Characterization of the Damping

of a Free Vibrating Piezoelectric Motor Stator

by Displacement Measurements

Federal Manufacturing & Technologies

S. S. Yerganian

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BY DISPLACEMENT MEASUREMENTS

S. S. Yerganian

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Topical Report

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Abstract

An eight-millimeter ring type traveling wave piezoelectric motor is being developed for use in the next generation of strong link safing mechanisms. These motors generate torque by the interaction of a rotor with a vibrating circular plate-like stator. This research is a step toward determining the mechanical energy loss in the vibrating stator itself (without the rotor) and neglects the electrical energy losses of the piezoelectric ceramic. Test samples were developed for measuring these energy losses, and the representations of the vibration of the stator and test samples were simplified to an equivalent single degree of freedom system. A measuring system was developed to determine the displacement amplitude of the stators and test samples, and the results of these

measurements were used to determine equivalent mechanical damping parameters for use in finite element analyses.

Summary

With the goal of ultimately producing an accurate finite element model of the entire motor, this research is a first step toward characterizing the energy loss and friction within the motor. It focuses only on the mechanical damping within the vibrating stator and limits the problem to the vibrating stator with the rotor removed. The objective of this research is to characterize the losses in the piezoelectric ceramic and metal materials that make up the bonded stator assembly when driven at the power levels used to drive the motor.

The first step taken to characterize the losses in the stator materials involved determining the correct method for incorporating the piezoelectric material properties into the finite element programs used. The analytical solution of a vibrating thin circular plate was then used to describe the vibration of the stators and of the circular disk samples used for damping property measurements. Because the vibration modes of the stators and circular disk samples were well isolated, the analytical solution for the thin vibrating plate was then simplified down to a single degree of freedom viscously damped mechanical system. The analogy between this simplified mechanical system and a series electrical circuit was used to represent the stators and disk samples as equivalent electrical circuits. The electrical quality factor was defined for the electrical circuit with an analogy developed for the mechanical system as a way of expressing the energy losses in the mechanical system. The structural damping of the stators and disk samples was expressed in terms of an equivalent viscous damping model to match the previously defined simple mechanical system and electrical circuit analogy. By taking advantage of the electrical circuit analogy, the stators and disk samples were tested on an impedance analyzer, and the low voltage damping properties were measured. To measure the high voltage damping in these structures, a laser displacement measuring system was developed and the stators and disk samples were measured at six different drive voltage levels. Finite element models were then made of the stators and disk samples, and comparisons were made between the displacement amplitudes predicted by the finite element models and the actual measured data. As expected, the displacement measurements showed the disk samples and stators to have an increase in energy losses at higher drive voltages. The effect on the energy losses due to the type of metal was found to be insignificant when compared to the effect due to the type of piezoelectric ceramic used. The shape of the frequency response curves of the displacement measurements and the increase in energy losses at higher energy levels were very similar to published results from previous research on both piezoelectric ceramic samples and bonded piezoelectric ceramic-to-metal samples. Based on the published literature, the energy losses increase with drive voltage and begin to level off at about three or four times the values found by low-level impedance testing. This results in an actual vibration amplitude one-third to one-fourth the value that would occur if the damping energy losses remained constant. Even when factoring in this three- to four-fold increase in damping into the finite element results of the bonded disk samples, the actual measured displacements of the samples fell far short of the expected results. This is believed to be due to problems with cracking and partial depoling of the samples during fabrication and testing. The actual measured displacements of the phosphor bronze and stainless steel stators correlated very well with the prediction from the finite element analyses, as the three- to four-fold increase in damping for the phosphor bronze and stainless steel stators was found to be 3.39 and 3.65, respectively.

Discussion

Scope and Purpose

The finite element method has been a valuable tool for understanding and optimizing the design of the ring type traveling wave piezoelectric motor being developed for the next generation of strong link safing mechanisms. Although this analysis has been beneficial in understanding the behavior of the motor, not enough is known about the amount of energy losses in the system to allow analyses which accurately predict the vibration amplitude and torque output of the motor. These energy losses during the operation of the motor are due to losses in the materials themselves and frictional losses in the rotor to stator interface. This work is a first step toward measuring the mechanical energy losses in the stator materials to be incorporated into a finite element analysis. The measurements were made on the stator without the rotor in order to omit the frictional losses between the stator and rotor.

Activity

Background

Many references explain in detail the theory of operation of the traveling wave piezoelectric motor including the creation of standing waves and the method by which two standing waves are superimposed to produce a traveling wave [1-6]. The principle components of the ring type traveling wave piezoelectric motor consist of a vibrating metal stator which interacts with a spinning rotor. The vibrations in the stator are induced by forces generated in a piezoelectric ceramic ring bonded to the bottom of the stator. This piezoelectric ring is plated with a segmented electrode pattern that allows some areas of the ring to be poled with a positive electric field, while others are poled with a negative electric field. After this poling operation is completed, the ring is bonded to the stator. By attaching electric leads to the ceramic ring, an electric voltage may be applied to the alternately poled segments. The fact that the segments were poled in different directions causes some segments of the piezoelectric ceramic ring to expand while other segments contract. It is this condition that causes the wavelike bending in the metal stator. A traveling wave is created in the stator by simultaneously applying a sine wave driving voltage to some segments and a cosine wave driving voltage to the remaining segments. The driving voltage frequency and the geometry of the segments around the piezoelectric ceramic ring are chosen to match a particular natural resonance mode of vibration of the stator. An illustration of the eight-millimeter outside diameter ring type traveling wave piezoelectric motor being developed at AlliedSignal Federal Manufacturing & Technologies (FM&T) is shown in Figure 1.

Figure 1. Traveling Wave Piezoelectric Motor

Although this type of piezoelectric motor is very simple in its operational principle and contains very few parts, the interdependencies between the various factors of the motor make it very difficult to model its behavior accurately. Research toward producing analytical models of the motor have focused on neglecting the rotor when determining the vibration frequency for driving the motor, and then developing an analytical contact model to describe the stator-to-rotor interaction [1-3,7-9]. Schmidt et al. [10] developed an analytical model which includes the rotor-to-stator contact in the vibration frequency calculation by modeling the stator geometry as an equivalent beam. Models of piezoelectric motors including rotor-to-stator contact have also been created using the explicit dynamic finite element method [4,11]. These finite element models have the advantage of requiring less simplifying assumptions with regards to the resonant frequencies and contact interactions than the analytical models, but the models are driven artificially due to the lack of piezoelectric materials in the explicit dynamic finite element software programs. Two factors which greatly affect the accuracy of either modeling approach are the damping energy losses within the materials as they are vibrated, and the amount of energy losses due to the friction between the rotor and stator. The energy losses in the piezoceramic material are highly dependent on the power levels used to drive the motor. This research focused on characterizing the energy losses, at different driving voltages, within the piezoelectric ceramic and metal materials that make up the bonded stator assembly.

Piezoelectric Material Modeling

Behavior of Piezoelectric Materials

The behavior of piezoelectric materials and the definition of terms is well documented [12-19]. For an anisotropic piezoelectric crystal (such as quartz) there exists a relationship in which mechanical loads applied to the material will generate electrical charges and conversely an applied voltage will create mechanical strains. Initially, piezoelectric ceramics are isotropic due to the random orientation of their polycrystalline structure and do not exhibit these piezoelectric effects. The asymmetric structure of the crystals in the piezoelectric material result in dipoles within the crystalline structure even in the absence of an external electric field. Areas with like orientation of dipoles are known as domains. The application of a large dc electric field at an elevated temperature causes a change in the orientation of many of the domains. This process is known as poling. The poling process results in a net polarization (known as the remnant polarization) of the material after the poling field is removed. An electric field of opposite polarity will tend to switch the polar direction of the domains. The value of the field of opposite polarity strong enough to eliminate the remnant polarization (depole the material) is known as the coercive field level. For a poled piezoelectric ceramic the remnant polarization will decrease with time (known as aging), but this effect tapers off such that all appreciable aging occurs within a short period of time from poling. In piezoelectric materials, the interdependence between electrical and mechanical behavior is so closely coupled that these relationships must be expressed in unique constitutive laws which include both the electrical and mechanical effects. The constitutive relationship between the strain and electric field is very nonlinear over a wide range of strains and electric fields. In most applications however, the applied electric fields, or resulting electric fields due to applied strains, are small enough with respect to the coercive field that the behavior is essentially linear within this narrow range of operation. The material properties which make up the constitutive laws are carefully expressed with respect to the conditions in which they are defined and measured. These constitutive laws result in a pair of constitutive equations commonly expressed in one of two ways: the first (known as the strain form) expresses the strain in terms of the applied stress and electric field and results in the following two constitutive equations:

$$S = s^E T + d_t E \quad (2-1)$$

$$D = dT + \varepsilon^T E \quad (2-2)$$

where:

S = total strain vector

S^E = elastic compliance tensor evaluated at constant

electric field

T = stress tensor

d = piezoelectric strain coefficient tensor

d_t = piezoelectric strain coefficient tensor transposed

ε^T = electrical permittivity tensor evaluated at

constant stress

E = electric field vector

The second form for expressing the constitutive relationship (known as the stress form) expresses the stress in terms of the applied strain and electric field and results in the following two constitutive equations:

$$T = c^E S - e_t E \quad (2-3)$$

$$D = eS + \varepsilon^S E \quad (2-4)$$

where:

c^E = elasticity tensor evaluated at constant electric field

e = piezoelectric stress coefficient tensor

e_t = piezoelectric stress coefficient tensor transposed

ε^S = electrical permittivity tensor evaluated at constant

strain

Although evaluation of the elements of the piezoelectric strain coefficient tensor d may be made under static or quasi-static

conditions, the best results are obtained by resonating the sample. The IRE/IEEE Standards [12-15] define the d^j coefficients by the partial derivatives of the strain form of the constitutive equations holding either stress or electric field constant. This results in the following definitions for the d^j coefficients:

$$d = \left(\frac{\partial S}{\partial E} \right)_T \text{ or } d = \left(\frac{\partial D}{\partial T} \right)_E$$

(where the subscript T or E represent constant stress or electric field, respectively)

Under most conditions, measurements made with a constant stress are made on an unloaded sample in which the constant stress is actually zero. Likewise, measurements under a constant electric field are usually performed with the electrical leads shorted together, setting the constant electric field to zero. The piezoelectric stress coefficient tensor e is also defined by the partial derivatives of the stress form of the constitutive equations holding either strain or electric field constant. This results in the following definitions for the e coefficients:

$$e = - \left(\frac{\partial T}{\partial E} \right)_S \text{ or } e = \left(\frac{\partial D}{\partial S} \right)_E$$

(where the subscripts S or E represent constant strain or electric field, respectively). In most cases however, the e coefficient material properties are not listed by the piezoelectric ceramic material manufacturers; instead, the e coefficients are calculated from the d^j coefficients and the material elasticity tensor c^E (evaluated at constant electric field). With $e = c^E d^j$. This can be demonstrated by substituting equation (2-1) into equation (2-3) and noting that $s^E = [c^E]^{-1}$

$$T = c^E [s^E T + d_t E] - e_t E \quad (2-5)$$

$$= c^E s^E T + c^E d_t E - e_t E$$

$$= T + c^E d_t E - e_t E$$

therefore,

$$c^E d_t E - e_t E = 0 \quad (2-6)$$

$$\text{or } [c^E d_t - e_t] E = 0 \quad (2-7)$$

requiring that

$$c^E d_t = e_t \quad (2-8)$$

making use of the general identity for matrices, $[AB]_t = B_t A_t$

$$e = d \left[c^E \right]_t \quad (2-9)$$

Taking advantage of the symmetry of c^E such that,

$$c^E = \left[c^E \right]_t$$

results in,

$$e = dc^E \quad (2-10)$$

As with the e coefficients, the manufacturer's literature seldom contains data for the permittivity values evaluated at constant strain ε^S . Instead, the permittivity values at constant stress ε^T are listed. The main reason for this lies in the ease of measurement of the constant stress values as compared to the constant strain values. To measure the ε^T values, the sample is simply measured in an unloaded (constant stress = zero) state. To measure the ε^S values, the sample must be fully constrained (constant strain = zero). The constant strain permittivity tensor may be evaluated from the constant stress values as follows:

Setting equation (2-4) equal to equation (2-2),

$$eS + \varepsilon^S E = dT + \varepsilon^T E \quad (2-11)$$

rearranging,

$$\varepsilon^S E = dT + \varepsilon^T E - eS \quad (2-12)$$

substituting in equation(2-3) for the stress term,

$$= \varepsilon^T E + d \left[c^E S - e_t E \right] - eS$$

$$= \varepsilon^T E + dc^E S - de_t E - eS$$

substituting e for dc^E from equation (2-10),

$$= \varepsilon^T E + eS - de_t E - eS$$

$$= \varepsilon^T E - de_t E$$

$$\varepsilon^S = \varepsilon^T - de_t \quad (2-13)$$

Modeling Piezoelectric Ceramics Using ABAQUS and ANSYS

The order of the matrices used to define the material properties of piezoelectric materials is according to the conventions established by the IRE which later became ANSI/IEEE standards [12-15]. These standards use the common practice of taking advantage of the symmetry of the stress and strain matrices and rewriting them as vectors. The standards also establish that for piezoelectric ceramics, the three direction always coincides with the poling axis. After poling, the originally isotropic material properties of the piezoelectric ceramic become direction dependent, and the material becomes orthotropic. Because the ceramic was originally isotropic, after poling, the material properties with respect to the one and two directions are equal. This symmetry of material properties in the plane normal to the poling axis (transverse directions with respect to the poling axis) is a special case of orthotropic behavior known as transversely isotropic behavior. For a general orthotropic material, the six-by-six elasticity or elastic compliance matrices contain only twelve non-zero elements, consisting of nine independent values. For the transversely isotropic case, only five of these values are independent [20]. The IRE/IEEE standards differ slightly from the ordering scheme used by most elasticity texts and the ABAQUS and ANSYS finite element codes. There is even a slight difference in the scheme used between ABAQUS [21-23] and ANSYS [24-28]. This results in differences in the ordering of the stress and strain vectors, the e and d coefficient matrices, and the C^E and S^E elastic property matrices. ANSYS also transposes the e and d coefficient matrices, such that equation (2-3) and equation (2-4) are written in the following form with respect to the ANSYS program:

$$T = C^E S - e E \quad (2-14)$$

$$D = e_t S + \varepsilon^S E \quad (2-15)$$

Table 1 makes a comparison of these three formats.

Table 1. Matrix and Vector Formatting Conventions

IRE/IEEE Standards	ABAQUS	ANSYS
from Auld [17,18]		

$$T = \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix} \quad S = \begin{Bmatrix} \gamma_{11} \\ \gamma_{22} \\ \gamma_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix}$$

Where: σ = stress

γ = strain

$$S^E = \begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{12} & s_{11} & s_{13} & 0 & 0 & 0 \\ s_{13} & s_{13} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} \end{bmatrix}$$

$$s_{66} = 2(s_{11} - s_{12})$$

$$T = \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} \quad S = \begin{Bmatrix} \gamma_{11} \\ \gamma_{22} \\ \gamma_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix}$$

Where: σ = stress

γ = strain

$$S^E = \begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{12} & s_{11} & s_{13} & 0 & 0 & 0 \\ s_{13} & s_{13} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{44} \end{bmatrix}$$

$$s_{66} = 2(s_{11} - s_{12})$$

Note: ABAQUS and ANSYS matrices are the same for transversely isotropic material

$$C^E = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}$$

$$c_{66} = \frac{(c_{11} - c_{12})}{2}$$

$$C^E = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{bmatrix}$$

$$c_{66} = \frac{(c_{11} - c_{12})}{2}$$

Note: ABAQUS and ANSYS matrices are the same for transversely isotropic material

Table 1 (continued). Matrix and Vector Formatting Conventions

IRE/IEEE Standard

ABAQUS

ANSYS

from Auld [17,18]

$$d = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix}$$

$$d = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{15} \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix}$$

$$d = \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & d_{33} \\ 0 & 0 & 0 \\ 0 & d_{15} & 0 \\ d_{15} & 0 & 0 \end{bmatrix}$$

$$e = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix}$$

$$e = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & e_{15} \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix}$$

$$e = \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{31} \\ 0 & 0 & e_{33} \\ 0 & 0 & 0 \\ 0 & e_{15} & 0 \\ e_{15} & 0 & 0 \end{bmatrix}$$

$$\varepsilon^r = \begin{bmatrix} \varepsilon^r_{11} & 0 & 0 \\ 0 & \varepsilon^r_{11} & 0 \\ 0 & 0 & \varepsilon^r_{11} \end{bmatrix} \quad \varepsilon^z = \begin{bmatrix} \varepsilon^z_{11} & 0 & 0 \\ 0 & \varepsilon^z_{11} & 0 \\ 0 & 0 & \varepsilon^z_{11} \end{bmatrix}$$

