SANDIA REPORT

SAND98-1887

Unlimited Release

Printed September 1998

Theoretical Analysis of the Sensitivity and Speed Improvement of ISIS over a Comparable Traditional Hyperspectral Imager

RECEIVED
OCT 8 1998
OSTI

Stephen M. Gentry and Brian R. Stallard

Prepared by
Sandia National Laboratories
Albuquerque, New Mexico 87185 and Livermore, California 94550

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under Contract DE-AC04-94AL85000.

Approved for public release; further dissemination unlimited.

Sandia National Laboratories

Issued by Sandia National Laboratories, operated for the United States Department of Energy by Sandia Corporation.

NOTICE: This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government, any agency thereof, or any of their contractors or subcontractors. The views and opinions expressed herein do not necessarily state or reflect those of the United States Government, any agency thereof, or any of their contractors.

Printed in the United States of America. This report has been reproduced directly from the best available copy.

Available to DOE and DOE contractors from
Office of Scientific and Technical Information
P.O. Box 62
Oak Ridge, TN 37831

Prices available from (615) 576-8401, FTS 626-8401

Available to the public from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Rd
Springfield, VA 22161

NTIS price codes
Printed copy: A03
Microfiche copy: A01

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

SAND98-1887
Unlimited Release
Printed September 1998

**THEORETICAL ANALYSIS OF THE SENSITIVITY AND SPEED
IMPROVEMENT OF ISIS OVER A COMPARABLE TRADITIONAL
HYPERSPPECTRAL IMAGER**

Stephen M. Gentry and Brian R. Stallard
Optics and Emerging Technologies
Sandia National Laboratories
P.O. Box 5800
Albuquerque, New Mexico 87185-0980

Abstract

The analysis presented herein predicts that, under signal-independent noise limited conditions, an Information-efficient Spectral Imaging Sensor (ISIS) style hyperspectral imaging system design can obtain significant signal-to-noise ratio (SNR) and speed increase relative to a comparable traditional hyperspectral imaging (HSI) instrument. Factors of forty are reasonable for a single vector, and factors of eight are reasonable for a five-vector measurement. These advantages can be traded with other system parameters in an overall sensor system design to allow a variety of applications to be done that otherwise would be impossible within the constraints of the traditional HSI style design.

Acknowledgements

This work was funded under Sandia Laboratory Directed Research and Development (LDRD) Funding from the Department of Energy, Contract No. DE-AC04-94AL85000. The authors recognize the following as key contributors to the development of the ISIS concept: C. Andy Boye, William Sweatt, Carter Grotbeck, of Sandia National Laboratory, and Michael Descour of the University of Arizona. We also thank CMO/SITAC for providing the HYDICE data used for example herein.

Table of Contents

1.0	Introduction.....	1
2.0	Purpose and Approach	3
3.0	SNR Gain Analysis.....	5
3.1	Signal-Independent Noise Limited Operation	5
3.1.1	Fundamental Pixel SNR for Signal-Independent Noise Limited Operation.....	5
3.1.1.1	Pixel Signal-to-Noise for Traditional HIS	6
3.1.1.2	Pixel Signal-to-Noise Ratio for ISIS	6
3.1.2	Filter-Vector Measurement SNR for Signal-Independent Noise Limited Operation ...	9
3.1.2.1	Filter Vector SNR for a HSI Instrument	10
3.1.2.2	Filter Vector SNR for an ISIS Instrument	11
3.1.2.3	Relative Improvement in the Filter Vector Measurement SNR: ISIS over HSI for Signal-Independent Noise Limited Performance.....	11
3.1.2.4	Relative Instrument Signal Loss Factor	12
3.1.2.5	Relative Net Measurement Noise Factor	12
3.1.2.6	Relative Net Responsivity Ratio.....	13
3.1.2.7	Relative Detector Noise Ratio.....	13
3.2	Signal-Dependent Noise Operation	14
3.3	Relative Speed Increase	15
4.0	Examples.....	15
4.1	HYDICE Tank Detection Example.....	15
4.2	SEBASS Tank Detection Example.....	16
4.3	Arbitrary Filter Monte Carlo Analysis.....	18
5.0	Conclusions	18

Table of Figures

Figure 1: Example ISIS Instrument.....	2
Figure 2: HYDICE Tank Spectral Matched Filter.....	3
Figure 3: Tank Spectral Matched Filter Vector Result.....	4
Figure 6: SEBASS IR Tank Detection Vector.....	16
Figure 7: Sample Monte-Carlo Results for 210 Band Vectors.....	19
Figure 8: Monte Carlo Filter Vector SNR Results	20
Figure 9: Monte Carlo Filter Vector Speed Results.....	20

Table of Tables

Table 1: HYDICE Vector Results.....	17
Table 2: SEBASS Vector Results.....	17

1.0 Introduction

Hyperspectral imaging¹ (HSI) is a remote sensing technique that is gaining significant attention for its unique ability to allow passive identification of materials in a scene. To achieve this, HSI systems acquire high spectral resolution simultaneous images in hundreds of spectral bands, generating a significant volume of data for a single image. This makes data acquisition and processing difficult and limits the ability to do real-time target detection with HSI. In addition, parsing the photons among a large number of spectral bins compromises signal-to-noise ratio (SNR). The Information-Efficient Spectral Imaging Sensor² (ISIS) is an optical system concept designed to significantly reduce data rates compared to HSI systems, increase net measurement SNR in many cases, and allow on-the-fly target discrimination with minimal processing overhead, while retaining similar high-resolution spectral specificity. ISIS also allows a number of orthogonal spectral basis vectors to be measured directly. These basis vectors can be a more optimal set than the comb function basis set used in traditional HSI systems.

