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Absorption of Laser Light in Overdense Plasmas by Sheath Inverse Bremsstrahlung

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The original sheath inverse bremsstrahlung model [P. J. Catto and R. M. More, 1977] is modified by including the $\mathbf{v} \times \mathbf{B}$ term in the equation of motion. It is shown that the present results are significantly different from those derived without the $\mathbf{v} \times \mathbf{B}$ term. The $\mathbf{v} \times \mathbf{B}$ term is also important in interpreting the absorption mechanism. If the $\mathbf{v} \times \mathbf{B}$ term were neglected, the absorption of the light would be incorrectly interpreted as an increase in the transverse electron temperature. This would violate the conservation of the transverse components of the canonical momentum, in the case of a normally incident laser light. It is also shown that both the sheath inverse bremsstrahlung and the anomalous skin effect are limiting cases of the same collisionless absorption mechanism. Finally, results from PIC plasma simulations are compared with the absorption coefficient calculated from the linear theory.

I Introduction

In recent years, there have been increasing interests in the interaction of short laser pulses with overdense plasmas¹⁻¹². For sufficiently short laser pulses, the hydrodynamics motion of the heated target does not play a dominant role, and the production of high-density plasmas with sharp density gradient become feasible. One topic of great interest is to study the dependence of the light absorption on the laser intensity and the plasma temperature in such plasmas. It has been observed in experiments¹⁻³ that, starting from sufficiently low intensity, the absorption rate increases as a function of laser intensity, until it reaches the "resistivity saturation"¹, a condition in which the electron mean free path reaches a minimum value. Further increase of the laser intensity and the plasma temperature will then cause an increase in the electron mean free path and a decrease in the absorption rate. When the electron mean free path is longer than the skin depth, theoretical studies^{13,14} suggest that, collisionless absorption mechanisms such as sheath inverse bremsstrahlung¹³ and the anomalous skin effect^{14,15} become important.

In this paper, we modify the original sheath inverse bremsstrahlung model¹³ by including the $\mathbf{v} \times \mathbf{B}$ term in the equation of motion. It will be shown that the present results are significantly different from those derived without the $\mathbf{v} \times \mathbf{B}$ term, except when the distribution function is isotropic [$f_0(\mathbf{v}) = f_0(|\mathbf{v}|)$]. For an isotropic distribution function, identical results will be obtained whether or not the $\mathbf{v} \times \mathbf{B}$ term in the equation of motion is included in the derivation. However, if the $\mathbf{v} \times \mathbf{B}$ term were neglected, the absorption of the light would be incorrectly interpreted as an increase in the transverse electron temperature, while the conservation of the transverse components of the canonical momentum requires that, after leaving the interaction region ($|x| \lesssim \delta$), an electron should have the same transverse velocity as before it entered the interaction region.

The present analysis also determines self-consistently the profile of the electromagnetic fields in the overdense plasma in the regime of validity for the sheath inverse bremsstrahlung model, i.e., $\omega^2 c^2 \gg \omega_p^2 v_e^2$. By deriving the absorption coefficients for both the sheath inverse bremsstrahlung and the anomalous skin effect from the same set of equations [Eqs. (20) and (21)], it is shown, in Sec. III, that both phenomena are limiting cases of the same collisionless absorption mechanism. The sheath inverse bremsstrahlung corresponds to the limit where $\omega^2 c^2 \gg \omega_p^2 v_e^2$, while the anomalous skin effect corresponds to $\omega^2 c^2 \ll \omega_p^2 v_e^2$.

Finally, numerical simulations of the light absorption in overdense plasmas have been carried out, using the PIC plasma simulation code ZOHAR¹⁶. The absorption coefficients observed in the numerical simulations agree qualitatively with the values calculated from the linear theory, as illustrated by the results shown in Sec. IV.

II Theoretical Model and Derivation of the Sheath Inverse Bremsstrahlung Absorption

The model consists of an overdense plasma filling the half-space $x > 0$, and electromagnetic fields of the following form

$$\begin{aligned} E_y(x, t) &= \text{Re}\{E_0 \exp[i(kx - \omega t)]\}, \\ B_z(x, t) &= \text{Re}\left\{\frac{ck}{\omega} E_0 \exp[i(kx - \omega t)]\right\}, \end{aligned} \quad (1)$$

where E_0 and ω are real-valued constants, while k is a complex number with a positive imaginary part. Immobile ions, with zero density for $x < 0$ and a constant density for $x > 0$, are assumed to form the neutralizing background. The plasma is assumed to be highly overdense ($\omega_p^2 \gg \omega^2$) and the fiducial thermal velocity v_e , which characterizes the electron distribution, is sufficiently small ($\omega^2 c^2 \gg \omega_p^2 v_e^2$). Except in the sheath regime near $x = 0$, the electron density is equal to n_0 for $x > 0$ and is zero for $x < 0$. When an electron hits the sheath ($x = 0$) from the right ($x > 0$), instantaneous

