Configuration Space Faddeev Calculations

Progress Report

for Period 1 November 1993 – 31 October 1994

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January 1995

Prepared for

THE U. S. DEPARTMENT OF ENERGY
AGREEMENT NO. DE-FG02-86ER40286

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ABSTRACT

The detailed study of few-body systems provides one of the most precise tools for studying the dynamics of nuclei and nucleons. Our research program consists of a careful theoretical study of few-body systems and methods for modeling these systems. During the past year we have completed several aspects of this program.

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Gerald L. Payne	40%
William H. Klink	40%
Wayne N. Polyzou	40%

On the following pages is a summary of the theoretical research which has been carried out under Contract No. DE-FG02-86ER40286 at the Department of Physics and Astronomy of The University of Iowa. The requirements of the contract have been fulfilled.

II. SUMMARY OF RESEARCH COMPLETED

Triton Calculations with the New Nijmegen Potentials

J. L. FRIAR, G. L. PAYNE, V.G.J. STOKS, AND J. J. DE SWART

In recent years one of the important problems of few-nucleon physics has been resolved. The non-relativistic Schrödinger or Faddeev equation can now be solved numerically for an arbitrary (energy-independent) potential model with negligible error. Such complete or "exact" solutions have been carried out at the level of 1% (or less) error for the ground states [1] of ^3He and ^3H (including a Coulomb interaction in the former case) for the ground state [2] of ^4He , for the low-lying (continuum) states [3] of ^5He , for the zero-energy scattering states of the n-d and p-d systems [4], for transitions [5] between these systems and the ground states, and for the n-d continuum states above breakup threshold [6].

The results of these calculations can be rather simply summarized. The ground states are underbound and their underbinding is correlated. The triton is underbound by an amount which varies from 0.2 to 1.1 MeV for a wide variety of potential models [1]. The sizes of these systems are strongly correlated with their binding, and reasonable extrapolations to the physical binding energies of ^3H and ^3He produce sizes which agree with experiment. The alpha particle is underbound by an amount which is correlated

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[1] C. R. Chen, G. L. Payne, J. L. Friar, and B. F. Gibson, Phys. Rev. C **31** (1985) 2266; **33** (1986) 1740; J. L. Friar, B. F. Gibson, and G. L. Payne, Phys. Rev. C **35** (1987) 1502; J. L. Friar, B. F. Gibson, and G. L. Payne, Phys. Rev. C **37** (1988) 2869; these references contain a representative set of potential model results.

[2] J. Carlson, Phys. Rev. C **36** (1987) 2026.

with the triton underbidding (by roughly a factor of four) and correcting one problem will usually correct the other. Low-energy n-d and p-d scattering and capture reactions are in reasonable agreement [4,5] with experiment (after extrapolating), except for the p-d scattering length, whose experimental value may be suspect.

One possible explanation of the disagreement between the theoretical predictions and the experimental values are three-nucleon forces [7]. Traditionally, the nuclear force is described by a sum of two-body (pairwise) potentials. However, more complicated three-body forces are present, which require the simultaneous specification of the spatial coordinates of all three nucleons, as well as their spin and isospin states. These three-nucleon forces are expected to contribute a rather small amount of binding, which is estimated to be 1–2 MeV to the triton potential energy (out of a total of roughly 50 MeV). Calculations of the triton binding which include three-nucleon forces are consistent [8] with this estimate.

While it is not yet possible to resolve problems with the three-nucleon forces, it should be possible to eliminate much of the uncertainty associated with the two-nucleon force. The Nijmegen group has constructed a Reid-like two-body potential model where each partial wave is parameterized independently. Introducing as many parameters as necessary, this model fit the nucleon-nucleon scattering data in the range 0–350 MeV almost as well as the Nijmegen phase-shift analysis. The Reid-like Nijmegen model constructed this way is denoted by Nijm I. A feature of relativistic origin in the Nijmegen

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[3] J. Carlson, (to be published).

[4] C. R. Chen, G. L. Payne, J. L. Friar, and B. F. Gibson, Phys. Rev. C 44 (1991) 50.

[5] J. L. Friar, B. F. Gibson, H. C. Jean, and G. L. Payne, Phys. Rev. Lett. 66 (1991) 1827.