Speed and SNR advantages are achieved in ISIS by utilizing optical designs that allow the spectral responsivity of the sensor system to be arbitrarily defined with hyperspectral resolution. An example of an ISIS system design is shown in Figure 1. In this design, light from an objective is focused on to a slit plane similar to traditional style hyperspectral imaging systems. The light is then spectrally dispersed onto an intermediate focal plane where a spatial light modulator (SLM) can weight the spectral components before recombination in the final focal plane. In this particular example, the light is split using polarization into two channels to allow use of liquid crystal type spatial-light modulators. In this case, different positive-only spectral functions can be programmed into each of the two arms or channels, or one bipolar spectral filter function can be programmed by splitting the function into positive and negative parts, sending one to each arm, and differencing the channel outputs. This optical system is therefore general in specifying arbitrary spectral responsivity. It can be used in a panchromatic mode by turning all the SLM elements on. It can be a multispectral system by programming band-pass filters into each of the arms, or it can be programmed with arbitrary bipolar spectral matched filter functions with hyperspectral resolution. Each mode has applications currently under investigation in the military, civil, and commercial communities.

There are three main advantages to the ISIS approach. First, the photons are not split into a large number of spectral channels prior to detection, providing a SNR advantage of ISIS relative to a traditional HSI system. Second, since the final focal plane is essentially an image of the first slit image plane, the output image is only spatial in nature, as opposed to a traditional HSI system that is spatially resolving in one direction and spectrally resolving in the other. This allows ISIS

concepts to utilize time-delay-and-integrate^{3,4,5} (TDI) focal plane technology, ultimately allowing ISIS designs to be significantly more sensitive than traditional HSI style systems. Third, ISIS data rates and data volumes are significantly lower than traditional HSI systems. In measuring a single spectral matched filter function for a scene, traditional HSI systems would typically capture 200 spectral samples for a given scene point before weighting them in a mathematic process. ISIS on the other hand only collects two samples per pixel to represent a completely general spectral filter function. These improvements in SNR and data rate can be used to trade other system properties when applications have limited photon budgets and speed constraints.

Figure 1: Example ISIS Instrument

This improvement does come at a cost relative to traditional HSI systems, which capture all the spectral information for a single pixel simultaneously. This HSI flexibility allows complex analysis and atmosphere removal to be done using features contained in the data set. Generally, the ISIS sensor approach requires that the user make some assumptions about the background, atmosphere, and target spectral properties prior to making a measurement. While this is a significant limitation, it is not unlike multispectral problems, which preceded HSI technology. The name 'Information-efficient Spectral Imaging Sensor' (ISIS) implies that the spectral measurement can be optimized to contain the spectral information required by the user while taking the minimum number of samples. By definition, the user must decide in advance what information is important,

and the most efficient set of spectral measurements to achieve that goal. In some cases HSI technology is the only way to achieve that goal. However, in many cases, ISIS technology can collect similar information with higher SNR and lower data rates. It is proposed that this design freedom will allow applications of ISIS hyperspectral technology where traditional HSI is limited by SNR and data rate.

2.0 Purpose and Approach

The purpose of this analysis is to derive an indication of the relative SNR and speed improvements of ISIS with respect to a similar capability HSI system. With this information in hand, realistic assessments of the use of ISIS technology can be made compared to traditional HSI style designs.

The fundamental basis of the ISIS concept is that by taking a measurement of a scene pixel, using one or a number of basis vectors, one can determine some desired information about the pixel at better SNR and lower data rate than a directly comparable HSI instrument. An example of such a vector developed to detect tanks in HYDICE imagery is shown in Figure 2. This vector is derived by spectral linear discriminant analysis⁶. The result of the application of this vector in HYDICE imagery is shown in Figure 3 after an appropriate threshold has been applied. Note that the tank is detected out of the background clutter.

Figure 2: HYDICE Tank Spectral Matched Filter

Figure 3: Tank Spectral Matched Filter Vector Result

The real test of the ISIS advantage hypothesis is to compare the final SNR of the single filter vector measurement using ISIS to the SNR of the *computation* of the same vector using a traditional HSI with all common factors held constant, e.g. integration time, aperture, detector responsivity, etc.) The resulting SNR increase then gives an accurate indication of the speed increase achievable for a particular target scenario where multiple vectors are required for adequate detection of a given target.

This analysis requires several steps:

- 1) Compute the fundamental detector pixel SNR obtained in each of the ISIS and HSI approaches.
- 2) Sum the pixel measurements in the weighting appropriate to each instrument and compute a new SNR for this summed result measurement for each instrument.
- 3) Ratio the resulting SNR's for the two approaches and remove common terms. This will result in a functional form for the advantage of ISIS relative to HSI with only the critical factors remaining.
- 4) The same analysis is repeated for two noise cases: signal-independent noise limit (e.g. electronic and thermal noise) and signal-dependent noise (e.g. shot or photon noise). These two conditions indicate the limits of operation of an instrument. If the signal-independent noise sources are reduced significantly, the ultimate performance limit is the signal-dependent noise limit. Therefore ISIS relative performance is bounded by these two limits.

