PROPOSED CRITERIA FOR THE STABILITY IN EARTHQUAKES OF NUCLEAR-MATERIAL SHIPPING CASKS

by

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ABSTRACT

A criterion based on the ratio of potential energy required to tip over the cask to the kinetic energy that a cask would obtain in an earthquake is proposed. The kinetic energy is estimated from the equation of motion for an inverted pendulum, the design basis ground velocities, and design basis spectral velocities. The previously known scaling effect which shows that larger structures are more stable than smaller ones of the same geometry is demonstrated.

INTRODUCTION

Using an energy method to evaluate overturning of rigid bodies (casks) in an earthquake leads to some useful results. The method predicts that for similar objects the larger ones are less likely to overturn than the smaller ones. It also predicts, what one would expect, that tall slender objects are less stable than short wide ones. Housner[6] analyzed the behavior of this type of system, after noting its apparent stability in earthquakes, calling it an "inverted pendulum." He determined that for similar objects the reason that the larger ones are more stable than the smaller ones is that the potential energy scales with the size of the object and the ground motion does not.

The method presented here assumes the object is rigid, rocks on its base in a plane and may be submerged in water. Site specific charts quantify the two results, stated above, by predicting the factor of safety against overturning for objects of a specific geometry. This paper presents an example chart, for solid cylinders, with no buoyancy. The method could be extended to other rigid objects, such as equipment cabinets, to develop criteria for anchorage.

Overturning occurs for a submerged object when the center of gravity of an object moves past the supporting edge of the object by an amount sufficient for the weight (W) to develop an overturning moment greater than restoring moment provided by buoyant force (B) (see Figure 1).

Static methods used to predict overturning determine the forces required for uplift but do not predict overturning. The forces acting on the object must act for some period of time, or over a series of pulses, to build up the energy required to cause the object to overturn.

STATIC ANALYSIS

Since a static analysis is easy, it should be done first to determine lift-off and sliding. This can be done using
the maximum horizontal base acceleration and 40% of the vertical base acceleration or visa versa, per ASCE 4-86[21, and using the most unfavorable condition. The 40% factor accounts for the fact that the maximum earthquake accelerations in each direction do not occur at the same time. The factor of safety against lift off is the ratio of the restoring moment to the overturning moment. Required factors of safety against overturning are given in the standard review plan (SRP)[3]. For DOE facilities load combinations associated with the SRP Safe Shutdown Earthquake (SSE) are usually used. Only one horizontal component of the earthquake should be used because casks are generally axisymmetric and the earthquake criteria are established such that the maximum horizontal component is specified.

The coefficient of friction required to prevent sliding before lift-off can occur should be determined, since sliding may pose problems.

If the factor of safety against lift-off is more than that required for overturning, then the cask does not lift off or overturn and the analysis is done. If it is less, then the cask lifts off and overturning may be investigated dynamically.

**DYNAMIC ANALYSIS**

The factor of safety against overturning is defined as the ratio of the potential energy required to overturn the cask to the kinetic energy the cask can obtain during the earthquake.

\[ FS = \frac{PE}{KE} \]

The potential energy of overturning is the weight of the cask times the height to which its center of gravity would be raised when overturned minus the work done by the buoyant force (see Figure 1). The potential energy is

\[ PE = W(r \cos \theta - h \sin \theta) - B(r \cos \theta - h \cos \theta + Bs) \]

From static analysis (see Figure 1) the cask is in equilibrium when

\[ W(r \cos \theta - h \sin \theta) = B(r \cos \theta - s \sin \theta) \]

so that the angle of tilt at equilibrium is given by

\[ \theta = \tan^{-1} \left( \frac{(W - B)r}{Wh - Bs} \right) \]

The kinetic energy of the cask rocking on a moving base is given by

\[ KE = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2) + \frac{1}{2} I_\theta \dot{\theta}^2 \]

where \( m \) is the mass of cask and \( I_\theta \) is the moment of inertia of the cask about the center of gravity (see Figure 2).

