and still be propagated as it leaves the taper and enters the straight section is given by Equation 38. The corresponding maximum total length L of the taper section is given by Equation 44. From Equation 11, $\tan\alpha = (R_o - R_i)/L$. Using this relation with Equation 44 provides an expression for the maximum taper length L in terms of the small diameter, $2R_i$, of the taper

$$L = \frac{2NR_i}{\tan(N\alpha + r) - N\tan\alpha} \quad (45)$$

Alternatively, L can be eliminated between Equations 11 and 44 to yield the following expression for the ratio of the large taper diameter to the small diameter.

$$\frac{R_o}{R_i} = \frac{\tan(N\alpha + r) + N\tan\alpha}{\tan(N\alpha + r) - N\tan\alpha} \quad (46)$$

Maximum Allowed Number of Reflections in a Taper

The maximum allowed number of total internal reflections that a ray, incident at angle θ on the large diameter of a tapered fiber, can make before the condition for total internal reflection at the core-cladding boundary is violated can be calculated from Equation 38. Typically, the values of core and cladding refractive indexes for silica optical fiber are $n = 1.4527$ and $n^1 = 1.4359$,

respectively, at the wavelength 850 nm . Generally, taper angles for tapered optical fiber are very small, with typical values of a few tenths of a milliradian. For example, the taper length of a tapered fiber with large face diameter $2R_o = 300\text{ }\mu\text{ m}$ and small-end diameter $2R_i = 100\text{ }\mu\text{ m}$ would be approximately 2 meters. A plot of N_{max} as a function of incident angle θ , calculated from Equation 38, is given in Figure 16 for three different values of the taper angle, α . For example, for a taper angle $\alpha = 0.1\text{ milliradian}$, the maximum number of reflections is $N_{\text{max}} = 550$ at an incident angle of $\theta = 3.5^\circ$. If the large diameter face is $2R_o = 300\text{ }\mu\text{ m}$, and the small diameter end is $2R_i = 100\text{ }\mu\text{ m}$, then the length of the taper is 1 meter. All rays incident at angle $\theta \leq 3.5^\circ$ will be passed. The value of N_{max} for a given value of incident angle θ , is quite sensitive to taper angle.

A plot of the maximum taper length, L , in terms of the small taper end diameter (radius, R_i) is given in Figure 17 where the results were calculated from Equation 45 for taper angle $\alpha = 0.1\text{ milliradian}$. For example, if the small diameter is $100\text{ }\mu\text{ m}$ ($R_i = 50\text{ }\mu\text{ m}$), then the maximum length of the taper to pass all light for $\theta \leq 5^\circ$ is 0.775 meter . The corresponding large diameter end is $2R_o = 155\text{ }\mu\text{ m}$, calculated from Equation 11.

The exact expression for the maximum length of the taper, as a function of the incident angle θ , is given by Equation 34. The exact relation is somewhat complicated by virtue of the double summations. A representative plot of the exact value of L , versus angle θ , calculated from Equation 34 is given by the solid curve in Figure 18. Because of the great range in length for

$0^\circ \leq \theta \leq 12.7^\circ$, the logarithm of (L/R_i) is plotted instead of L/R_i itself. The dots represent the values calculated from the approximate expression for maximum length L given by Equation 44. Note that the values calculated from the approximate expression agree well with those of the exact expression. This is true in general for typical tapered optical fibers for which $\alpha \ll 1$ and $N_{\text{max}} \gg 1$.

FIG 16

Figure 16. Maximum Number of Total Internal Reflections Within the Tapered Region for Which a Ray Will Propagate Through to the Small End, as a Function of Angle of Incidence θ on the Large Diameter Face of the Taper. (Results are plotted for three different angles of taper.)

FIG 17

Figure 17. Plot of the Maximum Taper Length L in Terms of the Small Taper Diameter $2R_i$, as a

Function of Angle of Incidence θ on the Large-Diameter Taper Face

Figure 18. Comparison of the Exact Value of the Ratio (L/R_i) Versus Incident Angle θ , Calculated From Equation 39, With the Value of L/R_i Calculated From the Approximate Expression in Equation 45

A schematic view of a tapered optical fiber cross section is given in Figure 19, showing the large diameter end, with radius R_o , the

small diameter end, with radius R_i , the taper length L , and the angles that a ray makes as it enters the large end and as it leaves the small end, for initial incident angle θ . Equation 45 provides an expression for the ratio R_o/R_i for maximum taper length L . The angle that the entering ray makes with the horizontal axis is r . The angle that the ray makes with the horizontal axis as it leaves the tapered portion is $\pi/2 - i$, which, from Equation 35, is

$$\beta' = 2(N+1)\alpha + r \quad (47)$$

Figure 19. Schematic Cross Section of a Tapered Optical Fiber Characterized by an Entrance Face Core Diameter $2R_o$ and Decreasing to Diameter $2R_i$ Over Distance L . (An entrant ray incident at angle θ on the large face makes angle r with the axis just inside and, after distance L , it makes angle β' with the axis.)

