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Orbit Stabilization of Nanosat

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ORBIT STABILIZATION OF NANOSAT

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Abstract

An algorithm is developed to control a pulsed ΔV thruster on a small satellite to allow it to fly in formation with a host satellite undergoing time dependent atmospheric drag deceleration. The algorithm uses four short thrusts per orbit to correct for differences in the average radii of the satellites due to differences in drag and one thrust to symmetrize the orbits. The radial difference between the orbits is the only input to the algorithm. The algorithm automatically stabilizes the orbits after ejection and includes provisions to allow azimuthal positional changes by modifying the drag compensation pulses. The algorithm gives radial and azimuthal deadbands of 50 cm and 3 m for a radial measurement accuracy of ± 5 cm and $\pm 60\%$ period variation in the drag coefficient of the host. Approaches to further reduce the deadbands are described. The methodology of establishing a stable orbit after ejection is illustrated in an appendix. The results show the optimum ejection angle to minimize stabilization thrust is upward at 86° from the orbital velocity. At this angle the stabilization velocity that must be supplied by the thruster is half the ejection velocity. An ejection velocity of 0.02 m/s at 86° gives an azimuthal separation after ejection and orbit stabilization of 187 m. A description of liquid based gas thrusters suitable for the satellite control is included in an appendix.

Table of Contents

	Page
I. Introduction	3
II. Atmospheric Drag Decelerations	3
III. Orbital Reaction to Variable Drag Deceleration	5
IV. Orbit Stabilization Algorithm	6
V. Deadband from Radial Measurement Errors and Drag Uncertainty	7
VI. Other Algorithms	9
VII. Controlled Azimuthal Positional Changes	10
VIII. Maneuvering Around MightySat	12
IX. Loss of ΔV Thruster Control	13
X. Conclusions	14
Appendix A (Ejection Stabilization)	15
Appendix B (Gas Thruster Design)	17
Acknowledgements	20
References	20

I. Introduction

The problem of controlling a satellite thruster to allow a satellite to fly formation with a host undergoing variable atmospheric drag deceleration can be addressed with various degrees of sophistication. Here we require the daughter be moved quickly into and be maintained in a stable orbit behind the mother satellite by one thruster after ejection. It is possible to achieve an azimuthal deadband of a few cm with four carefully selected thruster pulses per orbit. However, this requires precise knowledge of the radial displacement of the satellites, the differential time dependent drag deceleration between the satellites and the ability to project this forward based upon nodal regression, together with a sophisticated control algorithm to select the timing and amplitude of the pulses. An alternative is to have nearly continuous thrust such as available from a pulsed plasma thruster. The differential velocity could then be used as input to set thrust frequency or amplitude and give superb control.

Here I present a simple algorithm to perform this function using four drag compensation thruster pulses per orbit that are uncorrelated with periodic variations in the drag. It will be shown this algorithm gives good stability, reliability, and acceptable deadband. I test satellite response to the ΔV thruster control function using a Fortran program I have written to numerically evaluate thruster control algorithms. The program propagates the satellite orbits in the orbital ($r, r\theta$) plane by integrating the equations of motion with a 5 s time step to give 10 cm radial accuracy per orbit. This is accomplished in four steps; (1) determine the radial acceleration at the beginning of each time step from the difference between the centripetal and gravitational forces, (2) use the resultant change in radial position and velocity during the time step to determine the change in potential and radial kinetic energy, (3) use conservation of energy to determine the rotation energy and velocity at the end of the time step, and (4) obtain the final rotational velocity by subtracting the drag deceleration velocity. The ejection angle and velocity, altitude, and initial V_r are input parameters. The program alters the mother and daughter initial velocities at ejection due to recoil using conservation of momentum.

I designate the mother and daughter satellites MightySat and NanoSat in this study for a hypothetical flight by a SNL NanoSat with the USAF MightySat II.2 scheduled for launch in June 2002. The MightySat and NanoSat masses are 125 and 10 kg. The calculations were performed with $V_r = -10$ m/s at the apogee of the initial elliptical orbit ($r_a = 480$ km and $r_p = 445$ km) but the results were similar for circular orbits. The NanoSat was ejected upward at 0.02 m/s in these tests. Only upward ejections were considered because of the desire to maintain NanoSat in communication with GPS satellites during ejection. Electrical power and propellant constraints on NanoSat allow four ~ 10 mN-s gas thruster pulses for drag compensation and one pulse for orbit symmetrization per orbit for the formation flying

The methodology of obtaining a stable orbit after ejection and details of the operation of the algorithm are illustrated in Appendix A. The orbit stabilization is accomplished in the first two orbits after ejection. Appendix B describes some liquid gas thruster designs that could implement the algorithm.