![Figure 2. Cask Rocking on a Moving Base](image)

Substituting the expressions for \( x \) and \( y \),

\[ x = u + r \cos \theta - h \sin \theta \]
\[ y = v + r \sin \theta + h \cos \theta \]

and their time derivatives gives

\[ KE = \frac{1}{2} m(\dot{u}^2 + \dot{v}^2) + 2\dot{\theta}[(r \cos \theta - h \sin \theta)\dot{\theta} - (r \sin \theta + h \cos \theta)\dot{\theta}] + \frac{1}{2} I_\theta \dot{\theta}^2 \]
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No hydrodynamic mass moment on inertia values are given so $I_B$ is increased by the same percentage as the mass this results in

$$I_B' = 1.1 \times 1660 = 1830 \text{ k-sec}^2\text{-in}$$

Substituting these values into the expression for the kinetic energy, results in

$$KE = 425 \text{ k-in}.$$ 

The potential energy is calculated for the position of incipient tipover $\theta = 27.5^\circ$ giving $PE = 675 \text{ k-in}$. No hydrodynamic mass is included here, since it would not add to the potential energy. The factor of safety is then

$$FS = \frac{675}{425} = 1.6 > 1.1$$

therefore the cask is stable.

**DESIGN CHART**

To illustrate the method, a chart is developed below which is applicable to a specific location for solid cylinders with no buoyancy. Once a spreadsheet is developed to calculate the chart, new charts can be made for other locations and objects by changing the values of the ground and spectral velocities and $h/B^2$.

By defining $r = \beta h$ the potential energy is written

$$PE = Wh \left( \sqrt{1 + \beta^2} - 1 \right)$$

since $I_B = mr^2$ where $r_B$ is the radius of gyration about the base, the kinetic energy can be written

$$KE = \left( \frac{m}{2} \right) \left\{ u^2 + v^2 + 2 \left( \frac{h^2}{r_B^2} \right) \sqrt{((1+\beta^2)S_{ah}^2 + \beta^2 S_{av}^2)} (\beta u + \left( \frac{1+\beta^2}{2} \right) S_{ah}^2 + \beta^2 S_{av}^2 \right\}$$

A value for the hydrodynamic added mass is included. This is taken from Blevins Table 14-2. The total mass is then

$$m = \frac{103,000}{386} + 26 = 292 \text{ #sec}^2\text{lin}$$
Where \( I_B = I_c + m(r^2 + h^2) \) is the moment of inertia about the bottom edge of the cask.

The velocities \( \dot{u} \) and \( \dot{v} \) are the velocities of the base support. The velocity, \( \dot{\theta} \), is the angular velocity of the rocking cask. The former can be obtained from the ground velocity if the cask is sitting on the ground or from the structural analysis if it is sitting on a floor. The angular velocity, \( \dot{\theta} \), must be obtained by other means. A method of estimating it from the response spectrum of the base motion is explained below by using the equations of motion of the rocking cask on a moving base.

**EQUATIONS OF MOTIONS**

The equations of motion for the center of mass of a rocking cask (see Figure 3) on a moving base are

\[
\begin{align*}
H & = m\ddot{x} \\
V-W+B & = m\ddot{y} \\
H(\dot{y}-\dot{v}) - V(\dot{x}-\dot{u}) + B(h-s)\sin\theta & = I_c\dot{\theta}
\end{align*}
\]

By substituting expressions for \( H \) and \( V \) from the first two equations into the third and then substituting for \( x \) and \( y \), and their time derivatives into the resulting equation, the equation of motion for the rocking cask in terms of \( \dot{\theta} \) and the prescribed input motions, \( \dot{u} \) and \( \dot{v} \), is obtained:

\[
\dot{\theta} + \left( \frac{W}{I_B} \right) (r \cos \theta - h \sin \theta) - \\
\left( \frac{B}{I_B} \right) (r \cos \theta - s \sin \theta) = \\
\left( \frac{m}{I_B} \right) (r \sin \theta + h \cos \theta) \dot{u} - \\
\left( \frac{m}{I_B} \right) (r \cos \theta - h \sin \theta) \dot{v}
\]