For small taper angle, the expression $\sin \beta' / \sin r$ is very nearly equal to R_o/R_i . The ratio R_o/R_i was calculated from Equation 46 for $\alpha = 0.1$ milliradian for the interval $\theta = 1^\circ$ to $\theta = 12^\circ$, and the results are tabulated in Table 3. Also given in Table 3 are the corresponding calculated values of $\sin \beta' / \sin r$, with β' given by Equation 47 and r given by Equation 12. The agreement is very close. For small angles, the tangent is very nearly equal to the sine and, therefore, for $N \gg 1$ and $\alpha \ll 1$, it can be shown from Equation 46 that it can be written as

$$\frac{R_o}{R_i} = \frac{\sin \beta'}{\sin r} \quad (48)$$

Table 3. Comparison of the Expression for R_o/R_i in Equation 46 With the Expression for $\sin \beta' / \sin r$.

Angle of Incidence θ on Large Face of Taper	$\frac{R_o}{R_i} = \frac{\tan[\alpha N + r] + N \tan \alpha}{\tan[\alpha N + r] - N \tan \alpha}$	$\frac{\sin \beta'}{\sin r}$
1°	12.367	12.604
2°	6.247	6.302
3°	4.175	4.198
4°	3.137	3.149
5°	2.513	2.521
6°	2.099	2.104
7°	1.801	1.805
8°	1.577	1.580
9°	1.403	1.405
10°	1.265	1.267
11°	1.151	1.152
12°	1.057	1.058

Numerical Aperture of a Tapered Optical Fiber

The ratio $\sin \beta' / \sin r$ in Equation 48 can be expressed in terms of the numerical apertures of the taper and the straight-fiber portion of constant diameter $2R_i$ as follows. The ray making angle β' with the horizontal axis in Figure 19 will strike the core-cladding interface at an incident angle $\pi/2 - \beta'$ and, hence, from Snell's law, the relation

$$n_1 \sin\left(\frac{\pi}{2} - \beta'\right) = n_2 \sin W_c \quad (49)$$

holds, where W_c is the angle the ray makes with the normal at the point of incidence if it enters the cladding of refractive index n_2 .

For total internal reflection to occur, the limiting value of β' is such that $W_c = 90^\circ$, in which case the following is true from Equation 49

$$\cos \beta' = \frac{n_2}{n_1}, \quad (50)$$

and, therefore, the limiting value of $\sin \beta'_{\max}$ is

$$\sin \beta'_{\max} = (n_1^2 - n_2^2)^{1/2} / n_2 \quad . \quad (51)$$

Substituting the latter limiting value for $\sin \beta'$ into equation 48 gives

$$\frac{R_o}{R_i} = \frac{(n_1^2 - n_2^2)^{1/2}}{n_1 \sin r} \quad . \quad (52)$$

Let $r_{\max}$ be the limiting value of r corresponding to the limiting value of the angle β , i.e., $\beta'_{\max}$, in Figure 19. Then, according to Snell's law, the corresponding maximum angle of incidence, $\theta_{\max}$, on the taper face of diameter $2R_o$ is related to $r_{\max}$ through the relation $\sin \theta_{\max} = n_1 \sin r_{\max}$, and Equation 52, therefore, becomes

$$\frac{R_o}{R_i} = \frac{(n_1^2 - n_2^2)^{1/2}}{\sin \theta_{\max}} \quad . \quad (53)$$

The denominator on the right in the latter equation is merely the numerical aperture, NA_T , of the tapered fiber because $\theta_{\max}$ is the maximum incident angle in Figure 19 for which the angle β' will just allow total internal reflection as the rays enter the constant-diameter portion. For a fiber of constant diameter $2R_i$ located in air, the numerical aperture, NA , is given by $(n_1^2 - n_2^2)^{1/2}$ in Equation 1 and, therefore, Equation 53 can be written

$$NA_T = \frac{R_i}{R_o} NA \quad . \quad (54)$$

So, the numerical aperture of a tapered optical fiber characterized by large diameter $2R_o$ and small diameter $2R_i$ is smaller than that of a fiber of constant diameter, $2R_i$, by the ratio R_i/R_o .