II. Atmospheric Drag Decelerations

The difference between the atmospheric drag deceleration on NanoSat and MightySat is expected to vary by as much as $\pm 60\%$ during an orbit due to the variation in the atmospheric density between night and day and the orientation of the MightySat solar panels as they track the sun. An estimate of these variations is shown in Fig. 1 for a 480 km altitude orbit in June 2002 at a solar flux index of f10.7 (solar mean). Note that the atmospheric density changes by a factor of

two from 2:00 PM to 2:00 AM due to the variation in solar heating of the atmosphere. The atmospheric density by a rotation of the solar panels on MightySat modulate the drag from the daily variation in atmospheric density by an additional factor of 3.6. This occurs because the

Figure 1. (upper) Baseline deceleration of MightySat and NanoSat from atmospheric drag for July 2002 assuming an orbital plane aligned with the sun. The data shown are for one orbital period. The actual orbital plane will be at a 51.5° inclination to the equator of earth with the functionality varying with the nodal regression of the satellite.

(lower) The accumulated differences in momentum and resultant azimuthal error in separation obtainable with four carefully selected thruster pulses during one orbit for the deceleration shown in upper graph.

ballistic coefficient of the MightySat varies from ~ 0.14 to $\sim 0.054 \text{ m}^2/\text{kg}$ when the pair of 30 inch x 60 inch solar panels are changed from trimmed to untrimmed. The ballistic coefficient for NanoSat, which has an 8 inch x 10 inch cross section, is fixed at $.097 \text{ m}^2/\text{kg}$.

The variation is expected to be periodic but to vary slowly from orbit to orbit because of the nodal progression of the satellite orbits with respect to the sun due to the oblateness of the earth. The nodal progression is given by¹

$$\Omega = -9.9639[R/(R+a)]^{3.5} \cos(i)/[1-e^2]^2 \text{ (deg/mean solar day)}$$

where R is the earth radius, a is the altitude, i the inclination, and e the eccentricity of the orbit. Circular orbits at 480 km altitude and the anticipated 51.5° inclination have $\Omega = -4.8^\circ$. The earth orbits the sun on a yearly basis giving an additional 360°/365 daily progression. Therefore the satellites experience a daily regression of 5.8° with respect to the sun.

It is desirable to use a gas thruster with only four $\sim 10 \text{ mN}$, 1 s thrusts per orbit to conserve electrical power (see Appendix B). In principle this can result in an azimuthal control to 18 cm as shown by the curve with the plus and square symbol in lower Fig. 1. However, the thruster

control algorithm would need detailed and accurate knowledge of the time resolved drag deceleration to project forward the reactive effect of deceleration on orbital mechanics, thereby allowing the selection of the optimum timing and amplitude of the thruster pulses. The orbital "delay effect" explained in Section III illustrates why this is necessary to achieve optimal results.

III. Orbital Reaction to Variable Drag Deceleration

The deceleration of a satellite causes an effective increase in velocity that is delayed by approximately one fourth of an orbital period, $T/4$. This effect is shown in Fig. 2 for a calculation where MightySat is in a ballistic orbit with no drag and NanoSat is given a deceleration of $1 \mu\text{m/s}^2$ from $t = 0$ to $T/4$. The upper plot shows ΔV_θ is decreased initially as that is equal, but of the opposite polarity, to the input $d_{\text{NSat}} \times T/4$. The lower figure showing the NanoSat trajectory as seen in expected by the deceleration but that it is increased after $T/4$.

The resultant orbit eventually oscillates around a mean ΔV_θ that is equal but of the opposite

Figure 2. (upper) The azimuthal velocity, ΔV_θ , (continuous curve), radial displacement (dotted), and azimuthal displacement (dashed) of the daughter satellite in the reference frame of the mother satellite after the daughter is given a deceleration of $1 \mu\text{m/s}^2$ from 0 to $T/4$.

(lower) The trajectory of the daughter observed in the mother coordinate frame. The circles are plotted at 10 minute intervals starting at 5 minutes.

polarity, to the input $d_{\text{NSat}} \times T/4$. The lower figure showing the NanoSat trajectory as seen in the MightySat reference frame illustrates that NanoSat initially moves backwards for $\sim T/4$ but later

moves ahead of MightySat as it settles into at a lower mean orbit. This effect is caused by the interplay of kinetic and potential energy during orbital flight. The $> T/4$ time delay before the

mean resultant ΔV_θ is reached makes low deadband formation flying with a large variation in differential deceleration between the satellites a difficult task.