specular reflection is assumed, that is the y and z components of the momentum remain unchanged, while the x -component reverses with the amplitude unchanged. Since the typical time scale to reverse electron momentum in the sheath region is $1/\omega_p$, which is much shorter than both the wave period ($2\pi/\omega$) and the transit time in the skin depth ($c/v_e\omega_p$) in an overdense non-relativistic plasma, instantaneous reflection is a good assumption. Here, $\omega_p = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency. The assumption of specular reflection requires that the sheath be one-dimensional, i.e., the length scale of the transverse variation be much longer than the width of the sheath (approximately equal to the Debye length). The present analysis also assumes that the quiver velocity $v_{os} = eE_0/m_e\omega$ is much smaller than the fiducial thermal velocity v_e , so that the perturbation analysis is applicable.

For an electron located at $x'(t') = x > 0$, at the time $t' = t$, with velocity $\mathbf{v}'(t' = t) = \mathbf{v} = v_x\hat{e}_x + v_y\hat{e}_y + v_z\hat{e}_z$, the unperturbed orbits at any earlier time $t' < t$, in the absence of the electromagnetic fields ($E_0 = 0$), are

$$\begin{aligned} x^{(0)}(t') &= x + v_x(t' - t), \\ \mathbf{v}^{(0)}(t') &= \mathbf{v}, \end{aligned} \quad (2)$$

for $v_x < 0$, and

$$\begin{aligned} &\begin{cases} x^{(0)}(t') = x + v_x(t' - t), & t' > t_r, \\ x^{(0)}(t') = -x - v_x(t' - t), & t' < t_r, \end{cases} \\ &\begin{cases} v_x^{(0)}(t') = v_x, & t' > t_r, \\ v_x^{(0)}(t') = -v_x, & t' < t_r, \end{cases} \\ &v_y^{(0)}(t') = v_y, \quad v_z^{(0)}(t') = v_z, \end{aligned} \quad (3)$$

for $v_x > 0$, where $t_r = t - x/v_x$ is the time of reflection.

To calculate the electron orbits correctly to the first order in E_0 , the orbits are written in the following form

$$x'(t') = x^{(0)} + \text{Re}\delta x(t'), \quad v'_x(t') = v_x^{(0)} + \text{Re}\delta v_x(t'), \quad v'_y(t') = v_y^{(0)} + \text{Re}\delta v_y(t'), \quad (4)$$

where the perturbed orbits δx , δv_x , and δv_y satisfy the following linearized equations

$$\begin{aligned}\frac{d\delta v_y}{dt'} &= -\frac{e}{m_e} \left(1 - \frac{kv_x}{\omega}\right) E_0 \exp\{i[kx^{(0)}(t') - \omega t']\}, \\ \frac{d\delta v_x}{dt'} &= -\frac{ev_y}{m_e \omega} E_0 \exp\{i[kx^{(0)}(t') - \omega t']\}, \\ \frac{d\delta x}{dt'} &= \delta v_x(t').\end{aligned}\quad (5)$$

For an electron moving to the left with $v_x < 0$, Eqs. (2) and (5) readily give the perturbed orbits at the time $t' = t$

$$\begin{aligned}\delta v_y(t) &= \frac{eE_0}{im_e \omega} \exp[i(kx - \omega t)], \\ \delta v_x(t) &= \frac{eE_0}{im_e \omega} \frac{kv_y}{\omega - kv_x} \exp[i(kx - \omega t)].\end{aligned}\quad (6)$$

For an electron moving to the right with $v_x > 0$, the condition of specular reflection requires that $\delta v_y(t_r + \epsilon) = \delta v_y(t_r - \epsilon)$ and $\delta v_x(t_r + \epsilon) = -\delta v_x(t_r - \epsilon)$ for a small positive ϵ . Making use of Eq. (3) and the specular reflection conditions, Eq. (5) is solved to give the perturbed orbits for the right-moving particle ($v_x > 0$) at the time $t' = t$

$$\begin{aligned}\delta v_y(t) &= \frac{eE_0}{im_e \omega} \exp[i(kx - \omega t)], \\ \delta v_x(t) &= \frac{eE_0}{im_e \omega} \frac{kv_y}{\omega - kv_x} \exp[i(kx - \omega t)] + \frac{2eE_0}{m} \frac{ikv_y}{\omega^2 - k^2v_x^2} \exp\left[i\omega\left(\frac{x}{v_x} - t\right)\right].\end{aligned}\quad (7)$$

In the absence of the electromagnetic fields ($E_0 = 0$), the distribution function of the electron is independent of time and can be expressed as

$$F(x, \mathbf{v}, t) = \begin{cases} n_0 f_0(\mathbf{v}), & x > 0, \\ 0, & x < 0, \end{cases}\quad (8)$$

where f_0 satisfies the normalization condition $\int d^3v f_0(\mathbf{v}) = 1$ and the symmetry $f_0(-v_x, v_y, v_z) = f_0(v_x, v_y, v_z)$. In the presence of the electromagnetic fields, the induced current density can be calculated from the perturbed orbits in Eqs. (6) and