[6] W. Glöckle, H. Witała, and Th. Cornelius, Nucl. Phys. A508 (1990) 115c; W. Glöckle and H. Witała, Kerntechnik 57 (1992) 228.

potentials is the momentum-dependent part of the central potential, which follows from field theory. It gives rise to a nonlocal structure to the potential in configuration space. Such a term might be expected to behave differently in the triton, and for this reason two Reid-like versions of the Nijmegen potential were constructed: The nonlocal Nijm I potential, which contains these momentum-dependent terms (as do the Nijm78 and Nijm92 potentials), and a local Nijm II potential, where these terms were intentionally omitted. Also updates of two other *local* potentials were constructed: The Reid soft-core potential [9] was reparameterized using sums of regularized Yukawa functions and is denoted by Reid93. The Argonne potential [10] was extended to include charge-independence breaking in the phenomenological parameterization of the short-range interaction and is denoted by AV18.

The triton bound-state result for the nonlocal Nijm I potential for 34 channels was 7.72 MeV, which is nearly 800 keV lower than the experimental value of 8.48 MeV. The charge radii for ^3H and ^3He , and the Coulomb energy of ^3He were not significantly different from what one would expect for these binding energies. The results for this Nijm I potential are not significantly different from those of the original Nijm78 potential, whose binding energy for the triton is 7.63 MeV. The quality of the local Nijm II potential is equally as good as that of the nonlocal Nijm I potential, both having a χ^2 per datum of 1.03. Triton calculations with the Nijm II potential produced 7.62 MeV for 34 channels. This is a strong indication that replacing local structure in configuration space by a nonlocal one can affect the triton binding energy, even when the quality of the fit to the NN data remains unchanged. We also solved the triton bound state for the Reid93 and AV18 potentials. The Reid93 potential has an equally good fit to the NN data. The triton binding energy for this potential was 7.63 MeV, which is very close to that of the Nijm II potential. It is worth noting that the original Reid68 potential had a triton binding energy

of 7.35 MeV. The 300 keV difference in binding is the result of different quality fits to the NN data, and to a very different current data set than existed in 1968. The updated Argonne potential AV18 fits the NN data with a χ^2 per datum of 1.30, and produces a triton binding energy of 7.62 MeV.

The local potential results suggest (but obviously do not prove) that local potentials which fit the NN scattering data very well, bind the triton by a unique value of about 7.62 MeV. Should this prove to be true, the physics issues in the triton problem (besides the question of three-nucleon forces) then shift to the origin and presence of nonlocalities in the NN force. Indeed, the 100 keV difference between the Nijm I and Nijm II models is the effect of just such a nonlocality. These new Nijmegen potentials are the best ever constructed, and triton binding calculations with them provide a benchmark against which calculations with other potential models should be compared.

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[7] S. A. Coon, M. D. Scadron, P. C. McNamee, B. R. Barrett, D.W.E. Blatt, and B.H.J. McKellar, Nucl. Phys. A317 (1979) 242; S. A. Coon and J. L. Friar, Phys. Rev. C 34 (1986) 1060.

[8] L. D. Faddeev, Zh. Eksp. Teor. Fiz. 39 (1960) 1459 [Sov. Phys. - JETP 12 (1961) 1014].

[9] R. V. Reid, Ann. Phys. (N.Y.) 50 (1968) 411.

[10] R. B. Wiringa, R. A. Smith, and T. A. Ainsworth, Phys. Rev. C 29 (1984) 1207.

Relativistic and Nonrelativistic Models of Deuteron Structure Functions

W. N. POLYZOU AND W. GLÖCKLE

Hüber, Glöckle, and Bömelberg [1] studied quasi-elastic electron scattering in a nonrelativistic two-nucleon model. The purpose of this study was to understand the onset of scaling in a numerically solvable model. In this nonrelativistic model scaling properties of the longitudinal structure $S_L(|\vec{q}|, \nu)$, considered as a functions of the laboratory frame momentum transfer $|\vec{q}|$ and energy transfer ν , are studied. In the high-momentum transfer limit $|\vec{q}|^2 S_L(|\vec{q}|, \nu)$ and $|\vec{q}| S_L(|\vec{q}|, \nu)$ scale to functions of $\eta := 2m\nu/|\vec{q}|^2$ and $y := |\vec{q}|(\eta - 1)/2$, respectively. The results confirmed the analysis of West [2], with the scaling functions $\delta(1-\eta)$ and $P_l(y)$ where $P_l(y)$ is the probability distribution of momentum in the deuteron in the direction of the momentum transfer.

