The effect of small radiation divergence on measurements of the source size

P. Ilinski
Experimental Facilities Division, Argonne National Laboratory, Argonne, IL 60439

ABSTRACT

Third-generation storage rings have small particle beam emittances. The size of the particle beam can be determined from the measured image size of the focused radiation source. Undulators are preferred as radiation sources at third-generation storage rings, the radiation emitted by an undulator has both small size and divergence. The effect of the small beam divergence on determination of the source size is discussed here. The analysis is performed in an approximation of a Gaussian distribution of the radiation intensity. The paraxial approximation and matrix methods were also used for this derivation. It was found that the effect of the small beam divergence depends on the ratio of the angular acceptance of the source to the divergence of the emitted radiation.

Keywords: storage ring, undulator radiation, beam size, beam divergence, phase space representation, matrix method

1. DETERMINATION OF THE SOURCE SIZE

Third-generation storage rings have small particle beam emittances, which is a product of the beam size and divergence. The size of the particle beam can be determined using an imaging technique. The radiation source represented by the resulting undulator radiation is focused by a zone plate, and an image size is measured at the image plane (Fig. 1). The resulting undulator radiation is the convolution of the particle beam and the intrinsic undulator radiation. The resulting undulator radiation has a small divergence that affects the result of the determination. To calculate the source size from the image size, a phase space representation can be used. A matrix method is used to transform the beam through optical elements.

![Figure 1. Optical setup.](image)

The submitted manuscript has been created by the University of Chicago as Operator of Argonne National Laboratory ("Argonne") under Contract No. W-31-109-ENG-38 with the U.S. Department of Energy. The U.S. Government retains for itself, and others acting on its behalf, a paid-up, nonexclusive, irrevocable worldwide license in said article to reproduce, prepare derivative works, distribute copies to the public, and perform publicly and display publicly, by or on behalf of the Government.
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, make any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.
A particle beam in a well-behaved storage ring has a Gaussian distribution in the phase space and can be represented by a sigma contour that is the phase ellipse. It is convenient to describe an ellipse with a $c_i$ matrix. At the location where the beta function of the particle beam is minimal, the $\beta_0$ position, the beam phase ellipse is upright, with the maximum rms beam size and divergence $\sigma$ and $\sigma'$, respectively. When no divergence is introduced by the undulator, the phase ellipse of the focused Gaussian beam can be written as:

$$
\begin{pmatrix}
 c_1 & c_2 \\
 c_3 & c_4 \\
 0 & 1 \\
 0 & 1
\end{pmatrix}
= 
\begin{pmatrix}
 1 & 0 & 1 & L \\
 0 & 1 & -1/f & 1 \\
 0 & 1 & 0 & \sigma \\
 0 & 1 & 0 & \sigma'
\end{pmatrix}
\left(\begin{array}{c}
 z \\
 \end{array}\right)
\left(\begin{array}{c}
 \sigma \\
 \sigma'
\end{array}\right),
$$

where $z$ is the distance from the $\beta_0$ position to the center of the undulator, $L$ is the distance from the undulator center to the zone plate, $f$ is the zone plate focal length, and $d$ is the distance to the image plane. The first matrix represents the beam phase ellipse, transformed from the $\beta_0$ position to the undulator center. The next matrix makes a beam transformation by the distance $L$ to the location of the zone plate. The third matrix represents the focusing element with the focal length $f$; the last matrix transforms the beam by the distance $d$ to the image plane. Using the result of Eq. (1) and the properties of the ellipse $c_i$ matrix representation, the beam size can be written as a function of distance $d$:

$$
\sigma_{\text{image}}(d) = \sqrt{c_1^2 + c_2^2}.
$$

The image plane is located at the waist of the focused beam, that corresponds to the minimum of the $\sigma_{\text{image}}(d)$:

$$
d_{\text{image}} = -f \frac{(z + L)(f - z - L) - (\sigma / \sigma')^2}{(f - z - L)^2 + (\sigma / \sigma')^2}.
$$