Finally, the theoretically derived SNR improvements in these cases will be applied first to two examples of actual filter vectors, and then to a large number of Monte-Carlo derived filter vectors which are varied as a function of filter complexity and total filter bandwidth. These results will illustrate the projected gains achieved by using ISIS over HSI in a wide variety of conditions.

3.0 SNR Gain Analysis

3.1 Signal-Independent Noise Limited Operation

Signal-independent noise sources comprise a large class of limiting noise sources such as electronic noise, detector noise, thermal noise, dark current induced shot noise, quantization noise, and others. Fundamentally, these types of noise limit many detector designs. The common factor of all these noise sources is that their magnitude is independent of the level of the signal on the pixel. This assumption allows two independent equations to be derived for the signal and noise present in a given pixel measurement, and allows general conclusions about the specific factors that contribute to the final SNR improvement of ISIS relative to traditional HSI.

3.1.1 Fundamental Pixel SNR for Signal-Independent Noise Limited Operation

For either ISIS or HSI, the fundamental photonics of integration of light onto a pixel element have a similar representation, which is shown in Equation 1. Detector current is commonly chosen as a parameter for representing both signal and noise. Current or charge domain representations will achieve the same final SNR result.

$$i_d = \frac{A_l A_d}{f^2} \int_0^\infty \mathcal{R}(\lambda) \tau(\lambda) L_t(\lambda) d\lambda \quad (1)$$

i_d	=	instantaneous detector current (A)
A_l	=	lens entrance pupil area (m ²)
A_d	=	detector pixel area (m ²)
f	=	optical system effective focal length (m)
$\mathcal{R}(\lambda)$	=	detector spectral responsivity (A/W)
$\tau(\lambda)$	=	total transmission from all instrument sources
$L_t(\lambda)$	=	spectral radiance at the pupil of the optical system (W/cm ² /sr/μm)

This representation is common to many optical systems assuming that the object region is larger than the instantaneous pixel field of view.

3.1.1.1 Pixel Signal-to-Noise for Traditional HSI

For a traditional HSI system where the spectral bins are assumed to be small compared to the features in the target spectrum and the responsivity function, the signal level in a particular spectral sample pixel can be converted to a simple product of terms.

$$i_{dj} = \frac{A_l A_d}{f^2} \mathfrak{R}(\lambda_j) \tau_{inst} L_t(\lambda_j) \Delta\lambda \quad (2)$$

j	=	spectral pixel number
i_{dj}	=	current generated in pixel j
λ_j	=	mean wavelength associated with pixel j
$\Delta\lambda$	=	spectral bandwidth associated with pixel j (assumes uniform spectral sample intervals)
τ_{inst}	=	instrument spectral transmission (assumed to be constant over λ)

If one makes the assumption that the system of interest is electronic or detector noise limited and that the noise of the measurement is independent of the signal level, then the measurement noise can be represented as a single RMS noise current value, i_{nHSI} , and the HSI SNR can be written.

$$SNR_{HSIj} = \frac{i_{dj}}{i_{nHSI}} = \frac{A_l A_d}{i_{nHSI} f^2} \mathfrak{R}(\lambda_j) \tau_{inst} L_t(\lambda_j) \Delta\lambda \quad (3)$$

This is the SNR that a hyperspectral instrument can achieve for any given spectral sample j , and is the SNR representation that suffers from the wavelength division of the spectral sampling, $\Delta\lambda$.

3.1.1.2 Pixel Signal-to-Noise Ratio for ISIS

Equation 1 can be directly applied to the signal generated in an ISIS pixel for the corresponding scene point in each of the positive and negative channels.

$$i_{d\ pos} = \frac{A_i A_d}{2f^2} \int_0^\infty \Re(\lambda) \tau_{pos}(\lambda) L_t(\lambda) d\lambda \quad (4)$$

$$i_{d\ neg} = \frac{A_i A_d}{2f^2} \int_0^\infty \Re(\lambda) \tau_{neg}(\lambda) L_t(\lambda) d\lambda$$

$\tau_{pos}(\lambda)$ = total transmission from all sources in the positive channel

$\tau_{neg}(\lambda)$ = total transmission from all sources in the negative channel

The factor of two in the denominator arises from the fact that the signal photons must be split between the two channels. The positive and negative channels will have different SNRs based on the responsivity program set for each channel by the spatial light modulators.

If we assume a signal-independent noise source for the ISIS measurements, $i_{n\ ISIS}$, the SNR in each channel can be written.

$$SNR_{ISIS\ pos} = \frac{i_{d\ pos}}{i_{n\ ISIS}} = \frac{A_i A_d}{2i_{n\ ISIS} f^2} \int_0^\infty \Re(\lambda) \tau_{pos}(\lambda) L_t(\lambda) d\lambda \quad (5)$$

$$SNR_{ISIS\ neg} = \frac{i_{d\ pos}}{i_{n\ ISIS}} = \frac{A_i A_d}{2i_{n\ ISIS} f^2} \int_0^\infty \Re(\lambda) \tau_{neg}(\lambda) L_t(\lambda) d\lambda$$

The definition of $\tau_{neg}(\lambda)$ and $\tau_{pos}(\lambda)$ in the ISIS system require further clarification because the SLM's can only remove light to achieve the desired spectral responsivity defined by the filter-vector definition. The ISIS approach assumes that the system responsivity can be adjusted to achieve a net spectral responsivity that is proportional to the filter vector, $\tau_{FV}(\lambda)$, which can be split into positive and negative parts, $\tau_{FV\ pos}(\lambda)$ and $\tau_{FV\ neg}(\lambda)$ respectively.