Coupling From an Optical Fiber of Constant Diameter to a Tapered Optical Fiber

Measurements were made of the efficiency of coupling from a fiber of constant 100- μm core diameter to a 300/100- μm tapered fiber and also to a 600/100- μm tapered fiber. The experimental arrangement is shown in Figure 20. The measured results are tabulated in Table 4, along with the values of actual taper length and corresponding calculated taper angles.

FIG 20

Figure 20. Arrangement for Measuring Coupling Efficiency From Optical Fiber of Constant Core Diameter 100 μ m to Tapered Fiber Characterized by 300/100 μ m and 600/100 μ m

Table 4. Summary of Optical Power Coupling Measurements from a 100-μ m Constant-Diameter Optical Fiber to Tapered Optical Fibers, Using the Arrangement of

Figure 20.

		Actual Taper Length	Calculated Taper Angle	Transmitted Optical Power Coupling	Maximum Theoretical Angle of Incidence θ for Propagation Through Taper
Tapered Fiber	S/N				

300/100 μm	1	1.5 m	0.67×10^{-4} radian	3.878 μW	4.2°
	2	2.2 m	0.45×10^{-4} radian	4.142 μW	4.1°
	3	2.0 m	0.50×10^{-4} radian	4.518 μW	4.2°
600/100 μm	1	2.1 m	1.19×10^{-4} radian	1.902 μW	2.05°
	2	1.5 m	1.67×10^{-4} radian	1.935 μW	2.1°
	3	2.3 m	1.09×10^{-4} radian	1.406 μW	2.1°
	4	1.7 m	1.47×10^{-4} radian	1.558 μW	2.1°

The diameter of the straight portion of all the tapered fibers referred to in Table 4 is 100 μm , corresponding to $R_i = 5 \times 10^{-5}$ meter. Using the calculated taper angles given in Table 4, based on the actual taper lengths, the maximum taper length L for propagation versus angle θ was calculated for each tapered fiber. The results are given in Figures 21a and 21b for the 300/100- μm and 600/100- μm tapers, respectively. If the actual taper length for a given taper is located on the vertical scale, then the corresponding value of θ located on the horizontal scale is the maximum incident angle for which the taper fiber will transmit energy through to the small end. The last column in Table 4 gives the results for each of the tapered fibers. Basically, all the 600/100- μm tapers investigated will propagate for incident angles in the range $0 \leq \theta \leq 2.1^\circ$, while all the 300/100- μm tapers will propagate for $0 \leq \theta \leq 4.25^\circ$.

Figure 21. Maximum Calculated Taper Lengths, Versus Angle θ , for the 300/100- μm and 600/100- μm Tapered Fibers Listed in Table 4

From Table 4, it is seen that the average optical power transmitted by the tapers is $4.179 \mu W$ and $1.70 \mu W$ for the 300/100- μm and 600/100- μm taper fibers, respectively. That is, the 600/100- μm taper transmits only 40.7 percent as much optical power as the 300/100- μm tapers. This can be understood as follows. The distribution of energy emitted by the fiber of 100- μm constant-diameter, as a function of angle θ relative to the perpendicular to the fiber end face, is given by the "far-field" curve in Appendix A. The measured distribution is not quite symmetric, but a good approximation can be obtained by averaging the "+ θ " and "- θ " portions as has been done to provide the "far-field" profile in Figure 22. The total light energy distribution delivered by the 100- μm fiber into the tapers in Figure 20 is proportional to the volume under the curved surface obtained by revolving the surface in Figure 22 about the axis of symmetry. Dividing the θ -axis into 20 equal intervals and performing a numerical summation, the total volume under the distribution is $81.8 (2\pi)$ units. In the interval $0 \leq \theta \leq 2.1^\circ$, the volume is $11.79 (2\pi)$ units, while the volume for the interval $0 \leq \theta \leq 4.25^\circ$ is $28.13 (2\pi)$ units. As indicated in Table 4, the angular intervals of acceptance of the 600/100- μm and 300/100- μm tapers are 2.1° and 4.25° , respectively. So, the total energy delivered by the 100- μm constant-diameter in the range $0 \leq \theta \leq 2.1^\circ$ is only $11.79/28.13$, or 41.9 percent of that delivered in the interval $0 \leq \theta \leq 4.25^\circ$. The value, 41.9 percent, agrees fairly well with the ratio of the measured coupling of $1.7 \mu W/4.179 \mu W = 40.7$ percent.

