Yucca Mountain Site Characterization Project

A Computational Model for Three-Dimensional Jointed Media with a Single Joint Set

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Prepared by
Sandia National Laboratories
Albuquerque, New Mexico 87185 and Livermore, California 94550
for the United States Department of Energy
under Contract DE-AC04-94AL85000
"Prepared by Yucca Mountain Site Characterization Project (YMSCP) participants as part of the Civilian Radioactive Waste Management Program (CRWM). The YMSCP is managed by the Yucca Mountain Project Office of the U.S. Department of Energy, DOE Field Office, Nevada (DOE/NV). YMSCP work is sponsored by the Office of Geologic Repositories (OGR) of the DOE Office of Civilian Radioactive Waste Management (OCRWM)."

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A Computational Model for Three-Dimensional Jointed Media with a Single Joint Set

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Abstract

This report describes a three-dimensional model for jointed rock or other media with a single set of joints. The joint set consists of evenly spaced joint planes. The normal joint response is nonlinear elastic and is based on a rational polynomial. Joint shear stress is treated as being linear elastic in the shear stress versus slip displacement before attaining a critical stress level governed by a Mohr-Coulomb friction criterion. The three-dimensional model represents an extension of a two-dimensional, multi-joint model that has been in use for several years. Although most of the concepts in the two-dimensional model translate in a straightforward manner to three dimensions, the concept of slip on the joint planes becomes more complex in three dimensions. While slip in two dimensions can be treated as a scalar quantity, it must be treated as a vector in the joint plane in three dimensions. For the three-dimensional model proposed here, the slip direction is assumed to be the direction of maximum principal strain in the joint plane. Five test problems are presented to verify the correctness of the computational implementation of the model.
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SAND92-2625 was prepared under Yucca Mountain Project WBS number 1.2.4.2.3.1.

The data in this report was developed subject to QA controls in QAGRS124231B Revision 0, PCA 1.0 Task 1.1; the data is not qualified and is not to be used for licensing.
A Computational Model for Three-Dimensional Jointed Media with a Single Joint Set

Introduction

The behavior of rock structures is complicated by the presence of joints. Joints can be defined as fractures along which little or no shear displacement has taken place. The behavior of jointed media is discussed in various sources\textsuperscript{1-5}, and extensive work has been done to incorporate some of the analytic models describing joint behavior into two-dimensional finite element codes\textsuperscript{6-8}. This report describes work to develop a computational model for three-dimensional jointed media with a single set of joints. A three-dimensional model with a single set of joints is more restricted than some of the two-dimensional models incorporating two joint sets\textsuperscript{5-7}. Even though the three-dimensional model discussed in this report is more restricted in terms of the number of joint sets than some two-dimensional models, it does present the opportunity to examine issues that arise in constructing a fully three-dimensional model. Some of the concepts used in the two-dimensional models translate in a straightforward manner to the three-dimensional case while other concepts become more complicated in three dimensions. This report was written in part to discuss the complications that can arise as some well understood work in two dimensions is extended to three dimensions. It was also written to document a computational model that is useful for modeling layers of material where there are strongly directional properties.

The computational model described in this report uses a continuum approach; a continuum approach captures the gross response of jointed rock by distributing the individual responses of the joints throughout the rock mass. The normal joint response is nonlinear elastic and is based on a rational polynomial fit to experimental data. Joint shear stress is treated as linear elastic in the shear stress versus slip displacement before attaining a critical stress level governed by the Coulomb friction criterion. Beyond the critical stress value, a linear relation analogous to strain-hardening plasticity governs the rest of the shear stress versus slip-displacement relation. The rock matrix between the joints is treated as an isotropic linear-elastic material. The constitutive model is formulated in terms of a set of rate equations governing the stress-strain response of a jointed media.

The following sections describe the theory of the single joint set model for three-dimensional media and the computational implementation of the model in the finite element code JAC3D\textsuperscript{9}. Test problems are described to verify the computational implementation. Both numerical and analytic results for the test problems are presented in the verification sections. The test problems are simple enough so that certain key values can be calculated analytically to verify the computational results.
The single joint set model (Figure 1) assumes a single set of evenly spaced parallel joints.

![Diagram of joint planes and coordinate system](image)

Figure 1. Coordinate System for Single Joint Set Model in Three Dimensions.

These joints are associated with an $mnr$ coordinate system. The $m$ and $n$ axes lie in the joint planes, and the $r$ axis is perpendicular to the joint planes. Quantities in a global $xyz$ coordinate system can be transformed to the $mnr$ system by using proper coordinate transformation equations.

The normal joint behavior is characterized by

$$T_{rr} = \frac{-Au_r}{U_{max} - u_r} \quad (1)$$

where $T_{rr}$ is the joint normal stress, $A$ is the maximum tensile stress across the joint, $u_r$ is the normal joint displacement, and $U_{max}$ is the maximum closure of the joint. The maximum closure is a negative constant. Equation (1) sets an upper limit on the tensile stress that can be carried across the joint, and this maximum value is set by $A$. As the joint opening increases, the normal joint stress $T_{rr}$ approaches $A$ asymptotically. The value for
$U_{max}$ limits the maximum amount of closure that can be sustained by the joints. The value $u_r$ measures the amount of separation between the two faces of a given joint. The value for the normal joint stress, $T_{rr}$, as a function of the joint opening, $u_r$, is shown in Figure 2.

![Figure 2. Joint Normal Stress as a Function of Joint Opening.]

A bilinear shear stress-slip displacement response is assumed for the slip behavior of the joints. The onset of nonlinear response is assumed to be governed by a linear Mohr-Coulomb criterion. A scalar slip function is given as

$$F = |\tau| + \mu T_{rr} - C_0,$$  \hspace{1cm} (2)

where $T_{rr}$ and $\tau$ are, respectively, the normal and shear stress across the joint. In Equation (2), $\mu$ is the coefficient of friction and $C_0$ is the joint cohesion. The joint behavior is elastic if $F \leq 0$ and inelastic if $F > 0$. The joint shear stiffness is $G_s$ in the elastic range and $G_s'$ in the inelastic range. This behavior is shown in Figure 3.

Hooke's law for isotropic elastic materials can be combined with the normal and shear joint behavior to obtain strain rate versus stress rate relations. The strain rate versus stress rate equations are presented because of the nonlinear behavior of the joints. The strain rate versus stress rate relations in terms of the local joint sets are

$$\dot{\epsilon}_{mm} = \frac{T_{mm}}{2G} - \frac{K - 2G/3}{6KG} tr(T),$$  \hspace{1cm} (3)
Figure 3. Shear Stress Versus Slip Displacement Behavior for Joints.

\[
\dot{\varepsilon}_{nn} = \frac{\dot{T}_{nn}}{2G} - \frac{K - 2G/3}{6KG} \text{tr} (\ddot{T}) ,
\]

(4)

\[
\dot{\varepsilon}_{rr} = \frac{\ddot{T}_{rr}}{2G} - \frac{K - 2G/3}{6KG} \text{tr} (\ddot{T}) + \frac{\beta \ddot{T}_{rr}}{(A - T_{rr})^2},
\]

(5)

\[
\dot{\varepsilon}_{nm} = \frac{\ddot{T}_{mn}}{2G},
\]

(6)

\[
\dot{\varepsilon}_{nr} = \left[ \frac{1}{2G} + \frac{1}{2\delta G_k} \right] \ddot{T}_{nr},
\]

(7)

\[
\dot{\varepsilon}_{rm} = \left[ \frac{1}{2G} + \frac{1}{2\delta G_k} \right] \ddot{T}_{rm},
\]

(8)

where

\[
\text{tr} (\ddot{T}) = \ddot{T}_{mm} + \ddot{T}_{nn} + \ddot{T}_{rr},
\]

(9)

and

\[
\beta = -AU_{\text{max}}/\delta.
\]

(10)
In the preceding equations, the normal strain rates are denoted by \( \dot{\varepsilon}_{mm}, \dot{\varepsilon}_{nn}, \) and \( \dot{\varepsilon}_{rr}, \) and the associated normal stress rates are denoted by \( \dot{T}_{mm}, \dot{T}_{nn}, \) and \( \dot{T}_{rr}, \) respectively. (The subscripts on the normal quantities denote tensor directions and do not imply summation.) In the equations for the normal strain and stress rates, tensile stresses are positive and compressive stresses are negative. The shear strain rates are denoted by \( \dot{\varepsilon}_{mn}, \dot{\varepsilon}_{nr}, \) and \( \dot{\varepsilon}_{rm}, \) and the associated shear stress rates are denoted by \( \dot{T}_{mn}, \dot{T}_{nr}, \) and \( \dot{T}_{rm}, \) respectively. The dot above a given stress or strain component denotes differentiation with respect to time.