$$\Re(\lambda) \tau_{pos}(\lambda) - \beta \Re(\lambda) \tau_{neg}(\lambda) \propto \tau_{FV}(\lambda) = [\tau_{FV\ pos}(\lambda) - \tau_{FV\ neg}(\lambda)] \quad (6)$$

$$\beta \Re(\lambda) \tau_{pos}(\lambda) - \Re(\lambda) \tau_{neg}(\lambda) \propto \tau_{FV}(\lambda) = [\tau_{FV\ pos}(\lambda) - \tau_{FV\ neg}(\lambda)] \quad (6.1)$$

In this case, β or β' , are constants that match the actual responsivity of the positive and negative channels to the correct relative proportion. One of the two formulations is picked that achieves the highest net measurement responsivity. (For simplicity, equation 6 will be used for the remainder of this derivation with the understanding that if equation 6.1 is desired, the terms that indicate the positive and negative channels will be switched in all the following results.)

The factor β is determined after programming the transmission functions for the positive and negative channels to maximize their throughput. To do this, the channel transmission must be defined.

Equation 7 shows that the spectral transmission factor for each channel is defined in this derivation as the product of the bulk transmission losses in the channel, τ_{inst} , and the spectrally varying function imparted by the spatial light modulators, $\tau'(\lambda)$. The SLM transmissions, $\tau'_{pos}(\lambda)$ and $\tau'_{neg}(\lambda)$ are assumed to peak at unity transmission. The τ_{inst}^2 terms in these relations assume that ISIS optics require at least twice the bulk optical losses that a traditional HSI system can achieve in addition to the factor of two loss in the beam-splitter accounted for in equation 5.

$$\tau_{pos}(\lambda) = \tau_{inst}^2 \tau'_{pos}(\lambda) \quad (7)$$

$$\tau_{neg}(\lambda) = \tau_{inst}^2 \tau'_{neg}(\lambda)$$

The required spectral transmission functions for the SLM's are,

$$\tau'_{pos}(\lambda) = \frac{\tau_{FV pos}(\lambda) \mathfrak{R}_{pos max}}{\mathfrak{R}(\lambda)} \quad (8)$$

$$\tau'_{neg}(\lambda) = \frac{\tau_{FV neg}(\lambda) \mathfrak{R}_{neg max}}{\mathfrak{R}(\lambda)}$$

where the maximum measurement responsivity obtained in each channel after applying the filter vector are defined as below.

$$\mathfrak{R}_{pos\ max} = \left(\max \left(\frac{\tau_{FV\ pos}(\lambda)}{\mathfrak{R}(\lambda)} \right) \right)^{-1} \quad (9)$$

$$\mathfrak{R}_{neg\ max} = \left(\max \left(\frac{\tau_{FV\ neg}(\lambda)}{\mathfrak{R}(\lambda)} \right) \right)^{-1}$$

These relations indicate that the actual transmission function for the SLM, and therefore the added losses in the ISIS design over the HSI design will be driven by the relative spectral profiles of the natural system responsivity, $\mathfrak{R}(\lambda)$ and the desired spectral responsivity, $\tau_{FV}(\lambda)$.

Substituting for $\tau_{pos}(\lambda)$ and $\tau_{neg}(\lambda)$ into equation 5, the final ISIS elemental pixel measurement SNR is obtained for the positive and negative ISIS channels.

$$SNR_{ISIS\ pos} = \frac{i_{d\ pos}}{i_{n\ ISIS}} = \frac{A_t A_d \tau_{inst}^2 \mathfrak{R}_{pos\ max}}{2i_{n\ ISIS} f^2} \int_0^\infty \tau_{FV\ pos}(\lambda) L_t(\lambda) d\lambda \quad (10)$$

$$SNR_{ISIS\ neg} = \frac{i_{d\ neg}}{i_{n\ ISIS}} = \frac{A_t A_d \tau_{inst}^2 \mathfrak{R}_{neg\ max}}{2i_{n\ ISIS} f^2} \int_0^\infty \tau_{FV\ neg}(\lambda) L_t(\lambda) d\lambda$$

As expected, the relative spectral responsivity is entirely driven by the desired filter vector form.

The channel proportionality factor, β , is now defined to satisfy equation 6.

$$\beta = \frac{\mathfrak{R}_{pos\ max}}{\mathfrak{R}_{neg\ max}}. \quad (11)$$

3.1.2 Filter-Vector Measurement SNR for Signal-Independent Noise Limited Operation

Equations 3 and 10 now give us the SNR of the basic pixel measurements made by the HSI and ISIS systems respectively. These elemental results do not however give the final SNR comparison required to test ISIS with respect to traditional HSI. This is because ISIS is only able to measure the net filter vector dot product with the scene of interest, while HSI actually gives the samples that

are then weighted in the computer to achieve the same measurement. The final SNR in the HSI instrument is based on this weighted sum, and the final ISIS measurement SNR is based on the weighted sum of the positive and negative channels. To determine the actual performance increase of ISIS relative to HSI, the final measurement SNR of the two system approaches must be compared.