It is beyond the scope of this report to develop an algorithm to select the correct pulses based upon the forward projection of the effects of drag. Instead an algorithm will be presented that uses four equally time spaced drag compensation thrusts per orbit defined by the average difference in the radius of the satellites and one additional pulse to symmetrize the orbits based on the minimum and maximum in the differential radius. The symetrization routine uses ΔV_θ pulses denoted "Hohmann thrusts". This terminology is different from the normal space flight definition of such a transfer, where a Hohmann transfer² refers to a pair of $\pm \Delta V_\theta$ thrusts separated by one half orbital period, used to raise or lower a circular orbit.

IV. Orbit Stabilization Algorithm

The algorithm propagates one orbit of ballistic flight after ejection before automatic stabilization to allow measurement of the nominal difference in radii per orbit, Δr_{d0} , caused by differences in the atmospheric drag forces on the satellites. This is the largest perturbing effect and is expected to cause the MightySat orbit to drop approximately 10 m in radius per orbit compared to NanoSat. This loss must be known apriori to orbit stabilization because a correction for it must be added to the smaller corrections for differences observed in the concentricity and radii of the orbits. If Δr_{d0} could be determined with 10% accuracy before ejection, the selected ejection angle and velocity could be used to start the automatic stabilization one orbit earlier. This would reduce the azimuthal offset between the satellites before a stable orbit is achieved by approximately a factor of two.

The stabilization algorithm includes three operations; (1) $\Delta V_{\theta,H}$ Hohmann thrusts made a half period, $T/2$, after maximum or minimum in the differential radius, Δr_m , provided no such transform has been made the previous $T/2$, (2) $\Delta V_{\theta,d}$ a "sliding" drag correction each $T/4$ based on the average Δr displacement of the satellites the previous orbit, $\langle \Delta r \rangle$, and (3) updating the average drag loss correction, Δr_d , each $T/4$ based on one eighth the error in Δr the preceding $T/4$. Step 1 symmetrizes or ellipsizes the daughter orbit to the mother orbit. (Here I call a single ΔV_θ maneuver of the purposes of symmetrizing the orbits a Hohmann thrust.) Step 2 adjusts the average radius and uses an input averaged over T to avoid instabilities caused by elliptical orbits. The $T/4$ time for the "sliding" drag correction was chosen because four correction thrusts are desired per orbit. More frequent thrusts give only marginally better results and a minimum of two thrusts per orbit are necessary to avoid adding to the eccentricity of the orbit. Step 3 generates $\Delta r_d = \Delta r_{d0} + \int \Delta r dt / 8t$ to ascertain there is no drift in the azimuthal offset angle, $r\Delta\theta$. The initial value of Δr_{d0} is determined on the first ballistic orbit. Step 3 could be replaced by a gauge based upon the desired azimuthal offset but more instability is inherent with this procedure because this offset is based upon $\int \Delta r dt$ additional lag and.

The $\Delta V_{\theta,H}$ for the Hohmann thrust is given by

$$\Delta V_{\theta,H} = -V_o (\Delta r_m / r_o) / 4. \quad (a)$$

The $\Delta V_{\theta,d}$ for the radius error correction supplied four times per orbit at $T/4$ is expressed by

$$\Delta V_{\theta,d} = -V_o (\langle \Delta r \rangle + \Delta r_d) / 8r_o. \quad (b)$$

This is a factor of four smaller than the ΔV_θ necessary when only one correction is used per orbit and gives the correct radial restoring action when $\pm \Delta V$ Hohmann thrusts are being performed. Use of only $-\Delta V$ Hohmann thrusts, to avoid turning NanoSat 180° twice per orbit or adding a

second thruster, reduces the $\delta V_{\theta,d}$ by 1.5% to give

$$\Delta V_{\theta,d} = -0.993 (<\Delta r> + \Delta r_d) V_o / 8r_o \quad (c)$$

for the cases I considered.

With an accurate Δr measurement only one $\pm \Delta V$ Hohmann thrust need be executed during the second orbit after ejection to achieve a stable orbit. Therefore ejection stabilization can theoretically be achieved within two orbits as shown in Appendix A. With measurement and drag uncertainty more $\pm \Delta V$ Hohmann thrusts are needed because $2T$ are required to stabilize Δr_d . Therefore $\pm \Delta V$ Hohmann thrusts were allowed until the fifth orbit in this report but were rarely necessary. The operation of the algorithm is illustrated in the Appendix A.

V. Deadband from Radial Measurement Errors and Drag Uncertainty

The standard space flight terminology of "deadband" will be used to specify the nominal variation in the radial and azimuthal positions as a function of Δr and drag uncertainty. Calculations with no Δr measurement error or drag uncertainty gave ~ 2 and 10 cm deadbands, respectively, confirming the methodology of the stabilization algorithm. The azimuthal and radial deadbands produced by Δr uncertainty were determined by including a random variation in the input radii for relations (a) and (c). Drag uncertainty errors were simulated by placing random variations on the drag deceleration.

