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**Absence of Second
Stability in ATF**

N. Dominguez
V. E. Lynch

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ABSENCE OF SECOND STABILITY IN ATF

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July 1995

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CONTENTS

LIST OF FIGURES.....	v
ACRONYMS	vii
ABSTRACT	ix
1. INTRODUCTION.....	1
2. ATF VACUUM MAGNETIC FIELD CONFIGURATIONS	3
3. EQUILIBRIUM CALCULATION AND TRANSFORMATION	7
3.1 Calculation of 3-D equilibria.....	7
3.2 Transformation from VMEC to Boozer coordinates.....	9
4. BALLOONING MODE EQUATION AND MERCIER STABILITY CRITERION FOR 3-D GEOMETRIES	12
4.1 Ballooning mode equation	14
4.2 Mercier criterion.....	16
5. ATF STABILITY RESULTS	18
5.1 Comparison of 2-D and 3-D equilibrium calculations.....	18
5.2 Effects of shift in magnetic axis and quadrupolar magnetic fields	22
5.3 Effects of pressure profile	31
5.4 Comparison with other stability analyses.....	39
6. DISCUSSION	41
7. CONCLUSIONS.....	43
ACKNOWLEDGMENTS.....	45
APPENDIX A. BALLOONING MODE EQUATION.....	45
REFERENCES.....	48

LIST OF FIGURES

Figure		Page
1	Some ATF vacuum magnetic configurations for which the stability analysis was performed	5
2	Effect of quadrupole field on the rotational transform, $\beta = 0$	6
3	Spectral quantities for R (R_{mn}) and Z (Z_{mn}).....	11
4	Reconstruction of the flux surfaces corresponding to the case treated in Fig. 3	13
5	Mercier unstable regions for the inward-shifted (by 5 cm) configuration with zero current and $P \propto [1 - (\chi/\chi_b)]^2$ for (a) 3-D equilibria and (b) 2-D equilibria	19
6	Contributions to Mercier criterion with $\langle\beta\rangle = 2\%$	21
7	Mercier unstable regions for ATF configurations with $P \propto [1 - (\chi/\chi_b)]^2$ and zero current.....	23
8	Mercier unstable regions and rotational transform for different values of beta for $I_B = -0.13$	25
9	Stability boundaries and rotational transform for different values of beta for $I_B = 0.0$ (standard configuration)	26
10	Stability boundaries and rotational transform for different values of beta for $I_B = +0.13$	27
11	Unstable regions and rotational transform for different values of beta for $I_B = +0.19$	28
12	Unstable regions and rotational transform for different values of beta for $I_B = +0.23$	29
31	Growth rates for ballooning modes for the outward-shifted (by 2.5 cm) configuration with $\langle\beta\rangle = 4\%$	30
14	Toroidal shifts as a function of peak beta from calculations using 3-D VMEC equilibria and calculations using the 2-D equilibria obtained with the classical stellarator expansion	32
15	Pressure profile given in Eq. (10), rotational transform, and magnetic hill regions for $\beta_0 = 0.8\%$	33

16	Mercier criterion contributions for the pressure profile given in Eq. (10), with $\beta_0 = 0.8\%$	34
17	Mercier criterion for 2-D and 3-D equilibria for the pressure profile given in Eq. (10), for $\beta_0 = 0.8\%$	35
18	Ideal Mercier and ballooning unstable regions for the pressure profile given in Eq. (10) and zero current mode of operation	37
19	Growth rates of the Mercier-ballooning modes located in the unstable regions presented in Fig. 18	38
20	Stability boundaries and rotational transforms for the pressure profile given in Ref. [12]	41

ACRONYMS AND ABBREVIATIONS

3-D	three-dimensional
ATF	Advanced Toroidal Facility
2-D	two-dimensional
VF	vertical field
MHD	magnetohydrodynamic

ABSENCE OF SECOND STABILITY IN ATF

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ABSTRACT. Ideal Mercier and ballooning modes for three-dimensional (3-D) equilibria of the Advanced Toroidal Facility (ATF) [LYON, J.F., et al., Fusion Technol. **10** (1986) 179] are studied in detail. These modes are found to introduce instability limits at modest beta values for pressure profiles for which previous results, based on a two-dimensional (2-D) equilibrium approximation, predicted the existence of a second stability region. This absence of second stability is in agreement with an analysis by Cooper et al. [COOPER, W.A., et al., Nucl. Fusion **29** (1989) 617], who used a simplified model for the ATF plasma boundary. The increased instability is found to result from the 3-D feature of the Pfirsch-Schlüter current. Furthermore, a strong quadrupole field is found to decrease the beta limit of these modes, contrary to the results obtained using the 2-D equilibria. Although direct comparison with measured beta values is not possible owing to the absence of profile data for ATF plasmas, pressure profiles used previously to model ATF data as well as in the 2-D analysis are used to obtain our results. The details of the pressure profiles and the plasma configurations strongly influence the stability limits on average beta which remains below 3%.