3.1.2.1 Filter Vector SNR for a HSI Instrument

In this analysis, the filter-vector definition is assumed to be in the spectral radiance domain and not the responsivity domain. Therefore the actual HSI filter vector that operates on the HSI data, $\tau_{FV\ HSI}(\lambda)$, needs to be scaled to remove the natural sensor responsivity function.

$$\tau_{FV\ HSI}(\lambda) = \frac{\tau_{FV}(\lambda)}{\left(\frac{\mathfrak{R}(\lambda)}{\mathfrak{R}_{\max}}\right)} \quad (12)$$

The final measurement signal summation is therefore

$$I_{HSI} = \sum_{j=1}^{N_{HSI}} \tau_{FV\ HSI}(\lambda_j) i_{dj} = \sum_{j=1}^{N_{HSI}} \frac{\tau_{FV}(\lambda_j) \mathfrak{R}_{\max}}{\mathfrak{R}(\lambda_j)} i_{dj}, \quad (13)$$

and the net noise in equation 14 is a summation of unequally weighted noise sources (the variances add). Note that the weighting is not the direct filter vector, but the responsivity weighted vector.

$$I_{n\ HSI} = i_{n\ HSI} \sqrt{\sum_{j=1}^{N_{HSI}} \left| \frac{\tau_{FV}(\lambda_j) \mathfrak{R}_{\max}}{\mathfrak{R}(\lambda_j)} \right|^2} \quad (14)$$

The HSI system must weight the filter vector to compensate for the rolloff in responsivity at the shorter wavelengths. Therefore noise variance in these channels adds as the square of the final filter vector weighting function.

The final HSI filter vector measurement SNR can now be written using equations 2, 13, and 14.

$$SNR_{HSI} = \frac{I_{HSI}}{I_{n\ HSI}} = \frac{\frac{A_l A_d \tau_{inst} \Delta \lambda \mathfrak{R}_{\max}}{f^2} \sum_{j=1}^{N_{HSI}} \tau_{FV}(\lambda) L_i(\lambda_j)}{i_{n\ HSI} \sqrt{\sum_{j=1}^{N_{HSI}} \left| \frac{\tau_{FV}(\lambda_j) \mathfrak{R}_{\max}}{\mathfrak{R}(\lambda_j)} \right|^2}} \quad (15)$$

3.1.2.2 Filter Vector SNR for an ISIS Instrument

By definition, the vector summation processes for ISIS is depicted in equations 6 and 16.

$$I_{ISIS} = i_{d\ pos} - \beta i_{d\ neg} \quad (16)$$

The noise processes are assumed to be based on summation of unequally weighted Gaussian noise sources. Since the ISIS positive and negative channels are weighted in the final signal by the factor β , the noise resulting in the summation process is

$$I_{n\ ISIS} = i_{n\ ISIS} \sqrt{1 + \beta^2}. \quad (17)$$

Therefore, solving for the final measurement SNR for ISIS from equations 10, 11, 16, and 17 and simplifying yields,

$$SNR_{ISIS} = \frac{A_l A_d \tau_{inst}^2 \mathfrak{R}_{pos\ max}}{2 i_{n\ ISIS} \sqrt{1 + \beta^2} f^2} \int_0^\infty \tau_{FV}(\lambda) L_t(\lambda) \partial \lambda. \quad (18)$$

3.1.2.3 Relative Improvement in the Filter Vector Measurement SNR: ISIS over HSI for Signal-Independent Noise Limited Performance

The relative improvement in SNR between ISIS and HSI, K_{si} , can now be found by ratioing equations 18 and 15. The 'si' in the subscript indicates signal-independent noise.

$$K_{si} = \frac{\tau_{inst}}{2} \sqrt{\frac{\sum_{j=1}^{N_{HSI}} \left(\frac{\tau_{FV}(\lambda_j) \mathfrak{R}_{max}}{\mathfrak{R}(\lambda_j)} \right)^2}{1 + \beta^2}} \frac{\mathfrak{R}_{pos\ max}}{\mathfrak{R}_{max}} \frac{\int_0^\infty \tau_{FV}(\lambda) L_t(\lambda) \partial \lambda}{\Delta \lambda \sum_{j=1}^{N_{HSI}} \tau_{FV}(\lambda) L_t(\lambda_j)} \frac{i_{n\ HSI}}{i_{n\ ISIS}} \quad (19)$$

If we assume that the integral in the numerator can be represented by discrete sample summations over the known net pass-band of the instrument, and that both ISIS and the HSI have the same total spectral bandwidth, K_{si} can now be simplified. The summation over the radiance domain disappears leaving only terms that are related to the instrument responsivity and the filter vector.

$$K_{si} = \frac{\tau_{inst}}{2} \sqrt{\frac{\sum_{j=1}^{N_{HSI}} \left(\frac{\tau_{FV}(\lambda_j) \mathfrak{R}_{max}}{\mathfrak{R}(\lambda_j)} \right)^2}{1 + \beta^2}} \frac{\mathfrak{R}_{pos\ max}}{\mathfrak{R}_{max}} \frac{i_{n\ HSI}}{i_{n\ ISIS}} \quad (20)$$

(NOTE: This equation assumes the scaling of the negative channel to the positive channel as shown in equation 6. One can scale the positive channel to the negative channel as in Equation 6.1, resulting in a substitution of 'neg' for 'pos' in equations 11, 16, and 20.)

This relation can be split into four factors for clarity, one for basic increased instrument losses (A), one for the net relative improvement in noise due to the instrument design and the filter vector (B), one for loss in measurement responsivity (C), and one for the basic relative noise sources in the detector systems (D).

$$K_{si} = A B C D \quad (21)$$

Where A, B, C, and D are defined as follows.