(7), using the relation

$$\begin{aligned} j_y &= -n_0 e \int d^3 v \delta v_y f_0(\mathbf{v}) + n_0 e \int d^3 v v_y \frac{\partial \delta x_i}{\partial x_i} f_0(\mathbf{v}) \\ &= -n_0 e \int d^3 v \delta v_y f_0(\mathbf{v}) - n_0 e \int d^3 v v_y \frac{\partial \delta v_i}{\partial v_i} f_0(\mathbf{v}), \end{aligned} \quad (9)$$

for $x > 0$. Here, x_i and v_i ($i=1,2$, or 3) are the x , y , or z components of position ($\mathbf{x}$) and velocity ($\mathbf{v}$) vectors, and a repeated index implies a summation over the index. In deriving Eq. (9), use has been made of the relation $\frac{\partial \delta x_i}{\partial x_i} + \frac{\partial \delta v_i}{\partial v_i} = 0$, which can be derived from the Liouville's theorem. It is worth mentioning that the second term in the right-hand side of Eq. (9) is due to current bunching, which is characteristic of collective interactions between electromagnetic fields and plasmas (e.g., Weibel instability^{17,18} and cyclotron maser instability¹⁹). Substituting Eqs. (6) and (7) into Eq. (9) readily gives the induced current density

$$j_y = \frac{in_0 e^2 E_0}{m_e \omega} \left[1 + \int d^3 v \frac{k^2 v_y^2}{(\omega - kv_x)^2} f_0(\mathbf{v}) \right] \exp[i(kx - \omega t)] + j_R, \quad (10)$$

where j_R is defined by

$$j_R = -\frac{2in_0 e^2 E_0}{m_e} \int d^2 v_\perp \int_0^\infty dv_x f_0 \frac{kv_y^2}{\omega^2 - k^2 v_x^2} \left[\frac{2k^2 v_x}{\omega^2 - k^2 v_x^2} - i \frac{\omega x}{v_x^2} \right] \exp \left[i\omega \left(\frac{x}{v_x} - t \right) \right]. \quad (11)$$

In the regime where $\omega^2 c^2 \gg \omega_p^2 v_e^2$ and $\omega_p^2 \gg \omega^2$, the j_R term in Eq. (10) is small in comparison with the other term. Neglecting the j_R term, the induced current j_y can be substituted into the Maxwell's equations to determine the relation between k and ω , i.e.,

$$\omega^2 - c^2 k^2 = \omega_p^2 \left[1 + \int d^3 v \frac{k^2 v_y^2}{(\omega - kv_x)^2} f_0(\mathbf{v}) \right], \quad (12)$$

or equivalently

$$\omega^2 + \frac{c^2}{\delta^2} = \omega_p^2 \left[1 - \int d^3 v \frac{v_y^2 (\omega^2 \delta^2 - v_x^2)}{(\omega^2 \delta^2 + v_x^2)^2} f_0(\mathbf{v}) \right], \quad (13)$$

with $\delta = i/k$. In this regime [$\omega^2 c^2 \gg \omega_p^2 v_e^2$ and $\omega_p^2 \gg \omega^2$], the skin depth δ is

$$\delta = \delta_0 + \frac{\omega_p^2 \delta_0^3}{2c^2} \int d^3 v \frac{v_y^2 (\omega^2 \delta_0^2 - v_x^2)}{(\omega^2 \delta_0^2 + v_x^2)^2} f_0(\mathbf{v}), \quad (14)$$

where $\delta_0 = c/(\omega_p^2 - \omega^2)^{1/2}$. Making use of Eq. (11), the power transferring from the laser to the plasma, per unit area of laser-plasma interface, is readily obtained

$$\begin{aligned} P_{ab} &= \frac{1}{2} \int_0^\infty dx \operatorname{Re} \left\{ \left[E_0 \exp \left(-\frac{x}{\delta} - i\omega t \right) \right]^* j_R \right\} \\ &= \frac{2n_0 e^2 E_0^2 \delta^2}{m} \int d^2 v_\perp \int_0^\infty dv_x \frac{v_x v_y^2 (\omega^2 \delta^2 - v_x^2)}{(\omega^2 \delta^2 + v_x^2)^3} f_0(\mathbf{v}). \end{aligned} \quad (15)$$

It is worth mentioning that a similar derivation without the $\mathbf{v} \times \mathbf{B}$ term in the equation of motion will give

$$P_{ab} = \frac{n_0 e^2 E_0^2 \delta^2}{m} \int d^2 v_\perp \int_0^\infty dv_x \frac{v_x^3}{(\omega^2 \delta^2 + v_x^2)^2} f_0(\mathbf{v}). \quad (16)$$

For an isotropic distribution function [$f_0(\mathbf{v}) = f_0(|\mathbf{v}|)$], it can be shown, by carrying out the integration over the solid angle $\sin \theta d\theta d\phi$, that Eq.(16) is equivalent to Eq.(15). For general distributions function, however, Eq.(16) gives an absorption rate significantly different from Eq.(15). Moreover, if the $\mathbf{v} \times \mathbf{B}$ term had been neglected, the absorption of the light would be incorrectly interpreted as an increase in the transverse (y -direction) electron temperature, while the conservation of the transverse components of the canonical momentum requires that, after leaving the interaction region ($|x| \lesssim \delta$), an electron should have the same y -velocity as before it entered the interaction region.