**MAXIMUM ANGULAR VELOCITY**

The equation of motion looks like the forced vibration of a single degree of freedom system with the forcing terms on the right hand side of the equation. The coefficients of \( \dot{u} \) and \( \dot{v} \) can be interpreted as participation factors. These factors can be multiplied by the spectral velocities to obtain an estimate of the maximum angular velocities. These "participation factors" are not constant, so for conservatism the maximum values may be taken. The horizontal factor is maximum when \( r \sin \theta + h \cos \theta \) is maximum which is when \( \theta = \tan^{-1}(r/h) \) or when the tipover angle is reached, whichever is the smaller. The vertical factor is maximum when \( r \cos \theta - h \sin \theta \) is maximum or when \( \theta = 0 \). Since \( \dot{u} \) and \( \dot{v} \) are horizontal and vertical components, the corresponding values obtained for \( \dot{\theta} \) may be combined by the square root of the sum of the squares method to obtain an estimate of the maximum angular velocity.

**EXAMPLE**

A cask with the properties given below (see Figure 4) is resting on the bottom of a pool.

\[
\begin{align*}
W & = 103 \text{ k} \\
B & = 14 \text{ k} \\
I_B & = 1600 \text{ k-sec}^2-\text{in}
\end{align*}
\]
REFERENCES


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Dividing the PE by the KE gives

\[ FS = \frac{2gh}{\sqrt{(1+\beta^2)}} \left( \frac{h^2}{r_B^2} \right) \left\{ \frac{1}{2} \left( u^2 + v^2 \right) + \frac{h^2}{r_B} \left( \frac{h^2}{r_B^2} \right) \left( \frac{h^2}{r_B^2} \right) \right\} \]

Some insight about overturning can be gained by examining this expression. Notice that the mass (or weight) of the object does not appear, so that overturning is only a function of geometry. The geometric quantities are the half height, \( h \), the width to height ratio, \( \alpha h = \beta \), and the ratio, \( \frac{h^2}{r_B^2} \). The fact that the factor of safety is directly proportional to the height means that for the same geometry, \( \beta \), tall objects are more stable than short ones. Also for small width to height ratios one can see, by doing a series expansion for the square root, that the factor of safety is proportional to \( \beta^2 \). The third factor does not have a strong influence. These ideas are best illustrated by an example. Figure 5 shows the factor of safety for a solid cylinder with

\[ \frac{h^2}{r_B^2} = \frac{1}{4 + \left( \frac{5}{4} \right) \beta^2} \]

plotted as a function of \( h \) and \( \beta \). Notice that as the height goes up the safety factor goes up, and as the width to height ratio increases the height required for stability goes down.

**CONCLUSIONS AND RECOMMENDATIONS**

This analysis and the results of this example have implications for other types of equipment such as cabinets and control panels. Charts can be developed for each site so that the size and shape of equipment that needs to be anchored can be determined. Also, since \( h^2/r_B^2 \) does not differ much for various objects with rectangular cross sections, this chart can be used for an approximate evaluation of the overturning potential of other objects.

Two parameters used in this analysis, damping and hydrodynamic mass, are not well defined. Even though these parameters do not have a major influence on the factor of safety, they are important and should be investigated further. Some experimental work has been done and a computer program has been written to integrate the equations of motion. These data and the computer program could be used to investigate the validity of the proposed method and obtain appropriate damping values.

**Figure 5. Overturning Factor of Safety for a Solid Cylinder, without Bouyancy** (For Ground Velocities, in/sec, \( V_h = 25 \), \( V_v = 21 \) and Spectral Velocities: \( S_v = 25 \) and \( S_v = 18 \))

![Figure 5. Overturning Factor of Safety for a Solid Cylinder, without Bouyancy](image-url)