Figure 3 illustrates stabilization with the mean drag decelerations of 0.35 and $1.146 \mu\text{m/s}^2$ for NanoSat and MightySat from Fig. 1 together with a ± 5 cm random error in Δr . The ΔV_{θ} (dotted curve) is "saw-toothed" because of the jumps in NanoSat V_{θ} that occur when the drag compensation pulses are applied. The figure shows resultant Δr (continuous curve) and $r\Delta\theta$ (dashed curve) deadbands of ~ 10 cm and 1.3 m. The $r\Delta\theta$ deadband is close to the optimum theoretical value of $3\pi\Delta r$ achievable with good control. The deadbands scaled linearly with larger Δr measurement errors. Note that the Hohmann transfer ΔV pulses (closed circles) are small compared to the drag compensation ΔV pulses (diamonds) except for the second orbit where a pulse at 164 min. is off scale at -6.9 mm/s.

Figure 3. Orbit stabilization with d_{NSat} and d_{MSat} equal to 0.35 and $1.146 \mu\text{m/s}^2$. The continuous, dotted, dashed curves and show Δr , ΔV_{θ} , and $r\Delta\theta$ of NanoSat in the MightySat reference frame. The Hohmann and drag ΔV 's are indicated as circles with a dot and as diamonds.

Figure 4. Orbit stabilization with a stepwise variation in the MightySat drag deceleration from 0.5 to $2 \mu\text{m/s}^2$ with a period 10% shorter than the orbital period. The Hohmann and drag ΔV 's are indicated as closed circles and diamonds.

The deadbands for variation of atmospheric drag deceleration due to daylight modulation of the atmospheric density and rotation of the MightySat solar panels was simulated by putting a periodic variation in the MightySat deceleration. The results the MightySat deceleration varying from 0.5 to $2 \mu\text{m/s}^2$, with two complete oscillations every $0.9T$, are shown in Fig. 4 and indicate the deadbands increase to 30 cm and 3 m. The actual variation due to the 5.8° nodal regression with respect to the sun would have a period 8.5×10^{-4} smaller than T . A 10% more rapid change in periodicity of the deceleration was used in the calculation in Fig. 4 to show no ill effects occur when the modulation of the deceleration beats in and out of phase with the thruster pulses every 10 orbits. The $\pm 5 \text{ cm}$ $\langle \Delta r \rangle$ and Δr error was included in this and all subsequent calculations.

A random $\pm 0.125 \mu\text{m/s}^2$ (a 10% variation the mean deceleration) was placed on the stepwise periodic MightySat deceleration used for the calculation in Fig. 4 to better approximate what could be expected during formation flying. The results of this calculation are shown in Fig. 5 where the deadbands increased to ~ 2 and 16 m. The time resolved shape of the orbit perturbations scaled roughly linearly with the size of the variation of drag deceleration and the azimuthal deadbands were ~10 and 31 m for ± 5 and 20% variation in drag. A $\pm 10\%$ variation each $T/4$ without the stepwise variation gave a 13 m deadband. Although the amplitude of the drag could vary by 10% I do not expect it to be entirely random. Therefore the actual deadbands will probably be ~ 5 m.

Figure 5. Orbit stabilization with a $\pm 10\%$ variation added to the stepwise deceleration used on MightySat in Fig. 4. The $\pm 5 \text{ cm}$ Δr error is also included.

VI. Other Algorithms

Attempts to use other algorithms to improve the results shown in Figs. 3 to 5 were not successful. The use of time periods less than T to determine $\langle \Delta r \rangle$ were unsuccessful because unstable oscillations developed. This is illustrated by the calculation shown in Fig. 6 where a $T/4$ reference time window was used with a MightySat deceleration of $1.146 \mu\text{m/s}^2$ that was varied by a random $\pm 10\%$ every $T/4$. The instability occurs because the algorithm confuses Δr from elliptical orbits with $\langle \Delta r \rangle$.

Figure 6. A calculation of orbit stabilization with $\langle \delta r \rangle$ in the drag compensation determined over $T/4$.

Another attempt to reduce the deadband used the ΔV_θ between the satellites for an orbital correction as an augmentation to the successful algorithms listed above. The results for several choices in the use of ΔV_θ were unstable. The intuitive correction of raising the velocity when ΔV_θ is positive was tried first. The logic for this correction is that a positive ΔV_θ indicates NanoSat is in an orbit below MightySat and would need increased velocity to raise its orbit to achieve the desired lower velocity. (See Section III.) A test of this type ΔV_θ correction is shown in the upper plot of Fig. 7 where V_θ was changed each quarter period by $+15\%$ of the instantaneous ΔV_θ . The augmented ΔV_θ correction starts at the 3rd period. The results in Fig. 7 upper show that the Δr and ΔV_θ amplitudes oscillate with increasing amplitude after 500 minutes synchronous with the orbital period.