Vibration of Piezoelectric Structures

Disk Vibration and Simplification

The transverse vibration of both the stator of the piezoelectric motor and the disk samples used in the damping experiment are very similar to the vibration of thin circular plates. The theory of thin plates and shells has classically been used for the analytical solution of the vibration of thin circular and rectangular plates. The governing assumptions for these derivations require that the plate undergoes only small deflections, that it is thin with respect to its other dimensions, and that it is made of a homogeneous linearly elastic and isotropic material. The requirement that the plate be thin with small deflections allows simplifying assumptions that the plane sections remain plane after bending, and that transverse shear strains are negligible. Leissa [29] uses the same method and notation to derive the equilibrium equation for a transverse vibrating plate as Timoshenko et al. [30] used for the equilibrium of a statically loaded plate. Both of these derivations are based on the equilibrium of a differential element within the plate and are derived using a Cartesian coordinate system. Soedel [31] uses energy methods to develop a generalized solution for thin shell vibration problems in curvilinear coordinates, and then reduces the general solution down to the equilibrium equation for a thin plate. Regardless of the derivation method used, the resulting equilibrium equation for the transverse vibration of a thin plate becomes:

$$D \nabla^4(u_z) + \rho h \frac{\partial^2 u_z}{\partial t^2} = 0 \quad (3-1)$$

where:

u_z = displacement of midplane of plate as a function of position and time (x, y, t or r, θ, t depending on coordinate system)

$$D = \text{bending stiffness of plate} = \frac{Eh^3}{12(1-\nu^2)}$$

E = Young's modulus of elasticity for material

h = plate thickness

ν = Poisson's ratio

ρ = density of plate material

t = time

∇^4 is referred to as the biharmonic operator and is based on the Laplacian operator ∇^2 with

$$\nabla^4(\) = \nabla^2[\nabla^2(\)]$$

which is equivalent to

$$\nabla^4(\) = [\nabla^2(\)]^2$$

The form for ∇^2 and ∇^4 is dependent on the coordinate system used. For a Cartesian coordinate system:

$$\nabla^2(\) = \frac{\partial^2(\)}{\partial x^2} + \frac{\partial^2(\)}{\partial y^2}$$

and

$$\nabla^4(\) = \frac{\partial^4(\)}{\partial x^4} + \frac{\partial^4(\)}{\partial x^2 \partial y^2} + \frac{\partial^4(\)}{\partial y^4}$$

for circular plates it is advantageous to convert from Cartesian to cylindrical coordinates resulting in the following forms for the biharmonic and Laplacian operators:

$$\nabla^2(\) = \frac{\partial^2(\)}{\partial r^2} + \frac{1}{r} \frac{\partial(\)}{\partial r} + \frac{1}{r^2} \frac{\partial^2(\)}{\partial \theta^2}$$

and

$$\nabla^4(\) = \left[\frac{\partial^2(\)}{\partial r^2} + \frac{1}{r} \frac{\partial(\)}{\partial r} + \frac{1}{r^2} \frac{\partial^2(\)}{\partial \theta^2} \right]^2$$

For the harmonic motion of natural resonance, the time dependence of the displacement may be separated from r and θ and

expressed as $e^{j\omega_n t}$ such that,

$$u_z(r, \theta, t) = U_z(r, \theta) e^{j\omega_n t}$$

where:

$$j = \sqrt{-1}$$

ω_n = natural resonant frequency of a particular transverse vibration mode

Using this format for u_z and factoring out the $e^{j\omega_n t}$ term equation (3-1) becomes:

$$D \nabla^4 (U_z) - \rho h \omega_n^2 U_z = 0 \quad (3-2)$$

dividing through by D and expressing $\frac{\rho h \omega_n^2}{D}$ as λ^4

$$(\nabla^4 - \lambda^4) U_z = 0$$

This equation may be expressed in terms of ∇^2 instead of ∇^4 by writing it as

$$(\nabla^2 + \lambda^2)(\nabla^2 - \lambda^2) U_z = 0$$

Neglecting the trivial solution, $U_z = 0$, this is satisfied if

$$(\nabla^2 \pm \lambda^2) U_z = 0$$

The format of the solution may be chosen to be the product of separate functions of r and θ such that

$$U_z(r, \theta) = R(r) \Theta(\theta)$$

Substituting into equation (3-2) yields:

$$\nabla^2 (R\Theta) \pm \lambda^2 R\Theta = 0$$

Writing in terms of cylindrical coordinates yields:

$$\frac{\partial^2 (R\Theta)}{\partial r^2} + \frac{1}{r} \frac{\partial (R\Theta)}{\partial r} + \frac{1}{r^2} \frac{\partial^2 (R\Theta)}{\partial \theta^2} \pm \lambda^2 R\Theta = 0$$

Due to the independence of the $R(r)$ and $\Theta(\theta)$ functions, the partial derivatives of this equation are replaced with full derivatives resulting in:

$$\Theta \frac{d^2 R}{dr^2} + \frac{\Theta}{r} \frac{dR}{dr} + \frac{R}{r^2} \frac{d^2 \Theta}{d\theta^2} \pm \lambda^2 R \Theta = 0$$

Rearranging, to separate the variables, yields:

$$r^2 \left[\left(\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} \right) \frac{1}{R} \pm \lambda^2 \right] = \frac{-1}{\Theta} \frac{d^2 \Theta}{d\theta^2}$$

The only way that both sides of this equation may be equal for all values of r and θ is for both sides to be equal to the same constant. Designating this constant as ψ^2 results in

$$\psi^2 = \frac{-1}{\Theta} \frac{d^2 \Theta}{d\theta^2} \quad (3-3)$$

and

$$\psi^2 = r^2 \left[\left(\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} \right) \frac{1}{R} \pm \lambda^2 \right] \quad (3-4)$$

The first of these expressions may be rewritten as

$$\frac{d^2 \Theta}{d\theta^2} + \psi^2 \Theta = 0$$

The solution of this equation is of the form

$$\Theta = B \cos(\psi\theta - \phi)$$

where ϕ is a constant representing an arbitrary phase shift of the function with respect to the coordinate system. For a continuous circular plate, this function must repeat itself every 2π radians. This requires that ψ be an integer. The solution of $\Theta(\theta)$ becomes:

$$\Theta = B \cos(n\theta - \phi) \text{ with } n = 0, 1, 2, 3, \dots$$

and because equations (3-3) and (3-4) are equal, equation (3-4) becomes:

$$\psi^2 = n^2 = r^2 \left[\left(\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} \right) \frac{1}{R} \pm \lambda^2 \right] \text{ with } n = 0, 1, 2, 3, \dots$$

the solution of this equation is in terms of Bessel functions and takes on the form:

$$R(r) = A_1 J_n(\lambda r) + A_2 I_n(\lambda r) + A_3 Y_n(\lambda r) + A_4 K_n(\lambda r)$$

$J_n(\quad)$ and $I_n(\quad)$ are the first Bessel function and the modified first Bessel function of order n , respectively. $Y_n(\quad)$ and $K_n(\quad)$ are the second Bessel function and the modified second Bessel function of order n , respectively. A_1 , A_2 , A_3 , and A_4 are constants to be determined by the boundary conditions applied to the plate. A_3 and A_4 are only valid for annular plates, and are zero for solid plates.

The full solution that describes the shape of the vibration modes of the plate becomes:

$$U_z = R(r) \Theta(\theta) = [A_1 J_n(\lambda r) + A_2 I_n(\lambda r) + A_3 Y_n(\lambda r) + A_4 K_n(\lambda r)] B \cos(n\theta - \phi) \quad (3-5)$$

The mode shape itself is independent of amplitude and phase shift. Therefore, B and ϕ are arbitrarily chosen. The stator of the piezoelectric motor is similar to an annular plate that is free on the outside and clamped in the center (usually listed in the literature as the free-clamped boundary condition). For this situation, the boundary conditions at the center hole radius are zero slope with respect to the radial direction and zero deflection. The boundary conditions at the outer radius are zero radial bending moment per unit length and zero transverse shear per unit length. These four boundary conditions result in the following expressions (with r_i and r_o representing the inside and outside radii, respectively).

$$\frac{\partial U_z}{\partial r} = 0 \text{ at } r = r_i \quad (3-6)$$

$$U_z = 0 \text{ at } r = r_i \quad (3-7)$$

$$M_r = -D \left[\frac{\partial^2 U_z}{\partial r^2} + \nu \left(\frac{1}{r} \frac{\partial U_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 U_z}{\partial \theta^2} \right) \right] = 0 \text{ at } r = r_o \quad (3-8)$$

$$V_r = -D \left[\frac{\partial}{\partial r} (\nabla^2 (U_z)) + \left(\frac{1-\nu}{r^2} \right) \frac{\partial^2 U_z}{\partial \theta^2} \left(\frac{\partial U_z}{\partial r} - \frac{U_z}{r} \right) \right] = 0 \text{ at } r = r_o \quad (3-9)$$

By choosing values for B and ϕ , and inserting equation (3-5) into the boundary condition equations, a linear set of four

equations may be established to determine the constants A_i (A_1 through A_4). This set of linear equations may be written in matrix form as a four by four matrix $[g]$ multiplied by the vector A_i :

$$[g]\{A\} = 0 \quad (3-10)$$

The elements in the matrix $[g]$ are Bessel function expressions in terms of B , ϕ , λr , and n . With the exception of the trivial solution (in which all A_i values are zero), equation (3-10) is satisfied if the determinate of $[g]$ is zero. The result of this is that for every value of $n = 0, 1, 2, 3, \dots, \infty$, there are an infinite number of solutions for λr . This results in an infinite number of natural resonant vibration frequencies and corresponding mode shapes. To identify the mode shapes in terms of the value of n and λr , the naming convention of (s, n) is used where: s corresponds to the number of zero deflection concentric circles (nodal circles), and n corresponds to the number of zero deflection lines across the diameter of the circle (nodal diameter lines). The eight-millimeter piezoelectric stator is designed to drive the stator at natural resonance at its (0,3) mode of vibration. Figure 2 is an illustration of the (0,3) mode shape for the piezoelectric motor stator.

Figure 2. (0,3) Vibration Mode Shape for Piezoelectric Motor Stator

For the disk samples used in the vibration damping experiment, the disks were lightly supported at the nodal circle diameter for the (1,0) mode of vibration. This method of support held up the weight of the disks, but essentially added no other constraints, resulting in completely free boundary conditions. For the

completely free solid circular plate, constants A_3 and A_4 are zero, and the two relevant boundary conditions are those imposed by equations (3-8) and (3-9). Figure 3 is an illustration of the (1,0) "cupping" mode shape for the solid disk with free boundary conditions.

Figure 3. (1,0) Vibration Mode Shape for Free Disk

Although the amplitude of the displacement function that defines the mode shape at a resonant frequency is arbitrary, the physical amplitude of vibration of an actual structure is dependent on the amount of damping energy loss within the system. The stator geometry and segmented pattern on the PZT ring are very efficient at driving the stator in only the (0,3) mode of vibration. Likewise, the bonded disks used in the vibration damping study are very efficient at only driving the (1,0) mode of vibration. When driven with a sinusoidal drive voltage at the frequency that corresponds to the desired vibration mode, these structures respond as if they had only one vibration mode. This response mimics a simple single degree of freedom spring mass system. Treating the structure as an equivalent simple single degree of freedom system allows easy determination of the damping parameters of the structure.

Vibration of Single Degree of Freedom Mechanical System

For a single degree of freedom system consisting of a single spring, mass, and damper as shown in Figure 4, energy is traded back and forth between the kinetic energy of the moving mass and the elastic energy in the deforming spring. The loss of energy in the system is modeled as a viscous damper with the assumption that for low velocities, the force in the damper is proportional to the velocity of the mass.

Figure 4. Single Degree of Freedom Mechanical System

As derived by Vierck [32], if the system is driven by a harmonic force represented by

$$P(t) = P_m \sin(\omega_f t + \theta)$$

where:

$P(t)$ = imposed force as a function of time

P_m = maximum amplitude of force

ω_f = frequency of forcing function (radians/second)

θ = phase angle representing the initial conditions of

the forcing function with respect to time,

a summation of forces acting on the mass from the freebody diagram shown in Figure 4 results in the following equation of motion:

$$m\ddot{x} + c_d \dot{x} + kx = P_m \sin(\omega_f t + \theta) \quad (3-11)$$

where:

$x, \dot{x}, \ddot{x}$ = position, velocity, and acceleration of the mass,

respectively

m = mass

c_d = viscous damping coefficient

k = stiffness of spring

For the steady state response, it is convenient to choose the initial time reference such that the phase angle is zero and equation (3-11) becomes

$$m\ddot{x} + c_d\dot{x} + kx = P_m \sin(\omega_f t) \quad (3-12)$$

The solution to this differential equation is of the form

$$x(t) = X_m \sin(\omega_f t - \phi) \quad (3-13)$$

where:

$x(t)$ = position of mass as a function of time

X_m = maximum amplitude of motion

ϕ = phase shift between the position and forcing function (radians)

The value of the motion amplitude X_m is dependent on the magnitude of the forcing function P_m , the stiffness of the spring k , the amount of damping in the system, and how close the forcing frequency ω_f is to the system natural resonant frequency ω_n .

The natural resonant frequency ω_n is the frequency of motion for an equivalent free vibrating spring mass system without damping.

$$\omega_n = \sqrt{\frac{k}{m}}$$

The amount of damping in the system is best described by the critical damping ratio ζ . This value is the ratio of the damping coefficient c_d to $c_{d(critical)}$, where $c_{d(critical)}$ is the amount of damping which will prevent the system from oscillating in an exponentially decaying fashion once the forcing function is removed.

$$c_{d(critical)} = 2m \omega_n$$

$$\zeta = \frac{c_d}{c_{d(critical)}} = \frac{c_d}{2m \omega_n}$$

By substituting equation (3-13) into equation (3-12) and solving for X_m and ϕ , the maximum amplitude of vibration may now be expressed as,

$$X_m = \frac{\left(\frac{P_m}{k}\right)}{\sqrt{\left(1 - \left[\frac{\omega_f}{\omega_n}\right]^2\right)^2 + \left(2\zeta\left[\frac{\omega_f}{\omega_n}\right]\right)^2}}$$

and the phase angle expressed by:

$$\tan \phi = \frac{2\zeta\left[\frac{\omega_f}{\omega_n}\right]}{1 - \left[\frac{\omega_f}{\omega_n}\right]^2}$$

The term (P_m/k) may be thought of as the static deflection x_0 the system would undergo if subjected to a static load equal to the amplitude of the forcing function P_m .

$$x_0 = \frac{P_m}{k}$$

The magnification factor for the viscous damping case MF_v is the ratio between the dynamic amplitude of vibration and the static deflection.

$$MF_v = \frac{X_m}{x_0} = \frac{1}{\sqrt{\left(1 - \left[\frac{\omega_f}{\omega_n}\right]^2\right)^2 + \left(2\zeta\left[\frac{\omega_f}{\omega_n}\right]\right)^2}} \quad (3-14)$$

By substituting $\omega_f / \omega_n = 1$ into equation (3-14), the magnification factor at the natural resonant frequency becomes:

$$MF_{v(resonance)} = \frac{1}{2\zeta}$$

For small damping, the maximum amplitude occurs very near, but slightly below, the natural resonant frequency. This is because both terms being squared within the radical are a function of the drive frequency ratio ω_f / ω_n . Setting this ratio equal to one

minimizes the $\left(1 - \left[\omega_f / \omega_n\right]^2\right)^2$ term, but not the $\left(2\zeta\left[\omega_f / \omega_n\right]\right)^2$ term.

To find the frequencies for the maximum and minimum magnification factors, the derivative of equation (3-14) is taken with respect to the ratio ω_f / ω_n with the result set equal to zero. This results in the following expression for the maximum and minimum magnification factor values as a function of ω_f / ω_n and ζ :

$$\frac{\frac{\omega_f}{\omega_n} \left(1 - \left[\frac{\omega_f}{\omega_n}\right]^2 - 2\zeta^2\right)}{\left[\left(1 - \left[\frac{\omega_f}{\omega_n}\right]^2\right)^2 + \left(2\zeta\left[\frac{\omega_f}{\omega_n}\right]\right)^2\right]^{\frac{3}{2}}} = 0 \quad (3-15)$$

This results in three cases that make equation (3-15) true and define the maximum and minimum magnification factors.

The first case occurs when

$$\left(1 - \left[\frac{\omega_f}{\omega_n}\right]^2 - 2\zeta^2\right) = 0$$

which may be rewritten as:

$$\left[\frac{\omega_f}{\omega_n}\right] = \sqrt{1 - 2\zeta^2}$$

resulting in the equation defining the maximum magnification factor as long as $\zeta < \sqrt{1/2}$.

The second case occurs when $\omega_f / \omega_n = 0$. The result of this case is that regardless of the level of damping, the magnification factor approaches the value of one as the driving frequency approaches zero. This defines a local minimum point for the magnification factor if $\zeta < \sqrt{1/2}$, and the point of maximum magnification factor if $\zeta \geq \sqrt{1/2}$.

The third case occurs when $\omega_f / \omega_n = \infty$. This results in the magnification factor asymptotically tending toward zero as the driving frequency increases past the natural resonance frequency. Figure 5 is a plot of the magnification factor curve for viscous damping.

Figure 5. Magnification Factor Curve for Viscous Damping at Various Critical Damping Ratios

Harmonically Driven Series RLC Circuit

The series circuit consisting of a single resistor, capacitor, and inductor, as shown in Figure 6, is a direct analogy to the mechanical system previously discussed. In the electrical circuit, energy is traded back and forth between the capacitor and inductor. The energy stored and released as the capacitor is charged and discharged is a direct analogy to the energy stored in the mechanical spring as it is deflected and released. The energy built up in the magnetic field of the inductor and released as the field collapses is analogous to the buildup and release of kinetic energy with the changing velocity of the moving mass of the mechanical system. The energy loss in the electrical system is due to the resistor and, like the viscous damper, it drains energy with each cycle and disposes of the energy in the form of heat.

Figure 6. Series RLC Electrical Circuit

If the system is driven by a harmonic voltage represented by

$$v(t) = V_m \sin(\omega_f t + \theta)$$

where:

$$v(t) = \text{sinusoidal voltage as a function of time}$$

$$V_m = \text{maximum amplitude of voltage}$$

$$\omega_f = \text{frequency of voltage (radians/second)}$$

$$\theta = \text{phase angle representing the initial conditions of}$$

the voltage function with respect to time

for the steady state response, it is convenient to choose the initial time reference such that the phase angle of the drive voltage is zero such that:

$$v(t) = V_m \sin(\omega_f t)$$

Using the relationships for voltage drop across the inductor, resistor, and capacitor as:

$$v_L = L \frac{di}{dt}$$

$$v_R = Ri$$

$$v_C = \frac{1}{C} \int i \, dt$$

where:

$$v_L, v_R, v_C = \text{voltage drop across the inductor, resistor, and}$$

capacitor, respectively

$$i = \text{electrical current as a function of time}$$

$$L = \text{inductance}$$

R = resistance

C = capacitance

the application of Kirchoff's voltage law around the series loop in the diagram shown in Figure 6 results in the following equation:

$$L \frac{di}{dt} + Ri + \frac{1}{C} \int i \, dt = V_m \sin(\omega_f t)$$

This is the form most often used in electrical engineering [33], and is expressed in terms of voltage and current. To illustrate the analogy between the vibrating mechanical system and the electrical system, it is rewritten by expressing the current as the derivative of charge with respect to time.