The analysis of reference [1] showed that the y -scaling was strongly affected by the final-state interactions and, except at very high momentum transfers, was confined to a narrow region near the quasi-elastic peak. This analysis was nonrelativistic, and thus not appropriate for making conclusions about the scaling limits.

We formulated a relativistic model of this system in order to understand the how these conclusions are modified in a relativistic model and to understand the nonrelativistic limit. We studied these questions in a relativistic quantum mechanical model based on Hamiltonian light-front quantum mechanics with a conserved covariant current. The mass operator in the relativistic model was a rank on s-wave interaction between spinless nucleons designed to fit the deuteron binding energy and the N-N scattering length. This relativistic model is scattering equivalent to a nonrelativistic model with the same wave functions. The current operator is equivalent to one obtained in a relativistic impulse approximation, although in this case it is interpreted as the exact model current.

The results of this comparison were summarized below. This model has two independent structure functions, W^{++} and W^{22} . All of the predictions were tested numerically, and the numerical results were all consistent with the following theoretical predictions:

- In the relativistic model the scaling variable η is replaced by the relativistic scaling variable $\eta_r = m_d \nu / q^2$. In the nonrelativistic limit $\eta_r \rightarrow \eta$.
- In this model both $(q^2/m_d)W^{++}$ and $(q^2/m_d)W^{22}$ scale to functions of η_r . These are both sharply peaked functions that are maximal at the quasi-elastic peak ($\eta_r = 1$), but not delta functions. The scaling function associated with W^{++} is the light-front x -probability distribution in the deuteron and the scaling function associated with W^{22} is the x -distribution of the expectation value of the square of the component of the deuteron momentum in the $\hat{n} \times \vec{q}$ direction, where $\hat{n}$ defines the orientation of the light-front.
- Numerical Bjorken scaling is seen only for very high q^2 ($q^2 \gg m_d^2$).
- In the nonrelativistic limit W^{++} becomes the longitudinal structure function and all of the nonrelativistic results are recovered. The nonrelativistic and scaling limits do not commute.
- The relativistic model did not exhibit y scaling in the asymptotic limit. Three distinct choices of y were tried. These include the nonrelativistic choice, a relativistic choice that is obtained by looking at the leading correction to the Bjorken limit, and a hybrid of the two.

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[1] D. Hüber, W. Glöckle, and A. Bömelburg, Phys. Rev. C **42** (1990) 2342.

[2] G. West, Rep. C. **18** (1975) 262.

- All three choices of the y variables agreed with the nonrelativistic y in the window of momentum transfers $m_d^2 \gg q^2 \gg k^2$ where k^2 is the momentum scale of deuteron wave function. The relativistic model does exhibit y -scaling in all three variable in this window, provided final-state interactions are turned off. The y scaling fails about when Bjorken scaling sets in.

Relativistic Euclidean Quantum Models

F. COESTER AND W. N. POLYZOU

Time ordered Green functions are central to many “relativistic few-body models” motivated by local quantum field theory. In obtaining these models the field theory is truncated and some fundamental input is replaced by phenomenological input. Although it is often difficult to interpret these models as quantum theories of linear operators on a Hilbert space, the results of model calculations are often similar to results obtained in quantum mechanical models of the same system.

In many applications Euclidean Green’s functions are the primary objects, since the equations for the Euclidean Green’s functions are better behaved. As a first part of a more general study, we studied the quantum mechanical interpretation of model Euclidean Green’s functions. The methods used follow those used in the Osterwalder-Schrader reconstruction theorem [1], however our analysis was limited only to those properties that are essential for a quantum mechanical interpretation.

Given any ordering of the points in the model Euclidean Green’s function in which the first k space time-points correspond to final particles and the next $n - k$ points correspond to initial particles, the condition that the Hamiltonian be bounded from below was shown to require that the n -point Euclidean Green’s function be analytic in the $4(n - 1)$ successive difference variables associated with the given ordering. The domain of analyticity includes those points in which the real part of the relative Euclidean time is larger than the magnitude of the imaginary part of the relative spatial coordinate. The analyticity conditions must hold for all combinations of relative variables in which the Euclidean times of all of the final particles are greater than those of the initial particles.

