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DEMANDS PLACED ON WASTE PACKAGE PERFORMANCE TESTING AND MODELING BY SOME GENERAL RESULTS OF RELIABILITY ANALYSIS

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ABSTRACT

Waste packages for a U.S. nuclear waste repository are required to provide reasonable assurance of maintaining substantially complete containment of radionuclides for 300 to 1000 years after closure. The waiting time to failure for complex failure processes affecting engineered or manufactured systems is often found to be an exponentially-distributed random variable. Assuming that this simple distribution can be used to describe the behavior of a hypothetical single barrier waste package, calculations presented in this paper show that the mean time to failure (the only parameter needed to completely specify an exponential distribution) would have to be more than 10^7 years in order to provide reasonable assurance of meeting this requirement. With two independent barriers, each would need to have a mean time to failure of only 10^5 years to provide the same reliability. Other examples illustrate how multiple barriers can provide a strategy for not only achieving but demonstrating regulatory compliance.

INTRODUCTION

The development of "long-lived" or "robust" waste packages has been advocated by the Nuclear Waste Technical Review Board[1] (NWTRB) and others as a means of ensuring that the expected performance of a potential high-level nuclear waste repository at the Yucca Mountain Site will not only meet regulatory requirements but will be acceptable to the public as well. The fundamental premise underlying this position was stated by the NWTRB[1]: "... the Board believes that well-engineered structures are *less variable and more predictable than rock formations* [emphasis added]." This is certainly true when the time span for the prediction is comparable to the period over which we have experience with similar structures.

However, a unique feature common to all potential nuclear waste repositories is that confidence must be developed in the accuracy of performance predictions spanning 10,000 years or more. This time period is far longer than human history and about two orders of magnitude greater than the engineering experience span for most anthropogenic materials. Much of society's confidence in engineered structures is based upon observations of how well they perform their intended functions over their design lifetimes. Unfortunately, there are no such observations even for many of the component materials considered for engineered barriers, and certainly not for entire systems resembling a repository. Hence, an engineered barrier system (EBS) *alone* cannot be expected to instill the required degree of regulatory and public confidence in overall repository performance.

This is, of course, precisely the reason for proposing geologic repositories in the first place: if the scientific and engineering community, as well as the general public, were confident that the behavior of engineered materials and systems could be predicted with sufficient accuracy for the required period of time, then a pure "engineering fix" could be devised and licensed. No one (to the author's knowledge) seriously advocates a pure engineering solution to the nuclear waste problem; it is recognized by the NWTRB[1] and generally that the combination of geologic and engineered barrier systems (especially if a more robust EBS is included) could reduce the overall uncertainty in repository performance predictions.

In order to stimulate the conceptual development of a more robust EBS, the EBS Panel of the NWTRB posed the following four questions in January 1990 [emphasis added]:

1. Can a waste package be developed *that can be demonstrated to have reasonable assurance of lasting 10,000 years?*

2. What ambient conditions or factors need to be modified for a 10,000-year waste package to be attained if this, indeed, is not yet possible?

3. How would the probability of attaining a 10,000-year waste package be influenced if the as-emplaced heat generation rate of individual canisters were minimized?

4. How does the siting of the repository in an unsaturated zone, as opposed to a saturated zone, affect attaining a 10,000-year waste package?

The crucial problem in addressing these questions -- perhaps it is even the crucial problem for the entire nuclear waste program -- is *the demonstration with reasonable assurance* that the repository or any part of it will perform as intended. Developing the necessary scientific understanding of natural, altered, and manufactured components' behavior under current and projected conditions, performing the subsequent engineering design work, and actually constructing the repository and its subsystems are all familiar tasks, distinguished only by the physical and temporal scale of the undertaking (and perhaps by the rigor and pervasiveness of Quality Assurance requirements) from other large projects. What seems more daunting is making it all plausible when we're done. The considerations below are intended to suggest an approach which may ultimately help improve the credibility of more sophisticated and detailed calculations.

WAITING TIMES

In very general terms, a repository comprises a series of natural geologic, altered zone, and engineered "barriers." The unexcavated host rock, beyond the range of significant repository-induced changes in such properties as temperature and water content, comprises the *natural geologic barrier* system. The *engineered barrier* system, or EBS, includes components traditionally regarded as part of the waste package, such as the waste form itself, the pour cannister (for glass waste) or the fuel cladding (for spent fuel), and the actual emplacement cannister (which may itself have several layers of different materials). The *altered zone* includes unexcavated rock with properties that have either been deliberately changed during repository construction and pre-closure operations (*e.g.*, by changing the local chemistry) or that have been changed incidentally as a result of repository construction and operation (*e.g.*, by the drying of surrounding rock due to heat from radioactive decay and the movement of ventilation air).