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} \int \frac{dq}{dt} \, dt = V_m \sin(\omega_f t)$$

By simplifying the integral, this equation becomes

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q = V_m \sin(\omega_f t) \quad (3-16)$$

This is directly analogous to the mechanical system. The mechanical parameter and its analogy are as listed below (with the symbol $\Leftrightarrow$ used to represent the analogy between the electrical and mechanical parameters):

$$P(t) \Leftrightarrow v(t)$$

$$m \Leftrightarrow L$$

$$c_d \Leftrightarrow R$$

$$k \Leftrightarrow \frac{1}{C}$$

$$x \Leftrightarrow q$$

$$\dot{x} \Leftrightarrow \frac{dq}{dt} = i$$

$$\ddot{x} \Leftrightarrow \frac{d^2 q}{dt^2} = \frac{di}{dt}$$

As with the mechanical system, the solution to this differential equation is a sinusoidal function of the form:

$$i(t) = I_m \sin(\omega_f t - \phi)$$

where:

$i(t)$ = electrical current as a function of time

I_m = maximum amplitude of the current

ϕ = phase shift between the voltage and current function

(radians, but may be expressed in terms of degrees)

The electrical circuit has a natural resonant frequency similar to the natural resonant frequency of the mechanical system. This electrical natural resonant frequency is the frequency of oscillation of the circuit if the resistor were removed.

$$\omega_n = \sqrt{\frac{1}{LC}}$$

The response in terms of the magnitude of the current flow through the circuit is dependent on the magnitude of the driving voltage; the values of the resistor, inductor, and capacitor; and how close the driving frequency of the voltage is to the natural resonant frequency of the circuit.

In the electrical circuit, the amount of damping may be expressed, as was done with the mechanical system, in terms of a critical damping ratio ζ . Critical damping represents the value of the resistor which will prevent the system from oscillating in an exponentially decaying fashion once the forcing function is removed. For the series electrical circuit, critical damping occurs when:

$$R_{(critical)} = 2 L \omega_n$$

$$\zeta = \frac{R}{R_{(critical)}} = \frac{R}{2 L \omega_n} \quad (3-17)$$

A direct analogy to the displacement magnification factor of the mechanical system could be created for the electrical circuit by defining an electrical charge magnification factor. Instead, the admittance of the electrical circuit is commonly used as an indication of the frequency response of the electrical circuit. To find the admittance of the circuit as a function of frequency, the steady state harmonic electrical problem is usually solved by converting to the frequency domain. This effectively eliminates the time dependence of the response of the circuit, because the solution is always in terms of the point in time when the normally time-dependent voltage or current are at their maximum amplitude. In this method, the real drive voltage and response are converted to a complex drive function and response by the addition of a fictitious imaginary drive voltage and resulting imaginary response. This allows the drive voltage and response to be expressed in terms of Euler's identity,

$$Ae^{j\theta} = A \cos \theta + jA \sin \theta$$

In this format, the integrodifferential equations become complex algebraic equations. The complex algebraic equations are solved

in terms of the complex response, with the real portion of this response representing the actual solution to the problem. The shorthand phasor notation is often employed with this solution technique [33]. The relationships between the phasor voltage and current are either the phasor impedance or phasor admittance, such that:

$$\vec{V} = \vec{I} \vec{Z}$$

or equivalently,

$$\vec{V} \vec{Y} = \vec{I}$$

where:

$\vec{V}$ = phasor voltage

$\vec{I}$ = phasor current

$\vec{Z}$ = phasor impedance

$\vec{Y}$ = phasor admittance (reciprocal of impedance)

For the series circuit, the individual impedance values from the circuit elements add so that the total impedance becomes:

$$\vec{Z}_{Total} = \vec{Z}_R + \vec{Z}_L + \vec{Z}_C$$

where:

$\vec{Z}_{Total}$ = total impedance

$\vec{Z}_R = R$ = impedance of resistor

$\vec{Z}_L = j\omega_f L$ = impedance of inductor

$\vec{Z}_C = \frac{1}{j\omega_f C}$ = impedance of capacitor

$$j = \sqrt{-1}$$

The magnitude of the current I_m at a given driving frequency ω_f is equal to the magnitude of the voltage V_m times the magnitude of the admittance Y_m . The magnitude of the admittance is the reciprocal of the impedance magnitude Z_m such that:

$$\vec{Z} = R + j\omega_f L + \frac{1}{j\omega_f C}$$

$$= R + j\left(\omega_f L - \frac{1}{\omega_f C}\right)$$

$$Z_m = \sqrt{R^2 + \left(\omega_f L - \frac{1}{\omega_f C}\right)^2}$$

$$Y_m = \frac{1}{Z_m}$$

$$Y_m = \frac{1}{\sqrt{R^2 + \left(\omega_f L - \frac{1}{\omega_f C}\right)^2}} \quad (3-18)$$

Therefore, the value of the admittance magnitude is a function of the frequency and the values of the resistor, inductor, and capacitor. It is a good method for viewing the frequency response of the circuit and is similar to, but not directly analogous to, the magnification factor curve for the mechanical system. The admittance reaches its maximum value at the natural resonant frequency. At the natural resonant frequency, the only impedance is due to the resistor, and the magnitude of the admittance becomes equal to $1/R$. A typical admittance curve for a series electrical circuit is as shown in Figure 7.

Figure 7. Admittance Curve for Series RLC Circuit

Quality Factor for Electrical Circuit

As with the magnification factor curve of the mechanical system, the smaller the energy losses in the system, the narrower and taller the admittance curve will be. The sharpness of the admittance curve is often defined in terms of the quality factor Q_E defined as:

$$Q_E = \frac{2\pi(\text{total stored energy})}{\text{energy lost per cycle}} \quad (3-19)$$

For the previously defined series circuit, the energy stored in the capacitor w_C is

$$w_C = \frac{1}{2} C v_C^2$$

where v_C is the voltage across the capacitor

$$= \frac{1}{2} C \left[\int_0^t \frac{1}{C} i \, dt \right]^2$$

Since the time reference is arbitrary, the time reference is chosen such that the current function may be represented as

$$i(t) = I_m \cos \omega_f t$$

The energy stored in the capacitor becomes

$$\begin{aligned} w_C &= \frac{1}{2} C \left[\frac{1}{C} \int_0^t (I_m \cos \omega_f t) \, dt \right]^2 \\ &= \frac{C}{2} \left[\frac{I_m}{C \omega} \sin \omega_f t \right]^2 \end{aligned}$$

and finally,

$$w_C = \frac{I_m^2}{2C \omega_f^2} \sin^2 \omega_f t \quad (3-20)$$

The energy stored in the inductor w_L is

$$\begin{aligned} w_L &= \frac{1}{2} L i^2 \\ &= \frac{1}{2} L \left(I_m \cos \omega_f t \right)^2 \\ w_L &= \frac{1}{2} L I_m^2 \cos^2 \omega_f t \end{aligned} \quad (3-21)$$

The energy lost in the resistor per cycle is equal to the integral of the power through the resistor over one cycle such that

$$\begin{aligned} w_R &= \int_0^{\frac{2\pi}{\omega_f}} i^2 R dt \\ &= \int_0^{\frac{2\pi}{\omega_f}} \left(I_m \cos \omega_f t \right)^2 R dt \\ &= I_m^2 R \int_0^{\frac{2\pi}{\omega_f}} \cos^2 \omega_f t dt \end{aligned}$$

letting $u = \omega_f t$ and $du = \omega_f dt$, the integral reduces to

$$\begin{aligned} &= \frac{I_m^2 R}{\omega_f} \int_0^{2\pi} \cos^2 u du \\ &= \frac{I_m^2 R}{\omega_f} \left[\frac{u}{2} + \frac{1}{4} \sin 2u \right]_0^{2\pi} \\ w_R &= \frac{I_m^2 R \pi}{\omega_f} \end{aligned} \quad (3-22)$$

substituting equations (3-20),(3-21), and (3-22) back into equation (3-19):

$$Q_E = \frac{2\pi \left[\frac{I_m^2 \sin^2 \omega_f t}{2C\omega_f^2} + \frac{I_m^2 L \cos^2 \omega_f t}{2} \right]}{\frac{I_m^2 R \pi}{\omega_f}}$$

If the driving frequency ω_f equals the natural resonant frequency ω_n this simplifies down to:

$$Q_{E(resonance)} = \frac{L\omega_n}{R} \quad (3-23)$$

Comparing equation (3-17) to equation (3-23), it is evident that:

$$Q_{E(resonance)} = \frac{1}{2\zeta} \quad (3-24)$$

The higher the value of $Q_{E(resonance)}$, the taller and narrower the admittance curve will be. The width of the admittance curve is often described by a term known as the bandwidth with the points at the ends of the bandwidth commonly called the half power frequencies. The half power frequencies ω_i are defined as the driving frequencies that will result in the magnitude of the

admittance being $1/\sqrt{2}$ times the $1/R$ value that occurs at the natural resonant frequency. Setting equation (3-18) equal to $1/R\sqrt{2}$ to determine the half power frequencies:

$$Y_{m(halfpower)} = \frac{1}{R\sqrt{2}} = \frac{1}{\sqrt{R^2 + \left(\omega_i L - \frac{1}{\omega_i C} \right)^2}}$$

$$\frac{1}{R\sqrt{2}} = \frac{1}{\sqrt{R^2 + \left(\frac{R\omega_i L}{R} - \frac{R}{R\omega_i C} \right)^2}}$$

$$\frac{1}{R\sqrt{2}} = \frac{1}{\sqrt{R^2 + R^2 \left(\frac{\omega_i L}{R} - \frac{1}{R\omega_i C} \right)^2}}$$

$$Y_{m(halfpower)} = \frac{1}{R\sqrt{2}} = \frac{1}{R\sqrt{1 + \left(\frac{\omega_i L}{R} - \frac{1}{R\omega_i C}\right)^2}} \quad (3-25)$$

rewriting equation (3-23) as $R = L\omega_n / Q_E(\text{resonance})$ and substituting into equation (3-25),

$$\frac{1}{R\sqrt{2}} = \frac{1}{R\sqrt{1 + \left(\frac{\omega_i L Q_E}{L\omega_n} - \frac{Q_E}{L\omega_n \omega_i C}\right)^2}}$$

Substituting ω_n^2 for $1/LC$ yields:

$$\frac{1}{R\sqrt{2}} = \frac{1}{R\sqrt{1 + \left(\frac{\omega_i L Q_E}{L\omega_n} - \frac{Q_E \omega_n^2}{\omega_n \omega_i}\right)^2}}$$

simplifying,

$$\frac{1}{R\sqrt{2}} = \frac{1}{R\sqrt{1 + \left(Q_E \left(\frac{\omega_i}{\omega_n} - \frac{\omega_n}{\omega_i}\right)\right)^2}}$$

For this to be true, we have

$$Q_E \left(\frac{\omega_i}{\omega_n} - \frac{\omega_n}{\omega_i}\right) = 1 \quad \text{or} \quad Q_E \left(\frac{\omega_i}{\omega_n} - \frac{\omega_n}{\omega_i}\right) = -1$$

multiplying through by ω_i and moving all of the terms over to the left hand side,

$$Q_E \frac{\omega_i^2}{\omega_n} - Q_E \omega_n - \omega_i = 0 \quad \text{or} \quad Q_E \frac{\omega_i^2}{\omega_n} - Q_E \omega_n + \omega_i = 0$$

Each of these two equations has two possible solutions which may be found using the quadratic formula. Two of the solutions are positive, and two of the solutions are similar, but negative. Using the quadratic formula and solving for the two positive half power point frequencies (where ω_1 is the half power frequency below resonance and ω_2 is the half power frequency above resonance)

results in:

$$\omega_1 = \omega_n \left[\sqrt{1 + \left(\frac{1}{2Q_{E(resonance)}} \right)^2} - \left(\frac{1}{2Q_{E(resonance)}} \right) \right] \quad (3-26)$$

$$\omega_2 = \omega_n \left[\sqrt{1 + \left(\frac{1}{2Q_{E(resonance)}} \right)^2} + \left(\frac{1}{2Q_{E(resonance)}} \right) \right] \quad (3-27)$$

By subtracting equation (3-26) from equation (3-27), the bandwidth is found to be:

$$\omega_2 - \omega_1 = \frac{\omega_n}{Q_{E(resonance)}} \quad (3-28)$$

For larger values of $Q_{E(resonance)}$, the squared terms in equations (3-26) and (3-27) may be neglected resulting in the following approximate relations:

$$\omega_1 \approx \omega_n \left[1 - \left(\frac{1}{2Q_{E(resonance)}} \right) \right] \quad (3-29)$$

$$\omega_2 \approx \omega_n \left[1 + \left(\frac{1}{2Q_{E(resonance)}} \right) \right] \quad (3-30)$$

Quality Factor for Mechanical System

As with the electrical system, the sharpness of the mechanical system frequency response may be expressed in terms of a quality factor for a mechanical system Q_m defined as:

$$Q_m = \frac{2\pi(\text{total stored energy})}{\text{energy lost per cycle}} \quad (3-31)$$

The strain energy stored in the spring w_k is,

$$w_k = \frac{1}{2} kx^2$$

Since the time reference is arbitrary, the time reference is chosen such that the displacement function may be represented as:

$$x(t) = X_m \cos \omega_f t$$

The strain energy stored in the spring becomes:

$$w_k = \frac{1}{2} k [X_m \cos \omega_f t]^2$$

$$w_k = \frac{k X_m^2}{2} \cos^2 \omega_f t \quad (3-32)$$

The kinetic energy due to the moving mass w_m is,

$$w_m = \frac{1}{2} m \dot{x}^2$$

$$= \frac{1}{2} m \left[\frac{d}{dt} (X_m \cos \omega_f t) \right]^2$$

$$= \frac{1}{2} m [X_m \omega_f \sin \omega_f t]^2$$

and finally,

$$w_m = \frac{1}{2} m X_m^2 \omega_f^2 \sin^2 \omega_f t \quad (3-33)$$

The energy lost in the damper per cycle is found by summing up the work done by the damper over one cycle. The differential work $d w_d$ done as the damper moves through a differential distance dx is equal to the force in the damper times the differential distance,

$$d w_d = c_d \dot{x} dx$$

Writing dx as $dx = \dot{x} dt$,

$$d w_d = c_d \dot{x} \dot{x} dt$$

$$\frac{2\pi}{\omega_f}$$

Integrating to find the total lost energy over one period, where one period is equal to

$$w_d = \int_0^{\frac{2\pi}{\omega_f}} c_d \dot{x}^2 dt$$

Again, letting $x(t) = X_m \cos \omega_f t$ and $\dot{x} = -X_m \omega_f \sin \omega_f t$

$$w_d = \int_0^{\frac{2\pi}{\omega_f}} c_d \left(-X_m \omega_f \sin \omega_f t \right)^2 dt$$

$$= X_m^2 \omega_f^2 c_d \int_0^{\frac{2\pi}{\omega_f}} \sin^2 \omega_f t dt$$

Letting $u = \omega_f t$ and $du = \omega_f dt$, the integral reduces to

$$= \frac{X_m^2 \omega_f^2 c_d}{\omega_f} \int_0^{u=2\pi} \sin^2 u du$$

$$= X_m^2 \omega_f^2 c_d \left[\frac{u}{2} - \frac{1}{4} \sin u \right]_0^{2\pi}$$

$$w_d = X_m^2 \omega_f^2 c_d \pi \quad (3-34)$$

Substituting equations (3-32), (3-33), and (3-34) back into equation (3-31):

$$Q_m = \frac{2\pi \left[\frac{kX_m^2 \cos^2 \omega_f t}{2} + \frac{mX_m^2 \omega_f^2 \sin^2 \omega_f t}{2} \right]}{X_m^2 c_d \pi \omega_f} \quad (3-35)$$

If the driving frequency ω_f equals the natural resonant frequency ω_n , writing ω_n as $\sqrt{k/m}$ and making use of the identity

$\sin^2 \omega_f t + \cos^2 \omega_f t = 1$, equation (3-35) simplifies down to

$$Q_{m(resonance)} = \frac{k}{c_d \omega_n} \quad (3-36)$$

Writing ω_n^2 as k/m and writing c_d as $2m\zeta\omega_n$ allows equation (3-36) to be written as

$$Q_{m(resonance)} = \frac{1}{2\zeta} \quad (3-37)$$

For the special case when the drive frequency is equal to the natural resonant frequency, substituting ω_n in place of ω_f in equation (3-14) results in the relationship that the displacement magnification factor is equal to the mechanical quality factor at resonance as defined by equation (3-37).

The higher the value of $Q_{m(resonance)}$, the taller and narrower the vibration amplitude, velocity amplitude, and magnification factor curves will be. The bandwidth and half power frequencies may be defined for the mechanical system by defining a direct analog to the admittance of the electrical circuit. Utilizing the direct analogies between the mechanical system and the electrical circuit, the mechanical equivalent to the electrical admittance can be defined as,

$$\vec{Y}_{mechanical} = \frac{\dot{\vec{x}}}{\vec{P}}$$

where the terms for the mechanical equivalent to the electrical admittance, velocity, and force are all expressed in the frequency domain as phasors. By defining the bandwidth and half power points as done for the electrical admittance, the same relationships may be found for the half power frequencies of the mechanical system.

$$\omega_1 = \omega_n \left[\sqrt{1 + \left(\frac{1}{2Q_{m(resonance)}} \right)^2} - \left(\frac{1}{2Q_{m(resonance)}} \right) \right] \quad (3-38)$$

$$\omega_2 = \omega_n \left[\sqrt{1 + \left(\frac{1}{2Q_{m(resonance)}} \right)^2} + \left(\frac{1}{2Q_{m(resonance)}} \right) \right] \quad (3-39)$$

$$\omega_2 - \omega_1 = \frac{\omega_n}{Q_{m(resonance)}} \quad (3-40)$$

$$\omega_1 \approx \omega_n \left[1 - \left(\frac{1}{2Q_{m(\text{resonance})}} \right) \right] \quad (3-41)$$

$$\omega_2 \approx \omega_n \left[1 + \left(\frac{1}{2Q_{m(\text{resonance})}} \right) \right] \quad (3-42)$$

A nondimensional velocity magnification factor may be created similar to the displacement magnification factor defined earlier. This velocity magnification factor is defined as:

$$VMF = \frac{\dot{X}_m}{x_0 \omega_n} = \frac{\dot{X}_m}{\left(\frac{P_m}{k} \right) \omega_n}$$

Because the stiffness and the resonant frequency are constants with respect to the system, the nondimensional velocity magnification factor is actually equal to the magnitude of the mechanical analogy to the electrical admittance divided by a constant. Because the bandwidth and half power points are defined in terms of a ratio of response magnitudes, multiplying or dividing the mechanical analogy to admittance by a constant will result in the constant terms canceling each other out, and the bandwidth and half power points of the velocity magnification factor curve are also defined by equations (3-38) through (3-42).