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[1] K. Osterwalder and R. Schrader, *Comm. Math. Phys.* **31** (1973) 83; **42** (1975) 281.

Lorentz invariance and the spectral condition requires that the Euclidean Green's functions transform covariantly under the complex Euclidean group. The covariance under the complex Euclidean transformations can be used to extend each domain of analyticity described above to all points that can be reached by complex Euclidean transformations. The extended domains of analyticity corresponding to each permutation of the initial and final particles among themselves have common sets of real points. This extends the domain of analyticity.

In addition to the strong analyticity constraints that must be satisfied by the Euclidean Green's functions, the requirement that there be a Hilbert space on which the complex Euclidean group acts unitarily when restricted to the subgroup corresponding to real Lorentz transformations further restricts the Green's functions. This condition requires that the Euclidean Green's functions satisfy a subset of the Osterwalder-Schrader positivity conditions. Specifically if θ is the Euclidean time reversal operator, the Euclidean Green's function $E(\theta f_1^* \cdots \theta f_k^* : f_{k+1} \cdots f_n)$ should define a positive matrix when integrated against test functions f_e that vanish for negative Euclidean times. This must hold whenever the first k arguments correspond to final particles and the last $n - k$ correspond to initial particles.

Cluster properties require that the Euclidean Green's functions can be expanded as products of subsystem Green's functions with similar properties. The specific terms that appear in the expansion depend on the particle interpretation of the model.

We have constructed explicit representations for the Hilbert space using the Euclidean Green's functions and the infinitesimal generators for the Poincaré group.

The result of this analysis is that the requirement that a set of Euclidean Green's functions have a quantum mechanical interpretation imposes very strong requirements on these functions. Although they are weaker than the requirements imposed by a local

field theory, they are nevertheless very restrictive. The implications of these results are currently being explored.

Benchmark Solutions for n-d Breakup Amplitudes

J. L. FRIAR, G. L. PAYNE, W. GLÖCKLE, D. HÜBER, AND H. WITALA

Benchmark comparisons or “homework problems” have become increasingly common and useful in few-nucleon physics in recent years [1–5]. The remarkable successes in this field in solving seminal problems originally identified decades ago have been followed in some cases by the successful application of new techniques, which promise to have wider and easier applicability than some of the original methods. These efforts can be greatly aided by having benchmarked results for selected problems for purposes of comparison and analysis, and for establishing levels of accuracy.

Previously, we had done benchmark calculation [4] of scattering phase shifts, δ , and inelasticities, η , for n-d scattering above breakup threshold. In this work we complemented the elastic phase-shift parameters of Ref. [4] with breakup amplitudes for the reaction $n+d \rightarrow n+n+p$ at two initial neutron (lab) energies, 14.1MeV and 42MeV, corresponding to two of the three energies used in that reference. The third of those energies was so low that there was virtually no breakup. We solved the nonrelativistic Schrödinger equation containing the revised (and definitely not unique) Malfliet-Tjon I-III model s-wave potential [4]. The breakup amplitude is defined by the asymptotic three-nucleon Faddeev wave function in the center-of-mass frame, ψ_1 , which has the form [6]:

$$\psi_1(\mathbf{x}, \mathbf{y}) \sim e^{i\mathbf{p} \cdot \mathbf{y}} \phi_d(x) + \frac{\mathcal{A}(\theta) e^{iK\rho}}{(K\rho)^{5/2}} + \cdots + ,$$

where scattered elastic waves have been ignored, and the Schrödinger (total) wave function is given by ψ_1 and its Faddeev permutations: $\Psi \equiv \psi_1 + \psi_2 + \psi_3$. The standard Jacobi coordinates $\mathbf{x}$ and $\mathbf{y}$, as well as the usual hyperspherical coordinates $\rho = (\mathbf{x}^2 + \frac{4}{3}\mathbf{y}^2)^{1/2}$ and $\theta = \tan^{-1} \left[\frac{2\mathbf{y}}{\sqrt{3}\mathbf{x}} \right]$ determine the asymptotic form in terms of the initial neutron (lab) momentum, $\mathbf{p}$, and breakup momentum, $K(E = \hbar^2 K^2/M)$.