III Relation between the Sheath Inverse Bremsstrahlung and the Anomalous Skin Effect

In the usual treatment of the anomalous skin effect^{14,15}, the absorption coefficient is calculated by extending the plasma and the electromagnetic fields in the present model to the half space $x < 0$ with

$$E_y(-x) = E_y(x), \quad B_z(-x) = -B_z(x). \quad (17)$$

The discontinuity in B_z requires a current sheet $\mathbf{J} = \hat{e}_y J_0 \delta(x) \exp(-i\omega t)$, whose amplitude is determined by

$$B_z(x = 0^+) - B_z(x = 0^-) = 2B_z(x = 0^+) = -\frac{4\pi J_0}{c}. \quad (18)$$

Since an electron with $x > 0$ in the extended model will have the same orbits as the corresponding electron in the original model, the two models are equivalent as far as the region $x > 0$ is concerned. In the extended model, the electric field satisfies the following equation

$$\left(c^2 \frac{\partial^2}{\partial x^2} + \omega^2\right) E_y(x) = -4\pi i \omega [j_y(x) + J_0 \delta(x)], \quad (19)$$

where $j_y(x)$ is the current density induced by the electromagnetic fields. Performing the Fourier transform on Eq. (19), and making use of the well-known relation²⁰ between the induced current density j_y and the electric field E_y , it follows that

$$\tilde{E}_y(k) = \frac{-4\pi i \omega J_0}{D(\omega, k)}, \quad (20)$$

$$D(\omega, k) = \omega^2 - c^2 k^2 - \omega_p^2 \left[1 - \int d^3v \frac{kv_y^2}{\omega - kv_x} \frac{\partial f_0}{\partial v_x} \right]. \quad (21)$$

Here, $\tilde{E}_y(k)$ is the Fourier transform of $E_y(x)$, i.e.,

$$\tilde{E}_y(k) = \int_{-\infty}^{\infty} dx E_y(x) \exp(-ikx). \quad (22)$$

In the regime of the anomalous skin effect ($\omega^2 c^2 \ll \omega_p^2 v_e^2$ and $\omega_p^2 \gg \omega^2$), and for an isotropic distribution function [$f_0(\mathbf{v}) = f_0(|\mathbf{v}|)$], the function $D(\omega, k)$ can be approximated by

$$D(\omega, k) \simeq -c^2 \left(k^2 - \frac{i}{|k| \delta_{as}^3} \right), \quad \frac{1}{\delta_{as}^3} = \frac{2\pi^2 \omega_p^2 \omega}{c^2} \int_0^{\infty} dv v f_0(v). \quad (23)$$

Making use of Eq. (18), (20) and (23), the electric field at $x = 0$ is readily obtained

$$E_y(x = 0) = \frac{4i\omega J_0}{c^2} \int_0^{\infty} dk \frac{k \delta_{as}^3}{k^3 \delta_{as}^3 - i} = \frac{2B_z(x = 0^+) \omega \delta_{as}}{3\sqrt{3}c} (1 - i\sqrt{3}), \quad (24)$$

and so is the Poynting flux at $x = 0^+$

$$\frac{c}{8\pi} \text{Re}\{E_y(x=0)B_z^*(x=0^+)\} = \frac{\omega\delta_{as}|B_z(x=0^+)|^2}{12\sqrt{3}\pi}. \quad (25)$$

For a laser normally incident on a highly overdense plasma, the magnetic field at $x = 0$ is related to the incident electric field (E_{in}) by $|B_z(x=0^+)| = 2E_{in}$; therefore, the absorption coefficient η_{as} of the anomalous skin effect is

$$\eta_{as} = \frac{8\omega\delta_{as}}{3\sqrt{3}c}. \quad (26)$$

In the regime of the sheath inverse bremsstrahlung ($\omega^2 c^2 \gg \omega_p^2 v_e^2$ and $\omega_p^2 \gg \omega^2$), on the other hand, the following approximation is appropriate

$$\frac{1}{D(\omega, k)} \simeq -\frac{1}{c^2 \left(k^2 + \frac{1}{\delta^2}\right)} - \frac{\Delta}{c^2 \left(k^2 + \frac{1}{\delta^2}\right)^2} - \frac{\omega_p^2}{c^4 \left(k^2 + \frac{1}{\delta^2}\right)^2} \int d^3v \frac{kv_y^2}{\omega - kv_x} \frac{\partial f_0}{\partial v_x}, \quad (27)$$