The normal strain rate in the \( r \) direction is composed of a contribution due to the deformation of the rock matrix (the first two terms on the right hand side of Equation (5)) and a contribution due to the joint opening. If the effect of the joint opening is spread over the distance \( \delta \) defined by the joint spacing, then the joint opening can be treated as an additional strain component in a continuum model. The strain component due to the joint opening is obtained by rewriting Equation (1) so that the normal joint displacement is a function of the normal joint stress and then dividing the normal joint displacement by the joint spacing \( \delta. \) If \( (e_{rr})_j \) is the normal strain component due to the joint opening, then

\[
(e_{rr})_j = \frac{U_{max} T_{rr}}{(T_{rr} - A) \delta}. \tag{11}
\]

By taking the derivative of Equation (11) with respect to time, it is possible to obtain the normal strain rate component due to joint opening, which is the third term on the right hand side of Equation (5).

A process similar to the one just described for the normal joint opening is used to include the effects of slip on the joint planes in the expressions relating shear strain rates to shear stress rates. If the effect of the slip on the joint planes is spread over the distance defined by the joint spacing, then the joint slip can be treated as an additional strain component in a continuum model. The amount of slip in the \( n \) direction in the \( r \) plane as a function of the shear stress \( T_{nr}, \) for example, is given as

\[
u_{s,n} = \frac{T_{nr}}{G_k}. \tag{12}
\]

Dividing both sides of Equation (12) by \( 2\delta \) creates a shear strain component associated with the joint slip, \( (e_{nr})_s, \) consistent with the shear strain component due to shear deformation of the rock matrix. The expression for the shear strain component \( e_{nr} \) as a sum of the shear strain in the rock matrix, \( (e_{nr})_{\text{matrix}} \), and that attributable to the joint slip is
\[ e_{nr} = (e_{nr})_{\text{matrix}} + (e_{nr})_s = \frac{T_{nr}}{2G} + \frac{T_{nr}}{2G_k} . \]  

Taking the derivative of Equation (13) with respect to time yields Equation (7). A process similar to the one just used to obtain Equation (7) can be used to obtain Equation (8). Equations (7) and (8) for the shear strain rates, \( \dot{e}_{nr} \) and \( \dot{e}_{rm} \), assume that slip on the joint planes can occur in both the \( m \) and \( n \) directions, and that the joint slip in the \( m \) and \( n \) directions is uncoupled. In Equation (7), for example, the shear strain rate \( \dot{e}_{nr} \) has contributions resulting from shear in the rock matrix and slip on the joint plane. The calculation of \( \dot{e}_{nr} \) is independent of the calculation of \( \dot{e}_{rm} \). Equation (8) also assumes that \( \dot{e}_{rm} \) has contributions resulting from shear in the rock matrix and slip on the joint plane, and that its value is independent of the calculation of \( \dot{e}_{nr} \). Additional comments will be made in future sections about the inherent assumptions in Equations (7) and (8).

The mathematical description for joint behavior presented in the preceding sections has been used as the basis for developing a computational model for a single joint set in a three-dimensional media, and this model has been implemented in the code JAC3D. JAC3D solves nonlinear quasi-static solid mechanics problems using a conjugate gradient method. In order to implement the single set jointed rock model in JAC3D, it is necessary to calculate stress increments in terms of strain increments. The following sections describe the process for calculating stress increments in terms of strain increments.

**Implementation of Computational Model**

With the preceding description for joint behavior, the normal stress calculations are independent of the shear stress calculations. For the shear stress calculations, the normal stresses enter into the calculation of whether inelastic behavior occurs [Equation (2)], but other than this, the shear stress calculations do not depend on the normal stresses. Therefore, the normal stresses are calculated first, followed by the calculation of the shear stresses. The two sets of calculations are linked only via Equation (2).

**Normal Stress Calculations**

In order to calculate normal stress increments, first add Equations (3) through (5) to obtain the relation

\[ tr (\hat{T}) = 3K tr (\dot{\varepsilon}) - \frac{3K\beta \dot{T}_{rr}}{(A - T_{rr})^2} . \]  

where

\[ tr (\dot{\varepsilon}) = \dot{e}_{mm} + \dot{e}_{nn} + \dot{e}_{rr} . \]
If the expression for $tr(\dot{T})$ in Equation (14) is substituted into Equation (5), the result is

$$\dot{T}_{rr} + \frac{a_1 \beta T_{rr}}{(A - T_{rr})^2} = 2G \dot{\epsilon}_{rr} + a_2 tr(\dot{\epsilon}),$$  \hspace{1cm} (16)

where

$$a_1 = K + 4G/3$$  \hspace{1cm} (17)

and

$$a_2 = K - 2G/3.$$  \hspace{1cm} (18)

If the strain rates $\dot{\epsilon}_{mm}$, $\dot{\epsilon}_{nn}$, and $\dot{\epsilon}_{rr}$ are constant for some increment in time $\Delta t$, then it is possible to integrate Equation (16). Integrating Equation (16) from time $t$ to time $t + \Delta t$ yields Equation (19).

$$(2G \dot{\epsilon}_{rr} + a_2 tr(\dot{\epsilon})) \Delta t = T_{rr}^{t + \Delta t} - T_{rr}^t + \frac{a_1 \beta}{(A - T_{rr}^{t + \Delta t})} - \frac{a_1 \beta}{(A - T_{rr}^t)}$$ \hspace{1cm} (19)

The value for the normal stress, $T_{rr}$, at time $t$ is denoted by $T_{rr}^t$, and at time $t + \Delta t$ is denoted by $T_{rr}^{t + \Delta t}$. If the quantity $\gamma_r$ is defined as

$$\gamma_r = [2G \dot{\epsilon}_{rr} + a_2 tr(\dot{\epsilon})] \Delta t + T_{rr}^t + \frac{a_1 \beta}{(A - T_{rr}^t)},$$ \hspace{1cm} (20)

then Equation (19) can be rewritten as

$$(T_{rr}^{t + \Delta t})^2 - (A + \gamma_r) T_{rr}^{t + \Delta t} + A \gamma_r - a_1 \beta = 0.$$ \hspace{1cm} (21)

Equation (21) is quadratic in $T_{rr}^{t + \Delta t}$, and the roots of Equation (21) are given by

$$T_{rr}^{t + \Delta t} = \frac{-b \pm \sqrt{b^2 - 4c}}{2},$$ \hspace{1cm} (22)

where

$$b = -(A + \gamma_r)$$ \hspace{1cm} (23)

and
The desired root is the smaller of the two values of the real parts of \((- b - \sqrt{b^2 - 4c}) / 2\) and \((- b + \sqrt{b^2 - 4c}) / 2\). This is summarized in Equation (25).

\[ T_{rr}^{t + \Delta t} = \min \{ \text{real} \left[ \frac{-b + \sqrt{b^2 - 4c}}{2} \right], \text{real} \left[ \frac{-b - \sqrt{b^2 - 4c}}{2} \right] \} \]  

(25)

Now that a value for \( T_{rr}^{t + \Delta t} \) has been determined, it is possible to compute \( T_{mm} \) and \( T_{nn} \) at time \( t + \Delta t \). If the expression for \( tr (\dot{T}) \) in Equation (14) is substituted into Equations (3) and (4), the results are

\[
\dot{T}_{mm} + \frac{a_2 B \dot{T}_{rr}}{(A - T_{rr})^2} = 2G\dot{e}_{mm} + a_2 tr (\dot{e})
\]  