The Los Alamos/Iowa results were calculated in configuration space, where the Faddeev approach leads to a set of coupled integro-differential equations, whose unique solution is guaranteed by the boundary conditions specified by the asymptotic form of the wave function given above. The Bochum calculations were performed in momentum space, where the Faddeev approach leads to integral equations formulated directly for the off-shell breakup amplitudes [7]. The two methods have a totally different mathematical structure. The results obtained in the two approaches were in excellent agreement, typically better than 1%.

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- [1] C. Hajduk and P. U. Sauer, Nucl. Phys. **A369** (1981) 321. To the best of our knowledge this reports the first three-nucleon benchmark, of 5-channel Faddeev bound-state calculations in both momentum and configuration spaces. Since that time many comparisons of bound-state calculations have been made.
  - [2] C. R. Chen, G. L. Payne, J. L. Friar, and B. F. Gibson, Phys. Rev. C **44** (1991) 50; A. Kievsky, M. Viviani, and S. Rosati, Nucl. Phys. A (in press). These two references solve identical models of n-d and p-d scattering at zero energy.
  - [3] D. Hüber, W. Glöckle, J. Golak, H. Witała, H. Kamada, A. Kievsky, S. Rosati, and M. Viviani, Phys. Rev. C (in press).
  - [4] J. L. Friar, B. F. Gibson, G. Berthold, W. Glöckle, Th. Cornelius, H. Witała, J. Haidenbauer, Y. Koike, G. L. Payne, J. A. Tjon, and W. Kloet, Phys. Rev. C **42** (1990) 1838.
  - [5] Th. Cornelius, W. Glöckle, J. Haidenbauer, Y. Koike, W. Plessas, and H. Witała, Phys. Rev. C **41** (1990) 2538.
  - [6] W. Glöckle and G. L. Payne, Phys. Rev. C **45** (1992) 974.
  - [7] H. Witała, Th. Cornelius, and W. Glöckle, Few-Body Systems **3** (1988) 123; W. Glöckle, Lecture Notes in Physics **273** (1987) 3.

Assessment of Triton Potential Energy

J. L. FRIAR AND G. L. PAYNE

Substantial and numerous recent successes [1] in the area of few-nucleon physics indicate that it is appropriate to begin an assessment of our level of understanding of the physics underlying the dynamics of few-nucleon systems. A variety of computational and theoretical techniques have been successfully applied to the bound (or low-lying) states of $A = 2, 3, 4, 5$, and 6 and to the scattering states of the trinucleon systems. Attempts are underway to extend this program and are likely to be successful. These efforts have been very informative and generally produce excellent agreement with experimentation except for a few small, but potentially very significant, discrepancies. A vital question is whether we can now (or ever) understand the nuclear force well enough to accommodate all that we have learned and that we hope to learn from future progress and from these discrepancies. Thus, we have made an assessment of the dominant features contributing to the triton potential energy, with the objective of understanding qualitatively their origins and sensitivities. Relativistic effects, short-range repulsion, and OPEP dominance were reviewed. A determination of the importance of various regions of nucleon-nucleon separation was made numerically.

A major historical problem within the field, which has tainted the aforementioned successes, is the old perception that the nuclear force is too complicated to be understood in anything approaching fundamental terms and that perturbation theory, which underlies virtually all theoretical approaches, cannot be performed because the coupling constants are too large and therefore any systematic development is impossible. At least within

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[1] J. L. Friar, Summary talk presented at XIVth International Conference on Few-Body Problems in Physics, Williamsburg, VA, May 31, 1994, ed. by F. Gross, AIP Conference Proceedings (to appear). This talk discusses the status of the field and possible future directions.

the few-nucleon systems, where “exact” or “complete” nonrelativistic solutions of the Schrödinger equation are now routine and dynamical details can be tested, arguments were presented that this pessimism is largely unfounded. The evidence that the exchange of a pion (OPEP) is the dominant component of the nuclear force for low energies was reviewed. In particular, the importance of the Nijmegen phase-shift analysis of nucleon-nucleon scattering data [2] was discussed. This analysis study, which included an explicit OPEP tail for separations $r > 1.6$ fm and a phenomenological treatment of the shorter-range interaction, concluded that the NN data require the exchanges of a charged pion with mass $m_{\pi^+} c^2 = 139.4(10)$ MeV, and a neutral pion with mass $m_{\pi^0} c^2 = 135.6(13)$ MeV. Both results are consistent with the free-particle masses, while the very small error bars reflect the overall importance of OPEP.