Equivalent Circuit for Vibrating Piezoelectric Structure

A vibrating piezoelectric structure may be modeled as an equivalent electrical circuit, by taking advantage of the direct analogies between the behavior of a vibrating mechanical system and a series RLC circuit. This equivalent circuit contains two parallel legs as shown in Figure 8. The first leg of the circuit consists of the capacitor labeled C_p and the current labeled i_p . This leg represents the electrical capacitance of the piezoelectric element itself and the resulting current through the piezoelectric element due to the drive voltage. The second leg of the equivalent circuit physically consists of the mechanical effects of a spring, mass, and viscous damper, but may be represented electrically by an equivalent resistor R , inductor L , and capacitor C_s , respectively.

Figure 8. Equivalent Electrical Circuit for Piezoelectric Structure

The proportionality constant that represents the analogy between the mechanical system components and the equivalent electrical components is known as the force factor A . The force factor is the ratio between charge and displacement and also the ratio between force and voltage. Noting that the force factor is assumed to be independent of time, the following relationships exist for the force factor:

$$P(t) = Av(t)$$

$$x = \frac{q}{A}, \quad \dot{x} = \frac{\dot{q}}{A}, \quad \ddot{x} = \frac{\ddot{q}}{A}$$

Substituting into equation (3-12) results in the following relationship:

$$m \frac{\ddot{q}}{A} + c_d \frac{\dot{q}}{A} + kq = Av(t)$$

Dividing through by A results in:

$$m \frac{\ddot{q}}{A^2} + c_d \frac{\dot{q}}{A^2} + k \frac{q}{A} = v(t)$$

Comparing with equation (3-16) results in the following relationships for converting the mechanical system components into the electrical equivalent circuit components:

$$L = \frac{m}{A^2}$$

$$R = \frac{c_d}{A^2}$$

$$\frac{1}{C_s} = \frac{k}{A^2}$$

Due to the additional leg of the circuit, the impedance and admittance magnitude and phase angle curves of the equivalent circuit are different than previously defined by the series RLC circuit. The phase angle curve has two zero crossings known as the resonance and antiresonance frequencies. The resonant frequency is the first zero crossing. The frequency of maximum admittance and minimum impedance is near this frequency as well as the mechanical natural resonant frequency of the structure. (Due to the electrical analogy, this is also the natural resonant frequency of the series leg of the equivalent circuit.) The antiresonant frequency is the second zero crossing. The frequency of minimum admittance and maximum impedance is near this frequency as well as the parallel electrical natural

resonant frequency of both sides of the equivalent circuit. Because of the analogy between the vibrating piezoelectric structure and

the equivalent circuit, the piezoelectric structure appears to an electrical impedance analyzer as if it were a purely electrical device. An HP4194A impedance analyzer was used to measure the admittance of the disk samples and automatically fit the measured data to determine the equivalent circuit components. Figure 9 contains the measured admittance curve as well as the theoretical curve plotted using the equivalent circuit parameters of one of the disk samples.

Figure 9. Measured and Theoretical Admittance Amplitude From Impedance Analyzer Test Results

Accounting for Energy Losses

Structural Damping

The viscous damper model is commonly used to describe the energy losses due to its mathematical simplicity. For the piezoelectric structure, it has the added advantage of being analogous to the resistor in the equivalent circuit. The viscous damper model is a good approximation for an object moving in air at low

velocities [32], and may account for the energy loss of the vibrating piezoelectric structure due to air resistance. The majority of the energy loss in the vibrating piezoelectric structure is referred to as structural damping, and is due to the internal friction within the material itself as it vibrates. Structural damping may be highly nonlinear and very difficult to model analytically. Hysteretic damping is an approach to model internal material damping by experimental measurements of the energy loss within a hysteresis loop formed by cyclic loading of the material. The experimental test results show that the energy loss from material damping is independent of the frequency and is proportional to the square of the amplitude [32]. To account for the hysteretic loss, the stiffness may be expressed as a complex number, where the larger the imaginary portion of the number, the larger the loss. Nashif et al. [34] models the single degree of freedom spring mass system with hysteretic damping energy losses as shown in Figure 10:

Figure 10: Single Degree of Freedom Mechanical System With Hysteretic

Material Damping

If the system is driven by the same harmonic force used with the viscously damped system represented by,

$$P(t) = P_m \sin(\omega_f t + \theta)$$

a summation of forces acting on the mass from the freebody diagram shown in Figure 10 results in the following equation of motion:

$$m\ddot{x} + k(1 + j\eta)x = P_m \sin(\omega_f t + \theta)$$

For the steady state response, the phase angle may be arbitrarily chosen. It is convenient to choose the initial time reference such that the phase angle of the driving force function is the same as a cosine function with a zero phase angle. This results in the following equation of motion:

$$m\ddot{x} + k(1 + j\eta)x = P_m \cos(\omega_f t)$$

By taking advantage of Euler's identity, as was done for the impedance of the series electrical circuit, the forcing function and the displacement may be expressed as complex entities, with the actual solution being the real part of the complex solution. Using this method, the complex displacement and its derivatives become:

$$x(t) = X_m e^{j(\omega_f t + \phi)}, \quad \dot{x}(t) = jX_m \omega_f e^{j(\omega_f t + \phi)}, \quad \ddot{x}(t) = -X_m \omega_f^2 e^{j(\omega_f t + \phi)}$$

and the complex forcing function becomes:

$$P_m e^{j\omega_f t}$$

The equation of motion can now be expressed as:

$$\operatorname{Re}\left[-mX_m\omega_f^2 e^{j(\omega_f t + \phi)} + k(1 + j\eta)X_m e^{j(\omega_f t + \phi)}\right] = \operatorname{Re}\left[P_m e^{j\omega_f t}\right]$$

Rearranging yields:

$$\operatorname{Re}\left[-mX_m\omega_f^2 e^{j\omega_f t} e^{j\phi} + k(1 + j\eta)X_m e^{j\omega_f t} e^{j\phi}\right] = \operatorname{Re}\left[P_m e^{j\omega_f t}\right]$$

Converting this into the frequency domain by considering both the real and imaginary components and dividing through by $e^{j\omega_f t}$ results in the following expression:

$$-mX_m\omega_f^2 e^{j\phi} + k(1 + j\eta)X_m e^{j\phi} = P_m$$

Rearranging yields:

$$X_m e^{j\phi} (-m\omega_f^2 + k + jk\eta) = P_m$$

Solving for the displacement magnitude:

$$X_m = \frac{P_m}{\sqrt{(k - m\omega_f^2)^2 + (k\eta)^2}}$$

Dividing the numerator and denominator by k :

$$X_m = \frac{\left(\frac{P_m}{k}\right)}{\sqrt{\frac{(k - m\omega_f^2)^2 + (k\eta)^2}{k^2}}}$$

Rearranging the terms inside the radical,

$$X_m = \frac{\left(\frac{P_m}{k}\right)}{\sqrt{\left(1 - \frac{m}{k}\omega_f^2\right)^2 + \eta^2}}$$

and defining ω_n as done previously such that $\omega_n = \sqrt{k/m}$ and $1/\omega_n^2 = m/k$ yields:

$$X_m = \frac{\left(\frac{P_m}{k}\right)}{\sqrt{\left(1 - \left[\frac{\omega_f}{\omega_n}\right]^2\right)^2 + \eta^2}}$$

Defining a magnification factor as done previously, by dividing through by P_m/k , results in the magnification factor for the hysteretic damping case MF_h :

$$MF_h = \frac{X_m}{\left(\frac{P_m}{k}\right)} = \frac{1}{\sqrt{\left(1 - \left[\frac{\omega_f}{\omega_n}\right]^2\right)^2 + \eta^2}} \quad (4-1)$$

For hysteretic damping, the maximum value of the magnification factor occurs at the natural resonant frequency. This is because the denominator of (4-1) becomes a minimum at the natural resonant frequency. At the natural resonant frequency, the magnification factor for hysteretic damping becomes:

$$MF_{h(resonance)} = \frac{1}{\eta}$$

Figure 11 is a plot of the magnification factor curve for hysteretic damping.

Figure 11. Magnification Factor Curve for Hysteretic Damping for Various

Damping Coefficients

A comparison of Figure 5 and Figure 11 shows the similarity between the hysteretic damping model and the viscous damping model for the low damping cases near the natural resonant frequency. A comparison, at the natural resonant frequency, of the magnification factor for viscous damping, magnification factor for hysteretic damping, and the mechanical quality factor as defined for viscous damping yields:

$$Q_{m(resonance)} = MF_{v(resonance)} = MF_{h(resonance)} = \frac{1}{2\zeta} = \frac{1}{\eta} \quad (4-2)$$

Based on these comparisons, for the piezoelectric structure, vibrating very near the natural resonant frequency, the structural damping energy losses may be modeled as an equivalent viscous damper. This is accomplished by setting the critical damping coefficient of the equivalent viscous damper equal to one half the value of the hysteretic damping coefficient measured for the material, such that:

$$\zeta_{(equivalent)} = \frac{\eta}{2} \quad (4-3)$$

Incorporating Damping Into Finite Element Models

The damping properties added to either an ABAQUS or ANSYS finite element model are based on the equivalent viscous damping model [21-28]. Both programs allow some methods for the user to directly specify the critical damping ratio directly, but

these methods are not supported for some forms of analyses (such as direct integration solutions to either transient, or steady state dynamics problems). Both programs support the use of Rayleigh damping parameters as a method of specifying the equivalent viscous damping for all forms of dynamic analyses. In Rayleigh damping, two Rayleigh constants α and β are multiplied by the mass and stiffness of the system respectively to generate the equivalent viscous damping coefficient c_d .

$$c_d = \alpha m + \beta k \quad (4-4)$$

To determine the relationship between the Rayleigh constants and the equivalent critical damping ratio, both sides are divided by the critical damping coefficient

$$\frac{c_d}{c_{d(critical)}} = \frac{\alpha m}{c_{d(critical)}} + \frac{\beta k}{c_{d(critical)}}$$

$$\zeta = \frac{c_d}{c_{d(critical)}} \quad \text{and} \quad c_{d(critical)} = 2m\omega_n,$$

Substituting in the relationships that

$$\zeta = \frac{\alpha m}{2m\omega_n} + \frac{\beta k}{2m\omega_n} \quad (4-5)$$

Simplifying, and using the relationship $\omega_n^2 = k/m$ yields:

$$\zeta = \frac{\alpha}{2\omega_n} + \frac{\beta\omega_n}{2}$$

When incorporated into the finite element method, the Rayleigh constants α and β remain single scalar constants, but the damping coefficient, mass, and stiffness terms correspond to matrices, such that equation (4-4) becomes:

$$[c_d] = \alpha[m] + \beta[k]$$

The result of this is that there are separate values for the Rayleigh damping constants for each natural resonant frequency (eigenvalue) of the structure. Because of this, the values for the Rayleigh damping constants need to be determined near the frequency of the mode of vibration to be analyzed.

For a desired critical damping ratio, either the α term or the β term may be set to zero, resulting in the damping being a sole function of the mass or the stiffness, respectively. The α term most closely represents the effects of damping of a mass moving through a viscous fluid, while the β term most closely represents the effects of the internal friction of structural damping.

Although equation (4-5) corresponds to an amount of damping for a given natural resonant frequency, the fact that the α term is being divided by ω_n , while the β term is multiplied by ω_n , helps to keep the amount of damping relatively constant over a small frequency range. By substituting the upper and lower natural resonant frequencies, and their corresponding desired critical

damping ratios into equation (4-5), a pair of simultaneous equations may be solved to determine the appropriate α and β Rayleigh constants that work well over a reasonably sized frequency range.

Determination of Equivalent Damping Properties

Standard Measurement of Piezoelectric Materials

Traditionally the material properties (including damping properties) of piezoelectric materials have been determined according to the methods of the IRE/IEEE standards [12-15]. These measurements are based on the linear equivalent circuit model of piezoelectric materials in simple geometric shapes, and are performed by taking impedance or admittance measurements of test samples. These methods work well for piezoelectric crystals with a high value of Q_m such as quartz, and for ceramics when driven at low levels of vibration with low electric fields.

Holland [35] expanded on the standard representation of the losses within piezoelectric material properties by expressing them in terms of complex numbers (similar to the method used for hysteretic damping). Holland and EerNisse [36] then developed the admittance test methods for determining these complex material properties. Many authors [37-43] have built upon the use of the complex representation of the material properties, but this work is dependent on near linearity of the material and is only valid for piezoelectric ceramics at the low levels of drive voltage and vibration.

Belding and McLaren [44] measured material properties including Q_m using the IRE/IEEE standard methods for piezoelectric ceramics driven under high electric field conditions and loaded by being immersed in oil. Much of the research regarding the material properties of piezoceramic samples, when driven at high power levels, has been performed by displacement amplitude measurements [45-47]. Kugel et al. [48] performed displacement measurements on unimorph and bimorph bonded structures driven at high power levels. Research involving Hirose, Takahashi, Uchino, Aoyagi, Tomikawa, Sasaki, Yamayoshi, and Taga [49-51] utilized a constant current drive circuit to measure the material properties of piezoceramics at high power levels.

Experiment to Measure Equivalent Damping Properties

An experiment was designed to determine the equivalent damping properties of a simple vibrating structure with similarities to the bonded stator in the traveling wave piezoelectric motor. The experiment focused on vibrating the structure at the natural resonant frequency of an isolated transverse vibration mode and establishing the amount of damping based on the amplitude of the displacement. The displacements of the samples were measured using a non-contact laser displacement sensor.

A uniformly poled thin circular disk made by epoxy bonding a metal and a PZT ceramic disk together was chosen as the test specimen for the experiment. These structures are commonly referred to in the literature as unimorphs. These disk samples provided a simple structure that, when driven at the natural resonant frequency corresponding to the (1,0) mode of vibration, creates a well-isolated "cupping shape" vibration mode with a large smooth peak displacement at the center of the disk. The flattening slope of the smooth peak at the center of the disk simplified the alignment of the laser of the displacement sensor and only required the sensor to be aligned within about 0.005 inch of the center of the disk. This mode of vibration has the advantage of undergoing bending (to more accurately simulate the bending motion of the stator in the traveling wave piezoelectric motor), but with large enough displacements to allow displacement measurements when the disk is driven at both low and high voltage levels.

The variables in the experiment included the thickness of the PZT ceramic, the type of PZT ceramic, and the type of metal. The thickness of the PZT ceramic and the type of metal were varied to try to determine the relative influences made by the metal and the PZT to the damping. The two metals used consisted of 303Se condition B stainless steel and alloy 544 free machining phosphor bronze. The PZT ceramics were varied to compare a hard PZT

ceramic having a high mechanical quality factor with a soft PZT ceramic having a low mechanical quality factor. The PZT materials used were made by the EDO Corporation, Salt Lake City, Utah. The soft PZT was EC-76, and the hard PZT was EC-67 with mechanical quality factor specifications of 65 and 900, respectively. The metal and ceramic were both 0.595 inch in diameter, and were bonded using Ciba-Geigy Araldite CY179 epoxy. The metal disks were all 0.019 inch thick. The PZT disks were 0.015 inch and 0.020 inch thick. The metal disks were cut using an electrical discharge machining (EDM) system, and lapped to the final thickness. The PZT ceramic was supplied by the EDO Corporation in the form of slugs (pressed and sintered ceramic cylinders). The EC-76 cylinders were 0.595 inch in diameter, and the EC-67 cylinders were 0.670 inch in diameter. The EC-67 cylinders were ground to the 0.595-inch diameter using a diamond grinding wheel, and all of the ceramic disks were then cut off using a diamond grit cutting blade. The ceramic disks were then lapped to the final thickness, sputter coated with gold on top and bottom, and poled. The metal disks were also sputter coated with gold on one side to aid in the wetting and adhesion of the epoxy to the metal disk. The ceramic and metal disks were epoxy bonded under a 40 psi clamping pressure at 125 degrees Celsius. The Ciba-Geigy Araldite CY179 epoxy mixture was used because it becomes very thin and flows out at the curing temperature, providing a very thin bond thickness. By weighing the metal and ceramic disks before and after bonding, the amount of epoxy was calculated, resulting in a bond thickness of between 0.0002 and 0.0003 inch. This was verified to be reasonable by comparing to previously cross-sectioned samples of bonded stators that had an average bond thickness of about 0.0002 inch. Electrical leads, made of 0.010-inch diameter solid copper wire were attached to the top and bottom of the bonded disks using Ciba-Geigy Epibond 7002 conductive epoxy. These electrical leads provided both the connections for driving the disk electrically and a method of supporting the disk in a testing fixture.

The copper wire electrical leads were conductive epoxy bonded to the disk at the nodal circle diameter. The wires were attached with one wire on top and the other attached to the bottom and at the opposite end of the nodal circle. The wires were bent over to form long arms to be clamped into a test fixture. This arrangement acted well to support the weight of the disk without adding any appreciable bending stiffness to the structure, allowing the disk to act with essentially free boundary conditions. This arrangement was also chosen because it allowed clear access to the center of the disk for the laser displacement sensor. Figure 12 is an illustration of the disk sample as configured for the displacement testing as well as an end on view of a disk sample.

Figure 12. Disk Sample and Test Fixture

Method for Measuring Vibration Amplitude

The displacements of the disk samples were made using a Dynamic Controls Corporation LTS-15/1 laser displacement sensor. This sensor uses optical triangulation of a class 3b laser source which projects a 30-micron (0.0012-inch) diameter laser spot normal to the surface to be measured. The diffuse reflected laser light is picked up by two separate focusing optics and position-sensitive detector assemblies, which are offset at an angle to the light source. Due to the triangulation of the light, a change in height is detected as a lateral change in position on the position-sensitive detectors. The displacements detected from the position-sensitive detectors are averaged and the sensor output is supplied as a voltage with a resolution of 10 millivolts/micron. The total measurement range of the sensor is 1 mm (0.039 inch) centered about a focal point distance 15 mm (0.591 inch) below the bottom of the sensor. This displacement sensor was chosen for its fine resolution and its advantages over other measurement systems. The first advantage was that it has a high enough frequency response (-3 dB bandwidth of 500 kHz) to allow the measurements of the disk samples and traveling wave piezoelectric stators to be made without any significant roll off in the sensors output. The second advantage was that, because the sensing method is not based on the amount of light collected, this type of sensor does not require a calibration procedure for the surface of each new sample.