Making the correction based upon -15% of ΔV_θ also gave oscillations but they repeated every fourth orbital period as shown in Fig. 7 lower. These oscillations were caused by the Δr_d (step 3) portion of the algorithm chasing the reduction in radius that results from the reduction in velocity made when ΔV_θ is too large. Note that Fig. 7 upper shows increased radius in the 4th period when the low velocity in the 3rd orbit was increased and that Fig. 7 lower shows decreased radius at this time from the previously decreased the velocity. In each case a velocity change to NanoSat produces an opposite polarity velocity change approximately one orbit later. This reversal of the desired change occurs because of reactive effects of orbital mechanics caused by conservation of kinetic and potential energy explained in Section III.

Figure 7. (upper) A calculation for an augmented orbit correction every $1/4T$ based upon the instantaneous ΔV_{θ} . The correction was a positive 15% of ΔV_{θ} . The ΔV 's for the drag correction pulses are indicated as diamonds.

(lower) A calculation for an orbit correction every $1/4T$ based upon a negative 15% of the instantaneous ΔV_{θ} .

VII. Controlled Azimuthal Positional Changes

Azimuthal positional moves can be made efficiently by changing the size of the drag compensation thrust pulses. During such a move the Δr changes, therefore it is necessary to use the theoretical size of the Δr change as a reference to stabilize the orbit during the move. To avoid this complication here, I used the Δr_d previous to the onset of the pulse changes to determine the size of the baseline drag compensation pulses necessary during the move and for the following orbital period. The standard algorithm was then resumed. One to four Hohmann thrusts executed during an orbit, followed by similar opposite polarity thrusts the next orbit, give a clean azimuthal change. The $1/4T$ periodic $\Delta V_{\theta,d}$'s of ~ 1.1 mm/s (1.1 s duration 10 mN thrust) preclude larger thrusts without rotating the satellite 180° . A maneuver with one 1.1 mm/s Hohmann thrust per period gives an azimuthal change ~ 20 m. The azimuthal change can be reduced by decreasing the amplitude of the drag thrust modification. A larger change can be made by using four pulses per orbit, waiting one or more orbital periods before applying the reverse pulses, or adding more pulses in later orbits, as will be illustrated below.

The two period maneuver minimizes the time before the standard ΔV algorithm is resumed and therefore minimizes accumulation errors in Δr . The calculation for a maneuver with two 1.1 mm/s ΔV_θ pulses, with $\pm 5\%$ variation in the MightySat drag loss every $T/4$, is shown in Fig. 8 upper. Note the increase and decrease in the size of the drag pulses plotted as diamonds during the 10th and 11th periods. (Four equal drag compensation pulses/orbit based upon formula (c) were used for the baseline drag compensation in this calculation.) A plot of this maneuver and a similar opposite polarity maneuver initiated at the 20th orbit are plotted in the reference frame of MightySat in Fig. 8 lower. The reference frame moves to the right.

Figure 8. (upper) An azimuthal orbital change of 38 m produced by two +1.1 and two -1.1 mm/s ΔV_a pulses.

(lower) The path taken by NanoSat as it is ejected from MightySat according to the calculation shown in the upper plot of the figure. The circles are plotted at 10 minute intervals starting with the 5th minute. The first two large loops show the path of NanoSat before orbit stabilization. Drag compensation begins after the 1st orbit. Positive and negative 38 m azimuthal transfers starting on the 10th and 20th orbits are shown at left.

VIII. Maneuvering Around MightySat

The maneuver shown in Fig. 8 can be adjusted to allow NanoSat to circle MightySat. This could be accomplished by initiating the maneuver at a smaller $r\Delta\theta$ offset or by increasing the number of ΔV pulses in the sequences. A calculation of such a maneuver with sixteen 1 mm/s ΔV pulses is shown in Fig. 9. The circulation maneuver is illustrated in the MightySat coordinate frame shown in Fig. 10.

Figure 9. A calculation of an orbital maneuver with eight ± 10 mN-s impulses added to the drag compensation pulses after the 10th and 26th periods.

Figure 10. The path taken by NanoSat as it circles MightySat according to the coordinates shown in Fig. 9. The circles are plotted at 10 minute intervals starting at the 5th minute. The arrows represent the direction of motion. The dotted curve represents the initial two orbits after ejection prior to automatic orbit stabilization.

IX. Loss of ΔV Thruster Control

Loss of thruster control will cause NanoSat to quickly regress behind MightySat because NanoSat is assumed to have a smaller drag deceleration. The calculations for ejections at 0, 90, and 180° shown in Fig. 11 (upper), assume loss of thruster control after the 8th orbit. Note that no collisions occur in this scenario. In addition, there is no danger of a collision between the satellites if the thruster fails immediately after ejection at ejection angles from 0 to 90°.