Use of a Sphere Lens to Couple From a Fiber of Constant Diameter to a Tapered Fiber

It is clear from the foregoing analysis and measurements that the low coupling from the fiber of constant diameter to the tapered fibers is a result of the fact that the tapered fibers will not accept the full angular distribution delivered by the fiber of constant diameter. Furthermore, the tapered fiber of larger taper angle α will accept less energy than the smaller-angle taper; that is, the numerical aperture is reduced by the factor R_i/R_o . A single sphere lens can be used to transform the divergent beam from the 100- μm constant-diameter fiber into a convergent beam, with smaller included angle, incident on the taper fiber face. Based on the earlier analysis regarding the use of a single convergent sphere lens in the conjugate object/image relationship, it follows that the distance between the 100- μm fiber face must be less than the distance of the lens from the tapered fiber. The reduction in magnitude of the divergent angle to the convergent angle can be estimated from the ratio of the object/image distances. However, the best way to obtain an understanding is to measure the coupling efficiency as a function of the sphere lens location in relation to the fiber faces. Data was obtained as follows. The end of the 100- μm constant-diameter fiber and the large end of the tapered fiber were positioned on separate precision slides, such that they could be moved axially while maintaining co-axial alignment. A 3.0- mm diameter *LaSF9* sphere was mounted on a third slide and positioned between the two fiber faces, with its center located on the common axis of the fiber ends, as indicated in Figure 23. The distance between the sphere lens and constant diameter fiber face was measured, and then the taper fiber face was moved co-axially to obtain a maximum coupling of optical power. This was repeated for a number of positions of the sphere lens, and a plot of the measured coupling of optical

FIG 22

Figure 22. Averaged Far-Field Profile of the Light Energy Distribution Delivered by the 100- μ m Core Diameter Optical Fiber Into the Tapers in Figure 20

FIG 23

Figure 23. Arrangement for Measuring the Coupling of Optical Power From a Fiber of Constant Diameter to a Tapered Fiber With the Aid of a Sphere Lens

power as a function of distance between the $100\text{-}\mu\text{m}$ fiber face and the sphere lens is given in Figure 24. Note that the maximum measured coupling with the $300/100\text{-}\mu\text{m}$ fiber occurs for distance $a = 0.7\text{ mm}$ between the $100\text{-}\mu\text{m}$ fiber and the sphere lens, but the distance between the exit face of the lens and the tapered fiber face was 4.5 mm , as indicated in Table 5. With the $600/100\text{-}\mu\text{m}$ tapered fiber, the maximum measured coupling occurs with distance $a = 0.5\text{ mm}$ between the $100\text{-}\mu\text{m}$ fiber face and lens, with the corresponding distance between lens and tapered fiber equal to 5 mm .

FIG 24

Figure 24. Measured Optical Power Coupling From a Fiber of Constant Diameter 100- μm to 300/100- μm and 600/100- μm Tapered Optical Fibers Using the Arrangement of Figure 23 With a *LaSF9* Sphere Lens, and Using a Silica Sphere Lens

Table 5. Location of 3.0-*mm* Diameter Sphere Lenses for Maximum Measured Coupling in Data of Figure 24.

Sphere Lens Type	Taper Fiber	Distance Between 100 μm	Distance Between Large
		Fiber Face and Sphere	Taper Fiber Face and Sphere Lens
LaSF9	300/100 μm	0.7 <i>mm</i>	4.54 <i>mm</i>
	600/100 μm	0.52 <i>mm</i>	5.0 <i>mm</i>
Silica	300/100 μm	1.65 <i>mm</i>	7.43 <i>mm</i>
	600/100 μm	1.38 <i>mm</i>	12.26 <i>mm</i>