Each barrier introduces a delay or *waiting time* into a sequence of events that begins with waste emplacement and ends with release of a detectable quantity of a particular radionuclide to the accessible environment.

For example, consider a particle of liquid water starting at some point above the repository. The first waiting time is the time it takes this particle to reach the altered zone surrounding the repository (note that the time could be infinite if the flow path does not intersect any radionuclide inventory). Assuming it reaches the altered zone, it must then traverse it and contact the next barrier, which might be the waste package. It then has to penetrate one or more layers of the cannister, then the cladding or the pour cannister (depending upon the waste package design and the waste form), dissolve or entrain some radionuclide(s), work its way back through the EBS and the altered zone, traverse the vadose zone below the repository, and so on.

The sum of all these waiting times for one such sequence is one value (*i.e.*, *realization*) of the release time for a single radionuclide "particle;" it will be a random variable, because each individual term is a random variable. The ensemble of all such release time realizations for the entire initial inventory of radionuclides is directly related to the cumulative distribution function for radionuclide release. In essence, the calculation of repository performance reduces to the calculation of the probability distribution for a sum of waiting times, each of which is a random variable with an unknown probability distribution. From this perspective, a repository's sole purpose is to provide an acceptable distribution function for this sum -- that is, the probability of release for the entire period of regulatory concern must remain extremely low.

At present, not even the *form* of the probability distribution is known for any of these waiting times ; *a fortiori*, we know neither the nature or number of the parameters of these distributions, their values, or how these values might change with environmental conditions. In particular, nothing is presently known about the failure time distribution for any part of the engineered barrier system. It is this fact that makes the four questions raised by the NWTRB so extraordinarily difficult to answer. As will be illustrated below for multiple barriers assumed to obey a simple exponential failure time distribution, the demonstration of compliance with the regulatory requirement of "substantially complete containment" for periods of 300 to 1000 years will push the state-of-the-art in engineering and the frontiers of knowledge in science. Extending the prediction time to periods of 10,000 or more years, as required to defend "robust" designs, may be beyond our capabilities.

PROBABILITIES

Consider a single system which can fail at any time $t > 0$. To be specific, the system is considered to be the waste package component in a multiple-component EBS; however, it should be noted that a precise operational definition of "failure" for a waste package is far from obvious and may even depend upon the mode of release (*i.e.*, vapor phase versus liquid phase transport of radionuclides). Is a single corrosion pit resulting in a 5 micron hole through a cannister wall a "failure," or should there be more holes or larger holes before it is considered to have failed? The classical definition given by Harr[2], that *failure* denotes "...the inability of a system to perform its intended function," would in the case of the EBS mean that the EBS either has ceased to provide "substantially complete containment" or to comply with the controlled release requirement[3], which limits the fractional release for each radionuclide to 10^{-5} of its inventory remaining 1000 years after closure. Unfortunately, substantially complete containment is not defined precisely, hence we do not have a complete quantitative *regulatory* criterion for determining when failure has occurred.

The following discussion assumes that failure can be defined and that the resulting definition distinguishes unambiguously between a waste package that has failed and one that has not failed. The definition or even detection of the failure of an engineered component in a repository environment is by no means a trivial exercise, but further consideration of these subjects is beyond the scope of this paper.

Let $\mathbf{f}(t)$ represent the probability density function (**pdf**) for the random variable T, where T is the waiting time to failure for a single system being observed. In other words, the system is observed starting at time 0, and T is the time that has elapsed when failure occurs. Then $\mathbf{f}(t)dt$ is the probability that the observed value of T lies between t and $t + dt$. $\mathbf{F}(t)$ is the cumulative distribution function (**cdf**) and is defined by the integral:

$$\mathbf{F}(t) = \int_0^t \mathbf{f}(u)du.$$

The **cdf** is the probability that failure occurs on or before time t. Note that $1 - \mathbf{F}(t)$ is the probability that failure occurs after time t, i.e., the probability of *survival* until time t. Harr[2] refers to this function as the reliability function, $\mathbf{R}(t)$. It is sometimes useful to define the hazard function, $\mathbf{h}(t) = \mathbf{f}(t)/\mathbf{R}(t)$;

it is a *conditional* **pdf**, in that $h(t)dt$ is the conditional probability that the system will fail in the time interval from t to $t+dt$, given that it survived until time t.