Backward ejections are prone to collisions however. One such case occurs on the first orbit after ejection at 109° before the first thruster orbit stabilization pulse. This can be seen by extrapolating the results shown in Fig. 14 in the appendix for the 110° ejection example. There are also a series of nine ejection angles where collisions between the satellites occur after two to ten orbits for ejection angles between 110 to 180° if thruster control is lost at ejection. The trajectory for a collision after the second orbit for a 111° ejection is shown in Fig. 11 (center). Note that if the thruster fires properly for stabilization the minimum of closest approach is ~ 20 m and a collision is avoided.

The escape trajectories are only slightly different for circular and elliptical orbits during the first few orbits after ejection. Therefore no additional collision hazards occur for elliptical orbits. The trajectory for a 90° ejection from circular and elliptical orbits are shown in Fig. 11 (lower). The radial and azimuthal displacements for the elliptical orbit calculation show increasing oscillations at late time because the NanoSat and MightySat orbits get progressively farther out of phase at late times. The satellites are separated by 13.3 km one day after loss of thruster control.

Figure 11. The trajectories of NanoSat displayed in the MightySat reference frame for various losses of thruster control scenarios. (upper) Loss of control after the 8 orbital period for 0, 90, and 180° ejections. (center) A 110° ejection with loss of thruster control at ejection and after the 8 period. (lower) A 90° ejection from an elliptical and circular orbit. The apogee and perigee for the elliptical orbit were 480 and 445 km altitude

X. Conclusions

The algorithm described gives good orbit stabilization using only the Δr between the satellites as input. Four equally time spaced drag compensation and one orbit symmetrization pulse are used per orbit. When drag decelerations are constant the algorithm gives an azimuthal deadband of 1.3 m for a Δr measurement error of ± 5 cm. Approximating the variable deceleration expected for MightySat with a periodic stepwise deceleration of 0.5 to 2 $\mu\text{m/s}^2$ gives a deadband of 3 m. The addition of a $\pm 10\%$ random variation in the step size of this drag increases the azimuthal deadband to 16 m. The variation is not expected to be random so the final azimuthal deadband will be ~ 5 m.

It is shown that azimuthal positional changes and a circling maneuver around MightySat can be accomplished by changing the size of pairs of drag compensation pulses separated by one orbital period. These Hohmann transfers maximize the azimuthal change efficiency because thrust that is necessary for drag compensation is used to make the changes. The NanoSat would pass $r\Delta\theta/3\pi$ above/below MightySat for regressive/progressive moves, where $r\Delta\theta$ is the desired azimuthal change.

The orbit stabilization is now accomplished in the first two orbits after ejection and the optimum ejection angle to minimize stabilization thrust is 86° (See Appendix A). At this angle the stabilization velocity that must be supplied by the thruster is half the ejection velocity. An ejection velocity of 0.02 m/s at 86° gives an azimuthal separation after ejection and orbit stabilization of 187 m. A collision between the satellites during orbits with normal or loss of thruster control can be avoided by ejecting upwards between 0 to 100° from the orbital direction. Loss of thruster control would cause NanoSat to move progressively behind MightySat at a rate of ~ 13 km/day.

Appendix A (Ejection Stabilization)

The results of the stabilization of a typical ejection are shown in Fig. 12. The Hohmann thrusts at $T = 1.2$ and 1.7 are determined from $\Delta r_{\min}$ and $\Delta r_{\max}$. The drag compensation pulses are based on the change in differential radius during the first period, Δr_{d0} . The sizes of the Hohmann and drag compensation pulses are determined from relations (a) and (b) in Section IV. They begin after the first period. The azimuthal offset achieved versus ejection angle is shown in Figs. 13. The jump between 10 and 20° occurs when the minimum of the first period moves back in time to a point less than $T/2$. When this occurs the satellites must orbit twice before the first Hohmann pulse is executed to allow an accurate measure of Δr_{d0} . The timing of this minimum moves back as the angle is increased until at 180° the first Hohmann thrust occurs at the end of the first period. The cycle repeats between 190 and 370° . The example in Fig. 12 requires $3T/2$ for stabilization.

Figure 12. An orbit calculation with 0.02 m/s ejection velocity at 100° showing the times when the program measures the difference in radial position of the satellites, $\Delta r_{\max}$ and $\Delta r_{\min}$ and makes Hohmann thrust corrections $\Delta V_{\max}$ and $\Delta V_{\min}$.

Figure 13. The azimuthal offset $r\Delta\theta$ versus injection angle. The algorithm switches from one to two orbits before making the first Hohmann correction at 10 and 190° .