Coupling measurements of the foregoing type were also made using a 3.0-*mm* diameter silica sphere lens. Silica has a much lower refractive index (1.4527) at 850 *nm*, compared to 1.83 for *LaSF9*. As a result, it is possible to convert a given divergent beam into a convergent beam of smaller included angle, compared to *LaSF9*, although distances will be somewhat different. Because of this, it should be possible to achieve a slightly higher coupling efficiency with the silica sphere lens. Data were obtained with the 3.0-*mm* diameter silica lens, and the results are plotted as a function of sphere lens position in Figure 24 for both the 300/100- μm and 600/100- μm taper fibers. Note that the maximum measured coupling is approximately 7 percent greater for the 300/100- μm taper, compared to the *LaSF9* sphere, and 28 percent greater for the 600/100- μm fiber. The distances between the sphere-lens surface and the emitting (constant-diameter fiber) and receiving fiber (tapered) faces for maximum measured coupling are summarized in Table 5.

A 2-*mm* diameter *LaSF9* sphere-lens was also used to couple energy from the fiber of constant core diameter 100 μm to the 300/100- μm and 600/100- μm tapered optical fibers. The measured optical power coupling as a function of distance between the 100- μm fiber face and the 2-*mm* sphere is plotted in Figure 25. The maximum measured optical power coupling into the 300/100- μm taper is essentially the same as with the 3-*mm* sphere, as can be seen by comparing with Figure 24. However, the results with the 2-*mm* sphere lens differ from those with the 3-*mm* sphere lens in two regards. First, the measured coupling through the 600/100- μm taper is greater than through the 300/100- μm taper. Additionally, the coupling into the 600/100- μm taper is approximately 57 percent greater than that achieved with the 3-*mm* sphere lens.

FIG 25

Figure 25. Measured Optical Power Coupling From a Fiber of Constant Diameter 100 μm to Tapered Optical Fibers Using a 2.0-mm LaSF_9 Sphere-Lens in the Arrangement of Figure 23

Accomplishments

Measurements were made of the efficiency with which optical energy is coupled from one optical fiber to another, through an intervening space, using a pair of identical sphere lenses, and also using a single sphere lens. The sphere lens material was *LaSF9*, which has a relatively high refractive index value of 1.83 at the wavelength of test, $\lambda = 850 \text{ nm}$. The sphere lens diameters investigated were 2.0, 3.0, and 5.0 *mm*. For pairs of 3.0- and 5.0-*mm* diameter spheres, the measured coupling efficiency was essentially independent of the sphere separation, over the separation interval investigated. With a pair of 2.0-*mm* diameter spheres, the efficiency decreased for sphere separation greater than four times the focal length. Virtually all the coupling losses were accounted for on the basis of calculated Fresnel reflection loss at each interface and analysis of ray paths from one fiber, through the lens(es), and onto the second fiber. Experimental results showed that the coupling efficiency was higher with a pair of identical sphere lenses than with a single-sphere lens. This was shown to be consistent with the results of ray tracing.

All the foregoing measurements were made with optical fiber having a constant diameter along its length. Tapered optical fiber, having a diameter that decreases linearly with distance along a portion of the fiber length, was also investigated. Analytic expressions were derived giving the limiting angle of incidence for rays on the entrance face, for which propagation would take place through to the small diameter end of the tapered fiber. The sine of the maximum incident angle on the large diameter face of the tapered fiber is smaller than the sine of the maximum angle for a straight fiber by the ratio R_i/R_o , where R_o and R_i are the large and small diameters, respectively, of the tapered fiber. It was shown that the primary utility of a tapered optical fiber, compared to a fiber of constant cross-section diameter, is that a larger spot of light can be introduced on the larger-diameter face, compared to a fiber of constant diameter equal to the small portion of the taper diameter. However, the associated ease of alignment of light on the large taper face is offset by the fact that the included angle of the incident light must be smaller on the taper in order to be propagated. This was verified by experiments which showed that the coupling efficiency from a straight fiber of diameter 100 μm into a 300/100- μm tapered fiber was only 24 percent, compared to a 92 percent coupling from the 100- μm fiber to another 100- μm fiber. However, use of a single lens placed appropriately between the 100- μm input fiber and the 300- μm receiving taper fiber increased the coupling from 24 percent to 66 percent, by virtue of the reduction in included angle of the light incident on the taper, brought about by the action of the lens.

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Appendix A

The near field distribution of the 100- μm diameter fiber was measured using a 10-*mm* focal-length lens in the arrangement shown in Figure A-1a. The ratio of intensity at radial distance from the fiber center to the intensity at the fiber center is given in Figure A-2a.