(26)

and

\[
\dot{T}_{nn} + \frac{a_2 B \dot{T}_{rr}}{(A - T_{rr})^2} = 2G\dot{e}_{nn} + a_2 tr (\dot{e}).
\]  

(27)

By again relying on the assumption that the strain increments are constant over some time increment \( \Delta t \), it is possible to integrate Equations (26) and (27) to produce

\[
T_{mm}^{t + \Delta t} = \frac{-a_2 B}{A - T_{rr}^{t + \Delta t}} + \gamma_m
\]  

(28)

and

\[
T_{nn}^{t + \Delta t} = \frac{-a_2 B}{A - T_{rr}^{t + \Delta t}} + \gamma_n
\]  

(29)

where

\[
\gamma_m = [2G\dot{e}_{mm} + a_2 tr (\dot{e})] \Delta t + T_{mm}^{t} + \frac{a_2 B}{A - T_{rr}^{t}}
\]  

(30)

and

\[
\gamma_n = [2G\dot{e}_{nn} + a_2 tr (\dot{e})] \Delta t + T_{nn}^{t} + \frac{a_2 B}{A - T_{rr}^{t}}.
\]  

(31)
Equations (25), (28), and (29) give values for the normal stress components at time $t + \Delta t$. Calculation of the normal stresses for the single set three-dimensional jointed rock model is a simple extension of the concepts used for two-dimensional jointed rock models. Now that the normal stress components have been computed, the shear stresses can be determined.

**Shear Stress Calculations**

The calculation of shear stress increments for the single set three-dimensional jointed rock model relies on some of the concepts for the two-dimensional models. The transition from two to three dimensions introduces some considerations in the calculation of the shear stress increments that do not arise in two dimensions. This section examines calculation of the shear stress increments and the special situations that occur in a three-dimensional model. The first part of this section examines different schemes for calculating the shear stress increments for a given set of shear strain increments.

One method for calculating the shear stress increments assumes that the shear behavior on the $r$ plane is uncoupled in the $m$ and $n$ directions. The Mohr-Coulomb criterion would be used to determine if elastic or inelastic behavior governs the calculation of $T_{rm}^{t + \Delta t}$. Then, independent of these calculations, the Mohr-Coulomb criterion would be used to determine if elastic or inelastic behavior governs the calculation of $T_{nr}^{t + \Delta t}$. Equations (6) through (8) would be used to describe the shear strain-slip displacement under this assumption.

The method just described, although not unreasonable, does not seem to be desirable from an intuitive standpoint. Joint behavior is tied to a coordinate system that is arbitrarily oriented in the plane of the joints. A more reasonable scheme would seem to be one where joint slip occurs in some direction determined by a criterion using stress or strain. The concept of joint slip becomes slightly more complicated in a three-dimensional model as opposed to a two-dimensional model. A slip displacement increment in a two-dimensional model, $\Delta u_s$, is motion along a line that lies in the two-dimensional plane. The direction of motion along this line is indicated by the sign of $\Delta u_s$. A slip displacement increment in a two-dimensional problem can be treated as a scalar quantity with direction information implied by the sign of the slip increment. For a three-dimensional problem, a slip displacement increment is a vector quantity that lies in the joint plane. Going from two to three dimensions results, therefore, in a more complicated concept of slip on the joint plane. Because the concept of slip is more complicated in three dimensions, it becomes necessary to examine certain issues in the calculation of the shear strain increments that do not arise for the two-dimensional case.

To account for the fact that joint slip is a more complicated phenomenon in a three-dimensional problem, use a scheme to calculate the shear stress increments that assumes a slip direction is determined by some type of stress criterion. Since failure criteria are often stress based, a stress criterion for determination of a slip direction seems a reasonable approach for determining a slip direction.
As an initial proposal for calculating shear stress increments, assume that slip on the \( r \) plane occurs in the direction of the maximum shear stress on the \( r \) plane. The direction of the maximum shear stress is known at time \( t \), but it is not known at time \( t + \Delta t \) until the values of \( T_{rm}^{t+\Delta t} \) and \( T_{nr}^{t+\Delta t} \) have been calculated. The problem is that there is no a priori method of calculating the direction of the maximum shear stress at time \( t + \Delta t \) without knowing the value for \( T_{rm}^{t+\Delta t} \) and \( T_{nr}^{t+\Delta t} \). To use the shear stress values at time \( t + \Delta t \) to determine the slip direction would require an iterative process.

Rather than developing a scheme using the values \( T_{rm}^{t+\Delta t} \) and \( T_{nr}^{t+\Delta t} \) to determine a slip direction on the \( r \) plane, consider a scheme that uses the elastic estimates for shear stresses \( T_{rm} \) and \( T_{nr} \) at time \( t + \Delta t \). Although there is no theoretical basis for this approach for choosing the slip direction, it is a reasonable alternative that avoids the problem of having to iterate the shear stress calculations. The following sections outline the shear stress versus slip displacement calculations based on the assumption that the elastic estimates for \( T_{rm} \) and \( T_{nr} \) at time \( t + \Delta t \) determine the slip direction on the \( r \) plane.

All calculations are based on shear strain increments \( \Delta e_{mn} \), \( \Delta e_{nr} \), and \( \Delta e_{rm} \), which are defined by

\[
\Delta e_{mn} = \dot{e}_{mn}\Delta t, \quad (32)
\]

\[
\Delta e_{nr} = \dot{e}_{nr}\Delta t, \quad (33)
\]

and

\[
\Delta e_{rm} = \dot{e}_{rm}\Delta t. \quad (34)
\]

As has been done with the normal stress calculations, it is assumed that the shear strain rates are constant over the time \( \Delta t \).

The direction of the maximum shear stress is computed based on the elastic estimates for the shear stress components at time \( t + \Delta t \). These elastic shear stresses at time \( t + \Delta t \) are determined by

\[
(T_{mn})_{\text{elastic}}^{t+\Delta t} = T_{mn}^{t} + 2G\Delta e_{mn}, \quad (35)
\]

\[
(T_{nr})_{\text{elastic}}^{t+\Delta t} = T_{nr}^{t} + (2G\Delta e_{nr}) / [1 + G/ (\delta G_s)], \quad (36)
\]

and

\[
(T_{rm})_{\text{elastic}}^{t+\Delta t} = T_{rm}^{t} + (2G\Delta e_{rm}) / [1 + G/ (\delta G_s)]. \quad (37)
\]
Equations (35) through (37) produce values for the shear stresses assuming elastic shear stress versus slip displacement behavior. These equations assume uncoupled elastic shear stress versus slip displacement behavior in the $m$ and $n$ directions.

The angle associated with the maximum (or minimum) elastic shear stress value on the $r$ plane is

$$\theta = \left\{ \begin{array}{ll}
\tan^{-1} \left[ \frac{ (T_{nr})_{\text{elastic}}^{t+\Delta t} }{ (T_{rm})_{\text{elastic}}^{t+\Delta t} } \right], & (T_{rm})_{\text{elastic}}^{t+\Delta t} \neq 0 \\
\pi/2, & (T_{rm})_{\text{elastic}}^{t+\Delta t} = 0.
\end{array} \right. \quad (38)$$

A new coordinate system $m'n'r$ is obtained by a rotation of $\theta$ about the $r$-axis of the $mnr$ system. The shear stresses on the $r$ plane in the $m'n'r$ coordinate system are given by

$$T_{rm'} = T_{rm} \cos \theta + T_{nr} \sin \theta \quad (39)$$

and

$$T_{n'r} = T_{nr} \cos \theta - T_{rm} \sin \theta. \quad (40)$$

Because of the manner in which $\theta$ has been specified, Equation (38) yields a value of zero for $(T_{n'r})_{\text{elastic}}^{t+\Delta t}$ and $(T_{rm})_{\text{elastic}}^{t+\Delta t}$ takes on a maximum (or minimum) value.