A plot of the trajectories of NanoSat in the MightySat reference frame for 60 to 110° ejections where 90° is upward is shown in Fig. 14. The ejection velocity is 0.02 m/s. The oscillations correspond approximately to the orbital period. Note that for a 110° ejection NanoSat approaches within 8.6 m of MightySat 140 min. after ejection. The data from 42 periods is plotted to demonstrate the small deadband achieved with a $\pm 5\%$ measurement error in the δr used for the orbit stabilization. The scatter noted on the 60° ejection trajectory is from a small third orbit that occurred before stabilization. The absolute sum of the Hohmann transverse ΔV 's that occur during the first three orbits are shown in Fig. 15, along with the minimum of closest approach and azimuthal offset. The optimum ejection angle to minimize the initial Hohmann thrusts is 86°. Also note that the minimum of closest approach has a local maximum near 115° where NanoSat circles MightySat equidistantly for one orbit before stabilization is achieved.

Figure 14. A plot of the trajectories of NanoSat in the MightySat reference frame for 60 to 110° ejections where 90° is upward. The symbols are plotted at 10 min. intervals starting at 5 min.

Figure 15. The absolute sum of the Hohmann corrections for the 2 and 3 orbits, minimum of closest approach, and azimuthal offset for ejections at 50 to 110°.

Appendix B (Gas Thruster Design)

There are numerous review articles describing the trade-off between propulsion systems for small satellites. The comprehensive 1997 review article³ by Juergen Mueller is one of the best. His list of 82 references cover nearly all of the thruster concepts that would apply to the SNL NanoSat. Figure 6 shows the ratio of thrust to power for some typical electrically powered thrusters. The electrical efficiency of each device can be obtained from the theoretical curves shown in Fig. 16 for 100, 50 and 10% efficiency where a 100 % efficient thruster is given by $T/W = 2/9.8 * I_{sp}$. Note that the thrust/power ratio decreases with increasing specific impulse, I_{sp} .

It was very important to maximize the thrust to power ratio on the SNL NanoSat because the satellite design resulted in only ~ 5 W of electrical power. For this reason a cold gas thruster with $T/P > 1$ mN/W was recommended as the ΔV thruster. The Moog 58x125 thruster used for the Pluto Fast Flyby satellite⁴ is a possible basis for the NanoSat thruster. It supplies the desired thrust of ~ 10 mN at a chamber pressure of 10 psi and uses only ~ 2 W power for continuous operation and ~ 20W for the 10 ms opening time. Thrust durations as short as 20 ms are possible. A schematic of such a thruster with a propellant handling system is shown in Fig. 17.

Figure 16. The ratio of thrust to power for some electric powered thrusters. A 100% efficient thruster follow the curve $T/P = 2/9.8 * I_{sp}$.

Figure 17. A schematic of a gas thruster system

The specifications and performances of several gas propellants are compared in Table 1. Ammonia has the best overall performance but is ruled out for the SNL NanoSat because of toxicity. Nitrous oxide is very similar to CO_2 in specific and volumetric impulse, but has $10^\circ F$ more headroom below the critical temperature so would have less steep pressure-temperature dependence near $70^\circ F$. It also has a freezing temperature of $-131^\circ F$ compared to a sublimation temperature of $-109^\circ F$ for CO_2 , so freezing will be less of a problem with N_2O . Xenon is also an attractive fuel because of its extensive flight record and even lower freezing temperature of $-169^\circ F$. Propane is similar to several other combustible fuels and has many nice features such as low vapor pressure but has low volumetric impulse.

Fuel	Molecular Weight	Vapor Press. psi	Critical Temp. °F	Critical Press. psi	Freezing Temp. °F	Density @ <2000 psi g/cm ³	Isp ¹⁰ @68° F sec	I/V @72° F N-s/liter
Ammonia	17	130	270.4	1639	-214	0.61	96	630
N ₂	28	gas	-320	492	-346	0.16	68	119
CO ₂	44	750	87.8	1072	-109	0.725	61	433
N ₂ O	44	780	97.7	860	-131	0.736	61	449
Propane	44	105	206	617	-306	0.50	61	299
Xenon	131	gas	67.8	852	-169	1.70	26	433

Table 1. Cold gas propellant specifications and performances. The specific impulse, Isp, is a practical value 20% below the theoretical value.

The thermophysical properties⁵ of NH₃ (ammonia), CO₂, and N₂ are shown in Figs. 18, 19, and 20. These gases illustrate different types of fuels that could be used as a propellant in a gas thruster system. It is assumed that the propellants will be used at working temperature of ~ 70° F, because the Isp decreases with $T^{1/2}$. Ammonia is stored as a liquid far below (type 1), CO₂ near (type 2), and N₂ far above (type 3) the critical temperature. The data in the graphs allow an assessment of the fuel density that can be stored in a propellant tank and show the dependence of fuel pressure on temperature. The last quantity is important because tank mass and pressure regulator design depend on the propellant pressure and the pressure increases rapidly with temperature near the critical temperature. This could cause a tank explosion when maximum fuel loading is used with a type 2 fuel such as CO₂.