If N waste packages are emplaced at time 0, and their failure probabilities are independent (*i.e.*, common disasters such as meteorite impacts on a repository are not considered), then n, the total number of survivors at time t, is a discrete random variable obeying the familiar binomial distribution for the probability of n successes in N Bernoulli trials when the probability of success on each trial is $R(t)$:

$$P[n;N,R(t)] = N!/[(N-n)!*n!]*[R(t)]^n*[F(t)]^{(N-n)}$$

The mean number of survivors is $N*R(t)$, and the variance is $N*F(t)*R(t)$.

The probability that all N members survive until time t (equivalent to the event that no failures occur) is $[R(t)]^N$. Note that, for a repository designed to hold 70,000 metric tons of waste with about 2 metric tons per waste package, N is on the order of 35,000. For $R(t)$ equal to 0.9999, the probability that no failures occur by time t is $10^{-1.52}$, or approximately 0.03, and the mean (or expected) number of failures is 3.5.

Using a Bayesian approach, Harr[2] has shown that a sequence of 18 successes with no failures is the expected result for R = 0.95, and 98 successes with no failures corresponds to a reliability of 0.99. Since twenty to a hundred is probably the right order of magnitude for the number of actual similar systems in civil engineering practice, Harr[2] argues that 0.95 to 0.99 is the expected range of reliability for most civil engineering systems. To demonstrate an expected value of reliability of 0.9999, we would have to have a sequence of 9998 successes unbroken by a single failure. Achieving reliability this close to unity for long periods of time is possibly beyond the capabilities of engineering.

Of course, if the time is short enough, $R(t)$ will be arbitrarily close to unity, since it must equal unity at t=0. $R(t)$ is a monotonically decreasing function of time, asymptotically approaching zero as t tends to infinity. Without introducing explicit functions for the **pdf** and **cdf**, there is no way to determine the time at which the reliability function decreases to 0.9999, and hence we cannot compare the results given above with time-dependent regulatory requirements without assuming a probability distribution. In a following section, these calculations and comparisons are performed for a simple but commonly-observed failure-time distribution introduced in the next section.

THE EXPONENTIAL WAITING-TIME DISTRIBUTION

In order to gain some insight into the demands placed on waste package performance testing and analysis by regulatory requirements, a specific function must be assumed for the **pdf**. Perhaps the simplest **pdf** for failure time is the exponential distribution:

$$f(t) = \lambda e^{-\lambda t}$$

The single parameter λ is the reciprocal of the Mean Time To Failure, or MTTF. The **cdf** is $F(t) = 1 - e^{-\lambda t}$, and the reliability function is just $e^{-\lambda t}$. Not only is this distribution simple, it often provides a good representation for failure time probabilities when failures are rare events resulting from the complex interaction of many processes and mechanisms.

Examples of observed failure time statistics successfully described by this distribution are given in Figures 1 and 2, respectively, for the burnout of radar tubes (from Belz[4]) and for the collapse of oil-well casing due to salt-flow loading (from Chesnut and Goldberg[5]). The points are the observed reliability values and the lines show the reliability function for exponential distributions with the observed mean failure times.

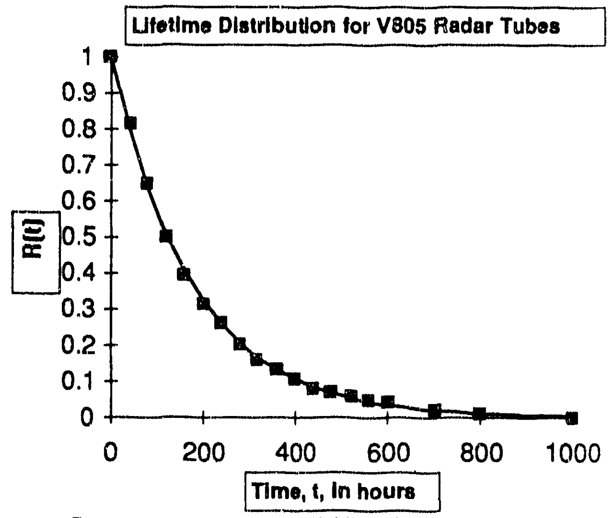

Figure 1. Comparison of observed reliability data for V805 Radar Tubes with the exponential distribution. A total of 903 failures was observed, and the mean time to failure was 179.3 hours[4].