where δ is defined by

$$\frac{c^2}{\delta^2} = \omega_p^2 - \omega^2 + \Delta c^2, \quad (28)$$

and Δ is a constant to be determined by the condition that the secular term in the final result should vanish. Substituting Eq. (27) into Eq. (20) and performing the inverse Fourier transform give

$$\begin{aligned} E_y(x > 0) &= \frac{1}{2\pi} \int_{\Gamma} dk \tilde{E}_y(k) \exp(ikx) = \frac{2i\omega J_0}{c^2} \int_{\Gamma} dk \frac{\exp(ikx)}{k^2 + \frac{1}{\delta^2}} + \frac{2i\omega J_0 \Delta}{c^2} \int_{\Gamma} dk \frac{\exp(ikx)}{\left(k^2 + \frac{1}{\delta^2}\right)^2} \\ &\quad + \frac{2i\omega \omega_p^2 J_0}{c^4} \int_{\Gamma} dk \frac{\exp(ikx)}{\left(k^2 + \frac{1}{\delta^2}\right)^2} \int d^3v \frac{kv_y^2}{\omega - kv_x} \frac{\partial f_0}{\partial v_x} \\ &= 2\pi i \frac{\omega \delta J_0}{c^2} \exp\left(-\frac{x}{\delta}\right) + \pi i \frac{\omega \delta^2 J_0 \Delta}{c^2} x \exp\left(-\frac{x}{\delta}\right) + \pi i \frac{\omega \delta^3 J_0 \Delta}{c^2} \exp\left(-\frac{x}{\delta}\right) \\ &\quad + E_A(x > 0), \end{aligned} \quad (29)$$

where the contour Γ of the dk integration goes from $k = -\infty$ to $k = +\infty$ along the real k -axis, with a small positive (negative) imaginary part when $\text{Re } k < 0$ ($\text{Re } k > 0$), and $E_A(x > 0)$ is defined by

$$E_A(x > 0) = \frac{2i\omega \omega_p^2 J_0}{c^4} \int_{\Gamma} dk \frac{\exp(ikx)}{\left(k^2 + \frac{1}{\delta^2}\right)^2} \int d^3v \frac{kv_y^2}{\omega - kv_x} \frac{\partial f_0}{\partial v_x}. \quad (30)$$

Reversing the order of the dk and d^3v integrations in Eq. (30), followed by closing the contour Γ in the upper complex- k plane ($\text{Im } k > 0$), it can be shown that

$$\begin{aligned}
E_A(x > 0) = & 4\pi \frac{\omega^2 \omega_p^2 J_0}{c^4} \int d^2 v_{\perp} v_y^2 \int_0^{\infty} dv_x \frac{\delta^4 v_x^2}{(\omega^2 \delta^2 + v_x^2)^2} \frac{\partial f_0}{\partial v_x} \exp\left(i \frac{\omega x}{v_x}\right) \\
& - \frac{\pi \omega \omega_p^2 J_0}{c^4} \int d^3 v \frac{\delta^2 v_y^2}{\omega \delta - i v_x} \frac{\partial f_0}{\partial v_x} x \exp\left(-\frac{x}{\delta}\right) \\
& - \frac{\pi \omega \omega_p^2 J_0}{c^4} \int d^3 v \frac{\delta^3 v_y^2}{\omega \delta - i v_x} \frac{\partial f_0}{\partial v_x} \exp\left(-\frac{x}{\delta}\right) \\
& + \frac{\pi \omega^2 \omega_p^2 J_0}{c^4} \int d^3 v \frac{\delta^4 v_y^2}{(\omega \delta - i v_x)^2} \frac{\partial f_0}{\partial v_x} \exp\left(-\frac{x}{\delta}\right). \quad (31)
\end{aligned}$$

It can be readily seen from Eqs (29) and (31) that, for the secular term $[x \exp(-x/\delta)$ term] to vanish, it requires that

$$\begin{aligned}
\Delta c^2 = \omega^2 - \omega_p^2 + \frac{c^2}{\delta^2} &= -i \omega_p^2 \int d^3 v \frac{v_y^2}{\omega \delta - i v_x} \frac{\partial f_0}{\partial v_x} \\
&= \omega_p^2 \int d^3 v \frac{v_y^2 v_x}{\omega^2 \delta^2 + v_x^2} \frac{\partial f_0}{\partial v_x}. \quad (32)
\end{aligned}$$

It can be shown, using integration by parts, that Eq. (32) is identical to Eq. (13).

