NUMERICAL ALGORITHMS AT NON- ZERO CHEMICAL POTENTIAL

April 27–May 1, 1999

Organizers

Tom Blum and Michael Creutz

RIKEN BNL Research Center

Building 510, Brookhaven National Laboratory, Upton, NY 11973, USA

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Preface to the Series

The RIKEN BNL Research Center was established this April at Brookhaven National Laboratory. It is funded by the “Rikagaku Kenkyusho” (Institute of Physical and Chemical Research) of Japan. The Center is dedicated to the study of strong interactions, including hard QCD/spin physics, lattice QCD and RHIC physics through nurturing of a new generation of young physicists.

For the first year, the Center will have only a Theory Group, with an Experimental Group to be structured later. The Theory Group will consist of about 12-15 Postdocs and Fellows, and plans to have an active Visiting Scientist program. A 0.6 teraflop parallel processor will be completed at the Center by the end of this year. In addition, the Center organizes workshops centered on specific problems in strong interactions.

Each workshop speaker is encouraged to select a few of the most important transparencies from his or her presentation, accompanied by a page of explanation. This material is collected at the end of the workshop by the organizer to form a proceedings, which can therefore be available within a short time.

T.D. Lee
July 4, 1997

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Numerical Algorithms at Non-Zero Chemical Potential

May 19, 1999

The RIKEN BNL Research Center hosted its 19th workshop April 27th through May 1, 1999. The topic was Numerical Algorithms at Non-Zero Chemical Potential.

QCD at a non-zero chemical potential (non-zero density) poses a long-standing unsolved challenge for lattice gauge theory. Indeed, it is the primary unresolved issue in the fundamental formulation of lattice gauge theory. The chemical potential renders conventional lattice actions complex, practically excluding the usual Monte Carlo techniques which rely on a positive definite measure for the partition function. This “sign” problem appears in a wide range of physical systems, ranging from strongly coupled electronic systems to QCD.

The lack of a viable numerical technique at non-zero density is particularly acute since new exotic “color superconducting” phases of quark matter have recently been predicted in model calculations. A first principles confirmation of the phase diagram is desirable since experimental verification is not expected soon.

At the workshop several proposals for new algorithms were made: cluster algorithms (Chandrasekharan, Wiese, and Brower), direct simulation of Grassman variables (Creutz), and a bosonization of the fermion determinant (Slavnov). All generated considerable discussion and seem worthy of continued investigation.

Several interesting results using conventional algorithms were also presented: condensates in four fermion models (Hands, Morrison), SU(2) gauge theory in fundamental and adjoint representations (Hands, Morrison), and lessons learned from strong coupling, non-zero temperature (Lombardo) and

heavy quarks (Laliena, Galante) applied to non-zero density simulations.

Azcoiti presented a stimulating discussion of how the Vafa-Witten theorem does not apply to QCD at finite density, and Rebbi presented recent results for the free energy of an SU(2) monopole-antimonopole pair.

We wish to express our sincere gratitude to Pam Esposito for her invaluable help with the logistics in organizing this workshop. We also wish to thank the participants for sharing their interesting ideas and providing unusually stimulating discussion, the central goal of the workshop.

–Tom Blum and Mike Creutz

Solution to Sign Problems using Cluster Algorithms

Shailesh Chandrasekharan¹

(Duke University)

In order to solve any sign problem, one has to be clever in choosing the right basis to expand the partition function. In other words one needs to find the right set of variables in which to write the path integral and observables. One systematic method is to regroup configurations in the old variable, such that a whole class can be averaged analytically. Once this is done the partition function and observables can be rewritten in terms of a new set of variables. In certain cases, if one is clever and perhaps fortunate, it can be shown that the average sign in these new variables is always positive.

Recently a new class of solutions to sign problems has emerged[1] with these ideas. The common step is to use cluster algorithms[2] in order to rewrite the path integral in terms of new variables. Typically, each cluster configuration has N_l clusters each with two orientations such that there are 2^{N_l} configurations that are degenerate. It is possible to find the sign of each of the 2^{N_l} configurations. If

1. The clusters contribute independently to the sign of the given configuration and
2. there is always at least one configuration among the 2^{N_l} cluster orientations that is positive.

it is possible to solve the sign problem completely. The clusters that change the sign of a configuration when they are flipped are referred to as meron clusters. If the above two properties are satisfied then it can be shown that the average sign is 1 only if there are no meron clusters in the configuration. Otherwise it is always 0.

In the context of fermions, the origin of the sign of a cluster in the background of other cluster orientations was understood recently. The sign depends the cluster topology and the number of times it hops over a fermion that belongs to another cluster. It is possible to avoid clusters that hop over fermions of other clusters in a class of models. The sign of the cluster then depend only on its topology and independent of the back ground orientations and satisfies the above two properties. Certain Z_2 fermionic gauge theories also fall in this class of models.

It is also likely that such methods are applicable to gauge theories in the Electric flux basis, where cluster algorithms seem plausible[3].

References

- [1] Bietenholz et. al., PRL75, 4524 (1995); S. Chandrasekharan and U.-J. Wiese cond-mat/9902128; see contributions by U.-J. Wiese to these proceedings.
- [2] F. Niedermayer, hep-lat/9704009 and references therein.
- [3] B. Beard et. al., hep-lat/9709120, Nucl.Phys.Proc.Suppl. 63 (1998) 775-789.

¹Work done in collaboration with Uwe-Jens Wiese(MIT)

SOLUTION STRATEGY

Regroup the configurations such that

(i) it is possible to average over each group analytically.

(ii) the weight of each group is always positive or zero.

$$\begin{aligned}
 Z &= \sum_{\{n\}} \exp(-S[n]) \text{Sign}[n] \\
 &= \sum_{\{l\}} W[l] d[l] \overline{\text{Sign}[l]}
 \end{aligned}$$

New variables $\nearrow$ $\{l\}$
 degeneracy factor up to signs $\nearrow$ $d[l]$
 Average sign $\nwarrow$ $\overline{\text{Sign}[l]}$
 $\overline{\text{Sign}[l]} = \left(\frac{1}{d[l]} \sum_{\{n \in \text{group}\}} \text{sign}[n] \right)$

We use cluster Algorithm techniques to accomplish this:

Each $\{l\}$ configuration has N_l clusters, each with two orientations.

Consider N_e clusters with two orientations
and assume they contribute indep. to Sign.

$$\text{Sign}[L] = \text{Sign}_s^1 \text{Sign}_s^2 \dots \text{Sign}_s^{N_e}$$

s : orientation of the object
= + or -

Two possibilities

$$\text{Sign}_+^k = - \text{Sign}_-^k \quad (\text{Meron cluster})$$

or

$$\text{Sign}_+^k = \text{Sign}_-^k \quad (\text{Non-Meron cluster})$$

2^{N_e} Conf: At least 1 reference conf is +ve

$$\overline{\text{Sign}[L]} = \frac{1}{2^{N_e}} \sum_{\{\text{all orientations}\}} \text{Sign}[L]$$

$$= 0 \quad (\text{If at least one object is a Meron})$$

$$= 1 \quad \text{otherwise.}$$

In such a case we can solve the Sign
problem completely!

Model:

$$H = \sum_{\langle xy \rangle} \left[-\frac{t}{2} (c_x^\dagger c_y + c_y^\dagger c_x) \eta_{xy} + U (n_x - \frac{1}{2})(n_y - \frac{1}{2}) \right]$$

$c_x^\dagger$: Fermion Creation Operator

c_y : Fermion Annihilation Operator

$n_x = c_x^\dagger c_x$: Fermion Number Operator.

$\eta_{xy} = \pm 1$ factors that can be a part of a $\mathbb{Z}_2$ gauge field.

↑
Can mimic staggered fermion phase factors.

→ Closely related to Spin $1/2$ quantum spin model.

$$c_x^\dagger \rightarrow S_x^+; \quad c_x \rightarrow S_x^-; \quad n_x \rightarrow S_x^3$$

$$\text{and } t=U: \quad H \rightarrow U \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$$

→ In the fermion occupation # basis it is possible to formulate a cluster algorithm without taking into account the sign due to fermion exchange.

For $U \geq |t| > 0$ we can solve the sign problem completely!

$$W[L] = \omega_1^5 \omega_2^2 \omega_3^5 2^{N_L} \overline{\text{Sign}[e]}$$

↑
Average of
8 spin
Conf.