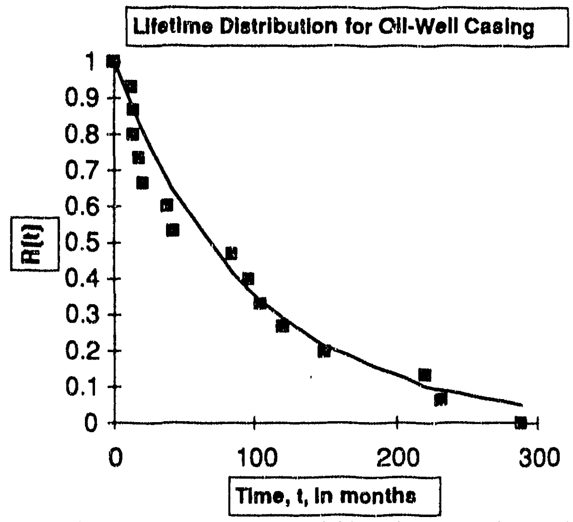

Figure 2. Comparison of observed reliability data for casing in oil wells on the Cedar Creek Anticline, Montana, with the exponential distribution. A total of 15 failures was observed, and the mean time to failure was 96.7 months[5].

These examples clearly must involve radically different failure mechanisms and processes, yet the distributions of failure times exhibit the same simple exponential form -- only the

MTTF is different. All the complexity one can envision from such sources as different mechanisms, environmental factors, manufacturing differences, material behavior, etc. merely affects the value of this single parameter. Precisely how the **MTTF** depends upon "deterministic" variables must be determined either by experiments combined with mechanistic theoretical analysis, or by statistical analysis of a sufficiently large number of failures.

In probability theory, the exponential distribution arises in the study of Poisson processes, in which the probability that an event will occur in the time interval from t to $t+dt$ is proportional to dt (for sufficiently small dt) and independent of t. The condition of time-independence means that there is no memory, or, in other words, no aging, so it is somewhat surprising that the exponential distribution fits the data so well in the two examples shown.

Perhaps the applicability of Poisson processes can be rationalized, albeit not proven, by considering a conceptual model for system failure known as the "bathtub distribution." In Figure 3 (see Harr[2]), the hazard function $\mathbf{h}(t)$ is sketched as a function of time.

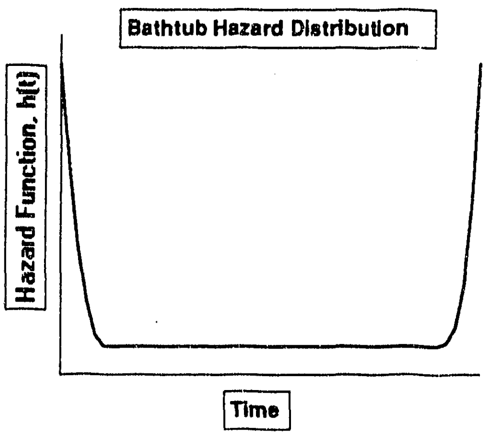

Figure 3. Sketch of the "Bathtub Distribution" for the hazard function vs. time. The early decreasing-hazard part is the "breaking-in" period, the central constant part is the period of design service life, and the late increasing-hazard part is the "wearing-out" period.

At early time (the "breaking-in" period), $\mathbf{h}(t)$ is expected to decrease with time. Intuitively, a high early failure rate would be expected to arise from defects in design, manufacturing, or construction. The late-time part of the "bathtub" shows an increasing hazard as components wear out. The central part of the distribution, with a constant value for $\mathbf{h}(t)$, is mathematically equivalent to an exponential distribution for the waiting time between failures. The constant value of the hazard function in the central part of the "bathtub" is equal to the reciprocal MTTF, as shown below:

$$\mathbf{f}(t) = \lambda e^{-\lambda t}$$

$$\mathbf{R}(t) = e^{-\lambda t}$$

$$\mathbf{h}(t) = \mathbf{f}(t)/\mathbf{R}(t) = \lambda$$

The waste package program will attempt to minimize the duration of the "breaking-in" period by stringent quality control and rigorous inspection, thereby hoping to eliminate most of the early failures and mitigate their consequences during the pre-closure period of repository operation. Ideally, this would allow the period of constant hazard to extend beyond the time period of interest, be it 300-1,000 years, 10,000 years, or longer. If this were achieved, the late-time period of increasing hazard need not be included in the analysis, since all of the design service life would have expired before it starts.

The achievement of constant hazard, and proving that it has been achieved, is a great scientific and engineering challenge. The analyses presented in the remainder of this paper assume that the challenge can be met, and we can accordingly assume the exponential distribution to be a reasonable functional form for the failure time distribution. The **MTTF** offers a reasonable and simple definition for the lifetime of a waste package or any other component of the EBS, whether or not the failure time distribution is exponential. Similar analyses can be performed for other distributions, such as the Weibull, but other parameters in addition to the mean would have to be specified in order to produce any quantitative results. If warranted by experience or theory, these extensions can easily be made.