Making use of Eqs. (18), (29), (31) and (32), we obtain the electric field

$$\begin{aligned}
E_y(x > 0) = & 2\pi i \frac{\omega \delta J_0}{c^2} \exp\left(-\frac{x}{\delta}\right) + 2\pi i \frac{\omega^3 \omega_p^2 \delta^5 J_0}{c^4} \int d^3 v \frac{v_y^2 v_x}{(\omega^2 \delta^2 + v_x^2)^2} \frac{\partial f_0}{\partial v_x} \exp\left(-\frac{x}{\delta}\right) \\
& + 4\pi \frac{\omega^2 \omega_p^2 J_0}{c^4} \int d^2 v_{\perp} v_y^2 \int_0^{\infty} dv_x \frac{\delta^4 v_x^2}{(\omega^2 \delta^2 + v_x^2)^2} \frac{\partial f_0}{\partial v_x} \exp\left(i \frac{\omega x}{v_x}\right), \quad (33)
\end{aligned}$$

and the Poynting flux at $x = 0^+$

$$\frac{c}{8\pi} \text{Re}\{E_y(x=0) B_z^*(x=0^+)\} = -\frac{\omega^2 \omega_p^2 \delta^4 |B_z(x=0^+)|^2}{4\pi c^2} \int d^2 v_{\perp} v_y^2 \int_0^{\infty} dv_x \frac{v_x^2}{(\omega^2 \delta^2 + v_x^2)^2} \frac{\partial f_0}{\partial v_x}. \quad (34)$$

Integrating by parts over dv_x , it is readily shown that Eq. (34) implies the same absorption rate as Eq. (15). Since the magnetic field at $x = 0$ is related to the incident electric field (E_{in}) by $|B_z(x=0^+)|^2 = 4E_{\text{in}}^2 c^2 / (\omega^2 \delta^2 + c^2)$, the absorption coefficient η_{sib} of the sheath inverse bremsstrahlung is

$$\eta_{\text{sib}} = -\frac{8\omega^2 \omega_p^2 \delta^4}{c(\omega^2 \delta^2 + c^2)} \int d^2 v_{\perp} v_y^2 \int_0^{\infty} dv_x \frac{v_x^2}{(\omega^2 \delta^2 + v_x^2)^2} \frac{\partial f_0}{\partial v_x} \quad (35)$$

$$= \frac{16\omega^2\delta^2}{c} \left[\int d^2v_{\perp} v_y^2 \int_0^{\infty} dv_x \frac{v_x(\omega^2\delta^2 - v_x^2)}{(\omega^2\delta^2 + v_x^2)^3} f_0(\mathbf{v}) \right] \left[1 - \int d^3v \frac{v_y^2(\omega^2\delta^2 - v_x^2)}{(\omega^2\delta^2 + v_x^2)^2} f_0(\mathbf{v}) \right]^{-1}.$$

Here, use has been made of Eq. (13) in deriving Eq. (35).

It can be seen from the derivations of Eqs. (26) and (35) that the sheath inverse bremsstrahlung and the anomalous skin effect are two limiting cases of a more general collisionless absorption mechanism described by Eqs. (20) and (21). The sheath inverse bremsstrahlung corresponds to the limit where $\omega^2 c^2 \gg \omega_p^2 v_e^2$, while the anomalous skin effect corresponds to $\omega^2 c^2 \ll \omega_p^2 v_e^2$. In the intermediate regime ($\omega^2 c^2 \sim \omega_p^2 v_e^2$), the absorption coefficient can be obtained by performing the inverse Fourier transform of Eqs. (20) and (21).

To further elucidate the result in Eq. (35), the absorption coefficient in a plasma with Maxwellian distribution function

$$f_0(\mathbf{v}) = \frac{1}{(2\pi v_e^2)^{3/2}} \exp\left(-\frac{|\mathbf{v}|^2}{2v_e^2}\right) \quad (36)$$

is obtained in a closed analytic form

$$\eta_{sib} = \frac{8}{\sqrt{2\pi}} \frac{v_e}{c} a [(a+1) \exp(a) E_1(a) - 1] \left[\frac{1}{2} + \frac{\sqrt{\pi a}}{2} \exp(a) \operatorname{erfc}(\sqrt{a}) \right]^{-1}, \quad (37)$$

where

$$a = \frac{\omega^2 \delta^2}{2v_e^2}, \quad E_1(a) = \int_a^{\infty} \frac{\exp(-t)}{t} dt, \quad \operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} \exp(-t^2) dt. \quad (38)$$

As can be seen from Eq. (37), the quantity $\eta_{sib} c / v_e$ depends on the system parameters only through the dimensionless parameter $\omega \delta / v_e$, which is approximately the ratio of the transit time in the collisionless skin depth over the period of the incident light. This result is to be compared with the more accurate one calculated from Eqs. (20) and (21).