The shear strain increments in the $m'n'r$ coordinate system are given by

$$\Delta e_{n'r} = \Delta e_{nr} \cos \theta - \Delta e_{rm} \sin \theta \quad (41)$$

and

$$\Delta e_{rm'} = \Delta e_{rm} \cos \theta + \Delta e_{nr} \sin \theta. \quad (42)$$

The calculations for the shear stress increments can now be made in the $m'n'r$ coordinate system. In this system, the shear stress increments $\Delta T_{m'n'}$ and $\Delta T_{rm'}$ are written simply as

$$\Delta T_{m'n'} = 2G \Delta e_{m'n'} \quad (43)$$

and

$$\Delta T_{rm'} = 2G \Delta e_{rm'}/\left[ 1 + G/(\delta G_{\delta}) \right]. \quad (44)$$

Now it is necessary to write some expression for the shear stress increment $\Delta T_{n'r}$ in terms of the shear strain increment $\Delta e_{n'r}$. For the particular coordinate transformation chosen, the strain increment $\Delta e_{n'r}$ is not necessarily zero. In general, $\Delta e_{n'r}$ can be a nonzero
quantity, and it becomes necessary to write some expression relating this shear strain increment to the shear stress increment \( \Delta T'_{nr} \). Since it is assumed that all of the slip displacement has taken place in the \( m' \) direction, it is not possible to use an expression similar to Equation (44) to relate \( \Delta e'_{n'r} \) and \( \Delta T'_{n'r} \) without violating the assumption about the direction of slip displacement on the joint plane. If the relation between \( \Delta e'_{n'r} \) and \( \Delta T'_{n'r} \) is written in the form

\[
\Delta T'_{n'r} = 2G\Delta e'_{n'r},
\]

then there will be a stress increment due to deformation of the matrix material only. This violates the general assumption that a shear strain increment in any direction is composed of both deformation of the matrix material and slip on the joint planes.

The method of determining the direction of slip on the joint planes given by Equation (38) leads to difficulties when trying to specify an equation to define \( \Delta T'_{n'r} \). This becomes more evident when the special case of \( \theta = 0 \) is considered. Suppose that Equation (38) predicts that the direction of slip on the joint plane is in the \( \theta = 0 \) direction. Furthermore, suppose that the shear stress versus slip displacement behavior is elastic. For this special case of slip in the \( \theta = 0 \) direction and elastic slip on the joint plane, the elastic shear stress increment in the \( m \) direction is given by Equation (44) and the total shear stress at time \( t + \Delta t \) is

\[
T'_{rm}^{t + \Delta t} = (T_{rm})^{t + \Delta t}_{\text{elastic}} = T'_{rm} + \left( (2G\Delta e_{rm}) / \left[ 1 + G/(\delta G_{n}) \right] \right).
\]

Since the direction of slip is in the \( \theta = 0 \) direction, Equation (38) requires that \( (T'_{nr})^{t + \Delta t}_{\text{elastic}} \) be zero. Consider the case where \( T'_{nr} \) is nonzero. For this particular situation, the value of the shear stress increment \( \Delta T_{nr} \) must be equal to and opposite in sign to the value of \( T'_{nr} \). If an equation similar to the form of Equation (44) is used to calculate the value of \( \Delta T_{nr} \), then the implication is that slip occurs in the \( n \) direction. If Equation (45) is used to determine the value of \( \Delta T_{nr} \), then the implication is that \( \Delta T_{nr} \) arises only from deformation of the rock matrix. With the current scheme for determining the direction of slip on the joint plane, there is no clear cut means to calculate the value of \( \Delta T_{nr} \) consistent with all of the assumptions about the direction of slip on the joint plane and the behavior of the jointed rock mass. The problem with this scheme is that the direction of slip is determined from calculations that first assume that slip behavior in the two coordinate directions \( m \) and \( n \) is independent. Shear stress increment calculations are then made on the basis that slip can occur in only one direction.

As an alternative to the assumption that the elastic estimates for \( T_{nr} \) and \( T_{rm} \) at time \( t + \Delta t \) determine the slip direction, assume that the direction of the maximum shear strain
increment on the $r$ plane determines the direction of slip. This assumption leads to kinematically consistent behavior; it does not lead to difficulties in the calculation of the shear stress increments like those obtained in the preceding sections. Furthermore, this particular method for determining the slip displacement direction leads to a relatively simple set of equations for $\Delta T_{rm}$ and $\Delta T_{nr}$.

The direction of the maximum shear strain increment in the joint plane ($r$ plane) is

$$\theta = \begin{cases} \atan (\Delta e_{nr}/\Delta e_{rm}), & \Delta e_{mr} \neq 0 \\ \pi/2, & \Delta e_{mr} = 0. \end{cases}$$  \hspace{1cm} (47)$$

A new coordinate system is again obtained by a rotation of $\theta$ about the $r$ axis of the $mnr$ coordinate system. The shear strain increments on the $r$ plane in the $m'n'r$ coordinate system are given by equations (41) and (42). Because of the manner in which $\theta$ is chosen, $\Delta e_{n'r}$ has a value of zero. The equations for the shear stress increments, therefore, take on the form

$$\Delta T_{m'n'} = 2G\Delta e_{m'n'},$$  \hspace{1cm} (48)$$

$$\Delta T_{n'r} = 0,$$  \hspace{1cm} (49)$$

and

$$\Delta T_{rm'} = 2G\Delta e_{rm'}/[1 + G/(\delta G_k)].$$  \hspace{1cm} (50)$$

The determination of slip direction based on the maximum shear strain increment in the $r$ plane eliminates problems with the specification of $\Delta T_{n'r}$.

Since $\Delta T_{n'r}$ is zero, the equations relating shear stress increments in the $mnr$ and $m'n'r$ coordinate systems become

$$\Delta T_{rm} = \Delta T_{rm'} \cos \theta$$  \hspace{1cm} (51)$$

and

$$\Delta T_{nr} = \Delta T_{rm'} \sin \theta.$$  \hspace{1cm} (52)$$

The equations to compute the shear stress increments for strictly elastic behavior have the form

$$\Delta T_{rm} = \frac{2G\Delta e_{rm'}\cos \theta}{1 + G/(\delta G_k)}$$  \hspace{1cm} (53)$$
and

$$
\Delta T_{nr} = \frac{2G\Delta e_{rm}\sin \theta}{1 + G/\delta G_s}.
$$

Equations (53) and (54) can be used to calculate the shear stress increments for strictly inelastic behavior by substituting $G_s'$ for $G_s$. Since the choice of slip direction given by Equation (47) does not lead to anomalous behavior and results in a relatively simple set of equations for the shear stress increments, it is used as the basis for determining the slip direction in the computational implementation of the single set jointed rock model. The computational implementation uses various forms of Equations (47) through (54) as a basis for calculating the shear stress increments.

Now that a coordinate transformation has been established for determining the direction of slip on the joint plane and equations derived for shear stress increments, the criterion for the onset of inelastic joint behavior can be defined. This criterion will be based on extensions of the ideas used for two-dimensional jointed rock models. A full description of the shear stress versus slip displacement behavior for the two-dimensional jointed rock models is given in Reference 10.

The onset of inelastic behavior is determined by a simple Mohr-Coulomb failure criterion. If the shear stress at time $t + \Delta t$ exceeds some limit, then inelastic behavior occurs. The limit used to determine if inelastic behavior occurs is the effective yield stress. The effective yield stress depends on two parameters, the yield stress at time $t + \Delta t$ and permanent offset on the joint plane at time $t$. The first parameter, the yield stress on the $r$ plane at time $t + \Delta t$, is defined by the relation

$$
T^{t+\Delta t}_y = C_0 - \mu T^{t+\Delta t}_{rr}.
$$

The yield stress varies over time because it depends on the normal joint stress, $T_{rr}$, which also varies with time.