Cluster Sign vs. Topology

The sign change due to a flip of a given cluster of spins in the fixed background of other spins is given by a sign factor S_c

$$S_c = 2 * \text{Mod} (N_T + N_{=}/2 + N_4 + N_h, 2) - 1$$

N_T : # of times the cluster touches a particular time slice

$N_{=}$: # of spin flip hops

N_4 : # of four same spin interactions such that all the spins belong to the cluster

N_h : # of hops over spin up when the spins do not belong to the cluster

Staggered Susceptibility

$$\chi = \frac{1}{V\beta} \left\langle \left[\bar{\epsilon} \sum_x (-1)^x (n_x - \frac{1}{2}) \right] \left[\epsilon \sum_y (-1)^y (n_y - \frac{1}{2}) \right] \right\rangle$$

$$= \frac{1}{V\beta} \left\langle \sum_c |O_c|^2 \delta_{N,0} + 2 |O_{c_1}| |O_{c_2}| \delta_{N,2} \right\rangle$$

$\underbrace{\hspace{10em}}_{\text{Zero Meson Sectors}} \quad \underbrace{\hspace{10em}}_{\text{2 Meson Sector}}$

$\langle \delta_{N,0} \rangle$

$|O_c| \equiv \text{Size of the cluster } c$

Results:

L	βU	$\langle \text{sign} \rangle(A_1)$	$\chi(A_1)$	$\chi(A_2)$
6	1.0	0.696(1)	13.44(2)	13.43(2)
8	1.0	0.536(3)	13.53(3)	13.52(3)
8	2.0	0.0164(5)	199(3)	203(2)
8	4.0	0.00051(7)	690(100)	729(9)
12	8.0	—	—	3110(12)

Merger-Cluster Algorithm for the 2-d $O(3)$ Model at non-zero chemical potential

U.-J. Wiese (MIT)

- Merger-Cluster algorithms and the nature of the sign problem
- Quantum antiferromagnets as a D-theory formulation of the 2-d $O(3)$ model
- Cluster algorithm for the Heisenberg model
- Cluster algorithm in an external magnetic field
- Cluster algorithm at non-zero chemical potential
- D-theory formulation of QCD

Systems with a Sign Problem

- Fermions in $d > 1$

- QCD at $\mu \neq 0$:

- Hubbard model at $\mu \neq 0$:

- Antiferromagnets at $B \neq 0$

- 2-d $O(3)$ -model at $\mu \neq 0$

- 2-d $O(3)$ -model at $\theta \neq 0$:

A. Pochinsky, W. Bietenholz and U.-J.W.,
Phys. Rev. Lett. 75 (1995) 4524

Partition function for a θ -vacuum :

$$Z(\theta) = \sum_n \exp(-S[n]) \exp(i\theta Q[n])$$

Topological charge is a sum over clusters :

$$Q[n] = \sum_c Q_c[n], \text{ for merons: } Q_c[n] = \pm \frac{1}{2}, \pm \frac{3}{2}, \dots$$

$$Z(\pi) = \sum_n \exp(-S[n]) (-1)^{Q[n]}$$

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receives contributions from configurations without merons only.

Nature of the Sign Problem

Fermionic partition function:

$$Z_f = \sum_n \text{Sign}[n] \exp(-S[n])$$

Fermionic observable:

$$\langle O \rangle_f = \frac{1}{Z_f} \sum_n O[n] \text{Sign}[n] \exp(-S[n]) = \frac{\langle O \text{Sign} \rangle}{\langle \text{Sign} \rangle}$$

Sign is exponentially small:

$$\langle \text{Sign} \rangle = \frac{1}{Z} \sum_n \text{Sign}[n] \exp(-S[n]) = \frac{Z_f}{Z} = \exp(-\beta V \Delta f)$$

Statistical error of $\text{Sign} = \pm 1$:

$$\frac{\Delta \text{Sign}}{\langle \text{Sign} \rangle} = \frac{\sqrt{\langle \text{Sign}^2 \rangle - \langle \text{Sign} \rangle^2}}{\sqrt{N} \langle \text{Sign} \rangle} = \frac{\exp(\beta V \Delta f)}{\sqrt{N}}$$

To accurately measure $\langle O \rangle_f$ one needs $N \sim \exp(2\beta V \Delta f)$ statistically independent field configurations.

If one can match every -1 with a 1, such that effectively $\text{Sign} = 0, 1$, the error reduces to

$$\frac{\Delta \text{Sign}}{\langle \text{Sign} \rangle} = \frac{\sqrt{\langle \text{Sign} \rangle - \langle \text{Sign} \rangle^2}}{\sqrt{N'} \langle \text{Sign} \rangle} = \frac{\exp(\beta V \Delta f / 2)}{\sqrt{N'}}$$

This solves half of the sign problem because one only needs $N' \sim \exp(\beta V \Delta f) = \sqrt{N}$ configurations.

Classical Spins

Classical spin
variable: $\vec{e}_x \in S^2$

Classical
Hamilton
function: $\mathcal{H}[\vec{e}] = - \sum_{\langle xy \rangle} \vec{e}_x \cdot \vec{e}_y$

$O(3)$ rotation
invariance: $\mathcal{H}[R\vec{e}] = \mathcal{H}[\vec{e}]$

Classical
partition
function: $Z = \int D\vec{e} \exp(-\beta \mathcal{H}[\vec{e}])$

2-d
spatial
lattice

Quantum Spins

Quantum spin
operator: $\vec{S}_x$
 $[S_x^i, S_y^j] = i \delta_{xy} \epsilon_{ijk} S_z^k$

Quantum
Hamilton
operator: $H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$

$O(3)$ rotation
invariance: $[H, \vec{S}] = 0$,
 $\vec{S} = \sum_x \vec{S}_x$

Quantum
partition
function: $Z = \text{Tr} \exp(-\beta H)$

2-d spatial lattice

1-d
Euclidean
time continuum

Dimensional reduction in slab geometry

Effective action of the dimensionally reduced 2-d classical $O(3)$ mod

$$S[\vec{\phi}] = \int_0^\beta dt \int d\vec{x} \left[\frac{1}{2} \dot{\vec{\phi}}^2 + \frac{1}{2} \nabla^2 \vec{\phi}^2 + \frac{1}{2} \nabla^2 \vec{\phi}^2 + \frac{1}{2} \nabla^2 \vec{\phi}^2 \right]$$

Effective coupling constant in 2-d : $\frac{1}{g^2} = \frac{1}{g^2} + \frac{1}{g^2}$

Mermin-Wagner-Coleman: no interacting Goldstone bosons in 2-d

$$\chi_{\text{H}_2\text{N}_2} \text{ formula: } \chi = \frac{1}{4\pi\epsilon_0} \exp \left(\frac{1}{4\pi\epsilon_0} \frac{1}{g^2} \right) + \frac{1}{4\pi\epsilon_0} \frac{1}{g^2}$$

S. Chakravarty, B.I. Halperin and D.E. Nelson, Phys. Rev. B39 (1989) 1234

E. Hasslerfratz and F. Niedermayer, Phys. Lett. B269 (1991) 121. 14

2-d $O(3)$ -model at $\mu \neq 0$

2-d target theory: $\partial_2 \vec{e} \rightarrow D_2 \vec{e} = \partial_2 \vec{e} + i \vec{\mu} \times \vec{e}$

$$S[\vec{e}] = \frac{\beta g_s}{2} \int d^2 x \left[\partial_1 \vec{e} \cdot \partial_1 \vec{e} + \partial_2 \vec{e} \cdot \partial_2 \vec{e} + \right. \\ \left. + 2i \partial_2 \vec{e} \cdot (\vec{\mu} \times \vec{e}) - \mu^2 + (\vec{\mu} \cdot \vec{e})^2 \right]$$

Effective (2+1)-d theory:

$$S[\vec{e}] = \int_0^\beta dt \int d^2 x \frac{g_s}{2} \left[\frac{1}{c^2} \partial_t \vec{e} \cdot \partial_t \vec{e} + \partial_\mu \vec{e} \cdot \partial_\mu \vec{e} + \right. \\ \left. + 2i \partial_2 \vec{e} \cdot (\vec{\mu} \times \vec{e}) - \mu^2 + (\vec{\mu} \cdot \vec{e})^2 \right]$$

Microscopic Hamiltonian:

$$H = J \sum_{\langle xy \rangle} \left[\frac{1}{2} (S_x^+ S_y^- e^{i\mu} + S_x^- S_y^+ e^{-i\mu}) + S_x^3 S_y^3 \right], \quad \vec{\mu} = (0, 0, \mu)$$

When one chooses $S_x^3 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ the cluster algorithm becomes very inefficient, because clusters that wrap around a "spatial" direction can not always be flipped.

When we choose $S_x^3 = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ loop clusters can break into open strings. All clusters can always be flipped, but there is a sign problem. Again, this problem can be solved with the meron cluster algorithm.

Equilibration of Cluster Algorithms

Old algorithm : Cluster algorithm with accept-reject step

New algorithm : Metron-cluster algorithm with sign problem

$$\beta = 10, \quad B = 0.75, \quad g^2 = 40$$

Outlook

The meron concept is universal but must be constructed for each individual case.

Guaranteed to work:

- Nonrelativistic fermions with short-range repulsion
- Relativistic staggered fermions with 4-fermi coupling
- Quantum magnets in an arbitrary magnetic field
- $O(3)$ model at $\mu \neq 0$

Perhaps doable and in progress:

- The Hubbard model
- Domain Wall fermions with 4-fermi coupling

When that works out we will try:

- QCD at $\mu \neq 0$

Transfer matrices at finite density

Michael Creutz

Hamiltonian uses
creation and annihilation operators

$$[a, a^\dagger]_+ \equiv aa^\dagger + a^\dagger a = 1$$

Path integral uses Grassmann variables

$$[\chi, \chi^\dagger]_+ = 0$$

Related via the transfer matrix.