3.1.2.4 Relative Instrument Signal Loss Factor

This factor depicts the additional optical losses in ISIS (not associated with the SLM filters) with respect to the hyperspectral design. Therefore in ISIS if you want good relative performance, remove the beam splitter (factor of 2) and maximize the channel transmission (τ_{inst}).

$$A = \frac{\tau_{inst}}{2} \quad (22)$$

3.1.2.5 Relative Net Measurement Noise Factor

This factor quantifies the relative net measurement noise resulting from summing the fundamental pixel measurements into a single number. Therefore the ISIS performance increase depends on how heavily the HSI has to weight the 'noisy' samples to compute a flattened filter vector, and how balanced the ISIS positive and negative channels are weighted.

$$B = \sqrt{\frac{\sum_{j=1}^{N_{HSI}} \left| \frac{\tau_{FV}(\lambda_j) \mathfrak{R}_{max}}{\mathfrak{R}(\lambda_j)} \right|^2}{1 + \beta^2}} \quad (23)$$

3.1.2.6 *Relative Net Responsivity Ratio*

This factor quantifies how much loss or gain in responsivity occurs in the ISIS filtering process relative to a HSI. To the extent that the filter vector requires high transmission where the normalized sensor responsivity is low, ISIS' relative performance will be reduced.

$$C = \frac{\mathfrak{R}_{pos\ max}}{\mathfrak{R}_{\max}} = \frac{\left(\max \left(\frac{\tau_{FV\ pos}(\lambda)}{\mathfrak{R}(\lambda)} \right) \right)^{-1}}{\mathfrak{R}_{\max}}. \quad (24)$$

The way to visualize this relative signal loss factor is to realize that the HSI system receives the photons based on some responsivity profile, but the final measurement amplifies lower responsivity channels by equation 15, so the final responsivity of the measurement always peaks at $\mathfrak{R}_{\max}$. ISIS is required to normalize each channel transmission to a maximum of one as seen in equation 8.

Depending on the actual responsivity and filter vector profiles, C can be greater or less than one.

3.1.2.7 *Relative Detector Noise Ratio*

HSI and ISIS differ in the fundamental nature of the focal planes they utilize. Traditional HSI systems require two-dimensional focal plane arrays (e.g. 512 spatial x 200 spectral) with fast readouts to get the data off the array quickly. ISIS on the other hand requires at a minimum only two linear arrays (e.g. 512 spatial x 2), or two TDI arrays for significant increases in SNR which can operate with readout rates as much as 100 times slower in many applications.

Santa Barbara Research Center (SBRC)⁷, manufacturer of high performance focal plane arrays, indicates that the decrease in complexity in focal plane design, from a two-dimensional array to a one-dimensional array, can significantly decrease ISIS limiting noise relative to HSI, making D below a significant factor.

$$D = \frac{i_{n\ HSI}}{i_{n\ ISIS}} \quad (25)$$

Specifically, SBRC indicates that noise could easily be reduced by a factor of two, by switching from a two-dimensional array to a one-dimensional array, while keeping the frame time constant. SBRC, who manufactures TDI arrays, indicates that a $\sqrt{N_{TDI}}$ in effective noise improvement can

be easily obtained for these style arrays. Current ISIS designs can employ up to 25 TDI stages without significant degradation in spectral resolution. *Given these factors, D can therefore range from 2 to 5.*

These four factors will be combined for real examples in a following section to illustrate the maximum gain in SNR achievable with ISIS.

3.2 Signal-Dependent Noise Operation

Section 3.1 assumed that the limiting noise in either an ISIS or HSI instrument was limited by something other than the signal itself. In the case that the signal contains or causes the limiting noise in the measurement, a different relationship for the gain factor K, needs to be developed. In this case, both the numerator and the denominator of equations 3 and 10 will now have a term related to the square root of the signal level. Noises of this type are signal induced shot noise or photon noise. Following the same formalism that is described in section 3.1, a SNR gain factor K_{sd} , is shown below in equation 26. The subscript 'sd' indicates that the limiting noise is signal-dependent.

$$K_{sd} = \sqrt{\frac{\tau_{inst}}{2}} \sqrt{\frac{\mathfrak{R}_{pos\ max}}{\mathfrak{R}_{max}}} \sqrt{\frac{\sum_{j=1}^{N_{HSI}} \tau_{FV}^2(\lambda_j) \frac{\mathfrak{R}_{max}}{\mathfrak{R}(\lambda_j)} L_{\lambda}(\lambda_j)}{\sum_{j=1}^{N_{HSI}} (\tau_{FV\ pos}(\lambda_j) L_{\lambda}(\lambda_j) + \tau_{FV\ neg}(\lambda_j) L_{\lambda}(\lambda_j))}} \quad (26)$$

Note that since the noise is now dependent on the signal level, the input radiance does not fall out of the equations as it did in the previous case. Again, the SNR gain is split into several terms. The first term is related to the added transmission losses that ISIS incurs relative to traditional HSI. The second term results from the different net responsivity between ISIS and HSI. The last term indicates the net noise gain or loss that ISIS has relative to a traditional HSI.

Note: Equation 26 assumes that this limiting noise has been achieved in both ISIS and HSI, and does not indicate how hard it is to achieve this level in actual systems. In reality, this noise level is very difficult to achieve in all but the most ideal conditions. Therefore, it is included here only as an indicator of the limit of performance. It will be shown in the examples below that in this limiting noise case, ISIS can be at a disadvantage relative to traditional HSI, due primarily to the added optical losses in the system.