The second parameter, the permanent offset, depends on the current shear stress versus slip displacement state. In order to define the permanent offset, first consider a special case where there is shear deformation only in the $m$ direction. At time $t$, the permanent offset is calculated from the relation

$$
u_{offset,m}^t = u_{s,m}^t - (T_{rm}^t / G_s).
$$

The quantity $u_{s,m}^t$ is the total slip on the joint plane in the $m$ direction at time $t$. The quantity $T_{rm}^t / G_s$ is the amount of slip on the joint plane in the $m$ direction that can be recovered by elastic unloading. If only elastic behavior has occurred up to time $t$, then the value for
will be zero. If inelastic behavior has occurred at any point prior to time $t$, the value for the offset will be nonzero. The difference between the total slip on the joint plane, $u_{s,m}'$, and the amount of slip on the joint plane that can be recovered by elastic unloading, $T_{rm}'/G_s$, is the permanent offset.

Because the determination of the permanent offset depends upon the joint slip, and the concept of joint slip is more complicated in three dimensions as previously indicated, the calculation of the permanent offset is not as straightforward in three dimensions as it is in two dimensions. There are several methods that can be used to calculate the total slip used in the calculation of the permanent offset.

For the first method for calculating total slip for determination of the permanent offset, assume that the total slip in the $m$ and $n$ directions is known at time $t$. The total slip in the $m$ direction at time $t$ is $u_{s,m}'$ and the total slip in the $n$ direction at time $t$ is $u_{s,n}'$. To define the permanent offset, use the slip and shear stress component in the $\theta$ direction on the $r$ plane, where $\theta$ is defined by Equation (47). The slip in the $\theta$ direction is given by

$$u_{s,m}' = u_{s,m}' \cos \theta + u_{s,n}' \sin \theta$$

(57)

and the shear stress component in the $\theta$ direction, $T_{rm}'$, is given by Equation (39). The permanent offset is defined as

$$u_{\text{offset}, m'} = u_{s,m}' - \left( T_{rm}' / G_s \right).$$

(58)

Equation (58) determines the permanent offset in terms of the total slip and the shear stress in the $m'$ direction, which is the direction of the slip on the joint plane. With this particular method for computing the permanent offset, any slip component or shear stress component orthogonal to the direction of slip does not affect the value of the permanent offset. Only the slip and shear stress components along the direction of slip enter into the permanent offset computations.

Another method for computing the permanent offset relies on a simple distance calculation. Assume, as before, that the total slip in the $m$ and $n$ directions is known at time $t$. Define the total amount of slip at time $t$ as

$$u_{s,m'} = \sqrt{(u_{s,m}')^2 + (u_{s,n}')^2}.$$

(59)

If $i$ and $j$ are unit vectors lying along the $m$ and $n$ axes, respectively, and the slip vector at time $t$ is defined as

$$\mathbf{u}' = u_{s,m}' i + u_{s,n}' j,$$

(60)
then Equation (59) represents the magnitude of the slip vector at time $t$. The slip vector at time $t$ is associated with an angle $\alpha$ defined by

$$\alpha = \text{atan}(u_{s,m}^t / u_{s,n}^t).$$  \hfill (61)

The angle $\alpha$ is the angle between the slip vector at time $t$, $y^t$, and the $m$ axis. The $mnr$ coordinated system can be transformed to the $m''n''r$ coordinate system by a rotation of $\alpha$ about the $r$ axis. This angle provides a basis for specifying an associated shear stress for the permanent offset calculations. The shear stress component in the joint plane in the $m''$ direction, $T_{r_m''}^t$, is defined by

$$T_{s,m''}^t = T_{rm}^t \cos \alpha + T_{nr}^t \sin \alpha.$$ \hfill (62)

The permanent offset, defined in terms of $u_{s,m''}^t$ and $T_{r_m''}^t$, is

$$u_{offset,m''}^t = u_{s,m''}^t - T_{r_m''}^t / G_s.$$ \hfill (63)

In general, the angle $\alpha$ is not necessarily the same as the angle $\theta$ specified by Equation (47). Therefore, the slip and shear stress components $u_{s,m''}^t$ and $T_{r_m''}^t$ are not necessarily the same as $u_{s,m'}^t$ and $T_{r_m'}^t$, and do not bear any direct relation to the transformation specified by Equation (47).

Finally, one could specify the total slip at some point on the joint plane in terms of the path traversed by that point. Suppose the slip at a given point on the joint plane over the $i^{th}$ time increment is given by

$$\left(\Delta u_s\right)_i = \sqrt{\left(\Delta u_{s,m}\right)_i^2 + \left(\Delta u_{s,n}\right)_i^2},$$ \hfill (64)

where $\left(\Delta u_{s,m}\right)_i$ and $\left(\Delta u_{s,n}\right)_i$ are the slip increments in the $m$ and $n$ directions, respectively, for the $i^{th}$ increment. The total slip up to time $t$ is given by

$$u_{s}^t = \sum_{i=1}^{k} \left(\Delta u_s\right)_i,$$ \hfill (65)

where the total number of time increments up to time $t$ is $k$. The quantity $u_{s}^t$ is a scalar quantity and is not associated with any particular direction on the joint plane. Because of this, it is difficult to define some related shear stress based on $T_{r_m}^t$ and $T_{nr}^t$ as in the previous schemes for computing the permanent offset.
There is no theoretical basis for choosing one of the above schemes for computing the permanent offset as opposed to another. For the purposes of the current computational work, the first scheme based on $u_{s, m}$ and $T_{s, m}$ has been selected as the means for computing the permanent offset. The main reason for choosing this particular scheme is that it is consistent with the calculation of the slip direction used in the calculation of the shear stress increments. In the implementation of the computational model, $u_{s, m}$ and $u_{s, n}$ are stored as state variables, and Equations (39), (47), (57), and (58) are used to calculate the permanent offset.

Now that a means of specifying the slip and an associated shear stress has been selected, it becomes possible to compute the effective yield stress. Consider the shear stress versus slip displacement history shown in Figure 4. The shear stress versus slip displacement at time $t$ is given by point A. Assume that a loading situation occurs over the time increment $\Delta t$. The yield stress at time $t + \Delta t$ is given by Equation (55). It is assumed that the shear stress versus slip displacement behavior will be elastic until point B is reached, which is the effective yield stress at time $t + \Delta t$. Inelastic behavior occurs when the effective yield stress rather than the yield stress is exceeded. This approach represents a translation of some basic ideas in strain hardening plasticity to the jointed rock model.

![Figure 4](image-url)
Point B lies on the shear stress versus slip displacement curve with a shear yield stress of $T_y + \Delta t$. The total slip displacement on the joint plane in terms of quantities defining the shear stress versus slip displacement curve at time $t + \Delta t$ can be written as

$$u_{s,m'}^{t+\Delta t} = T_y^{t+\Delta t}/G_s + [T_{ey}^{t+\Delta t} - T_y^{t+\Delta t}]/G_s'. \tag{66}$$

The total slip displacement on the joint plane in terms of the permanent offset shown in Figure 4 is

$$u_{s,m'}^{t+\Delta t} = u_{offset,m'}^{t} + T_{ey}^{t+\Delta t}/(G_s'). \tag{67}$$

By equating the right hand side of Equation (66) with the right hand side of Equation (67), it is possible to solve for the effective yield stress. The effective yield stress at time $t + \Delta t$ is

$$T_{ey}^{t+\Delta t} = T_y^{t+\Delta t} + G_s' u_{offset,m'}^{t}/(1 - G_s'/G_s). \tag{68}$$

The effective yield stress coincides with the yield stress if the value for the permanent offset is zero. The permanent offset is zero if there has been no inelastic behavior.

For the case of unloading, the effective yield stress at time $t + \Delta t$ is set to the yield stress at time $t + \Delta t$. The convention chosen for determining the effective yield stress results in the shear stress falling on or below the shear stress versus slip displacement curve at time $t + \Delta t$.