Substituting $d_{\text{image}}$ into Eq. (2) will result in the image size:

$$
\sigma_{\text{image}} = \frac{f}{\sqrt{(L - f)^2 + \frac{1}{\sigma^2}}}.
$$

When $L$ is much larger than $f$, the effect of the small radiation divergence depends on the ratio of the angular acceptance of the radiation source to the divergence of the emitted radiation. The rms beam size determined from the measured image size is:

$$
\sigma = \frac{L + z - f}{\sqrt{\frac{f^2}{\sigma_{\text{image}}^2} + \frac{1}{\sigma^2}}}.
$$

The effective distance to the source can be defined from the focusing condition of the zone plate, which is similar to the condition for a thin lens:

$$
\frac{1}{f} = \frac{1}{d_{\text{image}}} + \frac{1}{L_{\text{eff}}}.
$$

Solving this for $L_{\text{eff}}$, we obtain the effective distance to the source:

$$
L_{\text{eff}} = (L + z) + \frac{\sigma^2}{\sigma'^2} \frac{1}{L + z - f}.
$$
For a case in which the beam divergence is large, the distance to the source will become \( L_{\text{eff}} = L + z \), and the source size is defined by a simple magnification formula:

\[
\sigma = \sigma_{\text{image}} \frac{L + z - f}{f} = \sigma_{\text{image}} \frac{L + z}{d_{\text{image}}}.
\]  

Now we will introduce the intrinsic undulator radiation in our consideration. The resulting undulator radiation is a convolution of the undulator radiation emitted by one particle (the intrinsic undulator radiation) and the particle beam. The diffraction source size of the intrinsic undulator radiation in the x-ray energy range is small compared to the particle beam size and can be omitted, but the divergence of the intrinsic undulator radiation can be comparable to the beam divergence. For the on-axis observation, the angular distribution of the intrinsic undulator radiation at the fundamental harmonic energy can be approximated by a Gaussian distribution.\(^5\) The convolution of the beam phase ellipse with the intrinsic undulator radiation phase ellipse will result in a new phase ellipse that is at distance \( z \) from the \( \beta_0 \) position. As in Eq.(1), the matrix of the phase ellipse at distance \( d \) from the zone plate can be written as:

\[
\begin{pmatrix}
1 & d \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
-1/f & 1
\end{pmatrix}
\begin{pmatrix}
L \\
0
\end{pmatrix}
\begin{pmatrix}
\sigma \\
\sqrt{\sigma^2 + \sigma_u^2}
\end{pmatrix}

\]  

Performing the same calculations as in the previous case, we obtain the location of the image plane from the zone plate:

\[
d_{\text{image}} = f \left[ 1 - \frac{f \sigma' \left[ \sigma'_\text{tot} (f - L) - z \sigma' \right]}{\sigma^2 + \left( \sigma'_\text{tot} (f - L) - z \sigma' \right)^2} \right].
\]  

The effective distance from the zone plate to the source becomes:

\[
L_{\text{eff}} = f \left[ 1 - \frac{\sigma^2 + (\sigma'_\text{tot} (f - L) - z \sigma')^2}{\left( \sigma'_\text{tot} (f - L) - z \sigma' \right) f \sigma'_\text{tot}} \right],
\]  

where \( \sigma'_\text{tot} = \sqrt{\sigma^2 + \sigma_u^2} \). Finally the rms beam size is:

\[
\sigma = \frac{1}{\sqrt{\sigma^2 - \frac{1}{\sigma'_\text{tot}^2}}} \left[ L - f + \frac{z}{\sqrt{1 + \frac{\sigma_u^2}{\sigma^2}}} \right].
\]  

Compared to Eq. (5), the distance to the center of the undulator \( z \) has changed to the

\[
X = \frac{z}{\sqrt{1 + \frac{\sigma_u^2}{\sigma^2}}},
\]  

which represents the position of the effective undulator source.\(^6\)
2. ACKNOWLEDGMENTS

This work was supported by the U.S. Department of Energy, BES-Office of Energy Research, under Contract. No. W-31-109-Eng-38.

3. REFERENCES


Further author information: Email: ilinski@aps.anl.gov; Telephone: 630-252-0145