3.3 Relative Speed Increase

The final metric desired is the relative speed increase that a given SNR improvement allows. Since SNR typically varies as the square-root of the integration time, the speed increase, S , allowed for a constant signal to noise ratio, is proportional to the square of the SNR improvement.

$$S_{si} = K_{si}^2 \quad (27)$$

$$S_{sd} = K_{sd}^2$$

The speed increase is the final fundamental metric for comparison because it indicates how many filter-vectors ISIS can measure in the same time as a comparable traditional HSI could at the same SNR. The authors anticipate that most measurements of interest can be captured in five separate vector measurements when the vectors are properly designed. Therefore, if the speed increase is greater than a factor of five, the ISIS style instrument has an advantage over the traditional HSI system.

4.0 Examples

Several examples of the application of SNR gain equations are included in the following section. First are examples of two actual optimal vectors used to detect tanks in hyperspectral data sets, one from HYDICE, and one from SEBASS. These give an estimate of ISIS performance for a single vector measurement in real application. Since the actual SNR of ISIS is highly application dependent, and one cannot capture vectors for all applications, the third example shows the results picking a large number of random vectors of differing lengths and complexities, and computing the net SNR and speed increase. This analysis allows a general conclusion to be made concerning the net ISIS advantage as a function of these two parameters.

4.1 HYDICE Tank Detection Example

To determine an actual estimate of the ISIS SNR and speed gain, the HYDICE tank detection vector shown in Figure 2 will be used as an example. The following assumptions are made,

- use of a passivated InSb detector response² similar to that found in the HYDICE system,
- HYDICE spectral resolution and sampling,
- additional channel transmission factor is 0.85 relative to the traditional HSI system,
- scaling the positive ISIS channel achieves better performance.,

- signal spectrum is total solar spectral irradiance at sea level⁸.

The parameters, A, B, C, and D for the net SNR gain are found as shown in Table 1. Here it is seen that, for a single vector, *ISIS can measure this vector thirty seven times faster than the traditional HSI system*. If ISIS is required to make five spectral measurements to achieve the same application performance as a traditional HSI, *then ISIS can make the measurement more than seven times faster than the HSI system*.

4.2 SEBASS Tank Detection Example

A second example is a comparison to the SEBASS sensor developed by the Aerospace Corporation. The following assumptions are made:

- detection vector is defined as found by Schwartz et al.⁹, shown in Figure 6,
- use of a Si:As detector response¹⁰ similar to that found in the SEBASS system,
- SEBASS spectral resolution and sampling,
- additional channel transmission factor is 0.85,
- scaling the positive ISIS channel achieves better performance,
- the signal spectrum is assumed to be constant over wavelength.

The parameters, A, B, C, and D for the net SNR gain are found as shown in Table 2. In this case, *ISIS can measure this single vector eighteen times faster than a traditional HSI system*. If ISIS is required to make up to five similar spectral vector measurements to achieve the same application performance as a traditional HSI, then ISIS can still make the measurement more than 3.6 times faster than the *HSI system*.

Figure 6: SEBASS IR Tank Detection Vector

Table 1: HYDICE Vector Results

Parameter	Value	Explanation
N bands	53	The filter requires 53 HYDICE bands
β	1.08	The positive channel needs to be scaled up by 8%
A	$0.85/2 = 0.425$	Additional ISIS bulk system losses
B	3.93	ISIS has a factor of four noise improvement relative to traditional HSI.
C	0.73	ISIS net responsivity is 27% lower than the equivalent traditional HSI measurement.
D	5	Maximum TDI gain.
K_{si}	6.09	Signal-independent noise limited SNR gain.
S_{si}	37	Under signal-independent noise conditions, ISIS could have measured this vector in one thirty-seventh the time as the traditional HSI, or have measured 37 of similar vectors in the same time.
K_{sd}	0.53	Under signal-dependent noise conditions, ISIS would measure this vector at a half the SNR than traditional HSI.
S_{sd}	0.28	Under signal-dependent noise conditions, ISIS would have taken significantly longer to measure this vector at the same SNR as the traditional HSI system.

Table 2: SEBASS Vector Results

Parameter	Value	Explanation
N bands	46	The filter requires 46 SEBASS bands
β	1.11	The positive channel needs to be scaled up by 11%
A	$0.85/2 = 0.425$	Additional ISIS bulk system losses
B	2.24	ISIS has a factor of two noise improvement relative to traditional HSI.
C	0.89	ISIS net responsivity is 11% lower than the equivalent traditional HSI measurement.
D	5	Maximum TDI gain.
K_{si}	4.25	Signal-independent noise limited SNR gain.
S_{si}	18	Under signal-independent noise conditions, ISIS could have measured this vector in one eighteenth the time as the traditional HSI, or have measured 18 of similar vectors in the same time, at the same SNR as SEBASS.
K_{sd}	0.47	Under signal-dependent noise conditions, ISIS would again measure this vector at a half the SNR than traditional HSI.
S_{sd}	0.22	Under signal-dependent noise conditions, ISIS would again have taken significantly longer to measure this vector at the same SNR as the traditional HSI system.

4.3 Arbitrary Filter Monte Carlo Analysis

While the two previous examples are indicative of their specific applications, they are not indicative of all applications. In an effort to understand the general SNR and speed advantages of ISIS relative to traditional HSI systems, a Monte-Carlo filter vector study was conducted. In this study, a large number of arbitrary zero-mean vectors of varying complexity and length were generated and analyzed in the same manner as the HYDICE vector example above. An example of the actual scatter plot for the 210-band filter case is shown in Figure 7. In an effort to summarize the data, mean $\pm$ sigma curves were generated. These results were then summarized in Figures 8 and 9, for net SNR and speed gains of ISIS respectively, under a variety of filter length cases and in the presence of signal-dependent and signal-independent noise sources.