If the value for $(T_{ey}^{t+\Delta t})_{elastic}$ is less than or equal to the value for the effective yield stress $T^{t+\Delta t}_{ey}$, then elastic slip behavior occurs and Equations (53) and (54) are used to calculate the shear stress increment. If completely inelastic behavior occurs, Equations (53) and (54) are still applicable if $G_s'$ is substituted for $G_s$ in both equations. If there is a transition from elastic to inelastic behavior for a given strain increment, the expression for the stress increment $\Delta T_{rm'}$ becomes more complicated. In the transition from elastic to inelastic behavior, the strain increment due to elastic slip on the joint plane is given by

$$(T_{ey}^{t+\Delta t} - T_{rm'}^{t})/(2\delta G_s), \tag{69}$$

and the strain increment due to inelastic slip on the joint plane is given by

$$(T_{rm'}^{t+\Delta t} - T_{ey}^{t+\Delta t})/(2\delta G_s'). \tag{70}$$
The sum of the elastic and inelastic strain contributions in Equations (69) and (70), respectively, and the strain contribution from the matrix material gives the total strain increment $\Delta e_{rm}$. The expression for $\Delta e_{rm}$ is

$$2\delta \Delta e_{rm} = \delta \Delta T_{rm}/G + (T^l_{ey} + \Delta t - T^l_{rm})/G_s + (T^l_{rm} + \Delta t - T^l_{ey})/G_s' .$$  \hspace{1cm} (71)$$

By using Equation (71) it is possible to write the value for the stress increment $\Delta T_{rm}$ as

$$\Delta T_{rm} = \frac{2G\delta e_{rm} + (T^l_{ey} + \Delta t - T^l_{rm}) [G/(\delta G_s') - G/(\delta G_s)]}{1 + G/(\delta G_s')} .$$  \hspace{1cm} (72)$$

The shear stress increments $\Delta T_{rm}$ and $\Delta T_{nr}$ are obtained by using the transformations in Equations (51) and (52), respectively.

Now that $\Delta T_{rm}$ and $\Delta T_{nr}$ are known, it is possible to compute the slip displacements with the relations

$$\Delta u_{s,m} = \delta (2\Delta e_{rm} - \Delta T_{rm}/G) \hspace{1cm} (73)$$

and

$$\Delta u_{s,n} = \delta (2\Delta e_{nr} - \Delta T_{nr}/G) .$$  \hspace{1cm} (74)$$

In summary, a strain based criterion [Equation (47)] is used to determine a slip direction in the joint plane. The selection of the slip direction in Equation (47) leads to a set of equations relating shear strain increments to shear stress increments. An effective yield stress is then calculated to determine whether or not inelastic behavior occurs. The calculation of the permanent offset quantity used in the effective yield stress calculations also makes use of the slip direction determined by Equation (47).

**Verification of Computational Model**

A variety of problems are used to check the computational implementation of the single set jointed rock model. This section describes these verification problems and key analytic results from the problems that can be used to verify the computational implementation of the jointed rock model.

All of the test problems described are for a jointed media with a single set of joints and all of the test problems utilize the same set of material properties. The matrix material in the jointed media has a Young's modulus, $E$, of $1.0 \times 10^6$ psi and a Poisson's ratio, $\nu$, of 0.25. These two material properties correspond to a shear modulus, $G$, of $4.0 \times 10^5$ psi and a bulk
modulus, \( K \), of \( 6.6667 \times 10^5 \) psi. The maximum joint closure, \( U_{\text{max}} \), is -0.003 in. and the maximum tensile stress, \( A \), is \( 1.0 \times 10^3 \) psi. The joint shear stiffness, \( G_s \), is \( 1.0 \times 10^5 \) psi/in, the joint shear stiffness, \( G'_s \), is \( 1.0 \times 10^3 \) psi/in, the joint cohesion, \( C_0 \), is 250 psi, and the joint coefficient of friction, \( \mu \), is 0.7. The joint spacing, \( \delta \), is 0.5 in.

**Sample Problem 1: Simple Compression**

The first verification problem tests the behavior of the joint model in a direction normal to the joint planes. The geometry for this particular problem is a cube (Figure 5.) The cube, which has a length of 10 in. for its sides, is aligned with the \( xyz \) coordinate system, and the \( mnr \) coordinate system is aligned with the \( xyz \) coordinate system. The joints lie in planes that are normal to the global \( z \) axis. The planes of the cube perpendicular to the \( m \) axis (\( x \) axis) have a prescribed displacement of \( u_m = 0 \), and the planes of the cube perpendicular to the \( n \) axis (\( y \) axis) have a prescribed displacement of \( u_n = 0 \). The plane perpendicular to the \( r \)-axis at \( r = 0 \) has a prescribed displacement of \( u_r = 0 \), and the plane perpendicular to the \( r \)-axis at \( r = 10 \) has a prescribed displacement of \( u_r = -0.05 \). The displacement boundary conditions are shown in Figure 6. There are no initial stresses in the cube. The prescribed displacement on the top surface generates a compressive value for \( T_{rr} \). The
normal stresses, $T_{mm}$ and $T_{nn}$, are also compressive because of the Poisson effect. All of the shear stresses are zero.

The displacement on the top surface of the cube, $u_r |_{r = 10}$, can be viewed as the result of a constant strain rate $\dot{e}_{rr}$ for a given unit time. Using a unit time from the point where there is no downward displacement to the point where the displacement at the top reaches a maximum value is a convenient artifice for this particular test problem. It results in simple expressions for the strain rate and strain increments and simplifies presentation of results. The strain as a function of time is related to the total displacement by

$$e_{rr} = \frac{tu_r |_{r = 10}}{l} = -0.005t, \quad (75)$$

where $l$ is the length of the side of the cube. The strain increments for the unit time are

$$\Delta e_{rr} = -0.005 \quad (76)$$

and

$$\Delta e_{mm} = \Delta e_{nn} = \Delta e_{mn} = \Delta e_{nr} = \Delta e_{rm} = 0. \quad (77)$$

Since values for all of the strain increments are known from the prescribed boundary conditions, it is possible to calculate values for the stresses by using Equations (25), (28), and (29). At time $t = 1$, $T_{rr} = -1585$ psi and $T_{mm} = T_{nn} = -528$ psi. All of the shear stress components have a value of zero. The normal joint displacement in the $r$ direction is $u_j = 1.8395 \times 10^{-3}$ in. at time $t = 1$. The normal joint displacement is obtained by substituting the value for $T_{rr}$ at $t = 1$ into Equation (1). The process used to compute the stresses and joint closure at time $t = 1$ can be used for any time $0 \leq t \leq 1$. 

Figure 6. Displacement Boundary Conditions for Sample Problem 1.
This problem has been modeled with a 4 × 4 × 4 hexahedral element (eight nodes per element) mesh. There are four elements along each coordinate direction so that there are a total of sixty-four elements in the mesh. All of the elements are cubes of the same size (2.5in × 2.5in × 2.5in). There are eight interior elements in the model, i.e., there are eight elements that do not have a face on an exterior surface of the cube. Interior elements are chosen for results since they minimize the influence of edge effects, which is important for some of the sample problems, and seem to give the most accurate results.

Graphical results from the finite element calculations are shown in Figures 7 through 9 for an interior element. The normal stress value, $T_{rr}$, as a function of displacement of the top surface of the block in the $r$ direction is shown in Figure 7, and the normal stress values $T_{mm}$ and $T_{nn}$ as a function of displacement of the top surface of the block in the $r$-direction are shown in Figure 8. The curve for the normal stress, $T_{rr}$, in Figure 7 reflects the fact that the joint normal stress behavior is defined by a rational polynomial. The curves for $T_{mm}$ and $T_{nn}$ in Figure 8 show similar behavior to that for $T_{rr}$ since they result from the Poisson effect. The normal joint displacement, $u_j$, as a function of the displacement of the top surface of the block in the $r$ direction is shown in Figure 9. The results from the finite element model agree with the analytic predictions. The computer implementation gives values of $T_{rr} = -1585$ psi and $T_{mm} = T_{nn} = -528$ psi at time $t = 1$ for an interior element. The computed value for the joint closure is $u_j = 1.8395 \times 10^{-3}$ for an interior element.