Substituting the maxwellian distribution defined in Eq. (36) into Eq. (21) gives

$$D(\omega, k) = \omega^2 - c^2 k^2 + \omega_p^2 \frac{\omega}{\sqrt{2}|k|v_e} Z\left(\frac{\omega}{\sqrt{2}|k|v_e}\right), \quad (39)$$

where $Z(x)$ is the plasma dispersion function²¹. For a real-valued x , the function $Z(x)$ can be expressed as²¹

$$Z(x) = i\sqrt{\pi} \exp(-x^2) - 2 \exp(-x^2) \int_0^x dt \exp(t^2). \quad (40)$$

It is then readily shown by the inverse Fourier transform of Eq. (20) that

$$E_y(x=0) = -\frac{i\omega c}{\pi} B_z(x=0^+) \int_{-\infty}^{\infty} dk \frac{1}{c^2 k^2 - \omega^2 - \omega_p^2 \frac{\omega}{\sqrt{2}|k|v_e} Z\left(\frac{\omega}{\sqrt{2}|k|v_e}\right)}. \quad (41)$$

For a highly overdense plasma ($\omega_p^2 \gg \omega^2$), the magnetic field at $x=0$ is related to the incident electric field (E_{in}) by $|B_z(x=0^+)|^2 = 4E_{in}^2$. Changing the integration variable to $\xi = \omega/\sqrt{2}|k|v_e$, the absorption coefficient η_{ab} is then readily derived from Eq. (41)

$$\eta_{ab} = \frac{8\sqrt{2} v_e}{\pi c} \text{Im} \left[\int_0^{\infty} d\xi \frac{1}{1 - \frac{\xi^3}{b^2} Z(\xi)} \right], \quad (42)$$

where $b^2 = \omega^2 c^2 / 2\omega_p^2 v_e^2$.

Plotted in Fig. 1 are the quantities $\eta_{ab}c/v_e$ (solid line) calculated numerically from Eq. (42), $\eta_{sib}c/v_e$ (dotted line) calculated from Eq. (37), and $\eta_{as}c/v_e$ (dashed line) defined in Eq. (26). The curves are plotted versus $\omega^2 c^2 / \omega_p^2 v_e^2$. As mentioned earlier, the sheath inverse bremsstrahlung (SIB, dotted line) corresponds to the regime where $\omega^2 c^2 \gg \omega_p^2 v_e^2$, while the anomalous skin effect corresponds to $\omega^2 c^2 \ll \omega_p^2 v_e^2$ (ASE, dashed line). It can be seen that both $\eta_{sib}c/v_e$ and $\eta_{as}c/v_e$ are in good agreement with $\eta_{ab}c/v_e$, in their respective regimes of validity. It is worth reiterating that, for an isotropic distribution function [$f_0(\mathbf{v}) = f_0(|\mathbf{v}|)$], such as the one in Eq. (36), an identical absorption coefficient would be obtained whether or not the $\mathbf{v} \times \mathbf{B}$ term in the equation of motion was included in the derivation. However, neglecting the $\mathbf{v} \times \mathbf{B}$ term will lead to an incorrect interpretation of the absorption mechanism.

IV Numerical Simulations of the Light Absorption in Overdense Plasmas

To examine the validity of the absorption coefficient derived in the previous section, several runs of numerical simulations have been carried out, using the PIC plasma simulation code ZOHAR¹⁶. In all the simulations, the electric field of the incident light E_{in} satisfies $E_{in}^2 \ll \pi n_0 m_e v_e^2$ so that the quiver velocity of electrons in the overdense plasma is smaller than the thermal velocity, as was assumed in the analytical derivation of the absorption coefficient. The 1-D simulations are set up as follows:

- (i) The electromagnetic fields can vary only in the x -direction. The length of the simulation region is $l_x = 40c/\omega_p$.
- (ii) The boundary conditions of the electromagnetic fields at $x = 0$ correspond to a normally incident circularly polarized plane wave. The amplitude of the wave gradually increases from zero at the beginning of the simulation ($t = 0$) to a value of $eE_{in}/\omega m_e c$ at the end of the fifth wave period. The amplitude of the incident wave remains constant thereafter.
- (iii) The wave impedances of the boundaries are chosen such that the outgoing waves will be completely reflected at $x = l_x$ and completely transmitted at $x = 0$.
- (iv) The initial electron density is zero for $0 < x < l_x/2$, and is equal to a constant value n_0 for $l_x/2 < x < l_x$. Immobile neutralizing background charge density with the same profile is imposed to insure charge neutrality at $t = 0$.
- (v) At $t = 0$, simulation particles are loaded uniformly in the region $l_x/2 < x < l_x$, with Maxwellian distribution [Eq. (36)] in the velocity space.
- (vi) During the simulation, particles that hit the boundary at $x = 0$ leave the system, while those that hit the boundary at $x = l_x$ are re-emitted according to the Maxwellian distribution with the same thermal velocity v_e as the initial distribution function.

Three sets of simulations have been carried out with the initial thermal velocity v_e equal to $0.1c$, $0.05c$, and $0.025c$, respectively. Each set consists of simulations with

different values of ω/ω_p . The simulation parameters for the set with $v_e/c = 0.1$ are: (i) 400 equally spaced grid points for the electromagnetic fields; (ii) the steady-state value of laser electric field $eE_{in}/\omega m_e c = 0.05$; (iii) 12,800 simulation particles; (iv) the time step $\Delta t = 0.08/\omega_p$. Each time we reduced the value of v_e/c by a half (from $v_e/c = 0.1$ to $v_e/c = 0.05$, and then to $v_e/c = 0.025$), we doubled the number of simulation particles and grid points, and reduced the time step and the steady-state laser electric field eE_{in} by a half.