For signal-independent noise conditions, the final speed increase for moderate length filter vector designs (53 HYDICE bands) can be up to a factor of forty. As the vector length decreases and the filter complexity increases, the ISIS SNR and speed advantages are reduced.

For signal-dependent noise operation, ISIS generally will experience net reduction in SNR and speed performance over a comparable HSI system.

These results indicate that ISIS best applications will generally occur where:

- 1) signal-independent noise limits the system,
- 2) wide total wavebands are needed and,
- 3) spectral filter vector complexity is moderate.

These conditions are widely applicable to materials identification tasks where band features are wide and dispersed over wide wavebands. Ultra-narrow band applications, while they can be measured with ISIS, will not see significant advantage over traditional HSI.

5.0 Conclusions

The analysis presented herein predicts that ISIS-style designs can obtain significant SNR and speed increases relative to a comparable hyperspectral instrument under signal-independent noise limited operation. A speed increase of a factor of forty is reasonable for a single vector, and a factor of eight is reasonable for a five-vector measurement.

The ISIS speed and SNR advantages can be traded with other system parameters in an overall sensor system design to allow a variety of applications to be done that otherwise would be

impossible within the constraints of the traditional HSI style design. While these results are promising, the improved performance comes at a cost. The user must decide in advance what specific spectral measurements are required to achieve a particular task. This burden may be unacceptable to some that require all the spectral detail available in the traditional HSI design. The authors propose that after significant terrabytes of HSI data are collected and analyzed by the community, more information-efficient techniques for solving spectral sensing problems will be sought. Only at this time will utility of the ISIS approach be realized.

Figure 7: Sample Monte-Carlo Results for 210 Band Vectors

Figure 8: Monte Carlo Filter Vector SNR Results

Figure 9: Monte Carlo Filter Vector Speed Results

¹ Anderson, R., W. Malila, R. Maxwell, L. Reed, *Military Utility of Multispectral and Hyperspectral Sensors*, (Infrared Information Analysis Center, Environmental Research Institute of Michigan, Ann Arbor, Michigan, Report No. 246890-3-F, November 1994)

² Stallard, Gentry, Sweatt, Motomatsu, Boye and Grotbeck, *Principles and Satellite Applications of the Information-Efficient Spectral Imaging Sensor* (Sandia National Laboratories, SAND97-1493, June 1997)

³ Applications Note: "How TDI Works", "CCD Image Sensors and Cameras", *1992 DALSA Data Book*, (DALSA Inc., Waterloo, Ontario, Canada, p 78, 1992)

⁴ Hoffman, A., B. Krashefski, J. Stineman, J. Toman, J. Holt, *High Performance InSb Scanning Sensor Chip with 26 in TDI*, (Santa Barbara Research Center, Golita, CA, TAP92-0567, August 1992)

⁵ Wolfe and Zissis, *The Infrared Handbook* (Environmental Research Institute of Michigan, Ann Arbor, Michigan, pp.12-26, 12-28, 12-38, 1989)

⁶ Fukunaga, K., *Introduction to Statistical Pattern Recognition*, 2nd Ed. (Academic Press Inc., New York, p. 135, 1990)

⁷ Personal Communication, D. Richardson, (Santa Barbara Research Center, October 1997)

⁸ Wolfe and Zissis, *The Infrared Handbook* (Environmental Research Institute of Michigan, Ann Arbor, Michigan, Table 3-18, pp. 3-37 to 3-38, 1989)

⁹ Schwartz, Twede, and Cederquist, *Target Detection Analysis from SEBASS Imagery* (Environmental Research Institute of Michigan, Joint Multispectral Program Review, p. 128, March 1997)

¹⁰ Wolfe and Zissis, *The Infrared Handbook* (Environmental Research Institute of Michigan, Ann Arbor, Michigan, p. 11-85, 1989)

DISTRIBUTION:

<i>10</i>	<i>MS</i>	<i>0980</i>	<i>Stephen M. Gentry, 5725</i>
<i>1</i>	<i>MS</i>	<i>0980</i>	<i>Brian Stallard, 5725</i>
<i>1</i>	<i>MS</i>	<i>0603</i>	<i>William C. Sweatt, 1712</i>
<i>1</i>	<i>MS</i>	<i>0972</i>	<i>C. Andy Boye, 57252</i>
<i>1</i>	<i>MS</i>	<i>0971</i>	<i>Gary H. Mauth, 5703</i>
<i>1</i>	<i>MS</i>	<i>0980</i>	<i>Philip J. Green, 5725</i>
<i>1</i>	<i>MS</i>	<i>9018</i>	<i>Central Technical Files, 8940-2</i>
<i>2</i>	<i>MS</i>	<i>0899</i>	<i>Technical Library, 4916,</i>
<i>2</i>	<i>MS</i>	<i>0619</i>	<i>Review and Approval Desk, 12690, for DOE/OSTI</i>
<i>1</i>	<i>MS</i>	<i>0161</i>	<i>Patent and Licensing Office, 11500</i>
<i>1</i>	<i>MS</i>	<i>0149</i>	<i>LDRD Office, 4000</i>