SINGLE BARRIERS

We now assume that the exponential waiting time distribution can be used to describe the reliability of a hypothetical single-barrier waste package. The formulas given above for the **pdf** and **cdf** of the exponential distribution, along with the formulas for the binomial distribution, were used to calculate numerical values for the reliability, $\mathbf{R}(t)$, the expected number of survivors, $\mathbf{E}(n,t)$, and the probability of 100% survival, $\mathbf{P}_N(t)$ at t=40 years after emplacement and at t=1,000 years after emplacement, for assumed values of the **MTTF** equal to 1,000, 10,000, 100,000, 1,000,000, and 10,000,000 years. The choices of 40 and 1,000 years were made to represent, respectively, the time from waste emplacement to repository closure and the period during which regulations require "substantially complete containment" of radionuclides within the EBS.

Table 1. Reliability, expected number of survivors, and probability of 100% survival at 40 and 1,000 years for an initial population size of 35000 waste packages, with assumed values of the Mean Time to Failure ranging from 1,000 to 10,000,000 Years.

	t = 40 years			t = 1000 years		
MTTF	R	E(n)	P_N	R	E(n)	P_N
10^3 y	0.9608	33628	10^{-608}	0.3679	12876	10^{-15200}
10^4 y	0.9983	34939	$10^{-60.8}$	0.9048	31669	10^{-1520}
10^5 y	0.9996	34986	$10^{-6.08}$	0.9900	35652	10^{-152}
10^6 y	0.99996	34999	$10^{-.608}$	0.9990	35965	$10^{-15.2}$
10^7 y	>0.99999	34999.9	$10^{-.0608}$	0.99990	34996.5	$10^{-1.52}$

These results are summarized in Table 1. For all tabulated values of **MTTF** less than or equal to 100,000 years, the probability of observing at least one failure in 40 years (this probability is one minus the probability of 100% survival) is essentially unity, virtually assuring that at least one failure would occur within this relatively short time even for very "robust" designs.

Slightly less than 37% of the 1,000-year-MTTF packages would survive for 1,000 years, but almost 99% of the 100,000-year-MTTF packages would. Even for a MTTF of 1,000,000 years, the probability of at least one failure in the first 40 years is 0.75, and 0.1% of these packages would be expected to fail by 1,000 years after emplacement.

Note that the expected number of survivors at 40 years and 1,000 years in Table 1 is just the reliability function evaluated at 40 and 1,000, respectively, times the initial number of packages (35,000). The interested reader will note that only the 1,000,000-year-MTTF package has a reliability exceeding 0.9999 at 40 years, and its reliability is only 0.9990 at 1,000 years. Recalling that $R(t) = e^{-\lambda t}$, setting $t = 1{,}000$, $R(t) = 0.9999$ and solving for λ, one can show that attaining "four-nines" reliability with a single barrier subject to a Poisson failure process would require a MTTF of 10^7 years! Values for this MTTF are also given in Table 1.

PERFORMANCE CRITERIA

The discussion of single barrier failure probabilities clearly indicates the need for a quantitative measure of performance. There is no way to determine what value of reliability we need at any given time unless we consider the *consequences* of failure. After the period of "substantially complete containment" of 300 to 1,000 years, the regulatory requirement is to limit the fractional release of each radionuclide to 10^{-5} per year of its inventory remaining at 1,000 years after closure.

For the sake of this discussion, "substantially complete containment" will be defined arbitrarily by limiting the individual radionuclide fractional release rate during the first 1,000 years to a fraction, ε, of the controlled release allowance of 10^{-5} per year. We may require ε to be small, say 0.01, 0.001, or whatever seems both tolerable and achievable, but it cannot be set at zero if the repository program is to have any credibility. If we let $\delta_k(t)$ be the cumulative release of radionuclide species k, normalized by its 1,000-year inventory, and $\rho(t)$ be the allowed release requirement, then the system succeeds so long as

$$\delta_k(t) < \rho(t), \text{ where}$$

$$\rho(t) = 10^{-5}\varepsilon t, \qquad \text{for } 0<t<t_c$$

$$= 10^{-5}\varepsilon\, t_c + 10^{-5}(t-t_c), \text{ for } t > t_c,$$

and t_c is the time for which substantially complete containment is required (usually set to 1,000 years).