Because the displacement sensor uses a class 3b laser, the laser needed to be placed within a safety enclosure to protect the operator. To provide a method to align the laser over a particular spot and still protect the operator during the operation of the sensor, the displacement sensor was incorporated into a system consisting of a measuring microscope, rotary stage, micrometer position stages, two-position rail slide, and interlocked safety enclosure. The safety enclosure consists of a round bottom plate, as the bottom for the enclosure, and a hinged cover which encloses the laser when placed in the down position. Two separate interlocks allow electrical power to the laser only when the bottom plate is in position and the hinged cover is down. An adjustable stop on the two-position slide was adjusted so that when the sample is moved under the measuring microscope, the various rotary and positioning stages allow the area of interest on the sample to be positioned directly under the crosshairs. Another adjustable stop on the two-position slide was adjusted so that when the sample is moved under the laser, the point that was under the cross hairs of the microscope is directly under the laser. Figure 13 is an illustration of the displacement measurement system.

Figure 13. Displacement Measuring System

Although the laser displacement sensor had the resolution to detect the small displacements of the vibrating piezoelectric structures, the voltage output from the displacements was not much larger than the electronic noise within the internal circuitry of the sensor. Viewing the output on an oscilloscope showed the noise to be between 200 kHz and 300 kHz. To eliminate this noise, a filter circuit was developed using a Linear Technology Corporation LTC1069-7 linear phase 8th order lowpass filter. This filter circuit allowed the higher frequency noise to be eliminated from the sensor output without any appreciable attenuation of the lower frequency response from the vibrating structure. Before the sensor output could be filtered, the output needed to be amplified and an operational amplifier was used to amplify the output by a nominal gain factor of 27. By using a function generator to supply known amplitude sine waves, the actual gain through the filter circuit was measured as a function of frequency. This allowed the amplified displacement sensor output to be divided by the appropriate gain factor to yield the correct value for the measured displacement. Figure 14 contains a schematic of the filter circuit.

Figure 14. Filter Circuit for Displacement Measuring System

The analog voltage signal from the amplified and filtered output of the displacement sensor was viewed and measured using a Tektronix DSA 602 digitizing signal analyzer. To eliminate heating effects, the test samples were only run long enough to reach

steady state vibration. Depending on the mechanical quality factor Q_m of the sample at a particular drive voltage, the time required for the transient motion at startup to die-down to reach steady state vibration was between 15 milliseconds and 500 milliseconds. The split windowing option of the DSA 602 allowed the entire event (including the transient and steady state vibration) to be displayed in the top window with a separate window below. The bottom window represents a small portion of time within the steady state region in which the displacement measurements are made. The bottom window time scale was adjusted to be about ten periods long. This produced a nice scale for visualizing the sinusoidal steady state motion and for the DSA 602 to measure the voltage signal representing the displacement. In order to achieve the best accuracy, the RMS amplitude measurement of the voltage signal was averaged by the DSA 602 over the entire bottom window. The RMS measurement was chosen over a peak amplitude measurement because it is much less sensitive to any noise at the peaks of the sinusoidal response. Figure 15 is an example of the measurement screen of the DSA 602 testing a sample at a low power level with less energy losses in the device. The transient region at the startup is longer, and the time position of the bottom measuring window is moved out to occur in the steady state region. Figure 16 is of the same disk sample driven at a higher power level, creating more energy losses with the transient region dying out much sooner. Both of these figures contain traces of both the measured displacement and current. The identical shape to the traces is an illustration of the analogy between current (as related to the charge by the frequency for harmonic motion) and the displacement. To test each disk sample, the HP4194A impedance analyzer was used to find the natural resonant frequency and the equivalent circuit parameters for the disk when driven in the (1,0) vibration mode. The oscillator of the impedance analyzer was set to the 1-volt RMS (2.83 volts peak-to-peak) setting. The equivalent circuit parameters were used to

calculate the value of Q_m for a 2.83-volt peak-to-peak drive voltage. After testing on the impedance analyzer, the displacement tests were run at six different drive voltage levels. These drive voltage levels were 2.83, 5, 25, 50, 75, and 100 volts peak to peak. For each drive voltage, displacement data were gathered at enough frequencies to be able to plot the frequency response of the disk. To plot the frequency response, the displacement was measured at every 100 Hz over a 2-kHz range centered about the natural resonant frequency found by the impedance test. If the frequency response curve was sharp (due to low damping/high

Q_m), the response curve was refined by measurements at every 10 Hz over the 200 Hz range spanning the peak. If more refinement was necessary to determine the actual peak, more measurements were made at every 1 Hz over the 20-Hz range spanning the peak. For some of the disk samples, when tested at the higher drive voltage levels, the change in the frequency of the peak displacement required the measurement range to be widened below the original 2-kHz range with the measurement spacing beginning again at 100 Hz.

DSA 602 DIGITIZING SIGNAL ANALYZER

date: 27-OCT-98 time: 21:36:59

Figure 15. Sample Measurement at Low Drive Level From Screen of DSA 602

DSA 602 DIGITIZING SIGNAL ANALYZER

date: 27-OCT-98 time: 21:54:59

Figure 16. Sample Measurement at High Drive Level From Screen of DSA 602

Measurement Results and Comparison to Literature

The first three disks that were tested consisted of the thin EC-76 bronze, thin EC-67 stainless, and thick EC-67 stainless disks, respectively. Each of these disks was tested at each drive voltage level using the course 100-Hz increments over the 2-kHz range before any refinements in the testing frequency increment

size were made at any drive level. This posed no problem for the first disk sample tested (containing the soft PZT), but it did pose a problem for the second and third samples (containing the hard PZT).

When the second and third disks were tested using the refined

10-Hz increment size, the low voltage response of the hard PZT samples (after being driven at high power levels) changed significantly. This required complete re-testing, beginning again with 100-Hz increments of the two hard PZT disk samples. The frequency of the peak displacement had shifted down significantly, and the peak displacement amplitude was much lower than indicated by the previous test. To document this "before" and "after" phenomena (on all of the remaining disks), all of the low drive voltage testing (2.83 and 5 volts peak to peak) was performed on each of the remaining disks before they were subjected to higher drive voltage levels. Figure 17 contains an example of the "before" and "after" frequency response curves of a soft PZT disk sample when driven at the one volt RMS level. Figure 18 contains the same comparison for a hard PZT disk sample.

As a further comparison of the "before" and "after" effects, the same impedance test (done on each disk before displacement testing) was repeated on each sample after displacement testing. The "before and after" changes in the displacement measurement

data correlated precisely with the changes in the Q_m values calculated from the impedance analyzer tests. In all cases, the change in Q_m was dramatic for the hard PZT samples. The soft PZT samples had little or no change in Q_m , with some actually having a slightly higher value (due to the variability in the impedance analyzer testing). Table 2 compares the "before" and "after" values of Q_m for each disk sample as calculated by the equivalent circuit parameters of the HP4194A impedance analyzer.

Thin EC- 76 (soft PZT) on Stainless Steel Disk
2.83 Volts Peak-Peak (Before and After High Power Levels)

Figure 17. Frequency Response of Soft PZT Disk at Low Drive Level Before and After Being Driven at Higher Levels

Thick EC-67 (hard PZT) on Phosphor Bronze Disk
2.83 Volts Peak-Peak (Before and After High Power Levels)

Figure 18. Frequency Response of Hard PZT Disk at Low Drive Level Before and After Being Driven at Higher Levels

Table 2. Comparison of Mechanical Q Before and After Driving at High Power Levels

Sample	Mechanical Q	Mechanical Q
	"Before"	"After"
Thin 67 Bronze	968	387
Thin 67 Stainless	509	363
Thick 67 Bronze	644	328
Thick 67 Stainless	724	206
Thin 76 Bronze	162	151
Thin 76 Stainless	128	104
Thick 76 Bronze	149	156
Thick 76 Stainless	102	103

The change in the low drive voltage displacements and Q_m values, after being driven at high drive voltages, was inconsistent

with partial depoling of the PZT material. This is because the soft PZT samples did not experience the same degradation in performance, even though soft PZT is much more susceptible (due to a lower coercive field value) to the depoling effects of high electric fields.

To try to explain the sharp drop-off in Q_m of the hard PZT disk samples, all of the disk samples were inspected visually under 30x magnification, and no cracks were detected. To test for internal cracks, ultrasonic imaging was performed on all of the disk samples after displacement testing. This testing revealed that all of the hard PZT samples experienced serious cracks emanating from the high strain area at the center of the disk. The soft PZT disks also experienced cracks, but they appear to be emanating from flaws on the outer edges of the disk samples. Figures 19 and 20 are ultrasonic images of the hard and soft PZT samples.

Figure 19. Ultrasonic Imaging of Cracks in EC-67 (Hard PZT) Disk Samples

For the soft PZT samples, and the hard PZT samples before cracking, the peak displacement response matched very well to the natural resonant frequency found by the impedance analyzer tests. Also, for these low drive voltage tests, the frequency response curves (near the natural resonance frequency) were relatively sharp and symmetrical and were similar in shape to the magnification factor and admittance curves. As expected, because of the relative differences in Q_m , the disks with the soft PZT had wider and rounder frequency response curves than the hard PZT disks. The sharp responses of the "before" curves in Figures 17 and 18 are representative of the low voltage responses of the samples.

Figure 20. Ultrasonic Imaging of Cracks in EC-76 (Soft PZT) Disk Samples

At the higher drive voltages, the frequency of the peak displacement shifted lower for disks with both hard and soft PZT. For all of the soft PZT samples, the frequency response curve became wider and rounder and took on the backward slant characteristic of a strain softening vibrating mechanical system [52]. For all of the disks containing hard PZT, these effects were even more pronounced. All hard PZT samples experienced an unstable region resulting in the "jump phenomena" characteristic of systems undergoing large strain softening effects. These results are identical to the results found by Woollett and LeBlanc [45]. Figures 21 and 22 are the frequency response curves of the same samples shown in Figures 17 and 18 when driven at the 100-volt peak-to-peak drive level.

Figure 21. Strain Softening Response Curve of Soft PZT Disk Sample When Driven at High Power Levels

Figure 22. Strain Softening and "Jump Phenomena" Response Curve of Hard PZT Disk Sample When Driven at High Power Levels

At the higher drive levels, the displacement continues to increase with higher drive voltage, but at a much lower rate because Q_m (and consequentially the magnification factor) is dropping. This increase in energy losses can be seen as a "rolling off" effect when the maximum displacement amplitude of each disk sample is plotted as a function of the drive level as shown in Figure 23.

Another way of viewing the change in Q_m was based on the work of Kugel et al. [48], in which the ratio of the actual measured displacement to the expected displacement with the low voltage value of Q_m is plotted as a function of electric field. Figure 24 is a similar plot, with the exception that the changes in Q_m are plotted with respect to drive voltage instead of electric field. The expected displacements used in Figure 24 were based on the lower values of Q_m calculated from the impedance analyzer testing of each sample after it had been run at the high power levels. The shape of this data matches exactly with that reported in Kugel et al. This helps to explain why the type of metal used had no apparent effect on the displacement results of the disk samples. Kugel et al. incorporated displacement testing of both unimorph (metal bonded to PZT) samples and bimorph (positively poled PZT bonded to negatively poled PZT) samples in their research. The plotted results had the same shape for the bimorph and unimorph structures as that shown in Figure 24. This supports the conclusion that, unless the ratio of metal to PZT is much larger, or the strain levels in the metal become much larger, the energy losses due to internal structural damping within the metal are insignificant with respect to the losses occurring in the PZT ceramic. A further indication of this is that the research of other authors [44, 45, 50] was of samples of PZT only, but the plotted data with respect to Q_m had the same shape shown in Figure 24.

Figure 23. Maximum Displacement Amplitudes of Disk Samples as a Function of Drive Voltage

Figure 24. Change in Q_m of Disk Samples as a Function of Drive Voltage

Correlation of Measured Results to Finite Element Models

Disk Models

A steady-state harmonic response finite element analysis was run for each disk sample. The finite element meshes were created using the I-DEAS Master Series version 6 program from Structural Dynamics Research Corporation, Milford, Ohio. The disks were modeled using eight-node linear brick elements, with the copper wires modeled as linear beam elements. All degrees of freedom at the ends of the copper wires were fixed to simulate being clamped in the vibration test fixture. Figure 25 is an illustration of the finite element mesh of the thin PZT disk sample geometry. The ABAQUS version 5.8 program from Hibbitt, Karlson, and Sorensen Inc. Pawtucket, Rhode Island, was used to perform the analysis solution. To verify the problem setup, the harmonic analysis of one disk sample was also performed using the ANSYS version 5.4 program from Swanson Analysis Systems Inc. Houston, Pennsylvania.

Figure 25. Finite Element Mesh of Thin PZT Disk Sample

The published data available from most suppliers of the PZT ceramic do not include all of the necessary material properties for performing a finite element analysis. To further complicate matters, any material properties that exist from the manufacturer are measured at low drive signal conditions. Any changes in the material properties at higher drive signals are not accounted for in the data. A good source of complete low drive signal material properties for many PZT ceramic compositions is Technical Publication TP-226 from Morgan Matroc Inc. Electro-Ceramics Division, Bedford, Ohio. The EDO EC-67 and EC-76 PZT used in the disk samples and the Motorola 3195-HD PZT used in the piezoelectric motors were similar to Morgan Matroc PZT-8, PZT-5H, and PZT-5A ceramics, respectively. The elastic matrix properties from the Morgan Matroc ceramics were used as well as any other properties not available for the actual PZT ceramics. Table 3 contains the material data for the PZT ceramics. The values in parentheses are the values of the similar Morgan Matroc material. The values in bold type (of either the actual materials or Morgan Matroc materials) are the values used in the finite element models. Table 4 contains the material properties used for the metals in the disk and stator models.

Table 3. Material Properties of PZT Ceramics Used in Finite Element Analyses

Short Circuit Elastic Modulii (pa)	EC-67 (PZT-8)	EC-76 (PZT-5H)	3195HD (PZT-5A)
Y_{11}	93.0E+9 (89.0E+9)	64.0E+9 (60.0E+9)	69.0E+9 (62.0E+9)
Y_{33}	73.0 (75.0)	51.0 (48.0)	55.0 (53.0)
Dielectric Constant@1kHz			

K_{33}^T	1100 (1000)	3450 (3400)	1800 (1700)
Short Circuit Elastic Matrix Properties (pa)			
c_{11}	149.0E+9	126.0E+9	121.0E+9
c_{12}	81.1E+9	79.5E+9	75.4E+9
c_{13}	81.1E+9	84.1E+9	75.2E+9
c_{33}	132.0E+9	117.0E+9	111.0E+9
c_{44}	31.3E+9	23.0E+9	21.1E+9
c_{66}	34.0E+9	23.5E+9	22.6E+9
d coefficients			
(m/v)			
d_{13}	-107E-12	-262E-12	-179E-12
	(-97E-12)	(-274E-12)	(-171E-12)
d_{33}	241E-12	583E-12	390E-12
	(225E-12)	(593E-12)	(374E-12)
d_{15}	362E-12	730E-12	(584E-12)
	(330E-12)	(741E-12)	
Clamped Permittivity (f/m)			
ϵ_{11}^s	(7.97E-9)	(15.05E-9)	(8.11E-9)
ϵ_{33}^s	(5.31E-9)	(13.02E-9)	(7.35E-9)
Density (kg/m ³)			
ρ	7500 (7600)	7450 (7500)	7800 (7750)

Table 4. Material Properties of Metals Used in Finite Element Analyses

Young's Modulus (pa)	544 Phosphor Bronze	303Se Stainless Steel	Copper Wires
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Y	103.35E+9	193.0E+9	117.2E+9
Poisson's Ratio			
ν	.35	.29	.33
Density (kg/m ³)			
ρ	8890	7920	8914

The finite element models were based on the response of the disk samples when driven with the same one-volt RMS (zero-to-peak) signal used with the HP4194A impedance analyzer. The equivalent circuit parameters given by the impedance analyzer allow the calculation of Q_m for the disks at this drive level. For the damaged samples, which showed a large drop in Q_m after running at high drive voltage levels, the lower value of Q_m was used to define the damping parameters in the finite element models. The same damping values were added for each material in the ABAQUS input files in the form of Rayleigh parameters. For the ANSYS solution, the damping was added as a single critical damping coefficient for the entire model. With both ABAQUS and ANSYS, the direct integration method was used in the solution process. This method is slower, but has the potential of being more accurate than modal-based methods. A method similar to that used to find the peak displacement when measuring the disk samples was used to find the peak displacement of the finite element simulations. The model was run over a large range with 100-Hz increments and then refined with other runs using a 10-Hz increment size. These runs resulted in the expected frequency response curve shape, with the exception that the linear material properties of the finite element models will not display the strain softening behavior and jump phenomena associated with actual samples. Figure 26 is a typical response curve from a harmonic response finite element analysis of a disk sample.

Figure 27 is an illustration of the exaggerated deformed "cupping" shape of the disk sample analysis performed with ANSYS. Figure 28 is an illustration of the deformed shape of the same disk sample analyzed using ABAQUS. Notice that there is an error in the ABAQUS display method in which the display of the deformed shape does not contain phase information, and all deformations for a steady-state harmonic response analysis are viewed as positive. This results in the ends of the disk beyond the nodal circle being displayed as bending back upward sharply instead of down below the nodal circle as expected.