Example Hamiltonian

$$H = m(a^\dagger a + b^\dagger b) + \mu(a^\dagger a - b^\dagger b)$$

“mass” m

“chemical potential” μ

Want exact path integral for

$$Z = \text{Tr } e^{-\beta H}$$

Grassmann variables

$$\begin{aligned}\chi, \chi^* &\leftrightarrow a, a^\dagger \\ \xi, \xi^* &\leftrightarrow b, b^\dagger\end{aligned}$$

$$\int d\chi \, 1 = 0; \quad \int d\chi \, \chi = 1$$

Spinors

$$\psi = \begin{pmatrix} \chi \\ \xi^\dagger \end{pmatrix}; \quad \psi^\dagger = (\chi^\dagger \quad \xi)$$

“Dirac matrix”

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\bar{\psi} \equiv \psi^\dagger \gamma_0$$

Then

$$\bar{\psi} \psi = \chi^\dagger \chi + \xi^\dagger \xi$$

Wilson projection operators

$$P_\pm = \frac{1}{2}(1 \pm \gamma_0)$$

arise in hopping terms

$$\chi_i^\dagger \chi_j + \xi_i^\dagger \xi_j = \bar{\psi}_i P_+ \psi_j + \bar{\psi}_j P_- \psi_i$$

Particles only:

$$\text{Tr} : f(a, a^\dagger) := \int d\chi d\chi^\dagger e^{2\chi^\dagger \chi} f(\chi, \chi^\dagger)$$

generalizes to

$$\begin{aligned} \text{Tr} : f_1(a^\dagger, a) : : f_2(a^\dagger, a) : \dots : f_n(a^\dagger, a) : \\ = \int d\psi_1 d\psi_1^* \dots d\psi_n d\psi_n^* \\ f_1(\psi_1^*, \psi_1) \dots f_n(\psi_n^*, \psi_n) \\ \exp \sum \psi_t^* (\psi_t - \psi_{t-1}) \Big|_{\psi_0 = -\psi_n} \end{aligned}$$

Normal ordered exponentials:

$$\begin{aligned} e^{\alpha a^\dagger a} \\ = 1 + (e^\alpha - 1) a^\dagger a \\ =: e^{(e^\alpha - 1) a^\dagger a} : \end{aligned}$$

Combining particles and antiparticles:

$$\begin{aligned} \exp((\alpha + \rho) a^\dagger a + (\alpha - \rho) b^\dagger b) \\ = : \exp \left(\begin{pmatrix} a^\dagger & b^\dagger \end{pmatrix} (e^{\alpha + \rho \gamma_0} - 1) \begin{pmatrix} a \\ b \end{pmatrix} \right) : \end{aligned}$$

Time slices

$$Z = \text{Tr} \left(e^{-\beta H/N} \right)^N$$

Final result

$$Z = \int (d\psi d\psi^\dagger) e^S$$

with

$$\begin{aligned} S = \sum \bar{\psi}_t e^{-\beta m/N} e^{-\beta \mu \gamma_0/N} \psi_t \\ - \bar{\psi}_t P_+ \psi_{t-1} - \bar{\psi}_{t-1} P_- \psi_t \end{aligned}$$

Chemical potential inserts

$$e^{-\beta \mu \gamma_0/N}$$

in “mass” term.

$$S = \bar{\psi} M \psi$$

M not symmetric

Background gauge fields: $|M|$ not real

Alternative form

$$\begin{aligned}\Psi &\equiv e^{\frac{-\beta\mu\gamma_0}{2N}}\psi \\ \mathcal{S} &= \sum_{i=1}^n \bar{\Psi}_n e^{-\beta m/N} \Psi_n \\ &\quad - \bar{\Psi}_n P_+ e^{\beta\mu/N} \Psi_{n-1} \\ &\quad - \bar{\Psi}_{n-1} P_- e^{-\beta\mu/N} \Psi_n\end{aligned}$$

“Imaginary” $U(1)$ temporal gauge field

Spatial hoppings

$$\bar{\psi} \vec{D} \cdot \vec{\gamma} \psi,$$

unchanged: γ_0 anticommutes with $\vec{\gamma}$

Spatial Wilson term not invariant

“irrelevant”

Summary

Wilson projection operators natural

Chemical potential two ways:

1. $e^{\mu\gamma_0}$ in mass term
 2. $e^{\pm\mu}$ in forward/backward hopping
-

Order of operations?

$Z = \text{Tr} e^{-\beta H}$ is real and positive

Mix gauge and fermion integration?

$$\begin{aligned} \text{Tr} : f_1(b^\dagger, b) : : f_2(b^\dagger, b) : \dots : f_n(b^\dagger, b) : \\ = \int d\psi_1 d\psi_1^* \dots d\psi_n d\psi_n^* \\ f_1(\psi_1^*, \psi_1) \dots f_n(\psi_n^*, \psi_n) \\ \exp \sum \psi_t^* (\psi_t - \psi_{t-1}) \Big|_{\psi_0 = -\psi_n} \end{aligned}$$

- antiperiodic "time" boundaries $\psi_n = -\psi_0$
- "one sided derivative" $\psi_t^* (\psi_t - \psi_{t-1})$
- analogue of Wilson projection operator $1 + \gamma_\mu$

M. Creutz, Phys. Rev. D35, 1460 (1987)

Want multiple integrals

- $Z = \int (d\psi) e^{S(\psi)}$
- expand exponent
- keep terms with one factor of each ψ

Convert to operator manipulations

- introduce a different Fock space ($3d \rightarrow 4d$)
- $[a_i, a_j^\dagger]_+ = \delta_{ij}$
- one a_i for each ψ_i
- $a_i |0\rangle = 0$

Define the "full" state

- $|F\rangle \equiv a_n^\dagger \dots a_2^\dagger a_1^\dagger |0\rangle$

$$Z = \langle 0 | e^{S(a)} | F \rangle$$

- expand e^S
- one factor of a_i for each Fermion
- same rule as for Grassmann integration

Integrate sequentially

- pick one variable a_i
- define S_i as all terms involving a_i
- $\tilde{S}_i$ everything else
- $S = S_i + \tilde{S}_i$
- assume bosonic action so S_i and $\tilde{S}_i$ commute

$$Z = \langle 0 | e^{\tilde{S}_i} e^{S_i} | F \rangle$$

$\tilde{S}_i$ contains no a_i

- $\langle 0 | e^{\tilde{S}_i} n_i = 0$
- $n_i = a_i^\dagger a_i$
- insert projection operator $1 - n_i$

$$Z = \langle 0 | e^{\tilde{S}_i} (1 - n_i) e^{S_i} | F \rangle$$

$$a_i^2 = 0$$

- S_i is linear in a_i
- $e^{S_i} = 1 + S_i$

$$Z = \langle 0 | e^{\tilde{S}_i} (1 - n_i) (1 + S_i) | F \rangle$$

$1 - n_i$ is a projection

- $a_i (1 - n_i) = 0$
- $S_i (1 - n_i) = 0$
- $\tilde{S}_i (1 - n_i) = S_i (1 - n_i)$
- replace $\tilde{S}_i$ with full action S

$$Z = \langle 0 | e^S (1 - n_i) (1 + S_i) | F \rangle$$

Repeat for all sites

$$Z = \langle 0 | \prod_i (1 - n_i) (1 + S_i) | F \rangle$$

Algorithm:

- initialize with full state
- loop over sites:
 - apply $1 + S_i$
 - apply $1 - n_i$
 - store results in hash table
- return coefficient of empty state

With a local interaction

- finished sites empty
- new sites out of range filled
- nontrivial only in transverse volume
- sparse state space ideal for hash storage

Non-zero Density at Non-zero Temperature

Maria-Paola Lombardo

Istituto Nazionale di Fisica Nucleare,
Laboratori Nazionali del Gran Sasso, I-67010 Assergi (AQ)

I consider the two main problems encountered in non-zero density lattice QCD calculations 1) poor efficiency of reweighting – which causes the disappearance of physical phase transitions 2) “baryonic pions” – which cause the appearance of UNphysical phase transitions, and try to find dynamical situations where such problems might be lessened. In particular I discuss imaginary chemical potential and reweighting in two simplified, interrelated models: four dimensional QCD in the infinite coupling limit, and one dimensional QCD. By using mean field results and numerical simulations I study the performance of these methods as a function of temperature, and find that both of them are likely to be successful at high temperature. I argue that the information from these studies together with lattice results for the quark number susceptibility might be used to assess the candidate successful region in real QCD for imaginary chemical potential/reweighting calculations – apparently, a rather narrow window around T_c . I comment on the cosmological and experimental interest of these $T \simeq T_c$, small μ calculations.

Methods and Problems

- Analytic continuation from $\mu=0$
 - Ensemble does not overlap with the non-zero density state of interest
 - Physical Transitions may disappear
- Inclusion of conjugate quarks
 - Model contains unwanted particles with baryonic number (baryonic pions)
 - Unphysical Transitions may appear

$\text{Im} \mu$ at $T \approx 0$: almost no μ dependence
 $T \approx 0$

$\text{Im} \mu$ at high T :

$T = 220 \text{ MeV}$: NON TRIVIAL μ DEPENDENCE

- Rule of thumb for
Imaginary μ

$$\underbrace{\frac{2\pi}{3N_T}}_{\text{Periodicity}} > \underbrace{\mu_c \approx \frac{m_B}{3}}_{\text{Physical scale}}$$

seems satisfied only $T \lesssim T_c$

- Criterion: (once "detuned")

$$\frac{\partial m(\mu)}{\partial \mu} = \frac{\partial^2 \log(z)}{\partial \mu^2} \gg 0 \quad \parallel \text{quark number susceptibility}$$

• For Im μ

$$\partial m(\mu) = - \partial m(\mu) \text{ Real } \mu$$

Summary: -

At high T ($T \gtrsim T_c$)

- Indications that imaginary μ and reweighting "work"
Important to have two methods for cross checks
- It can be speculated/hoped that unphysical condensates melt below T_c [N.B T_c dipole condensation sensitive to details of the dynamics]

Checks:

- Work in a region where quark number susceptibility $\neq 0$
 - Compute QNS with reweighting QNS_2
 - Compute QNS with imaginary $\mu \rightarrow QNS_1 = -QNS_2$
- If
o.k
then
Topology
Spectrum?
- [
- Try quenched/modulus slight above, around T_c :
Stepan-Boltzmann? How many P.O.F.?