FIG A1

Figure A-1. Optical Arrangements for Measuring the (a) Near-Field Intensity Distribution and (b) the Far-Field Intensity Distribution From an Optical Fiber Output Face

FIG A2

Figure A-2. Near-Field (a) and Far-Field (b) Intensity Distributions From the 100- μ m Optical Fiber

Connected to the Source of the HP 8153A in Figure A-1.

The far field distribution from the 100- μm diameter fiber was measured using the arrangement shown diagrammatically in Figure A1-b, and the result is plotted in Figure A2-b.

Appendix B

Consider the arrangement in Figure B-1 in which the centers of two identical sphere lenses of radius R lie on the common axis of the centers of the ends of two identical optical fibers of core radius R_0 . Let the fiber on the left be the "sending" fiber and the fiber on the right the "receiving" fiber. Consider a point on the sending fiber face at distance h from the axis, and let a ray emerge at angle θ with respect to the perpendicular to the face as indicated. Let an (x,y) coordinate system be oriented in the plane of Figure B-1, with the x axis coincident with the common axis of the sphere and fiber centers, and let the origin of the (x,y) system coincide with the center of the sending fiber. The equation of the line representing the emergent ray is

$$y = h + x \tan \theta \quad (B1)$$

FIG B1

Figure B-1. Geometry of Ray-Tracing for Coupling From One Optical Fiber to Another With Two Identical Sphere Lenses

The equation of the circle representing the intersection of lens 1 with the plane of the figure is

$$[x - (a + R)]^2 + y^2 = R^2, \quad (\text{B2})$$

where a is the distance of the sending fiber face from the nearest point of sphere lens 1. To find the point of intersection (x, y) of the emergent ray on the lens 1 surface, Equations B1 and B2 must be solved simultaneously. This leads to a quadratic and, of the two possible roots, the smaller root is the one that applies. It is given by

$$x_1 = \frac{-B - [B^2 - AC]^{1/2}}{A} \quad (\text{B3})$$

where

$$A = 1 + \tan^2 \theta$$

$$B = h \tan \theta - a - R \quad (\text{B4})$$

$$C = h^2 + a^2 + 2aR$$

The corresponding y coordinate of the point of intersection is found from Equation B1.

$$y_1 = h + x_1 \tan \theta. \quad (\text{B5})$$

The normal to the lens surface at the point of intersection is merely the line joining the lens center with the point (x_1, y_1) . The angle that the incident ray makes with the normal is

$$i = \theta + w_1, \quad (\text{B6})$$

where

$$w_1 = \arcsin (y_1/R). \quad (\text{B7})$$

From Snell's law, the angle that the refracted ray makes with the normal to the lens surface as it enters the lens is given by

$$r = \arcsin((\sin i) / n) , \quad (\text{B8})$$

where n is the refractive index of the sphere lens material. The refracted ray travels in the sphere to a point (x_2, y_2) where it passes out into the air. The normal to the surface at this point is the line defined by the point and the sphere center. From the symmetry of the isosceles triangle formed by the path of the refracted ray in the sphere and the two normals, it is seen that the ray makes angle r with the normal at point (x_2, y_2) and is refracted away from the normal as it leaves the sphere lens, making angle i with the normal after exit. From the orientation of the isosceles triangle relative to the x axis, it can be seen that the (x_2, y_2) coordinates are given by

$$x_2 = a + R + R \cos(2r - w_1) \quad (\text{B9})$$

$$y_2 = R \sin(2r - w_1) .$$

The equation of the line representing the path of the ray after it emerges at point (x_2, y_2) is

$$y = y_2 + (x - x_2) \tan[2r - w_1 - i] . \quad (\text{B10})$$

This ray will intersect the second sphere lens at a point (x_3, y_3) which can be found by solving Equation B10 simultaneously with the equation of the circle representing the intersection of the lens with the plane of the figure. For x_3 , this leads to the quadratic

$$x_3 = \frac{-B^1 - \left[(B^1)^2 - A^1 C^1 \right]^{1/2}}{A^1} , \quad (\text{B11})$$

where

$$A^1 = 1 + \tan^2 \varphi$$

$$B^1 = y_2 \tan \varphi - a - b - R - x_2 \tan^2 \varphi \quad (\text{B12})$$

$$C^1 = (a + b + R)^2 + y_2^2 + x_2^2 \tan^2 \varphi - 2x_2 y_2 \tan \varphi - R^2 .$$