**Sample Problems 2 and 3: Rotation of the Joint Planes**

For the second and third test problems, the block geometry shown in Figure 5 is used again. The joint planes are rotated with respect to the $xy$ plane, however, to test the computational model in shear. For the second test problem, the joint planes are rotated 60 degrees counterclockwise about the $x$ axis. The bottom of the block is supported on a frictionless surface ($u_z = 0$ at $z = 0$), the sides are unconstrained, and the top of the block is given a downward displacement. The joint plane orientation for Problem 2 is shown in Figure 10 along with the displacement boundary conditions. There are no initial stresses. At any given time, the normal stresses, $\sigma_{xx}$ and $\sigma_{yy}$, and the shear stresses, $\tau_{xy}$, $\tau_{yz}$, and $\tau_{zx}$, are zero. Only the normal stress $\sigma_{zz}$ has a nonzero value.

The stresses $T_{rr}$ and $T_{nr}$, which are the two stresses in the joint coordinate system that are key to the calculation of the onset of inelastic joint slip behavior, are related to the stresses in the global coordinate system by the transformations

$$T_{rr} = \sigma_{zz} (\cos \theta)^2 + \sigma_{yy} (\sin \theta)^2 - 2\tau_{yz} \cos \theta \sin \theta$$

(78)

and
Figure 7. Joint Normal Stress $T_{rr}$ as a Function of the Displacement of the Top Surface in the $z$ Direction.
Figure 8. Normal Stresses $T_{mm}$ and $T_{nn}$ as a Function of the Displacement of the Top Surface in the $z$ Direction.
Figure 9. Joint Normal Closure $u_j$ as a Function of the Displacement of the Top Surface in the $z$ Direction.
\begin{equation}
T_{nr} = (\sigma_{zz} - \sigma_{yy}) \cos \theta \sin \theta + \tau_{yz} [ (\cos \theta)^2 - (\sin \theta)^2].
\end{equation}

Since \( \cos \theta = 1/2 \) and \( \sin \theta = \sqrt{3}/2 \), the expressions for \( T_{rr} \) and \( T_{nr} \) can be written as

\begin{equation}
T_{rr} = 1/4 \sigma_{zz},
\end{equation}

and

\begin{equation}
T_{nr} = \sqrt{3}/4 \sigma_{zz}.
\end{equation}

Since inelastic joint slip behavior occurs when the magnitude of \( T_{nr} \) is equal to \( \mu T_{rr} - C_0 \), the value for \( \sigma_{zz} \) at the onset of inelastic behavior can now be calculated using Equations (78) and (79). Inelastic behavior occurs when \( \sigma_{zz} = -968.9445 \) psi. Now that the value for \( \sigma_{zz} \) corresponding to the onset of inelastic joint slip is known, it is possible to calculate the corresponding value of the displacement at the top surface of the cube. Equations (78) and (79) can be used to obtain the values for \( T_{rr} \) and \( T_{nr} \) at the onset of inelastic slip on the joint plane. These values are \( T_{rr} = -242.2361 \) psi and \( T_{nr} = -419.5653 \) psi. Equation (82), which relates the stress component \( T_{nn} \) to the global stresses, can be used to compute the value for \( T_{nn} \) at the onset of inelastic slip. This value is -726.7084 psi. Since the sides of the cube are not constrained on the vertical planes, \( T_{mm} = 0 \) at all times.

\begin{equation}
T_{nn} = \sigma_{yy} (\cos \theta)^2 + \sigma_{zz} (\sin \theta)^2 + 2 \tau_{yz} \cos \theta \sin \theta
\end{equation}

Now that all of the stresses in the joint coordinate system are known at the onset of inelastic slip on the joint plane, the strain increments in the joint coordinate system required to reach this point can now be calculated. The strain increments, \( \Delta e_{mm} \), \( \Delta e_{nn} \), and \( \Delta e_{rr} \), can be
calculated by integrating Equations (3) through (5) with respect to time under the assumption of constant strain rates and substituting the values for the stresses at the onset of inelastic slip into the resulting equations. The strain increment $\Delta e_{nm}$ can be calculated by using Equation (54) (with $\Delta e_{rm}$ replaced by $\Delta e_{nr}$). The strain increments in the joint coordinate system corresponding to the onset of inelastic slip are $\Delta e_{mn} = 2.4224 \times 10^{-4}$, $\Delta e_{nn} = -6.6615 \times 10^{-4}$, $\Delta e_{rr} = -1.2306 \times 10^{-3}$, and $\Delta e_{nr} = -4.7201 \times 10^{-3}$. Now that the strain increments in the joint coordinate system at the onset of inelastic slip are known, it is possible to use the transformation

$$\Delta e_{zz} = \Delta e_{rr} (\cos \theta)^2 + \Delta e_{nn} (\sin \theta)^2 - 2\Delta e_{nr} \cos \theta \sin \theta$$

(83)

to obtain the critical strain displacement $\Delta e_{zz}$ in the global coordinate system. In Equation (83), $\theta = -60$ degrees since the transformation is being carried out from the joint coordinate system to the global coordinate system. Equation (83) gives a value of $\Delta e_{zz} = -4.8950 \times 10^{-3}$ at the onset of inelastic slip, which corresponds to a displacement at the top surface of the block of $u_z = -0.04895$ in.

This problem has been modeled using the mesh for Problem 1. Figure 11 shows a plot of $\sigma_{zz}$ as a function of the displacement at the top of the block in the $z$ direction for an interior element. Obtaining printed results that show the exact transition from elastic to inelastic behavior for this particular problem is difficult. The best means of verifying the results for this particular problem is with graphical output. The graphical output in Figure 11 indicates that onset of inelastic joint slip behavior in the computational model occurs at or near the values of $\sigma_{zz}$ and $u_z$ predicted by the analytic calculations. Once the point is reached where inelastic behavior occurs, the entire block of material becomes very soft. It becomes difficult to obtain solutions for the problem when the displacement at the top of the block exceeds -0.05 in., although it can be done. The conjugate gradient method implemented in JAC3D has difficulty converging to a solution when materials soften.

Problem 3 is a simple variation of test Problem 2. The joint planes are rotated 60 degrees clockwise about the $y$ axis as opposed to 60 degrees about the $x$ axis. A plot of $\sigma_{zz}$ as a function of the displacement at the top of the block for Problem 3 is identical to that for Problem 2 (Figure 11), which is to be expected. The value for $\sigma_{zz}$ at which inelastic joint slip behavior occurs is the same for this problem as it is for Problem 2.

**Sample Problems 4 and 5: Pure Shear**

The fourth and fifth problems test the model in pure shear behavior using the same geometry shown in Figure 5 and the same joint orientation shown in Figure 6. The boundary conditions are changed, however, so that the computational model can be tested in a pure shear mode. For the fourth sample problem, the top surface of the block at $r = 10$
Figure 11. Normal Stress in the z direction, \( \sigma_z \) as a function of the Displacement of the Top Surface in the z direction.
in. is displaced in the \( m \) direction by a distance of 0.2 in. The displacement in the \( m \) direction at the bottom of the cube is set to zero. The displacement \( u_m \) on the vertical sides of the cube with normals in the \( m \) direction is a linear function of \( r \). The displacement \( u_n \) on the vertical sides of the cube with normals in the \( n \) direction are set to zero. The boundary conditions for Problem 4 are shown in Figure 12. The top surface of the block reaches its maximum displacement in the \( m \) direction of 0.2 in. at time \( t = 1 \). The artifice of a unit time is used for this problem in a manner similar to that for Problem 1. The unit time convention again simplifies expressions for the strain and strain increments, and it also simplifies the presentation of results. Since the joint plane axes align with the global axes, the shear strain \( e_{rm} \) as a function of time is given simply by

\[
e_{rm} = \frac{1}{2} \left( \frac{0.2 t}{l} \right) = 0.01 t.
\]  

There is an initial stress state defined by \( T_{mm} = T_{nn} = -200 \) psi, \( T_{rr} = -500 \) psi, and \( T_{mn} = T_{nr} = T_{rm} = 0 \).