To study the contribution of various parts of the nucleon-nucleon potential to the triton binding energy, we numerically determined the contribution of the potential to the binding energy for inter-nucleon separations less than x . To do this we evaluated the matrix elements

$$\sum_{i=1}^3 \langle \Psi | \Theta(r_{jk} - x) V_i(r_{jk}) | \Psi \rangle.$$

For very large x , this approaches 100%. For the nucleon-nucleon interaction, the strong short-range repulsion is the dominant feature inside 1 fm NN separation. Although the net potential energy (tracked by the net kinetic energy) is repulsive inside 0.8 fm, the maximum accrual is quite small, $\sim 10\%$. The major accrual of attractive energy occurs between 1.0 and 2.0 fm, which is the domain of OPEP and possibly the TPEP tail. This calculation shows that the OPEP dominance results from barrier impenetrability at small NN separations. This suppresses the short-range contributions relative to what one might

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[2] R.A.M. Klomp, V.G.J. Stoks, and J. J. de Swart, Phys. Rev. C 44 (1991) R1258; V. Stoks, R. Timmermans, and J. J. de Swart, Phys. Rev. C 47 (1993) 512.

expect. It suggests that an even better understanding of the origins of the observed triton binding energy may be possible, allowing a credible separation of two-nucleon and three-nucleon force mechanisms.

Point Form Relativistic Impulse Approximation

W. H. KLINK

In point form relativistic quantum mechanics the interactions of many-body systems are put in the four-momentum operator and Lorentz transformations are kinematic. Matrix elements of the current operator are covariant in the sense that if invariant form factors are known, the current matrix element can be transformed to any reference frame without changing the one-body and many-body contributions from the current operator.

In Ref. [1] we have used point form relativistic quantum mechanics to obtain invariant form factors as reduced matrix elements of the Poincaré group for space-like momentum transfer that is valid for multiparticle final states, including breakup reactions. For systems in which the initial and final particles have the same mass and spin, the invariant form factors reduce to those of Yennie, Lévy, and Ravenhall [2]. In particular the various components of the current operator give the electric and magnetic form factors. Using the properties of the current matrix elements, a relativistic impulse approximation is formulated, in which the invariant form factors are computed with one-body current operators, all of which satisfy Poincaré invariance and current conservation. It is this impulse approximation that we propose to use in calculating deuteron form factors.

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[1] W. H. Klink, "Point Form Relativistic Quantum Mechanics and Electromagnetic Form Factors," preprint.

[2] D. R. Yennie, M. M. Lévy, and D. G. Ravenhall, *Rev. Mod. Phys.* **29** (1957) 144.

Analyzing pi-nucleon Production Data

W. H. KLINK AND M. ROGERS

Using point form relativistic quantum mechanics, it is possible to formulate a relativistic Lippman-Schwinger equation [1] in which the orbital and spin degrees of freedom can be coupled together in the same manner as is done nonrelativistically. This is of particular importance in analyzing reactions where resonances play an important role, because one can then couple the spin of the resonance to the various orbital angular momenta to obtain states diagonal in the total angular momentum.

It is possible to go further and define states in which all of the particles are coupled together simultaneously [2]. Instead of introducing stepwise coupled states with the awkward Racah coefficients needed to transform between different stepwise couplings, all the different stepwise couplings are related to the standard simultaneously coupled states.

We have applied this formalism to the analysis of pi-nucleon production data. Separable potentials with the correct threshold and resonance properties are used as input for exact solutions of the relativistic Lippman-Schwinger equation resulting in production amplitudes whose variables are directly connected with experimental data. We have begun analyzing pi-nucleon data where one pi-meson is produced and are comparing our results with those of Manley et al. [3].

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[1] W. H. Klink, Phys. Rev. C 41 (1990) 442.

[2] W. H. Klink, Ann. Phys. 213 (1992) 31.