The plot for NH₃ in Fig. 18 shows that it has a vapor pressure of ~ 130 psi at 72° F and at a density of 0.61 gm/cm³. The temperature dependence of pressure is fairly steep but the low pressure does not present a containment problem for temperatures < 150° F. The relatively high Isp of 96 s together with the liquid containment make this a desired fuel although its toxicity reduces its appeal.

The properties of carbon dioxide are shown in Fig. 19. The pressure curves that break above the continuous curve (upper left) are obtained from lineouts of the temperature contours shown in the lower graph. The CO₂ data shows that the density is limited to ~ 0.7 g/cm³ for a 2000 psi tank with T < 116° F. Use of a relief valve could allow filling the tank to ~ 800 g/cm³ if T < 95° F. A practical upper limit to the fuel density is 0.725 gm/cm³ that would give a pressure of 2000 psi at 110° F. At this density and T > 74° F the CO₂ in the tank would be filled with a liquid indistinguishable from a gas (the region labeled liquid in Fig. 19) with a density greater than liquid density.

Figure 18. The continuous curve is the equilibrium pressure of ammonia plotted versus temperature. The dashed and dotted curves are density of liquid and vapor ammonia.

Figure 19. (upper left) Thermophysical properties of carbon dioxide. Pressure versus temperature and average density. Liquid and vapor densities are shown as dashed and dotted curves.

(upper right) Average density versus temperature and percent of liquid in tank.

(lower left) Density versus pressure and temperature. Below the critical temperature liquid and vapor coexist in the tank and the ratio of the liquid to vapor depends on the amount of CO_2 into the tank. In the region labeled liquid the CO_2 is a gas with higher than liquid density.

Xenon and nitrous oxide, N_2O , are also type 2 gases with similar thermophysical properties to N_2O with critical temperatures 20°F below and 10°F above the value for CO_2 . The higher critical temperature of N_2O makes it more desirable from a propellant storage standpoint because it has a less steep pressure curve near the operating temperature of 72°F . Each of these fuels has low toxicity. These criterion combined with the fact that leakage is reduced with liquid fuels stored at moderated pressures made nitrous oxide the most desirable gas propellant for small satellites. The choice of liquid N_2O or CO_2 as an excellent propellant for small satellites is confirmed by others⁶ also.

The data for compressed nitrogen are shown in Fig. 20. This gas is a very common propellant for large satellite and re-entry vehicle guidance and control. The gas is normally used at 3500 to 5000 psi where a fuel density of $\sim 0.3\text{ gm/cm}^3$ can be stored. An advantage with N_2 is that the pressure increases smoothly with temperature and density so the amount of fuel remaining can be determined from the pressure if the temperature is measured. Also there will not be anomalous shifts in the center of gravity of the satellite that could occur with liquid fuels. A disadvantage is the severe fractional leakage problem with gas propellant stored at high pressures in the small containers used with small satellites.

Another propellant possibility is hydrogen peroxide, H_2O_2 , which can be stored as a liquid also at density of 1.4 gm/cm^3 . It is normally used with a silver catalyst to promote combustion into H_2O and O_2 . This fuel is flammable and when burned gives an Isp of 150 s. If used as a cold gas the Isp would be ~ 65 s. The freezing temperature of 20°F is a disadvantage.

Figure 20. Thermophysical properties of compressed nitrogen.

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References:

1. A. Chobotov, Ed. "Orbital Mechanics", AIAA Education Series, AIAA (1991), Formula 11.10,
2. C. D. Brown, "Spacecraft Propulsion", AIAA Educational Series, AIAA, Washington, DC. 1996.
3. Juergen Mueller, "Thruster Options for Microspacecraft: A review and Evaluation of Existing Hardware and Emerging Technologies", 33rd AIAA Joint Propulsion Conference and Exhibit, Seattle, WA, AIAA paper 97-3058, July 1997.
4. D. H. Morash, and L. Strand, "Miniature Propulsion Components for the Pluto Fast Flyby Spacecraft", AIAA Paper 94-3374, Indianapolis, IN, July 1994.
5. N. B. Vargaftik, "Handbook of Physical Properties of Liquids and Gases", 2nd Ed. Hemisphere Publishing Corporation, 1975.
6. J. Ward, S. Jason, and M. Sweeting, "Micorsatellite Constellation for Disaster Monitoring", 13th AIAA/USU Conf. on Small Satellites, paper SSC99-V-2, September 1999.

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