Consider the case of a single barrier and a radionuclide with decay constant α_k. As discussed in a previous section, $\lambda e^{-\lambda t}dt$ is the probability that the barrier will fail at a time between t and $t + dt$; the total repository inventory, $I_k(t)$ of radionuclide k is

$$I_k(t) = I_k(0)e^{-\alpha_k t},$$

where $I_k(0)$ is the total inventory of radionuclide k at emplacement. Then if we *assume* that failure means that release of radionuclides from the EBS occurs, the release from time t to time $t + dt$ is just $\lambda I_k(t)e^{-\lambda t}dt$. This expression can be integrated and then divided by the inventory at t_c to obtain the normalized cumulative release function $\delta_k(t)$:

$$\delta_k(t) = [\lambda/(\alpha_k + \lambda)]e^{\alpha_k t_c}[1 - e^{-(\alpha_k + \lambda)t}].$$

By expanding the last term at early time and retaining the linear term in t, the following approximation can be derived:

$$\delta_k(t) \cong \lambda e^{\alpha_k t_c} t.$$

Upon comparing this result with the "required" release rate $\rho(t)$ for $t<t_c$, and recalling that $1/\lambda = \text{MTTF}$, one can derive the following inequality:

$$\text{MTTF} > (1/\varepsilon)10^5 e^{\alpha_k t_c}.$$

Suppose we set ε to 0.01 (*i.e.*, the release rates during the substantially complete containment period are required to be 1% or less of the controlled-release rates). Note that the exponential term in the inequality is greater than unity for any non-zero value of the decay constant, and that it approaches unity for very long-lived radionuclides (the term is approximately 1.001 for a decay constant of 10^{-6}). By setting the exponential term equal to unity, we obtain a *lower bound* for the MTTF of 10^7 years. Smaller values of ε, or consideration of shorter-lived radionuclides, would require an even longer MTTF.

This analysis shows that we may indeed need a reliability function of 0.9999 at 1,000 years after placement in order to accomplish "substantially complete containment." Failures in such a robust single-barrier package would be so rare that they would almost certainly not be observed even in 100 years of testing. Even if such a package could be built, there is no apparent way to test it and demonstrate its performance.

MULTIPLE BARRIERS

The specific results given above obviously depend strongly upon the assumed form of the failure **pdf**, but the difficulty of testing a long-lived system remains even for other distributions. The shape of the exponential distribution is particularly troublesome, since the highest *rate* of failure occurs near time zero (even though the hazard function is constant), requiring an extraordinarily large MTTF to control early releases. There is some hope, however, if we consider multiple independent barriers. An important result of probability theory is the *Central Limit Theorem*,which, under fairly broad conditions, assures us that the sum of n independent random variables (no matter how they are distributed) is a random variable whose distribution approaches the normal (or Gaussian) distribution for sufficiently large values of n. The mean of the resulting normal distibution is the sum of the means of the individual variables and its variance is the sum of the variances.

What this means is that if we add up enough independent exponentially-distributed failure times -- one for each barrier -- the resulting *total* failure time will have its mode (i.e., maximum in the **pdf**) shifted away from the origin -- in fact it will be approximately centered on the mean, and the probability of failure at early time will be drastically reduced in comparison with the exponential distribution.

The **pdf** for the sum of n exponentially distributed variables can be obtained as an explicit function of time, using the characteristic function (**cf**) of the exponential distribution[6] and the general result of probability theory that the **cf** of the *sum* of n random variables is the *product* of the **cfs** of the individual variables (characteristic functions are similar to Laplace and Fourier transforms in this respect). The **cf**, $\phi_n(\tau)$, for n exponentially distributed random variables is

$$\phi_n(\tau) = \prod_{j=1}^{n} \left(1 - \frac{\iota\tau}{\lambda_j}\right)^{-1}$$

where ι is $(-1)^{1/2}$ and the reciprocal **MTTF** of variable j is λ_j. Two cases have to be considered:

1. If the λ_j are all distinct, then the product can be decomposed into a sum of terms, each of which is a coefficient times the **cf** of an individual exponentially-distributed variable. The coefficients can be expressed in terms of the λ_j using the method of partial fractions.

2. If the λ_j are identically equal to a fixed value, λ, for all j, then the product is just $\phi_n(\tau) = (1 - \iota\tau/\lambda)^{-n}$, which may be recognized as the **cf** for a Pearson Type III distribution (the Gamma distribution)[6].

Of course, it would be possible to have a mixture of cases 1 and 2, with some variables having identical values of λ and others having distinct values. The simplest expressions result for case 2, and will be used in the following analyses, since this will adequately illustrate the value of using multiple barriers.