Inelastic slip occurs for this problem when the magnitude of the shear stress, \( T_{mr} \), is equal to \( \mu T_{rr} - C_0 \), which is 600 psi for this problem. The time at which inelastic slip occurs is computed from the expression

\[
T_{rm} = 600 = \frac{2G e_{rm} \mu}{1 + G/(\delta G_s)} = \frac{2G0.01 t}{1 + G/(\delta G_s)},
\]

which is \( t = 0.675 \). At time \( t = 0.675 \), the top of the block has displaced by 0.1350 in. Up to time \( t = 0.675 \), the shear stress increment \( \Delta T_{rm} \) is 600 psi and the slip displacement increment, \( \Delta u_{s,m} \), is \( 6 \times 10^{-3} \) in. From time \( t = 0.675 \) to time \( t = 1 \), the shear strain increment is 0.00325. Equations (53) (with \( G_s' \) substituted for \( G_s \)) and (73) can be used.
to compute the shear stress increment and joint displacement increment, respectively, for the load increment corresponding to inelastic slip. For the inelastic slip phase, the shear stress increment $\Delta T_{rm}$ is 3.2459 psi and the slip displacement increment is $3.2459 \times 10^{-3}$ in. The total shear stress and slip displacements are, therefore, 603.2459 psi and $9.2459 \times 10^{-3}$ in., respectively, at time $t = 1$.

The same mesh used to model the first problem has also been used for the fourth test problem. Figure 13 shows the shear stress, $T_{rm}$, as a function of displacement in the $m$ direction at $r = 10$ in., and Figure 14 shows the slip displacement, $u_{s,m}$, as a function of displacement in the $m$ direction at $r = 10$ in. The results in Figures 13 and 14 are for an interior element. The finite element results agree with the analytic calculations. The graphical results indicate that the transition from elastic to inelastic behavior occurs at or near the time predicted by the analytic results. Obtaining a detailed print of calculations that shows the exact transition from elastic to inelastic behavior is a difficult process. The best means of verification of the time for the transition of elastic to inelastic behavior is the graphical output. The best means of verifying the overall accuracy of the computational model is to compare results with the analytic calculations at time $t = 1$. At time $t = 1$, $T_{rm}$ for an interior element has a value of 603 psi. This agrees with the analytic results for $T_{rm}$ to within 0.05%. At time $t = 1$, $u_{s,m}$ for an interior element has a value of $9.2505 \times 10^{-3}$ in., which agrees with the analytic results to within 0.05%.

The fifth test problem is a simple variation of the fourth test problem. Everything is essentially the same except for the fact that the pure shear of the block leads to displacement in the $n$ direction as opposed to the $m$ direction. The values computed for the fourth problem become applicable to $T_{nr}$ and $u_{s,n}$. The time histories for $T_{nr}$ and $u_{s,n}$ for an interior element are shown in Figures 15 and 16, respectively. They are similar to the time histories for $T_{rm}$ and $u_{s,m}$ for problem four, which is to be expected. The graphical output for $T_{nr}$ and $u_{s,n}$ is again the best means of verifying the time of transition from elastic to inelastic behavior. The value for $T_{nr}$ for an interior element is 603 psi at time $t = 1$. This agrees with the analytic results for $T_{nr}$ to within 0.05%. The value for $u_{s,n}$ for an interior element at time $t = 1$ is $9.2505 \times 10^{-3}$ in., which agrees with the analytic results to within 0.05%.

**Conclusion**

A computational model has been developed for three-dimensional jointed media with single set of joints. The computational model uses a continuum approach; that is, it captures the gross response of jointed rock by distributing the individual responses of the joints throughout the rock mass. This particular model is useful for modeling a large block of material where there are fairly uniformly distributed joint planes lying parallel to one another. It is also useful for modeling situations where there are two large blocks of
Figure 13. Shear Stress $T_{mr}$ as a Function of the Displacement of the Top Surface in the $x$ Direction.
Figure 14. Joint Slip in the $x$ Direction, $u_{5,m}$, as a Function of the Displacement of the Top Surface in the $x$ Direction.
Figure 15. Shear Stress $T_{nr}$ as a Function of the Displacement of the Top Surface in the $y$ Direction.
Figure 16. Joint slip in the $y$ Direction, $u_{s,n}$, as a Function of the Displacement of the Top Surface in the $y$ Direction.
material separated by a layer of material with a finite thickness that is relatively weak compared to the surrounding blocks of material.

The strain-stress relations for the single set jointed rock model are given in a rate form because of the nonlinear behavior of the joints. Under the assumption of a constant strain rate over some time, \( \Delta t \), it is possible to integrate the normal strain rate versus normal stress rate relations and obtain values for the normal stresses from simple algebraic equations. The concepts used for two-dimensional continuum jointed rock models translate easily to three dimensions when it comes to the normal stress calculations. Once the normal stresses are known, it is possible to begin the computation of the shear stresses. The normal stresses appear only in the Mohr-Coulomb equations used to determine whether the shear behavior is elastic or inelastic.

The computation of the shear stress increments in three dimensions becomes a more involved process than that in two dimensions. The main reason for this being that the concept of slip on the joint plane is more complicated in three dimensions than in two dimensions. In two dimensions, slip occurs along a line in a plane. It can be treated, for all practical purposes, as a scalar quantity. The sign on the slip indicates the direction of slip along the line, and slip will occur either in the positive direction or negative direction. In three dimensions, the slip is really a vector quantity lying in the joint plane. This more complex nature of slip in three dimensions complicates the computation of shear stress increments and forces the examination of certain issues that do not arise in two dimensions.

The shear stress rate versus shear strain rate suggest that the two shear stress increments in the joint plane could be calculated independently. This is really not a desirable approach, however, since the behavior of the system becomes tied to an arbitrarily oriented coordinate system in the joint plane. It becomes necessary to consider some scheme for determining a slip direction. Although it might appear that some type of stress-based criterion for determining the direction is slip is a reasonable approach, trying to implement such a scheme presents problems. A strain-based criterion for determining the direction of slip offers a much better scheme for determining slip direction.

Once the issue of direction of slip is addressed, some definition of the total slip must be determined in order to calculate an effective yield stress that determines whether or not elastic or inelastic behavior will occur. The definition of total slip becomes more complicated in three dimensions than in two dimensions simply because the concept of slip in general is more complex in three dimensions. For the computational model discussed in this paper, the definition of total slip was tied to the procedure used to calculate the direction of the slip on the joint plane for the shear stress increments. There is no particular theoretical basis for this choice of the definition of total slip outside of the fact it is tied to the scheme used to determine the direction of slip. The concept of total slip is one that may require further study.

Although the single set jointed rock model represents a more restricted model in terms of the number of joint planes used in the two-dimensional model discussed in Reference 5, it does serve the purpose of indicating the difficulties of translating certain well understood
concepts in two dimensions to three dimensions. It is probably indicative of the effort that will be required and the difficulties that will be encountered in the development of extensive three-dimensional geomechanics capabilities.

The five test problems presented in this report represent a reasonable minimum set for testing the model. The first sample problem verifies the joint normal behavior and its influences on the other two normal stresses. Sample problems 2 and 3 verify that the model is properly implemented in terms of the specification for the joint set rotation and the coupling of the normal and shear calculations via the use of the joint normal stress for determining the yield stress on the joint plane. Test problems 4 and 5 isolate the behavior of the shear stress versus slip displacement description so that its implementation can be easily verified. This set of problems again points out the complexity of moving from two to three dimensions. For sample problems 2 and 3, the graphical output of most interest \((\sigma_{zz} \text{ as a function of } u_r\big|_{r = 10})\) is the same for both problems. Both problems are required, however, for a thorough check of the model. Although test problem 4 is virtually identical to test problem 5 except for the fact that the critical motion is at right angles to each other for the two problems, both problems are necessary for a thorough check. All of the above test problems can be verified by analytic results. Test problems 2 and 3 are best verified by comparing graphical results from numerical calculations with analytic results. For test problems 1, 4, and 5, it is possible to make direct comparisons of some key values in the numerical calculations with analytic results. These direct comparisons show a high degree of accuracy for the computational model. Differences in numerical and analytic results for test problems 1, 4, and 5 are no greater than 0.05%.

References


APPENDIX

Information from the Reference Information Base
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