ABAQUS Finite Element Displacement Results
for Thin EC-76 (soft PZT) Stainless Steel Disk
2.83 Volts Peak-Peak

Figure 26. Frequency Response Curve From Results of Finite Element Analysis of a Disk Sample

Figure 27. Exaggerated Deformed Shape of Disk Sample From
ANSYS Harmonic Response Analysis

```
SDRC I-DEAS ABAQUS FILE TRANSLATOR 24-Jul-98 11:48:16
ODE: run_new_1vm76b2.odb ABAQUS/standard 5.8-1 Sat Mar 27 12:27:09 CST 1999

Step: Step 1 Increment 14: Frequency = 2.0230E+04
Deformed Var: U Deformation Scale Factor: +7.380e+02
```

Figure 28. Exaggerated Deformed Shape of Disk Sample From ABAQUS Harmonic Response Analysis

The displacement results of the finite element analyses of the disk samples were much larger than the actual displacement measurements of the disks made at the one-volt RMS drive level. Table 5 is a comparison between the finite element results and the actual displacement measurements made of the disk samples. The discrepancy between the finite element results and measurements is thought to be due to the cracking of the samples containing hard PZT and possible partial depoling of the samples containing soft PZT. Partial depoling is suspected for the soft PZT because the EDO EC-76 material has a low Curie temperature (190 degrees Celsius) in comparison to the Motorola 3195HD (350 degrees Celsius) and the EDO EC-67 (300 degrees Celsius) materials. The EDO EC-76 material was bonded under pressure using the same 125-degree Celsius process to cure the epoxy as commonly used for the Motorola 3195HD ceramic. It is believed that this temperature and pressure combination may have partially depoled the samples.

Table 5. Comparison of Disk Measurements to Finite Element Results

Sample	Maximum Measured Zero-Peak Displacement and Frequency from 2.83 Volt Peak-Peak Drive Signal After High Power Levels	Finite Element Results Zero-Peak Displacement and Frequency from 2.83 Volt Peak-Peak Drive Signal After High Power Levels
Thin 67 Bronze	1.61 μm @ 23.9 kHz	4.19 μm @ 22.6 kHz
Thin 67 Stainless	2.39 μm @ 25.9 kHz	3.72 μm @ 26.4 kHz
Thick 67 Bronze	1.08 μm @ 26.3 kHz	2.38 μm @ 25.9 kHz
Thick 67 Stainless	0.80 μm @ 29.0 kHz	1.49 μm @ 30.1 kHz
Thin 76 Bronze	1.16 μm @ 21.5 kHz	4.07 μm @ 20.2 kHz
Thin 76 Stainless	0.98 μm @ 24.3 kHz	2.36 μm @ 23.7 kHz
Thick 76 Bronze	0.85 μm @ 24.2 kHz	2.59 μm @ 23.2 kHz
Thick 76 Stainless	0.64 μm @ 24.9 kHz	1.72 μm @ 26.7 kHz

Stator Models

Stators for eight-millimeter piezoelectric motors were made of the same phosphor bronze and stainless steel as used in the disk vibration damping study. The only PZT used with this group of motors was Motorola 3195HD ceramic. This soft PZT is similar to Morgan Matroc PZT-5A with the material properties used for this ceramic as shown in the bold type of Table 3. As with the disk samples, the finite element mesh of the stator geometry consisted of eight node linear bricks created using I-DEAS Master Series version 6, and the analyses performed were steady-state harmonic response solutions using ABAQUS version 5.8, with one verification analysis performed using ANSYS 5.4. Figure 29 is an illustration of the finite element mesh of the eight-millimeter stator.

The stators were tested on the HP4194A impedance analyzer, and the Q_m calculated from the equivalent circuit. The stators were then driven in a traveling wave vibration by two sinusoidal drive signals ninety degrees out of phase with each other at 100 volts peak to peak. The peak displacement amplitudes were determined and enough data gathered for plotting by adjusting the drive

Figure 29. Mesh of Eight-Millimeter Stator

voltage frequency and narrowing in on the maximum response in the same manner as done with the disk samples. In the actual motor, adjacent segments on the PZT ceramic ring are poled with opposite polarity, and then one half of the motor is driven with a sine wave drive signal while the other half is driven with a cosine wave drive signal. To accomplish this with the finite element models, the nodes on the bottom of the PZT segments were grouped into one of four sets. One half of the motor contained a node set driven with a positive sine drive signal and another node set driven with a negative sine drive signal. Likewise, on the other half of the motor was a node set driven with a positive cosine drive signal and a node set driven with a negative cosine drive signal. The drive signal amplitude used with the finite element models was 100 volts peak to peak, but the damping values added

to the finite element model material properties was based on the Q_m measurements made by the low voltage impedance analyzer tests. The frequency response curve had the expected shape of a well-isolated single peak near the (0,3) vibration mode frequency. Figure 30 contains the frequency response curve of the phosphor bronze stator analysis performed using ABAQUS. Due to the lack of phase information included in the ABAQUS representation of the deformed shape, all of the teeth of the

Figure 30. Frequency Response Curve From Results of Finite Element Analysis of Phosphor Bronze Stator

stator were shown to have the same displacement. This actually serves to verify that the simulation was of a true traveling wave, since each tooth experiences the same displacement amplitude. Figure 31 is the representation of the exaggerated deflection shape from the ABAQUS analysis. As a further verification, the results of the ANSYS analysis were viewed to verify that the displacement shape from the harmonic response analysis was of an isolated (0,3) mode as seen in Figure 32.

Figure 31. Exaggerated Deformed Shape of Stator From ABAQUS Harmonic Response Analysis

Figure 32. Exaggerated Deformed Shape of Stator From ANSYS Harmonic Response Analysis

Because Q_m decreases dramatically for a piezoelectric structure driven at high power levels, and the finite element analyses were conducted using Q_m values from low level impedance analyzer tests, the finite element results were not expected to match the actual displacement measurements of the stators. Based on the degradation in Q_m of the disk samples shown in Figure 24 (and from the research of Kugel et al. [48]), the value for Q_m of the stators when driven at high power levels is expected to level off at about three or four times less than the value measured with the impedance analyzer. Due to the linearity of the finite element analyses, the results from the analyses were expected to be about three or four times greater than the measured displacement amplitudes. A comparison of the finite element results and measurements of the two stator samples is contained in Table 6, and seems to agree with this estimate.

Table 6. Comparison of Stator Measurements to ABAQUS Finite Element Results Using Damping Values From Low-Level Testing

Maximum Measured Zero-Peak Displacement and Frequency With 100 Volt Peak-Peak Drive Signal	Finite Element Results Zero-Peak Displacement and Frequency With 100 Volt Peak-Peak Drive Signal	Ratio of Finite Element Results to Measured Displacements
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Phosphor Bronze Stator 25D With Low Level $Q_m = 300$	3.49 μm @ 51.7 kHz	11.83 μm @ 50.6 kHz	3.39
Stainless Steel Stator 20A With Low Level $Q_m = 390$	2.73 μm @ 67.4 kHz	9.97 μm @ 70.0 kHz	3.65

Accomplishments

Traveling wave piezoelectric motors have the advantage of having a large torque output in a small package. They are well suited for intermittent duty applications requiring small electric motors. Unlike electromechanical motors, these motors generate their torque by the interaction of a rotor with a vibrating circular plate-like stator. This means that the performance of these motors is dependent on the vibration amplitude of the stator.

To better understand this behavior, the stator was approximated as a circular plate, and the classical derivation of the transverse vibration modes and frequencies of a circular plate was utilized. The solution of this derivation is in terms of complicated expressions of Bessel functions.

Once the vibration modes and frequencies are known, the complexity of the analysis of the vibration of the stator (or other circular plate structure) may be greatly simplified if the vibration modes and frequencies are sufficiently isolated from each other. If (due to the geometry and material) the vibration modes are well isolated, the entire analysis may be treated as a simple one degree of freedom vibrating mechanical system. A further benefit of this simplification lies in the ability to model the vibrating structure as an analogous electrical circuit, and utilize an impedance analyzer to determine the damping properties of the structure when driven at low levels.

If this system is driven at the resonant frequency for this vibration mode, the amplitude of vibration will be directly related to the amount of damping friction loss within the material. By taking advantage of this (for the stator and for simple geometry samples of bonded piezoelectric ceramic-to-metal disks) the damping properties of the materials used in the stator assembly could be evaluated. A displacement measuring system was developed using a fine resolution laser displacement sensor, to detect the small vibration amplitudes of the stator and disk samples. The disk samples were made of materials similar to those used in the stators, and measurements were taken by driving the disks at a number of power levels.

The result of this displacement testing demonstrated that the increase in friction losses within the piezoelectric ceramic was by far the major contributor in the damping of the vibrating structure. Furthermore, these ceramics have a nonlinear strain softening behavior when vibrated at high power levels. The correlation of the measured amplitudes of the disk samples to finite element analyses was very poor. This is probably due to damage to the soft PZT disk samples incurred during fabrication, and cracking of the hard PZT disk samples during vibration, coupled with softening nonlinearity unaccounted for in the finite element analyses.

The increase in the damping of the disk structures matched very well with previous research on PZT ceramics alone. The measured data and previous research show that, when driven at high power levels, the amount of damping in the PZT ceramic increases rapidly and then flattens out to a factor of about three or four times the level measured using a low-voltage impedance analyzer. When this trend is applied to finite element analyses of the stator, the displacement amplitude measurements and the finite element results correlate well.

Future Work

By performing this research and discovering the dominance of the piezoceramic with respect to energy losses in the stator, future research in this area will focus on measurements of the piezoceramics alone, with an emphasis on generating the nonlinear material property data for the specific ceramics used in the motors. Currently, the ANSYS and ABAQUS finite element programs

do not allow for fully nonlinear piezoelectric material properties. After generating the nonlinear data, some development will be needed to be able to incorporate these material properties into either of these finite element programs.

References

1. Sashida, T., Kenjo, T., *An Introduction to Ultrasonic Motors*, Oxford University Press, Oxford, U.K. 1993
2. Ueha, S., Tomikawa, Y., *Ultrasonic Motors, Theory and Practice*, Oxford University Press, Oxford, U.K., 1993
3. Cummings, J. R., "Analysis of the Performance and Contact Behavior of the Piezoelectric Traveling-Wave Motor," Thesis, University of Missouri Rolla, 1994
4. Halter, S., "Characterization of Friction Liner Materials for Piezoelectric Motors Using Finite Element Analysis," Thesis, University of Missouri Columbia, 1994
5. Pressly, R. B., Montesana, C. P., *Piezoelectric Motor Development at AlliedSignal Inc.*, KCP-613-5515, UNCLASSIFIED. AlliedSignal FM&T, Kansas City, MO, November 1994
6. "Ultrasonic Motor," Panasonic Technical Reference, Electric Motor Division, Matsushita Electric Industrial Company, Japan, 1986
7. Flynn, A. M., "Torque Production in Ultrasonic Motors," MIT Artificial Intelligence Laboratory, January 1994
8. Flynn, A. M., "Piezoelectric Ultrasonic Micromotors," Thesis, Massachusetts Institute of Technology, 1995
9. Hagedorn, P., Wallaschek, J., "Traveling Wave Ultrasonic Motors, Part I, Working Principle and Mathematical Modeling of the Stator," *Journal of Sound and Vibration*, 1992, Vol. 155, pp. 31-46
10. Schmidt, J. P., Hagedorn, P., Bingqi, M., "A Note on the Contact Problem in an Ultrasonic Traveling Wave Motor," *International Journal of Nonlinear Mechanics*, 1996, Vol. 31, No. 6, pp. 915-924
11. McMindes, D., "Development of a Rotary Piezoelectric Motor Finite Element Model," Project Report, University of Missouri Columbia, 1998
12. IRE Standards on Piezoelectric Crystals: Measurements of Piezoelectric Ceramics, 1961
13. IEEE Standard Definitions and Methods of Measurement for *Piezoelectric Vibrators*, IEEE Standard 177, Institute of Electrical and Electronics Engineers, May 1966
14. IEEE Standard Definitions of Primary Ferroelectric Terms, *ANSI/IEEE Standard 180-1986*, Institute of Electrical and Electronics Engineers, 1986
15. IEEE Standard on Piezoelectricity, *ANSI/IEEE Standard 176-1987*, Institute of Electrical and Electronics Engineers, January 1987
16. Jaffe, B., Cook Jr., W. R., Jaffe, H., *Piezoelectric Ceramics*, R.A.N. Publishers, Marietta, OH, 1971
17. Auld, B. A., *Acoustic Fields and Waves in Solids, Vol.1*, Second Edition, Krieger Publishing Company, Malabar, FL, 1990
18. Auld, B. A., *Acoustic Fields and Waves in Solids, Vol.2*, Second Edition, Krieger Publishing Company, Malabar, FL, 1990
19. Huebner, W., Pennell, M., *Characterization of High Q Piezoelectric Ceramics*, Final Report on Project Number 052G602078, University of Missouri Rolla, 1998
20. Chung, T. J., *Continuum Mechanics*, Prentice Hall, Englewood Cliffs, NJ, 1988
21. ABAQUS/Standard User's Manual Version 5.8, Vol. 1, Hibbitt, Karlsson, and Sorensen Inc., Pawtucket, RI, 1998
22. ABAQUS/Standard User's Manual Version 5.8, Vol. 2, Hibbitt, Karlsson, and Sorensen Inc., Pawtucket, RI, 1998
23. ABAQUS/Standard User's Manual Version 5.8, Vol. 3, Hibbitt, Karlsson, and Sorensen Inc., Pawtucket, RI, 1998
24. ANSYS Examples Supplement for Revision 5.1, Swanson Analysis Systems Inc., Houston, PA, 1994
25. ANSYS Analysis Guide for version 5.4, Swanson Analysis Systems Inc., Houston, PA, 1997
26. ANSYS Commands Manual for version 5.4, Swanson Analysis Systems Inc., Houston, PA, 1997
27. ANSYS Elements Manual for version 5.4, Swanson Analysis Systems Inc., Houston, PA, 1997
28. ANSYS Theory Manual for version 5.4, Swanson Analysis Systems Inc., Houston, PA, 1997
29. Leissa, A., *Vibration of Plates*, SP-160, NASA, Washington D.C., 1969
30. Timoshenko, S., Woinowsky-Krieger, S., *Theory of Plates and Shells*, Second Edition, McGraw-Hill, New York, NY, 1959
31. Soedel, W., *Vibrations of Shells and Plates*, Second Edition, Marcel Dekker, Inc. New York, NY, 1993
32. Vierck, R. K. *Vibration Analysis*, Second Edition, Harper and Row, New York, NY, 1979
33. Hayt Jr., W., Kemmerly, J. E., *Engineering Circuit Analysis*, Third Edition, McGraw-Hill, New York, NY, 1978
34. Nashif, A. D., Jones, I. G., Henderson, J. P., *Vibration Damping*, John Wiley and Sons, New York, NY, 1985
35. Holland, R., "Representation of Dielectric, Elastic, and Piezoelectric Losses by Complex Coefficients," *IEEE Transactions*

on *Sonics and Ultrasonic SU-14* (1967),
pp. 18-20

36. Holland, R., EerNisse, E. P., "Accurate Measurement of Coefficients in a Ferroelectric Ceramic," *IEEE Transactions on Sonics and Ultrasonics SU-16* (1969), pp. 173-181
37. Lethiecq, M., Tran-Huu-Hue, L. P., Patat, F., Pourcelot, L., "Measurement of Losses in Five Piezoelectric Ceramics Between 2 and 50 MHz.," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, May 1993, pp. 232-237
38. Martin, G., "Determination of Equivalent-Circuit Constants of Piezoelectric Resonators of Moderately Low Q by Absolute-Admittance Measurements," *Journal of Acoustical Society of America*, pp. 413-420, 1957
39. Smits, J. G., "Alternative Method for Accurate Determination of the Real and Imaginary Parts of the Materials Coefficients of Piezoelectric Ceramics," *IEEE Transactions on Sonics and Ultrasonic SU-23 No. 6* (November 1976)
40. Kelly, J., Ballato, A., and Safari, A., "Characterization of Loss Mechanisms in Piezoelectric Ceramic Microresonators," *IEEE Ultrasonics Symposium 1996*
41. Sherit, S., Gauthier, N., Wiederick, H. D., Mukherjee, B. K., "Accurate Evaluation of the Real and Imaginary Material Constants for a Piezoelectric Resonator in the Radial Mode," *Ferroelectrics*, 1991, Vol. 119, Gordon and Breach Science Publishers
42. Saitoh, S., Honda, H., Kaneko, N., Izumi, M., Suzuki, S., "The Method of Determining k_t and Q_m for Low Q Piezoelectric Materials"
43. Stanke, F. E., Lang, S. W., "Automated System for Measuring the Constants of Low Q Piezoelectric Materials," *IEEE Ultrasonics Symposium 1985*, pp. 656-661
44. Belding, J. H., McLaren, M. G., "Behavior of Modified Lead Zirconate-Lead Titanate Piezoelectric Ceramics Under High Electric Fields," *Ceramic Bulletin* vol. 49, pp.1025-1029, 1970
45. Woollett, R. S., and LeBlanc, C. L., "Ferroelectric Nonlinearities in Transducer Ceramics," *IEEE Transactions on Sonics and Ultrasonics SU-20* (1973), pp. 24-31
1. Li, S., Cao, W., and Cross, L. E., "The Extrinsic Nature of Nonlinear Behavior Observed in Lead Zirconate Titanate Ferroelectric Ceramic," *Journal of Applied Physics*, vol. 69, pp. 7219-7224, 15 May 1991
47. Zhang, Q. M., Wang, H., Zhao, J., "Effect of Driving Field and Temperature on the Response Behavior of Ferroelectric Actuator and Sensor Materials," *Journal of Intelligent Material Systems and Structures*, vol. 6, pp. 84-93, January 1995
48. Kugel, V. D., Zhang, Q. M., Xu, B., Wang, Q., Chandran, S., and Cross, L. E., "Behavior of Piezoelectric Actuators under High Electric Field," *Materials for Adaptive Structural Acoustic Control Conference*, Penn State University Period from February 1, 1996-January 31, 1997
49. Hirose, S., Takahashi, Uchino, K., Aoyagi, M., Tomikawa, Y., "Measuring Methods for High Power Characteristics of Piezoelectric Materials," *Material Research Society Symposium*, Vol. 360, Materials Research Society, 1995, pp. 15-20
50. Takahashi, S., Sasaki, Y., Hirose, S., Uchino, K., "Electro-Mechanical Properties of PbZrO_3 , PbTiO_3 , $\text{Pb}(\text{Mn}_{1/3}\text{Sb}_{2/3})\text{O}_3$, Ceramics Under Vibration Level Change," *Material Research Society Symposium*, Vol. 360, Materials Research Society, 1995, pp. 305-310
51. Hirose, S., Yamayoshi, Y., Taga, M., Shimizu, H., "A Method of Measuring the Vibration Level Dependence of Impedance Type Equivalent Circuit Constants," *Proceedings of the Eleventh Symposium on Ultrasonic Electronics*, Japanese Journal of Applied Physics, 1991, Vol. 30, pp. 117-119
52. Harris, C. M., Crede, C. E., *Shock and Vibration Handbook*, Second Edition, McGraw-Hill, New York, NY, 1976