Knowing x_3 allows y_3 to be calculated from Equation B10 to give

$$y_3 = y_2 + (x_3 - x_2) \tan \varphi , \quad (\text{B13})$$

where the angle φ is

$$\varphi = 2r - w_1 - i . \quad (\text{B14})$$

From the geometry of the triangle formed by the segment of length b and the intersections of the extensions of the normals through

points (x_2, y_2) and (x_3, y_3) , the angle of incidence i_3 of the ray as it enters the second sphere lens is found to be

$$i_3 = w_3 + 2r - i - w_1, \quad (\text{B15})$$

where the angle w_3 is

$$w_3 = \arcsin(y_3/R). \quad (\text{B16})$$

Based on Snell's law, the angle that the refracted ray makes with the normal inside the second sphere is

$$r_3 = \arcsin(\sin i_3/n). \quad (\text{B17})$$

By symmetry of the isosceles triangle in the sphere in the plane of the figure, the angle r_3 is also the angle of incidence the ray makes with the normal at point (x_4, y_4) where it emerges from the sphere. The coordinates (x_4, y_4) are given by

$$x_4 = a + R + b + R \cos(2r_3 - w_3) \quad (\text{B18})$$

$$y_4 = R \sin(2r_3 - w_3).$$

By symmetry, the angle the ray makes with the normal at point (x_4, y_4) as it leaves the sphere lens is i_3 , given in Equation B15.

Therefore, the equation of the line representing the path of the ray as it leaves the sphere at point (x_4, y_4) is

$$y = y_4 + (x - x_4) \tan \varphi^1, \quad (\text{B19})$$

where the angle φ^1 is defined by

$$\varphi^1 = 2r_3 - w_3 - i_3. \quad (\text{B20})$$

Let the receiving fiber face be placed at distance L beyond the second sphere lens as indicated in Figure B1. This corresponds to the distance $x = a + 2R + b + L$. The distance y^1 from the axis at which the ray will strike the receiving fiber face can therefore be obtained by substituting the latter value of x into Equation B19 to yield

$$y^1 = y_4 + (a + b + 2R + L - x_4) \tan \varphi^1, \quad (\text{B21})$$

where φ^1 defined in Equation B20 is the angle that the ray makes with the normal to the receiving fiber face as it enters the fiber.

So, a ray leaving the sending fiber face at distance h from the center, at angle θ relative to the normal to the fiber face, will arrive at distance y^1 from the center of the receiving fiber face, making an angle φ^1 with the normal to the face. For propagation through the receiving fiber to occur, it is necessary that the magnitude of y^1 be less than the fiber radius R_o and that the sine of the angle φ^1

be less than the numerical aperture of the fiber, i.e., $\sin \varphi^1 \leq \left(n_1^2 - n_2^2 \right)^{1/2}$, where n_1 and n_2 are the refractive indexes of the fiber core and cladding, respectively.

Through straightforward but tedious manipulations of Equations B1 through B21, it can be shown that

$$y^1(-h, -\theta) = -y^1(h, \theta) \quad (\text{B22})$$

and that

$$y^1(-h, \theta) = -y^1(h, -\theta) \quad . \quad (\text{B23})$$

Appendix C

Figure C-1 shows the arrangement in which a single sphere lens of radius R is aligned with its center on the common axis of two optical fibers, each of core radius R_o . Let a be the distance of the sending fiber from the nearest point on the sphere lens. For a ray leaving the sending fiber at distance h from the center, making an angle θ with the normal, all of the Equations B1 through B10 apply to define the ray path through the lens in Figure C-1. If the receiving fiber face is placed at distance L beyond the sphere lens, then the distance y^1 at which the ray will strike the receiving fiber face is obtained by substituting $x = a + 2R + L$ into Equation B10 to yield

$$y^1 = y_2 + (a + 2R + L - x_2) \tan \Phi \quad , \quad (\text{C1})$$

where the angle Φ is the argument of the tangent function in Equation B10, defined by

$$\Phi = i - 2r + w_1 \quad . \quad (\text{C2})$$

Consider the tapered fiber shown in the schematic cross-section in Figure 1

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Consider the tapered fiber shown in the schematic cross-section in Figure 1

Figure C-1. Geometry of Ray-Tracing for Coupling From One Fiber to Another With a Single Sphere Lens

The angle Φ is the angle that the ray, incident on the receiving fiber face at distance y^1 from its center, makes with the normal to the fiber face.