With an obvious change in notation, the **pdf** corresponding to the **cf** for case 2 is found in Ref. 6, page 930:

$$\mathbf{f}_n(t) = [\lambda/(n-1)!][\lambda t]^{n-1} e^{-\lambda t}.$$

By integrating, the **cdf** and reliability function are obtained:

$$\mathbf{F}_n(t) = 1 - e^{-\lambda t} \sum_{j=1}^{n-1} [\lambda t]^j / j! = 1 - \mathbf{R}_n(t)$$

Note that these expressions reduce to the **pdf** and **cdf**, respectively, for a single barrier when $n = 1$. Figure 4 shows how the shape of the **pdf** changes as n increases. In Fig. 4, the total mean failure time is kept constant at 1,000 years; hence, each individual mean failure time is 1000/n and gets shorter as n increases. As shown, the *mode* of the distribution shifts more and more toward the mean with increasing n, and the distribution also becomes more symmetric and more sharply peaked about the mean, lending some graphic plausibility to the operation of the Central Limit Theorem. Also, the area under the curves from t=0 to t=total **MTTF** is reduced as n increases -- i.e., the reliability at early time is increased relative to the single barrier case.

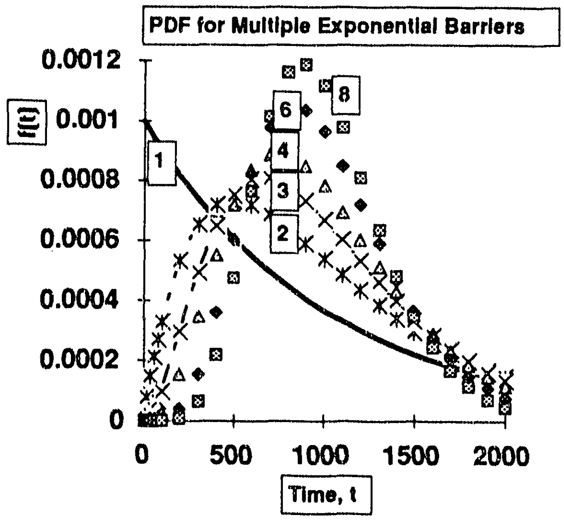

Figure 4. Probability Density Functions (pdfs) for multiple barriers having a constant total Mean Time to Failure of 1,000 years. Each curve is labeled with the corresponding total number of barriers. The waiting time distribution for each barrier is exponential, with MTTF of 1,000/n.

MULITPLE-BARRIER NORMALIZED RELEASE

The analysis for the single-barrier normalized release of radionuclides is easily extended by using the multiple-barrier **pdf** in place of the single-barrier **pdf**. The resulting expression is:

$$\delta_k(t) = [\lambda/(\lambda + \alpha_k]^n e^{\alpha_k t_c}\{1 - [e^{-(\lambda+\alpha_k)t} \sum_j [(\lambda+\alpha_k)t]^j / j!]\},$$

where the summation index j ranges from 0 to n-1, and n is the total number of barriers.

This function and $\rho(t)$ were evaluated for t ranging from 1 to 10,000 years, with t_c fixed at 1000 years and ε fixed at 0.01, for n = 1, 2, 3, 4, 6, and 8. Figure 5 shows the resulting plots of $\delta_k(t)$ and $\rho(t)$ for nine combinations of values for the parameters α and λ. Only combinations for which $\delta_k(t)$ never exceeds $\rho(t)$ during the period of regulatory concern successfully meet the "requirements." Note that *none* of the cases with 100-year **MTTF** individual barriers meet even the statutory controlled release requirement, and that eight 1,000-year **MTTF** barriers are not quite adequate.

SUMMARY AND CONCLUSION

The results developed in this paper may be summarized in the following statements (some of these conclusions are qualitatively correct for failure distributions other than exponential):