Non working ideas in finite density QCD

Vicente Azcoiti

Abstract

I analyse and discuss in which cases the Vafa-Witten theorem on vector symmetries in vector-like theories can be extended to complex actions.

NON WORKING IDEAS IN FINITE DENSITY QCD

0.- P and CT (V. Azcoiti, A. Galante
HEP-TH/9901068)

1.- VAFA-WITTEN THEOREM

2.- COMPLEX ACTIONS

3.- QUARK PROPAGATOR IN A
BACKGROUND GAUGE FIELD

4.- SUMMARY

If $|X(A)| < C \quad \forall A \text{ gauge field}$

$$|\langle X(A) \rangle| \leq \frac{\int [dA] e^{-S(A)} |X(A)|}{\int [dA] e^{-S(A)}} < C$$

$J(x)$ any operator with non zero
isospin or baryon number

(for instance $\bar{q} \gamma_\mu \vec{\tau} q(x)$ the isospin
current)

$$C(x, A) = \langle J(x) J^*(0) \rangle_A$$

correlation function in the background

$$\Delta(A) = m I + i \Lambda(A)$$

$i \Lambda(A)$ massless antihermitian
Dirac operator

$$\Delta^{-1}(x|0, A) = \langle x | \Delta^{-1}(A) | 0 \rangle =$$

$$= \int_0^\infty dz e^{-mz} \langle x | e^{-i z \Lambda(A)} | 0 \rangle$$

LOCALITY

UNITARITY

In case i. (θ vacuum) the massless Dirac operator at any gauge field configuration $i\Lambda(A)$ does not depend on θ .

$\Rightarrow$ Vafa-Witten bounds hold for " c ", or more rigorously

$$P(x, c) = 0 \quad \text{if} \quad |c| \geq A e^{-Bx}$$

$$\Rightarrow |\langle J(x) J^*(0) \rangle| < A e^{-Bx}$$

VECTOR SYMMETRIES CAN NOT BE SPONTANEOUSLY BROKEN

What about chemical potential?

3. QUARK PROPAGATOR IN A BACKGROUND GAUGE FIELD

The massless Dirac operator $i\Lambda(A, \mu)$ at finite chemical potential μ is no longer an antihermitian operator

$$\Lambda^\dagger(A, \mu) = \Lambda(A, -\mu)$$

$\therefore$ Vafe-Witten procedure to get bounds for the quark propagator breaks-down in this case

What can we learn about Dense Matter using Standard Algorithms?

Simon Hands (University of Wales Swansea)

The talk reviewed simulations of dense baryonic matter which are feasible using existing techniques, namely Hybrid Monte Carlo, Hybrid Molecular Dynamics, and Multi-Bosonic algorithms. The focus was on finding signals for condensation of diquark pairs from the vicinity of the Fermi surface at high density, leading to either a superfluid or superconducting ground state. Simulations using conventional algorithms generically contain both quarks q and conjugate quarks q^c , and will not reproduce realistic physics if light qq^c composites carrying non-zero baryon number can form. The models simulated to date, namely four-fermion models of the Gross - Neveu - Nambu - Jona-Lasinio type, in which qq^c bound states form at the scale of the constituent quark mass rather than the pion mass, and SU(2) gauge theory with fundamental quarks, in which light qq^c states are physical, both appear to realise the superfluid alternative. Numerical results from the four-fermion model provide evidence for diquark condensation from both 1-point and 2-point correlation functions, although further understanding of both volume effects and spectroscopy is needed. The global symmetries of SU(2) lattice gauge theory with staggered fermions in both fundamental and adjoint representations are analysed (see attached transparencies) – the pattern of symmetry breaking in the presence of either chiral $\langle\bar{q}q\rangle$ or diquark $\langle qq\rangle$ condensates is distinct from that of the continuum theory. Possible diquark condensates and resultant phase diagrams in the (m, μ) plane are outlined (temperature T is assumed zero). For the adjoint case with one staggered flavor a superconducting phase at high density is plausible, although simulations of this model may well be hampered by a sign problem.

Further details are published in:

Phys. Rev. **D59**:116002 (1999)
and hep-lat/9902011, 012, 033, 034

SU(2) Lattice Gauge Theory

The action remains real even once $\mu \neq 0$, so simulations can be made with existing algorithms (Maria, Susan)

Will consider quarks in both fundamental (**2**)
and adjoint (**3**) representations
with $N=1$ staggered lattice fermions

(i) Fundamental

$$S_{\text{kin}} = \sum_{x, \mu} \gamma_{\mu}^{(x)} \left[\bar{\chi}(x) U_{\mu} \chi(x+\hat{\mu}) - \bar{\chi}(x) U_{\mu}^{\dagger} \chi(x-\hat{\mu}) \right]$$

2 manifest global symmetries: $U(1)_V: \chi \mapsto e^{i\alpha} \chi; \bar{\chi} \mapsto \bar{\chi} e^{-i\alpha}$
 $U(1)_A: \chi \mapsto e^{i\alpha \epsilon} \chi; \bar{\chi} \mapsto \bar{\chi} e^{i\alpha \epsilon}$

due to pseudoreality of **2** (ie. $\tau_2 U_{\mu} \tau_2 = U_{\mu}^*$)

can rearrange:

$$S_{\text{kin}} = \sum_{x \text{ even } \mu} \gamma_{\mu}^{(x)} \left[\bar{X}_e(x) U_{\mu} X_o(x+\hat{\mu}) - \bar{X}_e(x) U_{\mu}^{\dagger} X_o(x-\hat{\mu}) \right]$$

with $\bar{X}_e = (\bar{\chi}_e, -\chi_e^{\text{tr}} \tau_2); X_o^{\text{tr}} = (\chi_o^{\text{tr}}, \bar{\chi}_o \tau_2)$

The full global symmetry is now seen to be $U(2)$:

$$X_o \mapsto V X_o; \bar{X}_e \mapsto \bar{X}_e V^{\dagger}$$

The $U(2)$ symmetry is broken spontaneously to $U(1)$ by either

chiral condensate $\langle \bar{\chi} \chi \rangle \neq 0$
 or
 diquark condensate $\langle \chi^T \tau_2 \chi \rangle \neq 0$

} both gauge invariant scalars related by $U(2)$

lattice
 $U(2) \rightarrow U(1)$

cf. continuum $SU(2)$ g.t.
 $SU(2N_f) \rightarrow Sp(2N_f)$
 (Peskin)

Goldstone Count

$$\langle \bar{\chi} \chi \rangle \neq 0$$

$$0^-, 0^+, 0^+$$

$$\langle \chi^T \tau_2 \chi \rangle \neq 0$$

pion, 2 scalar diquarks

pseudoscalar diquark
 scalar diquark
 scalar meson

presence of light diquarks promotes early onset

For $m, \mu \neq 0$ $U(2)$ symmetry explicitly broken

$\langle \chi^T \tau_2 \chi \rangle$ breaks $U(1)_V$ spontaneously
 ⇓ true order parameter

1 exact scalar Goldstone mode

Proposed Phase Diagram

(ii) Adjoint now the **3** is real

$$\Rightarrow S_{\text{kin}} = \sum_{\text{x even } \mu} \frac{g_{\mu}^2(x)}{2} \left[\bar{\Psi}_e(x) U_{\mu}(x) \Psi_o(x+\hat{\mu}) - \bar{\Psi}_e(x) U_{\mu}^{\text{tr}}(x-\hat{\mu}) \Psi_o(x-\hat{\mu}) \right]$$

with $\bar{\Psi}_e = (\bar{\chi}_e, \chi_e^{\text{tr}})$; $\Psi_o^{\text{tr}} = (\chi_o^{\text{tr}}, \bar{\chi}_o)$

The $U(2)$ global symmetry $\Psi \mapsto V\Psi$; $\bar{\Psi} \mapsto \bar{\Psi}V^{\dagger}$

can be broken by a chiral condensate $\langle \bar{\chi} \chi \rangle \neq 0$

$$\left\langle \frac{1}{2} \left[\bar{\Psi}_e \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \bar{\Psi}_e^{\text{tr}} - \Psi_o^{\text{tr}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \Psi_o \right] \right\rangle \neq 0$$

breaks

$$U(2) \longrightarrow Sp(2)$$

Cf. continuum pattern $U(2N_f) \rightarrow O(2N_f)$
(Peschke)

The Goldstone count yields a single O^{-} state:

the pion

For $N_f = 4$

$\Rightarrow$ there should be no early onset
in the chiral limit

(Cf. QCD)

Local Gauge Non-invariant Condensate

$$\chi^{\dagger}(x) \propto t_a \chi(x) \Rightarrow \text{spacetime scalar}$$

↑
generators of 3 are antisymmetric

Globally: $\langle \chi^{\dagger} \chi \rangle \neq 0$ breaks $U(2) \rightarrow U(1)$