Displacement Measurement Data of Disk Samples

Thin EC-67 (hard PZT) Phosphor Bronze Disk
2.83 Volts Peak-Peak (Before and After High Power Levels)

Thin EC-67 (hard PZT) Phosphor Bronze Disk
5.0 Volts Peak-Peak (Before and After High Power Levels)

Thin EC-67 (hard PZT) Phosphor Bronze Disk
100.0 Volts Peak-Peak

Thin EC-67 (hard PZT) Stainless Steel Disk
2.83 Volts Peak-Peak (After High Power Levels)

Thin EC-67 (hard PZT) Stainless Steel Disk
5.0 Volts Peak-Peak (After High Power Levels)

Thin EC-67 (hard PZT) Stainless Steel Disk
25.0 Volts Peak-Peak

Thick EC-67 (hard PZT) Phosphor Bronze Disk
2.83 Volts Peak-Peak (Before and After High Power Levels)

Thick EC-67 (hard PZT) Phosphor Bronze Disk
5.0 Volts Peak-Peak (Before and After High Power Levels)

Thick EC-67 (hard PZT) Phosphor Bronze Disk
25.0 Volts Peak-Peak

Thick EC-67 (hard PZT) Stainless Steel Disk
2.83 Volts Peak-Peak (After High Power Levels)

Thick EC-67 (hard PZT) Stainless Steel Disk
5.0 Volts Peak-Peak (After High Power Levels)

Thick EC-67 (hard PZT) Stainless Steel Disk
25.0 Volts Peak-Peak

Thin EC-76 (soft PZT) Phosphor Bronze Disk
2.83 Volts Peak-Peak (After High Power Levels)

Thin EC-76 (soft PZT) Phosphor Bronze Disk
5.0 Volts Peak-Peak (After High Power Levels)

Thin EC-76 (soft PZT) Phosphor Bronze Disk
25.0 Volts Peak-Peak

Thin EC-76 (soft PZT) Stainless Steel Disk
2.83 Volts Peak-Peak (Before and After High Power Levels)

Thin EC-76 (soft PZT) Stainless Steel Disk
5.0 Volts Peak-Peak (Before and After High Power Levels)

Thin EC-76 (soft PZT) Stainless Steel Disk
25.0 Volts Peak-Peak

Appendix B

Sample ABAQUS Input Data for Disk Analysis

ABAQUS Input File for Thin EDO EC-67 Stainless Steel Disk (1vm67s2.inp)

*HEADING

SDRC I-DEAS ABAQUS FILE TRANSLATOR 24-Jul-98 11:48:16

kyle:/users/u54500/bond_disks/nodal_circle/1vm67s2.inp

Scott Yerganian 1/12/99

This is a model of an EDO PZT disk bonded to a 0.019 inch

thick metal disk. The disk diameter is 0.595 inches.

It has 0.010" diameter copper wires attached at the nodal circle for the (1,0)

mode of a free-free disk. This Analysis is an effort to duplicate the results

from low voltage impedance testing of this disk.

The ends of this wire are fixed in an effort to simulate the cue-ball

connections.

The damping numbers used correspond to the Q found from the impedance test.

This analysis is a harmonic analysis of the disk subjected to one volt RMS

(equivalent to 1.42 volts zero to peak amplitude).

The details of this analysis are as follows (thin PZT=0.015", thick PZT=0.020"):

Thin PZT, EDO67, Stainless, $Q=363$, Impedance Test Resonance=25.9kHz

Zoom in range 26.3-26.5kHz

***** Read in and define the material properties for PZT *****

*INCLUDE, INPUT=edo67_mat.inc

*DAMPING,ALPHA=237.2913,BETA=0.79717E-08

***** Read in and define the material properties for metal *****

*INCLUDE, INPUT=303Se_mat.inc

*DAMPING,ALPHA=237.2913,BETA=0.79717E-08

***** Read in and define the material properties for the copper wires *****

*INCLUDE, INPUT=cu_wire_mat.inc

***** Read in the node and element definition data *****

*INCLUDE, INPUT=19metal_15pzt.inc

*****Fix the nodes at the end of the wires and restrain the y-direction for

*****the node at the top center of the disk*****

*BOUNDARY

1775, 2,,

12356, 1, 6,

12359, 1, 6,

**Couple all of the top nodes of PZT together and set them all equal to

node 2203***

*EQUATION

2

TOPNDS,9,1.0,2203,9,-1.0

**Couple all of the bottom nodes of PZT together and set them all equal to

node 7001***

*EQUATION

2

BOTNDS,9,1.0,7001,9,-1.0

***** Set the voltage at the nodes at the top equal to zero as

***** part of the model data *****

*BOUNDARY

2203, 9, 9,

*****End of Model Definition*****

*****Begin Step Definition*****

***** Steady State Dynamic Step *****

*STEP,PERTURBATION

*STEADY STATE DYNAMICS,DIRECT

26300,26500,21,1,1

****Comment out the restart command to eliminate creating large files****

**RESTART,WRITE,FREQUENCY=1

**** Apply a 1.42 volt sinusoidally (real component) voltage to bottom nodes****

*BOUNDARY,OP=MOD

7001, 9,9, 1.42

*NSET,NSET=NOUT

1775, 7001, 2203

*NODE FILE,NSET=NOUT,FREQUENCY= 1

U, RCHG, PHCHG

*EL FILE,FREQUENCY= 0

*NODE PRINT,NSET=NOUT,FREQUENCY= 1

U, RCHG, PHCHG

*EL PRINT,FREQUENCY= 0

*END STEP

EDO EC-67 Material Properties (edo_67_mat.inc)

*SOLID SECTION,ELSET=PZT,MATERIAL=EDO67

*MATERIAL,NAME=EDO67

*ELASTIC,TYPE=ORTHO

14.9E+10,8.11E+10,14.9E+10,8.11E+10,8.11E+10,13.2E+10,3.40E+10,3.13E+10

3.13E+10

*PIEZOELECTRIC,TYPE=E

0.,0.,0.,0.,362.E-12,0.,0.,0.

0.,0.,0.,362.E-12,-107.E-12,-107.E-12,241.E-12,0.

0.,0.,

*DIELECTRIC,TYPE=ORTHO

0.079686E-07,0.079686E-07,0.053124E-07

*DENSITY

7.500+03

303Se Stainless Steel Material Properties (303Se_mat.inc)

*SOLID SECTION,ELSET=METAL,MATERIAL=303Se

*MATERIAL,NAME=303Se

*ELASTIC,TYPE=ISOTROPIC

1.93E+11 0.29

*DENSITY

7.920E+03

*EXPANSION,TYPE=ISO,ZERO=21.85

17.30E-06

Copper Wire Material Properties (cu_wire_mat.inc)

*BEAM SECTION,MATERIAL=CU_WIRE ,SECTION=CIRC,ELSET=BOTWIRE

1.270000E-04,

-1.000000E+00, 0.000000E+00, 0.000000E+00

*BEAM SECTION,MATERIAL=CU_WIRE ,SECTION=CIRC,ELSET=HORZWIRE

1.270000E-04,

0.000000E+00, 0.000000E+00,-1.000000E+00

*BEAM SECTION,MATERIAL=CU_WIRE ,SECTION=CIRC,ELSET=TOPWIRE

1.270000E-04,

1.000000E+00, 0.000000E+00, 0.000000E+00

*MATERIAL,NAME=CU_WIRE

*ELASTIC,TYPE=ISOTROPIC

1.172E+11 3.300E-01

*DENSITY

8.914E+03

*EXPANSION,TYPE=ISO,ZERO=21.85

1.170E-05

*CONDUCTIVITY,TYPE=ISO

4.500E+01

Model Definition (19metal_15pzt.inc)(abbreviated)

***** Definition of Nodes *****

*NODE, SYSTEM=R

1, 0.0000000E+00,-1.5113000E-03, 4.8260000E-04

2, 0.0000000E+00,-1.8135600E-03, 4.8260000E-04

.

... Removed majority of node definition data....

.

12452, 2.1638745E-02, 0.0000000E+00,-2.5400000E-03

12453, 2.2143974E-02, 0.0000000E+00,-2.5400000E-03

*****Element Definition*****

*ELEMENT,TYPE=B31 ,ELSET=BOTWIRE

12249, 9579, 12397

12251, 12398, 12399

12252, 12399, 12358

12250, 12397, 12398

*ELEMENT,TYPE=B31 ,ELSET=HORZWIRE

12294, 12440, 12441

12293, 12439, 12440

.

... Removed majority of element definition data....

.

12202, 12446, 12447

12201, 12445, 12446

*ELEMENT,TYPE=B31 ,ELSET=TOPWIRE

12214, 12363, 12355

12212, 12361, 12362

12211, 12360, 12361

12210, 937, 12360

12213, 12362, 12363

*ELEMENT,TYPE=C3D8E ,ELSET=PZT

7097, 7100, 7102, 7144, 7142, 7101, 7103, 7145, 7143

7048, 2253, 2255, 2297, 2295, 7050, 7052, 7094, 7092

.

... Removed majority of element definition data....

.

7096, 2303, 2305, 2347, 2345, 7100, 7102, 7144, 7142

10999, 11080, 10720, 7922, 8344, 11081, 10721, 7923, 8345

*ELEMENT,TYPE=C3D8I ,ELSET=METAL

100, 2304, 2306, 2348, 2346, 2305, 2307, 2349, 2347

102, 2306, 2308, 2350, 2348, 2307, 2309, 2351, 2349

.

... Removed majority of element definition data....

.

1, 1, 2, 23, 22, 2202, 2204, 2246, 2244

99, 52, 53, 74, 73, 2304, 2306, 2348, 2346

*****Node Set Definitions*****

*NSET,NSET=TOPNDS

2205, 2207, 2209, 2211, 2213, 2215, 2217, 2219, 2221, 2223

2225, 2227, 2229, 2231, 2233, 2235, 2237, 2239, 2241, 2243, 2245

.

... Removed majority of node set definition data....

.

6251, 6253, 6255, 6257, 6259, 6261, 6263, 6265, 6267, 6269, 6271

6273, 6275, 6277, 6279, 6281, 6283

*NSET,NSET=BOTNDS

7003, 7005, 7007, 7009, 7011, 7013, 7015, 7017, 7019, 7021

7023, 7025, 7027, 7029, 7031, 7033, 7035, 7037, 7039, 7041, 7043

.

... Removed majority of node set definition data....

.

11049, 11051, 11053, 11055, 11057, 11059, 11061, 11063, 11065, 11067, 11069

11071, 11073, 11075, 11077, 11079, 11081

Appendix C

Sample ABAQUS Input Data
for Stator Analysis

ABAQUS Input File for Phosphor Bronze Stator (stator_brnz_25D3.inp)

*HEADING

heartland:/home/u54500/stator/stator_brnz_25D3.inp

Scott Yerganian 3/25/99

This is the second model of a phosphor bronze stator with a 0.005" thick soft

PZT This analysis is an effort to compare the results of this model driven at

100 volts peak to peak with the actual results from testing of sample 25D

at 100 volts peak to peak.

The damping numbers used correspond to the Q of 300 found from the low voltage
impedance test.

The details of this analysis are as follows:

Motorola 3195, Bronze, Q=300, Impedance Test Resonance=51.69kHz

Range 50.5-50.7kHz

***** Read in and define the material properties for PZT *****

*INCLUDE, INPUT=3195_mat.inc

*DAMPING,ALPHA=529.874,BETA=5.2423E-09

***** Read in and define the material properties for metal *****

*INCLUDE, INPUT=544PhsBrz_mat.inc

*DAMPING,ALPHA=529.874,BETA=5.2423E-09

***** Read in the node definition data *****

*INCLUDE, INPUT=stator_nodes.inc

***** Read in the element definition data *****

*INCLUDE, INPUT=stator_elements.inc

***** Read in the node set definitions *****

*INCLUDE, INPUT=nsets.inc

*****Fix the nodes in the center of the disk*****

*BOUNDARY

CNODES, 1, 3,

**Couple all of the top nodes of PZT together and set them all equal to

node 113***

*EQUATION

2

TNODES,9,1.0,113,9,-1.0

**Make node set out of all nodes to be driven with a positive sine wave **

**** and couple their voltage to node 2301 *****

*NSET,NSET=PSINE

NSET3, NSET4, NSET11, NSET12

*EQUATION

2

PSINE,9,1.0,2301,9,-1.0

Make node set out of all nodes to be driven with a negative sine wave *

**** and couple their voltage to node 121 *****

*NSET,NSET=NSINE

NSET13, NSET2

*EQUATION

2

NSINE,9,1.0,121,9,-1.0

Make node set out of all nodes to be driven with a positive cosine wave *

**** and couple their voltage to node 6665 *****

*NSET,NSET=PCSINE

NSET5, NSET8, NSET9

*EQUATION

2

PCSINE,9,1.0,6665,9,-1.0

Make node set out of all nodes to be driven with a negative cosine wave *

**** and couple their voltage to node 8537 *****

*NSET,NSET=NCSINE

NSET6, NSET7, NSET10

*EQUATION

2

NCSINE,9,1.0,8537,9,-1.0

***** Set the voltage at the nodes at the top equal to zero as

***** part of the model data *****

*BOUNDARY

113, 9, 9,

*****End of Model Definition*****

*****Begin Step Definition*****

***** Steady State Dynamic Step *****

*STEP,PERTURBATION

*STEADY STATE DYNAMICS,DIRECT

50500,50700,21,1,1

****Comment out the restart command to eliminate creating large files*****

**RESTART,WRITE,FREQUENCY=1

**** Apply 50 volt (zero to peak amplitude) voltages to bottom nodes***

*BOUNDARY, LOAD CASE=1

6665, 9,9, 50.0

8537, 9,9, -50.0

*BOUNDARY, LOAD CASE=2

2301, 9,9, 50.0

121, 9,9, -50.0

*NSET,NSET=NOUT

25831, 26767

*NODE FILE,NSET=NOUT,FREQUENCY= 1

U, RCHG, PHCHG

*EL FILE,FREQUENCY= 0

*NODE PRINT,NSET=NOUT,FREQUENCY= 1

U, RCHG, PHCHG

*EL PRINT,FREQUENCY= 0

*END STEP

Motorola 3195 PZT Material Properties (3195_mat.inc)

*SOLID SECTION,ELSET=PZT,MATERIAL=3195

*MATERIAL,NAME=3195

*ELASTIC,TYPE=ORTHO

12.1E+10,7.54E+10,12.1E+10,7.52E+10,7.52E+10,11.1E+10,2.26E+10,2.11E+10

2.11E+10

*PIEZOELECTRIC,TYPE=E

0.,0.,0.,0.,584.0E-12,0.,0.,0.

0.,0.,0.,584.0E-12,-179.0E-12,-179.0E-12,390.0E-12,0.

0.,0.,

*DIELECTRIC,TYPE=ORTHO

0.08110E-07,0.08110E-07,0.07349E-07

*DENSITY

7.800+03

Phosphor Bronze Material Properties (544PhsBrz_mat.inc)

*SOLID SECTION,ELSET=METAL,MATERIAL=BRONZE

*MATERIAL,NAME=BRONZE

*ELASTIC,TYPE=ISOTROPIC

1.0335E+11, 0.35

*DENSITY

8.890E+03

*EXPANSION,TYPE=ISO,ZERO=21.85

17.28E-06

Stator Node Definition Data (stator_nodes.inc)(abbreviated)

*NODE, SYSTEM=R, NSET=TOTALN

13,-1.277950E-03,0.000000E+00,3.810000E-04

14,-1.408913E-03,0.000000E+00,3.810000E-04

.

... Removed majority of node definition data....

.

27567,-3.997960E-03,1.396162E-04,1.524000E-03

27568,-3.997960E-03,1.396162E-04,1.651000E-03

Stator Element Definition Data (stator_elements.inc)(abbreviated)

*ELEMENT,TYPE=C3D8I ,ELSET=METAL

102, 13, 30, 31, 14, 151, 183, 185, 153

103, 151, 183, 185, 153, 152, 184, 186, 154

.

... Removed majority of element definition lines....

.

21350, 25866, 25872, 25875, 25869, 25867, 25873, 25876, 25870

21351, 13012, 13016, 13056, 13048, 25844, 25847, 25877, 25871

*ELEMENT,TYPE=C3D8E ,ELSET=PZT

12200, 11794, 11810, 11808, 11792, 11931, 11939, 11938, 11930

12427, 12259, 12275, 12273, 12257, 12260, 12276, 12274, 12258

.

... Removed majority of element definition lines....

.

256, 115, 123, 122, 114, 349, 365, 363, 347

13724, 13929, 13937, 13936, 13928, 14261, 14277, 14275, 14259

Node Set Definitions (nsets.inc)(abbreviated)

**** Define the center nodes for clamped restraint ****

*NSET,NSET=CNODES

13

455

.

... Removed majority of node set definition lines....

.

23199

23699

*** Define node set 2 with the exception of node 121 *****

*NSET,NSET=NSET2

122

123

.

... Removed majority of node set definition lines....

.

2183

2184

*** Define node set 3 with the exception of node 2301 *****

*NSET,NSET=NSET3

2302

2303

.

... Removed majority of node set definition lines....

.

4551

4552

*** Define node set 4 *****

*NSET,NSET=NSET4

4669

4670

.

... Removed majority of node set definition lines....

.

6547

6548

*** Define node set 5 with the exception of node 6665 *****

*NSET,NSET=NSET5

6666

6667

.

... Removed majority of node set definition lines....

.

8419

8420

*** Define node set 6 with the exception of node 8537 *****

*NSET,NSET=NSET6

8538

8539

.

... Removed majority of node set definition lines....

.

10415

10416

*** Define node set 7 *****

*NSET,NSET=NSET7

10533

10534

.

... Removed majority of node set definition lines....

.

12287

12288

*** Define node set 8 ***

*NSET,NSET=NSET8

12405

12406

.

... Removed majority of node set definition lines....

.

14283

14284

*** Define node set 9 ***

*NSET,NSET=NSET9

14401

14402

.

... Removed majority of node set definition lines....

.

16155

16156

*** Define node set 10 ***

*NSET,NSET=NSET10

16273

16274

.