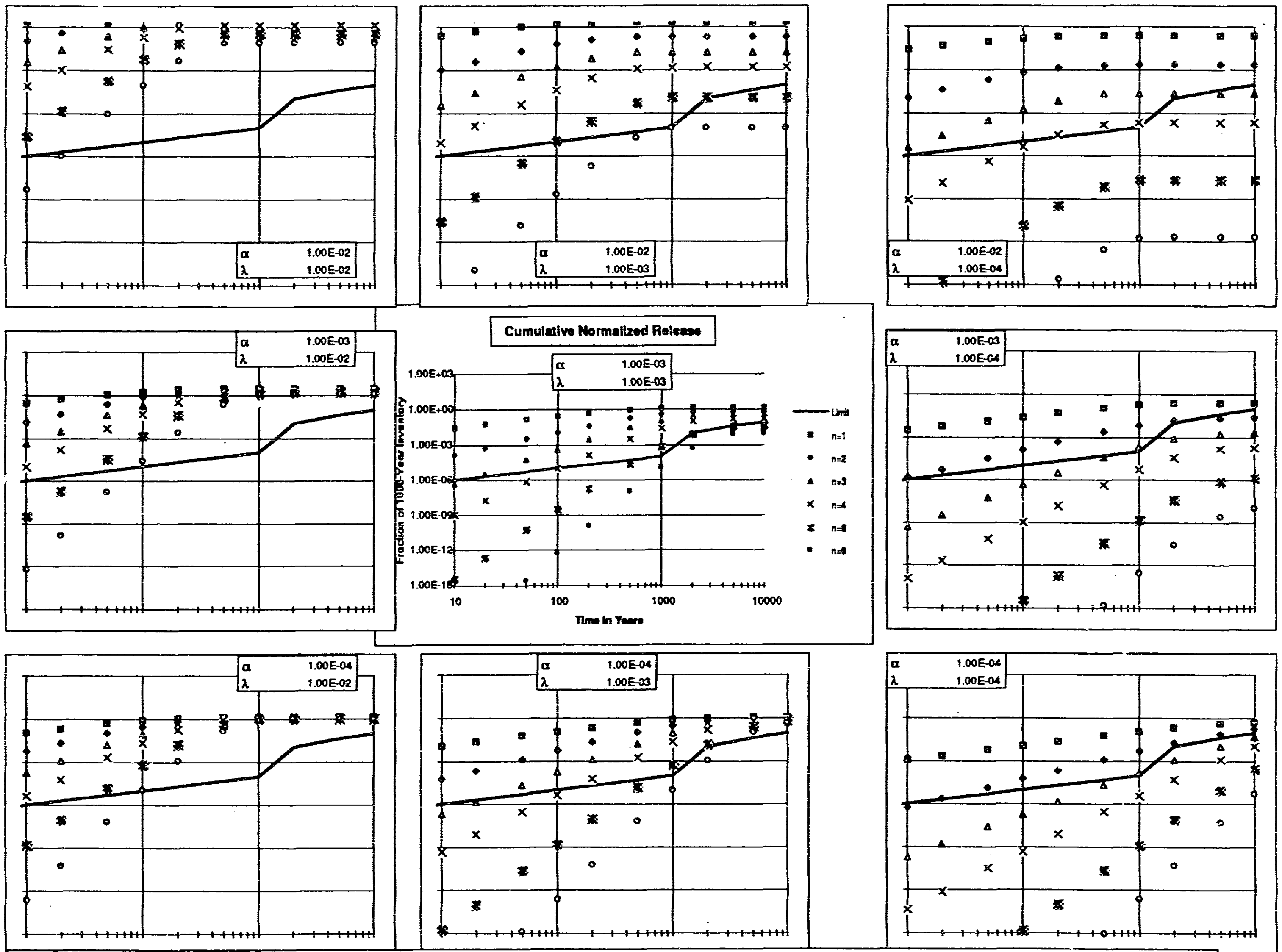

Figure 5. Sensitivity of the Cumulative Normalized Release to the number of Barriers, the Reciprocal MTTF of the Individual Barriers, and the Radionuclide Decay Constant

1. Even if radioactive decay is ignored, a single barrier must have a mean lifetime on the order of 10^7 years to meet even the controlled release requirement.

2. For a non-zero decay constant (*e.g.*, a hypothetical radionuclide with mean lifetime of 1,000 years), even a 10^7-year single barrier would not meet the release criteria used.

a. With two barriers, the single-barrier **MTTF** required would be on the order of 100,000 years or greater.

b. Three barriers with a 10,000-year **MTTF** each would almost succeed, but fail if their individual **MTTFs** were reduced to 1,000 years.

c. More than six barriers would be required for individual **MTTFs** of 1,000 years.

3. Six 1,000-year **MTTF** barriers would not meet the modified release criteria for a radionuclide mean lifetime of 100 years, but 8 would. If their **MTTFs** are reduced to 100 years each, then the release criteria would *not* be met.

4. Although an endless variety of cases could be run, varying not only the parameters considered above but t_v and ε as well, the cases presented serve adequately to demonstrate that the development of a safe nuclear waste repository is far from being a conventional large-scale construction project. It is vital to understand the details governing the performance of both the natural and the engineered barrier systems.

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