1. INTRODUCTION

The Advanced Toroidal Facility (ATF) [1] is an $\ell = 2$ torsatron [2], where ℓ is the poloidal multipolarity, with $M = 12$ toroidal field periods and an average plasma aspect ratio of 7.8. The magnetic field is produced by two helical coils with a winding law

$$\phi = \phi_0 + \frac{\ell\omega}{M}$$

where ϕ and ω are the geometric toroidal and poloidal angles, respectively, and ϕ_0 is a constant. The currents in these helical coils produce the vacuum rotational transform ι . The device also has three pairs of axisymmetric vertical field (VF) coils to provide a vertical field and to give flexibility to the configuration of the plasma, which can be changed by changing the external currents flowing in the VF coils. For vacuum magnetic fields, the rotational transform increases monotonically as a function of radius. The rotational transform profile changes with beta in order to satisfy the condition of zero net toroidal current. For finite beta, the currents in the VF coils can be adjusted to generate a rotational transform which increases monotonically as a function of radius. The sequence of equilibria generated by such adjustments is called a 'controlled flux-conserving sequence of zero-current equilibria' [3]. Studies of the equilibrium and stability properties of this sequence using two-dimensional (2-D) average methods [3-6] made ATF appear to be a very attractive device, because a path to high beta seemed possible for the 2-D equilibria.

In stellarators, the maximum value of beta is set either by the equilibrium toroidal axis shift or by instabilities due to pressure-gradient-driven magnetohydrodynamic (MHD) modes. In 1982, a simplified analysis based on a study of tokamak stability [7] concluded that ideal ballooning mode instability should not exist in stellarators [8]. This conclusion was countered in 1983 by results showing, in a simple numerically obtained

equilibrium, the existence of this instability in stellarators [9]. In 1987 it was clearly demonstrated, analytically and numerically, that three-dimensional (3-D) effects not taken into account in Ref. [8] are responsible for the existence of this instability [10].

Notwithstanding the results of Refs [9–11], the debate on ballooning instability in stellarators continued. However, in Ref. [12] Cooper et al. demonstrated that in self-consistent 3-D numerical equilibria there is ideal ballooning instability. Using a simple plasma model for the ATF standard configuration, Cooper et al. showed that a stability beta limit exists and is determined by the ideal ballooning modes. Other independent calculations [13] corroborate this conclusion for ATF.

Stability studies for the ATF design were based on low- n and Mercier stability calculations done with the stellarator expansion approach [14,15]. Bauer et al. performed the first stability studies of ATF using 3-D numerical equilibria obtained from the BETA code [16,17]. More recent work [12] describes the stability beta limits to Mercier and ballooning modes of a 3-D configuration modeled on the standard ATF configuration. With this model, the rotational transform monotonically increases as a function of radius for low beta and becomes double valued as beta increases.

Here we carry out a numerical evaluation of the stability of the ATF device to ideal Mercier and ballooning modes, using 3-D equilibria obtained from the VMEC equilibrium code [18–20] for different plasma configurations in ATF. The main motivation for this work was to determine whether the ideal ballooning instabilities persisted for high beta 3-D ATF equilibria with rotational transform monotonically increasing as a function of radius. Such equilibria can be obtained using the quadrupolar fields of ATF.

The approach taken here differs from that used in Ref. [12] in two major ways. First, our force-free (zero-pressure) equilibria are more realistic because they are obtained by fitting the fields determined from a field line following code. Second, we have taken into account the flexibility of the ATF device and analysed a broad range of plasma

configurations, including those with strong quadrupole fields. The implications of these differences for the stability picture of ATF will be discussed.

In Section 2, we describe the ATF configurations studied. In Section 3, we discuss how the equilibria were obtained and transformed from VMEC coordinates to a straight magnetic field line coordinate system. In Section 4, we present the ballooning equation and the Mercier criterion. In Section 5, we show the stability boundaries of various ATF configurations, using different pressure profiles. In particular, we examine a pressure profile used in an earlier study of ATF experimental results [21] and the pressure profile used in Ref. [12]. In Section 6, we discuss our results and the implications of their differences from the results of Ref. [12] for the stability picture of ATF. We give our conclusions in Section 7.

2. ATF VACUUM MAGNETIC FIELD CONFIGURATIONS

A large number of ATF configurations can be obtained by changing the currents in the VF coils. For example, the magnetic axis can be horizontally displaced with respect to the geometric center of the coils; as a consequence, the magnetic field line curvature (the magnetic dipole moment of the configuration) is controlled. The other available control parameter is the quadrupole magnetic moment, which is changed by changing the current in the mid-VF coils; this allows variation of the ellipticity of the magnetic surfaces, changing the rotational transform and the magnetic shear. These changes are performed within the constraint of zero net toroidal current on each flux surface [1,3]. The configuration used in a particular experiment depends on the phenomenon to be studied. However, in all cases the configuration must be ideally MHD stable. Thus, the results obtained here can be used to find configurations in which other non-MHD phenomena can be studied. Studies of the neoclassical transport properties of ATF

configurations are reported in Ref. [22], and analysis of the MHD stability of these configurations (using the stellarator expansion) is reported in Refs [1,3–6].