... Removed majority of node set definition lines....

.

18151

18152

*** Define node set 11 ***

*NSET,NSET=NSET11

18269

18270

.

... Removed majority of node set definition lines....

.

20023

20024

*** Define node set 12 ***

*NSET,NSET=NSET12

20141

20142

.

... Removed majority of node set definition lines....

.

22019

22020

*** Define node set 13 *****

*NSET,NSET=NSET13

22137

22138

.

... Removed majority of node set definition lines....

.

23891

23892

**** Define the nodes on the top of the PZT (except node 113) to be grounded ****

*NSET,NSET=TNODES

114

115

.

... Removed majority of node set definition lines....

.

23875

23876

Appendix D

Sample ANSYS Input Data for Disk Analysis

ANSYS Input File for Thin EDO EC-76 Phosphor Bronze Disk (an1vm76b2.dat)

!*****

!southpark:/users/u54500_southpark/ansysPiezo/bond_disks/an1vm76b2.dat

!Scott Yerganian 3/28/98

!This is a harmonic analysis of a 0.020" thick EDO76 PZT

!disk bonded to a 0.019" thick Phosphor bronze disk

!with a sinusoidal voltage of 25 volts applied. I want to

!run this analysis and then compare its results to the same

!analysis done using ABAQUS

/FILNAM,run_an1vm76b2

/PREP7

/UNITS,SI

CSYS,0, \$CLOCAL,13,0,0.,0.,0.4826E-03,270.,90.,90.

/COM

/COM NODE DEFINITIONS

CSYS,0

!***** Read in the node definitions *****

/INPUT,metal_15pzt.input

!***** Read in the PZT material properties *****

/INPUT,edo_76_mat.input

!***** Read in the metal material properties *****

/INPUT,PhsBrz544_mat.input

!***** Read in the wire material properties *****

/INPUT,cu_wire_mat.input

!***** Read in the element definitions *****

/INPUT,elements.input

!*****

!***** Read in couple.input to couple the voltage on the all of the nodes****

!***** on the top surface of the PZT together, and then couple the voltage***

!***** of all of the nodes on the bottom surface of the PZT together*****

/INPUT,couple.input

!*****

!**Fix the Ends of the Copper Wires & top node in the y direction**

D,1775,UY,0,,,

D,12356,UX,0,,,,,UY,UZ,ROTX,ROTY,ROTZ

D,12359,UX,0,,,,,UY,UZ,ROTX,ROTY,ROTZ

!*****Finish the Model Definition*****

FINISH

!*****Begin Solution Definition*****

/SOLUTION

ANTYPE,HARMIC

HROPT,FULL

HARFRQ,20200,20390

DMPRAT,0.003086

NSUBST,19

KBC,1

OUTRES,ALL,ALL

DCUM,ADD

D,2203,VOLT,0.0

D,7001,VOLT,1.4142

SOLVE

FINISH

Node Definition (metal_15pzt.input)

!*** Node Definition *****

CSYS,0

N,1,0.,-0.15113E-02,0.4826E-03

N,2,0.,-0.181356E-02,0.4826E-03

N,3,0.,-0.211582E-02,0.4826E-03

N,4,0.,-0.241808E-02,0.4826E-03

.

... Removed majority of node definition lines....

.

N,12452,0.216387447853604E-01,0.,-0.254E-02

N,12453,0.221439743096124E-01,0.,-0.254E-02

EDO EC-76 Material Properties (edo_67_mat.input)

!*****PIEZO MATERIAL (EDO67)*****

TB,ANEL,2

TBDATA,1,14.9E+10

TBDATA,2,8.11E+10

TBDATA,3,8.11E+10

TBDATA,7,14.9E+10

TBDATA,8,8.11E+10

TBDATA,12,13.2E+10

TBDATA,16,3.4E+10

TBDATA,19,3.13E+10

TBDATA,21,3.13E+10

DENS, 2, 7500.0

!*****

! Piezo Property Definition

MP,PERX, 2, 0.0797E-07

MP,PERY, 2, 0.0797E-07

MP,PERZ, 2, 0.0531E-07

/COM ANSYS Piezoelectric "e" matrix

TB,PIEZ,2

TBDATA, 3, -5.0756000

TBDATA, 6, -5.0756000

TBDATA, 9, 14.456600

TBDATA, 14, 11.330600

TBDATA, 16, 11.330600

!*****

Copper Wire Material Properties (cu_wire_mat.input)

!*****Copper Wire Properties*****

EX, 1, 1.1721E+11

NUXY, 1, 3.3000E-01

DENS, 1, 8.9139E+03

ALPX, 1, 1.1700E-05

KXX, 1, 4.5000E+01

!*****

Element Definition (elements.input)

!*****Definition of Beam Element Properties (copper wires)*****

R,1,0.50670746E-07,0.20431709E-15,0.20431709E-15,0.25399998E-03,0.25399998E-03,0.

RMORE,0.,0.40863418E-15,1.1282052,1.1282052,,

ET, 1, 4, 0, 0, 0, 0, 0, 0

KEYOPT, 1, 10, 0

!*****Definition of Solid Element Properties*****

ET, 2, 5, 0, 0, 0, 0, 0, 0

!*****Definition of Metal Elements*****

TYPE, 2 \$ MAT, 3

ESYS, 0

EN,1,1,2,23,22,2202,2204,2246,2244

EN,2,2202,2204,2246,2244,2203,2205,2247,2245

.

... Removed majority of element definition lines....

.

EN,3999,2201,1974,483,715,6282,5922,3124,3546

EN,4000,6282,5922,3124,3546,6283,5923,3125,3547

!*****Definition of PZT Elements*****

TYPE, 2 \$ MAT, 2

ESYS, 0

EN,7000,2203,2205,2247,2245,7000,7002,7044,7042

EN,7001,7000,7002,7044,7042,7001,7003,7045,7043

.

... Removed majority of element definition lines....

.

EN,10998,6283,5923,3125,3547,11080,10720,7922,8344

EN,10999,11080,10720,7922,8344,11081,10721,7923,8345

!*****Definition of Copper Wire Elements*****

TYPE, 1 \$ REAL, 1 \$ MAT, 1

ESYS, 0

EN,12200,12444,12445

EN,12201,12445,12446

.

... Removed majority of element definition lines....

.

EN,12296,12442,12443

EN,12297,12443,12444

CSYS,0

Node Set Definitions and Coupling (couple.input)

!*****Couple the Voltage on the Top of the PZT****

CP,1,VOLT,2203,2205

CP,1,VOLT,2207,2209,2211,2213,2215,2217,2219,2221

.

... Removed majority of coupled node definition lines....

.

CP,1,VOLT,6255,6257,6259,6261,6263,6265,6267,6269,6271

CP,1,VOLT,6273,6275,6277,6279,6281,6283,,,

!*****Couple the Voltage on the Bottom of the PZT*****

CP,2,VOLT,7001,7003

CP,2,VOLT,7005,7007,7009,7011,7013,7015,7017,7019

CP,2,VOLT,9541,9543,9545,9547,9549,9551,9553,9555,9557

CP,2,VOLT,9559,9561,9563,9565,9567,9569,9571,9573,9575

.

... Removed majority of coupled node definition lines....

.

CP,2,VOLT,11053,11055,11057,11059,11061,11063,11065,11067,11069

CP,2,VOLT,11071,11073,11075,11077,11079,11081

Appendix E

Sample ANSYS Input Data for Stator Analysis

ANSYS Input File for Stainless Steel Stator (an_stator_303_20A3.dat)(abbreviated)

!*****

!southpark:/users/u54500_southpark/ansysPiezo/stator/an_stator_303_20A3.dat

!Scott Yerganian 4/10/99

!This is an third full harmonic analysis of the CX1 stator 20A in an attempt

!to match the results I got from the ABAQUS stator_303_20A.inp analysis

!*****

/FILNAM,run_an_stator_303_20A3

/PREP7

!*****

!Mechanical Properties of Material 1 (Stainless Steel Elements)

MP,EX, 1, 1.93E+11

MP,NUXY, 1, 0.29

MP,DENS, 1, 7920.0

!*****

!Material Properties of Material 2 (Real Piezo Elements)

/INPUT,PZT_3195_mat.input

!*****

!Mechanical Properties of Material 3 (Stainless Steel Pseudo Piezo Elements)

MP,EX, 3, 1.93E+11

MP,NUXY, 3, 0.29

MP,DENS, 3, 7920.0

!*****

! Electrical Properties of Material 3 (Stator Psuedo Piezo Elements)

MP,PERX, 3, 0.0

MP,PERY, 3, 0.0

MP,PERZ, 3, 0.0

/COM ANSYS Piezoelectric "e" matrix

TB,PIEZ,3

TBDATA, 3, 0.0

TBDATA, 6, 0.0

TBDATA, 9, 0.0

TBDATA, 14, 0.0

TBDATA, 16, 0.0

!*****

!Node Definition

!*****

!Node coordinates

CSYS,0

/INPUT,stator_node_coords.input

!*****

!Element Definition

!*****

!Stator Elements

R,1,,,,,

ET, 1, 45, 0

TYPE, 1 \$ REAL, 1 \$ MAT, 1

ESYS, 0

EN,102,13,30,31,14,151,183,185,153

EN,103,151,183,185,153,152,184,186,154

.

... Removed majority of element definition lines....

.

EN,22351,27557,27563,27566,27560,27558,27564,27567,27561

EN,22352,27558,27564,27567,27561,27559,27565,27568,27562

CSYS,0

!*****

!*****

!Real Piezo Elements

R,2,,,,,

ET, 2, 5, 0

TYPE, 2 \$ REAL, 2 \$ MAT, 2

ESYS, 0

EN,254,114,122,121,113,347,363,361,345

EN,255,347,363,361,345,348,364,362,346

.

... Removed majority of element definition lines....

.

EN,20279,23874,23890,23888,23872,119,127,126,118

EN,20280,23876,23892,23890,23874,120,128,127,119

CSYS,0

!*****

!*****

!Stator Psuedo Piezo Elements

R,3,,,,,

ET, 3, 5, 0

TYPE, 3 \$ REAL, 3 \$ MAT, 3

ESYS, 0

EN,240,114,113,105,106,347,345,329,331

EN,241,347,345,329,331,348,346,330,332

.

... Removed majority of element definition lines....

.

EN,20272,23874,23872,23856,23858,119,118,110,111

EN,20273,23876,23874,23858,23860,120,119,111,112

CSYS,0

!*****

!Node selection from ABAQUS translation file "NSET1" (center nodes)

NSEL,S,NODE,,13

NSEL,A,NODE,,455

.

... Removed majority of node set definition lines....

.

NSEL,A,NODE,,23199

NSEL,A,NODE,,23699

CM,CNODE,NODE,,

NSEL,ALL,,

!*****

!Node selection from ABAQUS translation file "NSET2" (Bot nodes of NSET N000030)

NSEL,S,NODE,,121

NSEL,A,NODE,,122

.

... Removed majority of node set definition lines....

.

NSEL,A,NODE,,2183

NSEL,A,NODE,,2184

CM,NSET2,NODE,,

NSEL,ALL,,

!*****

!Node selection from ABAQUS translation file "NSET3" (Bot nodes of NSET N030060)

NSEL,S,NODE,,2301

NSEL,A,NODE,,2302

...

... Removed majority of node set definition lines....

NSEL,A,NODE,,4551

NSEL,A,NODE,,4552

CM,NSET3,NODE,,

NSEL,ALL,,

!*****

!Node selection from ABAQUS translation file "NSET4" (Bot nodes of NSET N060090)

NSEL,S,NODE,,4669

NSEL,A,NODE,,4670

...

... Removed majority of node set definition lines....

NSEL,A,NODE,,6547

NSEL,A,NODE,,6548

CM,NSET4,NODE,,

NSEL,ALL,,

!*****

!Node selection from ABAQUS translation file "NSET5" (Bot nodes of NSET N090120)

NSEL,S,NODE,,6665

NSEL,A,NODE,,6666

...

... Removed majority of node set definition lines....

NSEL,A,NODE,,8419

NSEL,A,NODE,,8420

CM,NSET5,NODE,,

NSEL,ALL,,

!*****

!Node selection from ABAQUS translation file "NSET6" (Bot nodes of NSET N120150)

NSEL,S,NODE,,8537

NSEL,A,NODE,,8538

.

... Removed majority of node set definition lines....

.

NSEL,A,NODE,,10415

NSEL,A,NODE,,10416

CM,NSET6,NODE,,

NSEL,ALL,,

!*****

!Node selection from ABAQUS translation file "NSET7" (Bot nodes of NSET N150180)

NSEL,S,NODE,,10533

NSEL,A,NODE,,10534

.

... Removed majority of node set definition lines....

.

NSEL,A,NODE,,12287

NSEL,A,NODE,,12288

CM,NSET7,NODE,,

NSEL,ALL,,

!*****

!Node selection from ABAQUS translation file "NSET8" (Bot nodes of NSET N180210)

NSEL,S,NODE,,12405

NSEL,A,NODE,,12406

.

... Removed majority of node set definition lines....

.

NSEL,A,NODE,,14283

NSEL,A,NODE,,14284

CM,NSET8,NODE,,

NSEL,ALL,,

!*****

!Node selection from ABAQUS translation file "NSET9" (Bot nodes of NSET N210240)

NSEL,S,NODE,,14401

NSEL,A,NODE,,14402

.

... Removed majority of node set definition lines....

.

NSEL,A,NODE,,16155

NSEL,A,NODE,,16156

CM,NSET9,NODE,,

NSEL,ALL,,

!*****

!Node selection from ABAQUS translation file "NSET10" (Bot nodes of NSET N240270)

NSEL,S,NODE,,16273

NSEL,A,NODE,,16274

.

... Removed majority of node set definition lines....

.

NSEL,A,NODE,,18151

NSEL,A,NODE,,18152

CM,NSET10,NODE,,

NSEL,ALL,,

!*****

!Node selection from ABAQUS translation file "NSET11" (Bot nodes of NSET N270300)

NSEL,S,NODE,,18269

NSEL,A,NODE,,18270

.

... Removed majority of node set definition lines....

.

NSEL,A,NODE,,20023

NSEL,A,NODE,,20024

CM,NSET11,NODE,,

NSEL,ALL,,

!*****

!Node selection from ABAQUS translation file "NSET12" (Bot nodes of NSET N300330)

NSEL,S,NODE,,20141

NSEL,A,NODE,,20142

.

... Removed majority of node set definition lines....

.

NSEL,A,NODE,,22019

NSEL,A,NODE,,22020

CM,NSET12,NODE,,

NSEL,ALL,,

!*****

!Node selection from ABAQUS translation file "NSET13" (Bot nodes of NSET N330000)

NSEL,S,NODE,,22137

NSEL,A,NODE,,22138

.

... Removed majority of node set definition lines....

.

NSEL,A,NODE,,23891

NSEL,A,NODE,,23892

CM,NSET13,NODE,,

NSEL,ALL,,

!*****

!*****

! Define Top Nodes of PZT and Couple Their Values

NSEL,S,NODE,,113

NSEL,A,NODE,,114

.

... Removed majority of node set definition lines....

.

NSEL,A,NODE,,23875

NSEL,A,NODE,,23876

CM,TOP_NDS,NODE,,

NSEL,ALL,,

!*****

!Couple the Appropriate Nodes for Applying Common Voltages

!*****

!Couple the Top Nodes of the PZT Elements (Nodes in contact with stator)

CP,1,VOLT, TOP_NDS

!*****

!Couple the Nodes for applying the Positive Sine Driving Voltage

!(lowest node = 2301)

ALLSEL,ALL

CMSEL,S,NSET11

CMSEL,A,NSET12

CMSEL,A,NSET3

CMSEL,A,NSET4

CP,2,VOLT,ALL

ALLSEL,ALL

!*****

!Couple the Nodes for applying the Negative Sine Driving Voltage

!(lowest node = 121)

CMSEL,S,NSET13

CMSEL,A,NSET2

CP,3,VOLT,ALL

ALLSEL,ALL

!*****

!Couple the Nodes for applying the Positive Cosine Driving Voltage

!(lowest node = 6665)

CMSEL,S,NSET5

CMSEL,A,NSET8

CMSEL,A,NSET9

CP,4,VOLT,ALL

ALLSEL,ALL

!*****

!Couple the Nodes for applying the Negative Cosine Driving Voltage

!(lowest node = 8537)

CMSEL,S,NSET6

CMSEL,A,NSET7

CMSEL,A,NSET10

CP,5,VOLT,ALL

ALLSEL,ALL

!*****

!Apply the Fixed Restraint to the Center Nodes

!*****

D,CNODE,UX,0.0,,UY,UZ

!*****

!Finish model definition

FINISH

!

!

!*****

!*****

!Start of Solution Definition

!*****

!*****

/SOLUTION

!Harmonic analysis of traveling wave of stator driven in three wave

!mode using piezo electric applied voltages of 50 volts amplitude

!(100 volts peak to peak) with steps at odd 70.1kHz,70.3kHz,70.5kHz ...71.9kHz

!and using damping corresponding to a mechanical Q of 390

ANTYPE,HARMIC

HROPT,FULL

DMPRAT,0.001282

HARFRQ,69900,71900

NSUBST,10

OUTRES,ALL,ALL

DCUM,ADD

KBC,1

D,TOP_NDS,VOLT,0.0

D,2301,VOLT,0.0,50.0

D,121,VOLT,0.0,-50.0

D,6665,VOLT,50.0,0.0

D,8537,VOLT,-50.0,0.0

SOLVE

FINISH

Stator Node Definition Data (stator_node_coords.input)

N,13,-1.277950E-03,0.000000E+00,3.810000E-04

N,14,-1.408913E-03,0.000000E+00,3.810000E-04

N,15,-1.539875E-03,0.000000E+00,3.810000E-04

N,16,-1.670837E-03,0.000000E+00,3.810000E-04

N,17,-1.801800E-03,0.000000E+00,3.810000E-04

N,18,-1.932788E-03,0.000000E+00,3.810000E-04

N,19,-2.063750E-03,0.000000E+00,3.810000E-04

.

... Removed majority of node definition data....

.

N,27566,-3.997960E-03,1.396162E-04,1.397000E-03

N,27567,-3.997960E-03,1.396162E-04,1.524000E-03

N,27568,-3.997960E-03,1.396162E-04,1.651000E-03