Goldstone count: $0, 0^-, 0^-$ pion
2 pseudoscalar diquarks

Locally $\langle \chi^{\dagger} \chi \rangle \neq 0$ breaks $SU(2)_c$ via Higgs mechanism
ie $\langle qq \rangle$ is an adjoint Higgs field
 $\Rightarrow$ superconducting solution

N.B. - there are fermions in the spectrum in both "phases"

Will need new techniques to probe phase diagram

SU(2) Simulations at Nonzero Chemical Potential and Diquark techniques

Susan Morrison (University of Wales Swansea)

I outlined three classes of models amenable to simulations at nonzero chemical potential using currently available lattice algorithms. Models such as Gross-Neveu and Nambu-Jona-Lasinio can be formulated with a real and positive measure for $\mu \neq 0$ however since the pion has a non-zero disconnected contribution the physics of chiral symmetry breaking is very different from that in QCD. We can simulate quenched QCD but the theory is pathological due to the presence of light states formed from a quark and conjugate quark which are degenerate with the pion but have non-zero baryon number. The third class of model amenable to simulation is SU(2) with dynamical quarks which has a real and positive measure for quarks in the fundamental representation. The $SU(2)$ theory has a symmetry between quarks and conjugate quarks which means that the lightest baryon and the pion are truly degenerate. Another special feature of the model is that in the chiral limit at zero chemical potential the chiral condensate can be rotated to a diquark condensate. I presented techniques for measuring a diquark condensate in a hybrid Monte-Carlo simulation and presented data to show how the diquark condensate and the chiral condensate varied as a function of a diquark source term at fixed bare quark mass. I also presented simulations for $SU(2)$ with quarks in the adjoint representation where the measure is real. We have still to establish whether there is a sign problem in this model. For $SU(2)$ with both fundamental and adjoint quarks the HMC simulations require very short timesteps and trajectory lengths when the chemical potential is switched on. This is likely to be associated with the existence of small eigenvalues. I presented simulations of the model with adjoint fermions using the Two Step Multibosonic Algorithm where measurements of the average plaquette seem to decorrelate more quickly than they do with the HMC algorithm. I outlined the importance of developing a full understanding of the $SU(2)$ models at non-zero chemical potential.

Theories we CAN Simulate at $\mu \neq 0$

- ① L-fermion theories eg Gross-Neveu
NJL

BUT realisation of Goldstone mechanism is different from QCD in these models.

- ② QUENCHED QCD

BUT theory is "pathological" due to presence of BARYONIC PION in spectrum

- ③ Gauge theories with dynamical fermions
using SU(2) colour group

What can we learn about physics
at $\mu \neq 0$

Well defined questions + SURPRISES!

Second Standard Scenario

Two colors QCD - will discuss simulation later

PHYSICS OF FIELD THEORIES AT $\mu \neq 0$ IS
CRUCIALLY DEPENDENT ON THE GAUGE
GROUP

Fundamental Rep. of $SU(2)$ is PSEUDOREAL

$\Rightarrow q$ and $\bar{q}$ are related by a symmetry

$\therefore$ BARYONS and MESONS are essentially identical!
cf QCD where baryon and meson spectra are
very distinct!

ALSO

diquark condensates in $SU(2)$ theory can be
gauge singlets

at $\mu = 0$ $\langle \bar{q}q \rangle$ can be 'rotated' into $\langle qq \rangle$
without Energy input to system

PCAC $m_\pi \propto \sqrt{m}$ $m \equiv$ bare or current
quark mass
 $SU(2)$ $m_{1b} \propto \sqrt{m}!$ GOLDSTONE BARYON

$\therefore$ may be no sensible distinction between
baryons and mesons

Two Colors QCD

$\beta=1.5, m=0.05, 6^4$ lattice

Minimum eigenvalue on sample of 100 configs

 $\beta=2.3, N_f=4, m=0.1, 8^3 4$ lattice

$SU(2)$ is a theory we CAN simulate using available algorithms.

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$SU(2)$ with Fundamental Fermions

- Real, Positive action $\therefore$ why do we need small dt and short traj. length for $\mu \neq 0$?
- Verify $\mu_0 \sim \sqrt{m}$ ie Zero onset in chiral limit
- Explore behaviour of DIQUARK condensate as function of bare quark mass. and μ .

$SU(2)$ with Adjoint Fermions

explain small eigenvalues

- Action is real BUT is it positive $\downarrow$
Why are simulations numerically intensive
- Verify that $\mu_0 \neq 0$ in chiral limit
ie QCD-like theory
- Which DIQUARK condensate forms?
- Asymptotic freedom lost for $N_f > \frac{11}{4}$
explore. $N_f = 1$ and $N_f = 4$.

11

Towards a bosonic positive measure representation for the phase of complex fermion determinants

A. A. Slavnov
Steklov Math. Inst., Moscow

I construct a representation
for $\exp\{\varphi\}$, where φ is the
phase of a complex fermion
determinant, in terms of a
bosonic path integral with a
positive action.

Complex fermion determinant:

$$\det D = \int \exp \left\{ \sum_{n,m} \bar{\Psi}_n D^{nm} \Psi_m \right\} d\bar{\Psi} d\Psi$$

$D = A + iB$, A, B are Hermitian operators.

$$\det D = |\det D| e^{i\varphi}$$

$$\langle O \rangle = \int O e^{i\varphi} |\det D| e^{-S(A)} dA =$$

$$= \int O \cos \varphi |\det D| e^{-S(A)} dA$$

For $\varphi \ll 1$

$$\cos \varphi = 1 - \frac{\varphi^2}{2} + O(\varphi^4) \approx 2 - \text{ch } \varphi.$$

may be relevant for:

- 1) Models with small chemical potential.
- 2) Standard model - the phase $\sim g_w^2$

$$\ln(\det D) = \text{Tr} \ln D = \ln |\det D| + i\varphi.$$

$$\text{Tr} \ln D = \int_0^1 \text{Tr} \frac{iB}{A + iB\alpha} d\alpha + \text{Tr} \ln A$$

$$2i\varphi = \text{Tr} \ln D - \text{Tr} \ln D^* =$$

$$= \int_0^1 \text{Tr} \frac{i[A, B]_+}{A^2 + i[B, A]_- \alpha + B^2 \alpha^2} d\alpha$$

Discretized version:

$$2i\varphi \approx \sum_{n=0}^N N^{-1} \text{Tr} \frac{i[A, B]_+}{A^2 + i[B, A]_- \frac{n}{N} + B^2 \frac{n^2}{N^2}}$$

$$\varphi = \sum_{n=0}^N \text{Tr} \ln \left[\left(A + \frac{B}{2N} \right)^2 + i[B, A]_- \frac{n}{N} + B^2 \frac{n^2}{N^2} \right] -$$

$$- \sum_{n=0}^N \text{Tr} \ln \left[\left(A^2 + i[B, A]_- \frac{n}{N} + B^2 \frac{n^2}{N^2} \right) \right] + \mathcal{O}\left(\frac{1}{N}\right)$$

$$e^{24} = \prod_{n=0}^N \det \frac{[\tilde{A}^2 + i[B, \tilde{A}] - \frac{n}{N} + B^2 \frac{n^2}{N^2}]^2}{[A^2 + i[B, A] - \frac{n}{N} + B^2 \frac{n^2}{N^2}]^2}$$

$$\tilde{A} = A + \frac{i}{2N} B.$$

e^{24} is expressed as a product of fermionic and bosonic determinants of nonnegative operators.

A bosonized representation for e^{24} may be written using the construction (N.S.S. Phys. Lett B 338 (1996) 147, B 366 (1996) 253).

$$\prod_n \det \frac{\tilde{K}_n^2}{K_n^2} = \lim_{\mu \rightarrow 0, b \rightarrow 0, L \rightarrow \infty} \int e^{-S_{\mu, b, L}[U, \psi, \chi]} d\psi^\dagger d\psi d\chi^\dagger d\chi$$

$S_{\mu, b, L}$ is a five dimensional local positive definite fermionic action.

D Theory :

Constructing Bosons from Fermions and Fermions From Spins

Rich Brower

Shailesh Chandrasekharan

Uwe-Jens Wiese

- D THEORY CONSTRUCTION KIT
 1. Exact Symmetries for Quantum $\hat{H}$ operator
 2. Flux Basis & Deformed Algebras
 3. Spin alignment & Cluster Rules
- U(1): WARM UP FOR FULL QCD
 1. U(1) clusters: Flux Sheets
 2. Adding Miron Cluster for Staggered Fermions
- SIGN PROBLEM FOR DOMAIN WALL FERMIONS
 1. Trouble with Ions
 2. New role for 4 Fermi terms?

LATTICE HAMILTONIAN FOR COMPACT QED.

THE KOGUT-SUSKIND HAMILTONIAN:

$$H_{KS} = \frac{1}{2} \hat{E}_{x,\mu}^2 + \frac{1}{e_0^2} \hat{U}_{x,\mu} \hat{U}_{x+\mu,\nu} \hat{U}_{x+\nu,\mu}^\dagger \hat{U}_{x,\nu}^\dagger$$

LOCAL GAUGE INVARIANCE ON EACH LINK $(x, x + \mu)$:

$$[\hat{E}, \hat{U}] = -\hat{U} \quad [\hat{E}, \hat{U}^\dagger] = \hat{U}^\dagger \quad [\hat{U}, \hat{U}^\dagger] = 0$$

$\hat{E}$ AND $\hat{U}^\dagger \hat{E} \hat{U}$ ARE LEFT AND RIGHT GAUGE GENERATORS.