[3] D. M. Manley and E. M. Saleski, Phys. Rev. D 45 (1992) 4002; D. M. Manley et al., Phys. Rev. D 30 (1984) 904; R. S. Longacre et al., Phys. Rev. D 17 (1978) 1795.

Quantum Anharmonic Oscillators and Particles in Curved Magnetic Fields

T. ALLEN AND W. H. KLINK

All quantum mechanical anharmonic oscillators have an underlying nilpotent group structure that can be used to relate anharmonic oscillators to other quantum mechanical systems such as particles in nonconstant electric and magnetic fields [1,2]. We have analyzed the simplest such example, namely quartic anharmonic oscillators resulting in magnetic fields of the form $\vec{B} = (\beta_1 + \beta_2 x)\hat{z}$. For his master's thesis, Tom Allen is analyzing the low-lying levels of the quartic anharmonic oscillator numerically and will use the resulting energy eigenfunctions to compute the wave functions for a particle moving under the influence of the magnetic field given above.

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[1] P.E.T. Jorgensen and W. H. Klink, Publ. RIMS 21 (1985) 969.

[2] W. H. Klink, "Nilpotent Groups and Anharmonic Oscillators," in *Noncompact Lie Groups and Their Physical Applications*, NATO ASI Series, edited by R. Wilson and E. A. Tanner (Dordrecht: Kluwer Academic Publishers, 1994).

III. PAPERS PUBLISHED, ACCEPTED OR SUBMITTED

Papers Published

(1 November 1993 – 30 November 1994)

1. H. C. JEAN, G. L. PAYNE, and W. N. POLYZOU
The Balian Brezin Method in Relativistic Quantum Mechanics
Few-Body Systems 16, 17, 1994
2. F. COESTER and W. N. POLYZOU
Vacuum Structures in Hamiltonian Light-Front Dynamics
Found. Phys. 24, 387, 1994
3. W. H. KLINK
Nilpotent Groups and Anharmonic Oscillators
Proceedings of the NATO Advanced Workshop on Noncompact Lie Groups
and Their Physical Applications, San Antonio, Texas, January 4–89, 1993
edited by Elizabeth A. Tanner and Raj Wilson (Kluwer Academic Publishers,
Dordrecht, 1994) p. 301

Papers Accepted or Submitted
(as of 30 November 1994)

1. J. L. FRIAR and G. L. PAYNE
Proton-Deuteron Scattering and Reactions
The Coulomb Problem, edited by Frank Levin and David Micha
[submitted]
2. J. L. FRIAR, G. L. PAYNE, W. GLÖCKLE, D. HÜBER, and H. WITALA
Benchmark Solutions for n-d Breakup Amplitudes
Phys. Rev. C [submitted]
3. J. L. FRIAR and G. L. PAYNE
Assessment of Triton Potential Energy
Few-Body Systems [submitted]
4. W. H. KLINK
Form Factors and Representations of the Poincaré Group
[to be published]
5. W. H. KLINK and T. TON THAT
Casimir Operators of Semidirect Products of Semisimple with
Heisenberg Groups
[submitted]
6. W. H. KLINK
Point Form Relativistic Quantum Mechanics and Electromagnetic
Form Factors
[submitted]

Meetings Attended

G. L. Payne

Few-Body Conference, Williamsburg, Virginia, May 1994

W. H. Klink

Few-Body Conference, Williamsburg, Virginia, May 1994

Conference on Interacting Boson Model, Padua, Italy, June 1994

Workshop talk, International Institute for Theoretical and Applied Physics,
Ames, Iowa, August 1994

Midwest Nuclear Theory Conference, Argonne National Laboratory,
Chicago, Illinois, October 1994

W. N. Polyzou

Few-Body Conference, Williamsburg, Virginia, May 1994

Invited Talks

W. H. Klink

“Applications of Point Form Relativistic Quantum Mechanics,”
Iowa State University, Ames, Iowa

W. N. Polyzou

“Scaling of Deuteron Structure Functions in a Relativistic
Light-Front Model,” C.E.B.A.F., October 1994

Visitors

Dr. Walter Glöckle, May 1994

Dr. Hsien-Chih Jean, September 1994

Dr. Charles Benesch, November 1994