Figure 2 shows the absorption coefficients observed in the simulations in comparison with the absorption coefficients calculated numerically from Eq. (42). Qualitative agreement between the simulations and the linear theory can be seen in Fig. 2. For the simulations with the parameter $\omega^2 c^2 / \omega_p^2 v_e^2$ close to unity, the observed absorption coefficients are higher than the estimates from the linear theory. A possible cause of the discrepancy is that the linear theory assumes instant reflection in the sheath region, which may not be a good approximation in those simulations. This explanation is supported by the trend that the simulations with smaller v_e/c agree better with the linear theory, as smaller v_e/c for a given $\omega^2 c^2 / \omega_p^2 v_e^2$ means shorter reflection time (of order $1/\omega_p$) relative to the wave period ($2\pi/\omega$).

In the simulations, the flows of the electromagnetic energy and the electron kinetic energy through the boundaries at $x = 0$ and $x = l_x$ are taken in to account; therefore, what is shown in Fig. 2 corresponds to the net transfer of the electromagnetic energy into the electron kinetic energy. For most of the simulations with $v_e/c = 0.1$ or $v_e/c = 0.05$, the total (electromagnetic plus kinetic) energy is conserved very well in the simulation, with the change in the total energy (ΔE_{total}) less than 10% of the net increase in the electron kinetic energy (ΔE_e). The exceptions are the cases with $\omega^2 c^2 / \omega_p^2 v_e^2 = 20.0$ and $\omega^2 c^2 / \omega_p^2 v_e^2 = 0.05$, for which $\Delta E_{total} / \Delta E_e \sim 20\%$. For the simulations with $v_e/c = 0.025$, $\Delta E_{total} / \Delta E_e$ is also about 20%.

V Summary and Discussion

The sheath inverse bremsstrahlung model¹³ was modified by including the $\mathbf{v} \times \mathbf{B}$ term in the equation of motion. The results are significantly different from those derived without the $\mathbf{v} \times \mathbf{B}$ term, except when the distribution function is isotropic [$f_0(\mathbf{v}) = f_0(|\mathbf{v}|)$]. For an isotropic distribution function, identical results would be obtained whether or not the $\mathbf{v} \times \mathbf{B}$ term in the equation of motion was included in the derivation. However, if the $\mathbf{v} \times \mathbf{B}$ term had been neglected, the absorption of the light would be incorrectly interpreted as an increase in the transverse electron temperature, while the conservation of the transverse components of the canonical momentum requires that, after leaving the interaction region ($|x| \lesssim \delta$), an electron should have the same transverse velocity as before it entered the interaction region.

It was also shown that the sheath inverse bremsstrahlung is related to the anomalous skin effect in the sense that both are limiting cases of a more general collisionless absorption mechanism described by Eqs. (20) and (21). The sheath inverse bremsstrahlung corresponds to the limit where $\omega^2 c^2 \gg \omega_p^2 v_e^2$, while the anomalous skin effect corresponds to $\omega^2 c^2 \ll \omega_p^2 v_e^2$. In the intermediate regime ($\omega^2 c^2 \sim \omega_p^2 v_e^2$), the absorption coefficient can be obtained by performing the inverse Fourier transform of Eqs. (20) and (21).

Numerical simulations of the light absorption in overdense plasmas have been carried out, using the PIC plasma simulation code ZOHAR¹⁶. The absorption coefficients observed in the numerical simulations are in qualitative agreement with the linear-theory values. However, some simulations showed absorption coefficients significantly higher than the linear-theory results. A possible explanation of the discrepancy is that the assumption of instant reflection made in the linear theory may not apply very well in those simulations. It should be reiterated that the present analysis assumes that the electric field of the incident light E_{in} satisfies $E_{in}^2 \ll \pi n_0 m_e v_e^2$, so that the quiver

velocity of electrons in the overdense plasma is smaller than the thermal velocity. Moreover, the present model assumes normal incidence for the laser light. Studies of the cases of oblique incidence and of intense incident laser light ($E_{in}^2 \gg \pi n_0 m_e v_e^2$) will be subjects of future investigations.

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Figure Captions

Fig. 1 The quantities $\eta_{ab}c/v_e$ (solid line) calculated numerically from Eq. (42), $\eta_{sib}c/v_e$ (dotted line) calculated from Eq. (37), and $\eta_{as}c/v_e$ (dashed line) defined in Eq. (26), plotted versus $\omega^2 c^2 / \omega_p^2 v_e^2$.

Fig. 2 The absorption coefficients times c/v_e observed in the PIC simulations are plotted in comparison with the linear-theory results.

Fig. 1

Fig. 2