THESE CR'S IMPLY INFINITE DIMENSIONAL REPRESENTATIONS:

$$\hat{U} \rightarrow \exp[-i\theta] \quad \hat{E} \rightarrow i \frac{d}{d\theta}$$

AN EQUIVALENT "RISHON REPRESENTATION" CAN BE USED:

INTRODUCE FERMIONIC WINDING MODES, $\{c_l, c_l^\dagger\} = \delta_{l,l'}$:

$$c_l |\theta\rangle = e^{-il\theta} |\theta\rangle \implies \psi(\theta) = \sum_l c_l e^{il\theta - l\tau}$$

FOR $l = 0, \pm 1, \pm 1, \dots \pm \infty$.

QUANTUM LINKS FOR FULL QCD

U(N) SOLUTION: CONSTRUCTED BY HORN(1981), ORLAND-ROHRLICH (1990) AND CHANDRASEKHARAN-WIESE(1996) AND EXTENDED TO SU(2).

BUT NO WAY TO DO SU(3) OR FERMIONS

SU(3) GAUGE ALGEBRA IS REALIZED AS A $SU_R(3) \times SU_L(3)$ SYMMETRY ON THE LINKS OPERATORS U ,

$$[\hat{R}_\alpha, \hat{U}_{ij}] = \hat{U}_{ik} \lambda_{kj}^\alpha \quad , \quad [\hat{L}_\alpha, \hat{U}_{ij}] = -\lambda_{ik}^\alpha \hat{U}_{kj} \quad ,$$

THE SU(3) GENERATORS OBEY THE LIE ALGEBRA,

$$[\hat{R}_\alpha, \hat{R}_\beta] = 2if_{\alpha\beta\gamma} \hat{R}_\gamma \quad , \quad [\hat{L}_\alpha, \hat{L}_\beta] = 2if_{\alpha\beta\gamma} \hat{L}_\gamma \quad , \quad [\hat{R}_\alpha, \hat{L}_\beta] = 0 \quad .$$

ANSATZ (MOTIVATED BY U(1) CASE):

$$\hat{U}_{i,j} = \delta_{i,j} \hat{U}_0 + i\lambda_{ij}^\alpha \hat{U}_\alpha = \delta_{i,j} \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} + i\lambda_{ij}^\alpha \begin{vmatrix} 0 & -i\hat{\Lambda}_\alpha \\ 0 & 0 \end{vmatrix}$$

AND

$$\hat{R}_\alpha = \begin{vmatrix} \hat{\Lambda}_\alpha & 0 \\ 0 & 0 \end{vmatrix} \quad , \quad \hat{L}_\alpha = \begin{vmatrix} 0 & 0 \\ 0 & \hat{\Lambda}_\alpha \end{vmatrix} \quad ,$$

THIS WORKS IF THE OPERATORS $\hat{\Lambda}$ OBEY THE SAME ALGEBRA AS THE GELL-MANN LAMBDA'S!

RISHON REPRESENTATIONS OFFER A NATURAL SOLUTION

INTRODUCE (N) AND $(\bar{N})$ FERMIONIC OPERATORS a_i AND $b_i^\dagger$ FOR $i = 1, \dots, N$ WITH THE COMMUTATORS,

$$\{a_i, a_j^\dagger\} = \delta_{i,j} \quad , \quad \{b_i, b_j^\dagger\} = \delta_{i,j}$$

ON EACH LINK $(x, x + \mu)$ TO REPRESENT GAUGE FIELDS $\hat{U}_{x,\mu}$.

$$\hat{U}_{i,j} = \sum_{\alpha=0}^{N^2-1} \lambda_{ij}^\alpha a_n^\dagger \lambda_{mn}^\alpha b_m \rightarrow \hat{U}_{i,j} = a_i^\dagger b_j$$

AND GAUGE GENERATORS,

$$\hat{L}_\alpha = \sum_i a_i^\dagger \lambda_{i,j}^\alpha a_j \quad , \quad \hat{R}_\alpha = \sum_i b_i^\dagger \lambda_{i,j}^\alpha b_j$$

SINCE THE LINK NUMBER IS CONSERVED,

$$\mathcal{N} = \sum_i a_i^\dagger a_i + \sum_i b_i^\dagger b_i,$$

SO THE 2^{2N} DIMENSIONAL REP ARE REDUCIBLE OF $SU(2N)$.

$U(1)$: $\mathcal{N} = 0, 1, 2 \Rightarrow 1, 2, 1$ DIM REPS. (ONLY $\mathcal{N} = 1$ IS NON-TRIVIAL)

$U(2)$: $\mathcal{N} = 0, 1, 2, 3, 4 \Rightarrow 1, 4, 6, 4, 1$ DIM REPS.

$SU(3)$: $\mathcal{N} = 0, 1, 2, 3, 4, 5, 6 \Rightarrow 1, 6, 15, 20, 15, 6, 1$ DIM REPS. (ONLY $\mathcal{N} = 3$ HAS $\hat{U}^3 \neq 0$).

ADDING QUARKS

$$\begin{aligned}
H = & J \sum_{x,\mu \neq \nu} \text{Tr}[U_{x,\mu} U_{x+\hat{\mu},\nu} U_{x+\hat{\nu},\mu}^\dagger U_{x,\nu}^\dagger] + J' \sum_{x,\mu} [\text{DET} U_{x,\mu} + \text{DET} U_{x,\mu}^\dagger] \\
& + \frac{1}{2} \sum_{x,\mu} [\Psi_x^\dagger \gamma_5 \gamma_\mu U_{x,\mu} \Psi_{x+\hat{\mu}} - \Psi_{x+\hat{\mu}}^\dagger \gamma_5 \gamma_\mu U_{x,\mu}^\dagger \Psi_x] + M \sum_x \Psi_x^\dagger \gamma_5 \Psi_x \\
& + \frac{r}{2} \sum_{x,\mu} [2 \Psi_x^\dagger \gamma_5 \Psi_x - \Psi_x^\dagger \gamma_5 U_{x,\mu} \Psi_{x+\hat{\mu}} - \Psi_{x+\hat{\mu}}^\dagger \gamma_5 U_{x,\mu}^\dagger \Psi_x]. \quad (1)
\end{aligned}$$

HERE $\Psi_x^\dagger$ AND Ψ_x ARE QUARK CREATION AND ANNIHILATION OPERATORS,

$$\{\Psi_x^{i\alpha}, \Psi_y^{j\beta\dagger}\} = \delta_{xy} \delta_{ij} \delta_{\alpha\beta}, \quad \{\Psi_x^{i\alpha}, \Psi_y^{j\beta}\} = \{\Psi_x^{i\alpha\dagger}, \Psi_y^{j\beta\dagger}\} = 0,$$

AND THE TOTAL GAUGE GENERATOR IS

$$\vec{G}_x = \sum_\mu (\vec{R}_{x-\hat{\mu},\mu} + \vec{L}_{x,\mu}) + \Psi_x^\dagger \vec{\lambda} \Psi_x,$$

THE QUARK SPINOR,

$$\Psi_x = \begin{pmatrix} \Psi_{Rx} \\ \Psi_{Lx}^\dagger \end{pmatrix}. \quad (2)$$

ANNIHILATES THE VACUUM: $\Psi_x |0\rangle = 0$.

$$Z = \text{Tr} \langle 0 | \exp(-\beta H) | 0 \rangle. \quad (3)$$

WITH THE TRACE ONLY ON THE GLUONIC HILBERT SPACE.

Ph. de Forcrand and V. Laliena

The Role of the Polyakov Loop in Finite Density QCD

1.- Introduction.

2.- Toy model.

3.- Correlation between the phase of the determinant and the imaginary part of the Polyakov Loop.

* Toy model.

** Static QCD. $\longrightarrow$ (Blum, Hetrick, Toussaint).

*** QCD with light quarks.

4.- Conclusions.

1. Introduction.

QCD partition function:

$$Z = \text{Tr} \left[\exp(-\beta(\hat{H} - \mu \hat{B})) \hat{P}_g \right] = \sum_{\substack{\text{gauge} \\ \text{inv. states}}} e^{-\beta(E - \mu N_B)}$$

- Hamiltonian formulation: positive measure, but impractical
- Path integral representation: sign problem.

There might be a positive measure (?)

Task: Find a bunch of configurations representative of the subset of phase space contributing to the partition function.

Idea: constrain some intensive quantities to its averaged value.

Polyakov Loop:

$$\text{HPE} = \log \det \Delta = - \sum_{\ell=1}^{\infty} \frac{\kappa^{\ell}}{\ell} B_{\ell}$$

Lowest order term sensitive to μ :

$$K^{N_T} \cosh(N_T \mu) \sum_x (\text{Tr } P_x + \text{Tr } P_x^{\dagger}) + \\ + K^{N_T} \sinh(N_T \mu) \sum_x (\text{Tr } P_x - \text{Tr } P_x^{\dagger})$$

Phase of $\det \Delta$ proportional to $\text{Im} \sum_x P_x$

No sign problem if $\text{Im} \sum_x P_x = 0$

$6^3 \times 4$ $\beta = 5.0$ $c = 1/0.6$ 16000 events

No cut in Im Pol.

$6^3 \times 4$ $\beta = 5.0$ $C = 1/0.6$ 6272 events

$\text{Im } P_0 \in [-0.02, 0.02]$

4. Conclusions

- * The Polyakov loop is correlated to the phase of the determinant, at least for heavy quarks.
- * The fluctuations of the phase are minimal when $\text{Im Pol} = 0$.
- * The harmless constraint $\text{Im Pol} = 0$ will improve the numerical computation, at least in small lattices.
- * We have studied:

(a) Toy model ($U_0(x,t) = \bar{U}_0$; $U_k(x,t) = \mathbb{1}$)

- $\det \Delta$ real if $\text{Im Pol} = \text{Im Tr } \bar{U}_0^N = 0$
- Effect of T similar to effect of μ .
(Both tend to order the system).

(b) Static QCD

- Very strong correlation $\text{phase}(\det) - \text{Im Pol}$
- Very strong improvement of the numerical computation using the constraint $\text{Im Pol} = 0$

(c) QCD with light quarks (preliminary results).

- Correlation $\text{phase} - \text{Im Pol}$ less clear.
- It seems that the constraint $\text{Im Pol} = 0$ improves things.

* What else can be constrained in order to improve, or solve, the sign problem?

Free energy of an SU(2) monopole-antimonopole pair

Ch. Hoelbling, C. Rebbi, V. Rubakov

builds on work by 't Hooft, Mandelstam, Mack + Pethoua, Yaffe, Ukawa, Wulkeny + Guth, Srednicki + Susskind
relevant for center vortices (Cornwall, Kovacs + Tomboulis)

$$\beta \rightarrow -\beta$$

$$S(\beta, \beta') = \frac{1}{2} \left(\beta \sum_{p \in L} T_2(U_p) + \beta' \sum_{p \in L} T_2(U_p) \right)$$

$$Z(\beta, \beta') = \sum_c e^{-S(\beta, \beta')}$$

$$F = -T \ln \frac{Z(\beta, -\beta)}{Z(\beta, \beta)}$$

Multihistogram method

$$\frac{h_i(E)}{n_i} = p_i(E) \frac{e^{-\beta'_i E}}{Z_i} \Rightarrow p_i(E) = Z_i \frac{h_i(E)}{n_i} e^{\beta'_i E}$$

approximate with 1 at first step

$$p(E) = \frac{1}{N} \sum_i p_i(E)$$

$$Z_i = \sum_E p(E) e^{-\beta'_i E}$$

iterate

Figure 2: The overlap of histograms at $\beta = 2.6$ and different β' . The monopole separation is $2a$ and $N_t = 6$.

Monopole pair potential in confined phase

Monopole pair potential in deconfined phase

Classical vs. quantum potential

Temperature dependence of monopole potential

Abelian projection
classical conf.

Abelian projection
quantum case,
in the neighborhood
of $M \bar{H}$

" "
in the rest of
the lattice

Learning from infinite mass QCD

QCD at infinite bare mass and chemical potential provides an analytically solvable model where some ideas on the finite density behavior of the theory may be tested. In this limit the contribution of the phase of the fermion determinant shows an unexpected temperature dependence: it is absent in the zero temperature limit and gives finite contribution for $T > 0$. Comparison between quench approximation, modulus QCD and the complete theory suggests also that the failure of the former has to be ascribed to the non incorporation of Pauli exclusion principle. Infinite mass results are compared with available numerical simulations and evidences of inconsistencies in published data are presented. At the end we investigate the large but finite mass regime in regions of $\beta - \mu$ plane where the contribution of the phase is negligible. We get evidence for a transition line that, starting from the critical point at $\mu = 0$, moves towards smaller β with increasing μ .

A. Galante

U. Zangola

try to extend the $m \rightarrow \infty$ results to large but finite m

- evaluate $\langle e^{i\phi} \rangle_{\mu}(\beta, \mu)$

- exclude the regions of parameter space where

$$\langle e^{i\phi} \rangle_{\mu}(\beta, \mu) < \frac{1}{\nu}$$

- look for some signal

- the phase can be relevant or not depending on T

this should not be the case if the modulus fails because of the formation of spurious qq^* states

- the quench approximation is very bad not incorporating the Fermi-Dirac statistics may have dramatic effects
- $\beta=0$, large m QCD may be a workbench for testing numerical algorithms.

the $\beta \neq 0$ case is not so simple

$$Z \propto \int [dV] e^{-\beta S_G(V)} \prod_{i=1}^{V_S} (c^3 + 1 + c \text{Tr}(L_i^*) + c^2 \text{Tr}(L_i))$$

$$Z(\beta, \mu) = Z_{PG}(\beta) \cdot Z(\beta=0, \mu) \cdot R(\beta, \mu)$$

$$R(\beta, \mu) = \frac{\int [dV] e^{-\beta S_G} \prod_{i=1}^{V_S} \left(1 + \frac{c \text{Tr}(L_i^*) + c^2 \text{Tr}(L_i)}{c^3 + 1} \right)}{\int [dV] e^{-\beta S_G}}$$

$$T = 0 \quad L_S = L_+ \longrightarrow \infty$$

$$1 + \frac{c \text{Tr}(L_i^*) + c^2 \text{Tr}(L_i)}{c^3 + 1} = 1 + O(e^{-L_+})$$

$$R(\beta, \mu) \longrightarrow 1$$

For $\beta=0$ we get factorization

$$\rho = \mu + \frac{1}{3L_+} \ln \left(\left(\frac{zm}{e^\mu} \right)^{3L_+} + 1 \right)$$

$$n(\mu) = \frac{1}{1 + e^{3L_+(\ln(zm) - \mu)}}$$

L_+ finite ($T \neq 0$)

$$\rho_\phi \neq 0$$

no discontinuities

$L_+ \longrightarrow \infty$ ($T=0$)

$$\frac{1}{L_+} \ln \left[\frac{\left(\frac{zm}{e^\mu} \right)^{3L_+} + 1}{\left(\left(\frac{zm}{e^\mu} \right)^{L_+} + 1 \right)^3} \right] \leq \rho_\phi \leq 0 \longrightarrow \rho_\phi = 0$$

1st order Transition

$$\mu_c = \ln(zm)$$

$$\mu_c \simeq \frac{1}{3} m_B$$

$$Z \Delta = z m I + e^{\mu} G + e^{-\mu} G^{\dagger} + V$$

$G (G^{\dagger}) \rightarrow$ forward (backward) Temporal links

$V \rightarrow$ forward and backward spatial links

$m \gg 1$ Blum et al PRL 76 (1996) 1019

$$\det \Delta = e^{3V\mu} \prod_{i=1}^{V_s} \det(c + L_i)$$

$$c = \left(\frac{zm}{e^{\mu}} \right)^{L_+}$$

$$L_i = \prod_{j=1}^{L_+} U_4(j)$$

$$Z = \left(\frac{1}{z} \right)^{3V} e^{3V\mu} \int [d\mu] e^{-\beta S_G(\mu)} \prod_{i=1}^{V_s} \det(c + L_i)$$

Finite Temperature QCD

- Lattice QCD with Wilson and staggered fermions gives:

$$T_c \sim 160-170 \text{ MeV} \quad (2 \text{ flavors})$$

Transition not first order (2 flavors)

- Both Wilson and staggered fermions distort symmetries at finite lattice spacing.

$\Rightarrow$ try domain wall fermions

- Questions?

Will left/right modes stay separated?

How large an L_s ?

Will algorithms work?

$8^3 \times 4, L_s=12, m_l(\text{sea})=0.10$

PDWF

$8^3 \times 4, \beta=5.20, m_0=1.90$

PDWF

$8^3 \times 4, \beta=5.45, m_0=1.90$

PDWF, 2 flavors, $\beta = 5.2$, $8^3 \times 4$
QCDSP

dyn DWF with Wilson gauge action

$m_0=1.9, L_s=24, m_f=0.02$

Numerical Algorithms at Non-Zero Chemical Potential

April 27 - May 1, 1999

Organizers: Thomas Blum and Michael Creutz

List of Registered Participants

<u>Name</u>	<u>Affiliation</u>	<u>Mailing Address</u>	<u>E-mail Address</u>
Mark Alford	MIT	Massachusetts Institute of Technology 77 Massachusetts Avenue Cambridge, MA 02139 USA	alford@ctp.mit.edu
Roberto Aloisio	Univ. L'Aquila and INFN Gran Sasso	Physics Department University of L'Aquila via Vetoio 67100 L'Aquila Italy	roberto.aloisio@lngs.infn.it
Vicente Azcoiti	Univ. of Zaragoza	Departamento de Fisica Teorica Facultad de Ciencias Universidad de Zaragoza 5009 Zaragoza Spain	azcoiti@azcoiti.unizar.es
Thomas Blum	RBRC	RBRC, Physics Dept., Bldg. 510A Brookhaven National Laboratory Upton, NY 11973 USA	tblum@bnl.gov
Richard Brower	Boston Univ.	Physics Department 590 Commonwealth Avenue Boston University Boston, MA 02215 USA	brower@bu.edu
Shailesh Chandrasekharan	Duke Univ.	Dept. of Physics, Box 90305 Duke University Durham, NC 27707 USA	sch@phy.duke.edu
Michael Creutz	BNL	RBRC, Physics Dept., Bldg. 510A Brookhaven National Laboratory Upton, NY 11973 USA	creutz@bnl.gov
George Fleming	Columbia Univ.	Columbia University 538 West 120th Street New York, NY 10027 USA	gfleming@cuphyb.phys.columbia.edu
Philippe de Forcrand	ETH Zurich	SCSC, RZ-F8 ETH Zurich CH-8092 Zurich Switzerland	forcrand@scsc.ethz.ch
Hirotsugu Fujii	Univ. of Tokyo	Institute of Physics University of Tokyo at Komaba 3-8-1, Meguro Tokyo 153-8902 Japan	hfujii@phys.c.u-tokyo.ac.jp
Angelo Galante	Univ. of Zaragoza	Department of Theoretical Physics University of Zaragoza 50009 Zaragoza Spain	galante@azcoiti.unizar.es
Simon Hands	Univ. of Wales Swansea	Department of Physics University of Wales Swansea Singleton Park Swansea SA2 8PP United Kingdom	s.hands@swansea.ac.uk
Dmitri Kharzeev	RBRC	RIKEN BNL Research Center Physics, Bldg 510A Brookhaven National Laboratory Upton, NY 11973 USA	kharzeev@bnl.gov
John Kogut	Univ. of Illinois	University of Illinois 1110 W. Green Street Urbana, IL 61801-3080 USA	j-kogut@uiuc.edu
Victor Laliena	ETH Zurich	CSCS/SCSC; RZ F 9 Claussiusstrasse 59 ETH- Zentrum CH-8092 Zurich Switzerland	laliena@scsc.ethz.ch
Maria-Paola Lombardo	INFN-LNGS	Istituto Nazionale di Fisica Nucleare Laboratori Nazionali del Gran Sasso S.S. 17 bis Km 18,910 I-67010 Assergi (AQ) Italy	lombardo@lngs.infn.it

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List of Registered Participants

<u>Name</u>	<u>Affiliation</u>	<u>Mailing Address</u>	<u>E-mail Address</u>
Robert Mawhinney	Columbia Univ. and RBRC	Department of Physics Columbia University 538 West 120th Street New York, NY 10025 USA	rdm@phys.columbia.edu
Susan Morrison	Univ. of Wales Swansea	Department of Physics University of Wales Swansea Singleton Park Swansea SA2 8PP United Kingdom	s.morrison@swansea.ac.uk
Shigemi Ohta	KEK and RBRC	RIKEN BNL Research Center Physics, Bldg 510A Brookhaven National Laboratory Upton, NY 11973 USA	shigemi.ohta@kek.jp
Claudio Rebbi	Boston Univ.	Physics Department Boston University 590 Commonwealth Avenue Boston, MA 02215 USA	rebbi@bu.edu
Shoichi Sasaki	RBRC	RIKEN BNL Research Center Physics, Bldg 510A Brookhaven National Laboratory Upton, NY 11973 USA	ssasaki@bnl.gov
Thomas Schaefer	Inst. for Advanced Study	Institute for Advanced Study Olden Lane Princeton, NJ 08540 USA	schaefer@ias.edu
Robert Shrock	SUNY, Stony Brook	ITP State University of New York Stony Brook, NY 11794-3840	robert.shrock@sunysb.edu
Donald Sinclair	ANL	ANL 9700 South Cass Avenue Argonne, IL 60439 USA	dk@hep.anl.gov
Andrei Slavnov	Steklov Mathematical Inst.	Steklov Mathematical Institute Gubkina st. 8 117966 Moscow GSP-1 Russia	slavnov@mi.ras.ru
Misha Stephanov	SUNY, Stony Brook	ITP State University of New York Stony Brook, NY 11794-3840	misha@insti.physics.sunysb.edu
Raju Venugopalan	BNL	Physics, Bldg. 510A Brookhaven National Laboratory Upton, NY 11973 USA	raju@bnl.gov
Uwe-Jens Wiese	MIT	Center for Theoretical Physics Building 6-314 MIT 77 Massachusetts Avenue Cambridge, MA 02139 USA	wiese@mitlns.mit.edu
Matt Wingate	RBRC	RIKEN BNL Research Center Physics, Bldg 510A Brookhaven National Laboratory Upton, NY 11973 USA	wingate@bnl.gov
Lingling Wu	Columbia University	Department of Physics Columbia University 538 West 120th Street New York, NY 10025 USA	lwu@phys.columbia.edu
Yuriy Zhestkov	Columbia University	Department of Physics Columbia University 538 West 120th Street New York, NY 10025 USA	zhestkov@phys.columbia.edu

RIKEN BNL Research Center

Workshop on Numerical Algorithms at Non-Zero Chemical Potential

April 27–May 1, 1999

Physics Department - Room 2-160

Organizers:

Tom Blum and Michael Creutz

AGENDA

Tuesday, April 27

- 09:00–09:50 Registration
- 10:00–10:15 Welcome/announcements
- 10:20–11:10 Solution to Sign Problems Using Cluster Algorithms
 [Shailesh Chandrasekharan, Duke]
- 11:10–11:30 Break
- 11:30–12:20 Meron-Cluster Algorithms for the 2-d $O(3)$ Model with Chemical Potential
 [Owe-Jens Wiese, MIT]
- 12:30–14:00 Lunch
- 14:00–14:50 Transfer Matrices and Grassmann Integrals
 [Michael Creutz, BNL]
- 15:00–15:30 Break
- 15:30–17:00 Discussion
- 17:00–19:00 Welcome party

Wednesday, April 28

- 09:30–10:20 On-Zero Density at Non-Zero Temperature
 [Maria-Paola Lombardo, INFN Gran Sasso]
- 10:30–11:00 Break
- 11:00–11:50 Non Working Ideas in Finite Density QCD
 [Vicente Azcoiti, Zaragoza]
- 12:00–13:30 Lunch
- 13:30–14:30 Philip Page (HET/RKEN Theory Seminar)
 (no proceedings package)
- 15:00–15:30 Lunch
- 15:30–17:00 Discussion

Thursday, April 29

- 09:30–10:20 What Can We Learn About Dense Matter from Standard Algorithms
[Simon J. Hands, U. of Wales, Swansea]
- 10:30–11:00 Break
- 11:00–11:50 Successful Simulations and Diquark Techniques at Nonzero Density
[Susan E. Morrison, U. of Wales, Swansea]
- 12:00–14:00 Lunch
- 14:00–14:50 Towards a Bosonic Positive Measure Representation for the Phase of Complex
Fermion Determinants
[Andrey Slavnov, Stekov Mathematical Inst.]
- 15:00–15:30 Break
- 15:30–17:00 Discussion
- 17:30–19:30 Dinner

Friday, April 30

- 09:30–10:20 Quantum Links, Fermionic Glue, and Bosonic Quarks
[Richard Brower, Boston U.]
- 10:30–11:00 Break
- 11:00–11:50 The Role of the Polyakov Loop in Finite Density QCD
[Victor Laiena, ETH, Zurich]
- 12:00–14:00 Lunch
- 14:00–15:15 Free Energy of an $SU(2)$ Monopole Anti-Monopole Pair
[Claudio Rebbi, Boston U.]
- 15:15–16:00 Break
- 16:00–17:00 Kip Thorne (BSA Distinguished Lecture)
(no proceedings package)

Saturday, May 1

- 09:30–10:20 Learning from Infinite Mass QCD
[Angelo Galante, U. of Zaragoza]
- 10:30–11:00 Break
- 11:00–12:00 On the RIKEN/BNL Supercomputer
[Robert Mawhinney, Columbia U. and RBRC]
- 12:30–14:00 Discussion over lunch

Departures

Forthcoming RIKEN BNL Center Workshops

Title: **Gauge Invariant Observables in Gauge Theories**
Organizers: P. Orland/P. van Baal
Dates: May 25–29, 1999

Title: **OSCAR II: Predictions for RHIC**
Organizers: Y. Pang/M. Gyulassy
Dates: July 8–16, 1999

Title: **Coulomb and Pion-Asymmetry Polarimetry and Hadronic Spin Dependence at RHIC Energies**
Organizer: E. Leader
Date: August 18, 1999

Title: **RHIC Spin**
Organizer: G. Bunce
Dates: Oct. 6–8, 1999

For information please contact:

Ms. Pamela Esposito
RIKEN BNL Research Center
Building 510A, Brookhaven National Laboratory
Upton, NY 11973, USA
Phone: (516)344-3097 Fax: (516)344-4067
E-Mail: rikenbnl@bnl.gov
Homepage: <http://penguin.phy.bnl.gov/www/riken.html>

RIKEN BNL RESEARCH CENTER

NUMERICAL ALGORITHMS AT NON-ZERO CHEMICAL POTENTIAL

APRIL 27–MAY 1, 1999

Li Keran

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*Nuclei as heavy as bulls
Through collision
Generate new states of matter.*
T. D. Lee

Speakers:

V. Azcoiti
M. Creutz
V. Laliena
S.E. Morrison
A. Slavnov

R. Brower
A. Galante
M.-P. Lombardo
P. Page
O.-J. Wiese

S. Chandrasekharan
S.J. Hands
R.D. Mawhinney
C. Rebbi

Organizers: Tom Blum and Michael Creutz