Figure 1 (taken from Ref. [1]) illustrates the effect of changing the external currents in the VF coils on vacuum magnetic flux surfaces in the $\phi = 0^\circ$ plane. These configurations were obtained with a current of 0.875 MA in each of the $\ell = 2$ helical coils. The vacuum magnetic field is close to 1 T at the magnetic axis. In Fig. 1, the position of the magnetic axis is indicated on the x -axis, and the current in the mid-VF coils is indicated on the y -axis by the parameter I_B , which is the ratio of the current in the mid-VF coils to that in one of the helical coils. Additional information on these configurations is available in Ref. [1].

The so-called “standard configuration” appears at the center of Fig. 1; the magnetic axis is at 2.1 m and the average minor radius is 27 cm. Shown at left and right are the effects of shifting the magnetic axis of the standard configuration inwards and outwards by 5 cm; shown from top to bottom are the effects of changing I_B . As the radius R_{AXIS} increases, the magnetic well increases and the magnetic shear decreases, and as I_B increases, the magnetic well decreases and the magnetic shear increases.

In Fig. 2 we show profiles of the vacuum rotational transform ι for configurations with different values of I_B versus an average radius for the flux surfaces $\rho = \sqrt{\Phi/\Phi_b}$, where Φ is the magnetic toroidal flux divided by 2π and Φ_b is the value of Φ at the boundary. For the standard configuration with $I_B = 0$, the rotational transform $\iota = 0.334$ at the axis and $\iota = 0.98$ at the edge. These values were selected to avoid dangerous low order resonances in the plasma, at the radial positions where $\iota = 2/5$, $1/2$ and $3/4$, and to allow a stabilizing magnetic shear contribution at the edge. The importance of a particular resonance depends on the specific configuration, pressure profile, beta value and mode of operation. The standard configuration was chosen to place $\iota = 1/2$ inside the magnetic well, making the resonance innocuous to low- n modes, as determined using 2-D equilibria [1, 3–6].

Fig.1. Some ATF vacuum magnetic configurations for which the stability analysis was performed. The vacuum magnetic flux surfaces are shown at the geometrical toroidal angle $\phi = 0^\circ$. These configurations are obtained by changing the currents in the VF coils.

Fig. 2. Effect of quadrupole field on the rotational transform, $\beta = 0$.

3. EQUILIBRIUM CALCULATION AND TRANSFORMATION

The evaluation of ideal ballooning instabilities is numerically difficult because it involves the calculation of 3-D equilibria, which must be very accurate for stability studies. In addition, once the equilibria are obtained, the analysis of stability to ballooning modes must be carried out in a system with straight magnetic field line coordinates, requiring transformation to a coordinate system in which the number of harmonics is much larger than that in the original coordinate system.

One of the major problems for realistic configurations has been the lack of fast codes capable of finding 3-D numerical equilibria with good numerical accuracy. With the implementation of the VMEC code [18–20], the problem of analysing the MHD stability of stellarators became tractable.

3.1. Calculation of 3-D equilibria

VMC uses an energy principle [18] to obtain the solution of the MHD equilibrium equation,

$$\vec{J} \times \vec{B} = \nabla P \quad (1)$$

Here, $\vec{J}$, $\vec{B}$ and P are the plasma current, the magnetic field and the plasma pressure, respectively. Toroidally nested magnetic flux surfaces are assumed; they are expressed in terms of a condensed representation of harmonics in a Fourier series using the VMEC angles as

$$R(s, \Theta, \phi) = \sum_{mn} R_{mn}(s) \cos(m\Theta - n\phi) \quad (2)$$

$$Z(s, \Theta, \phi) = \sum_{mn} Z_{mn}(s) \sin(m\Theta - n\phi) \quad (3)$$

Here R and Z are the cylindrical coordinates of the flux surface with label s . We have chosen s to be proportional to the magnetic toroidal flux, $s = \Phi/\Phi_b$, where $\Phi_b = 1/2$. In this non-straight magnetic field line coordinate system, ϕ is the geometric toroidal angle

and Θ is a poloidal-like angle. The pressure is assumed to be isotropic and to depend only on s .

To find an equilibrium configuration, VMEC uses an iteration scheme to minimize the MHD energy. Iteration is continued until the total volume-averaged force squared [18] is less than 10^{-9} . For the calculations discussed here, we used the fixed boundary version of the code, in which Eq. (1) is solved in the region enclosed by a prescribed boundary. The boundary used for this work corresponds to the last closed flux surface, which was obtained by using a field line following code [23]. This code finds the vacuum magnetic field by using the Biot-Savart law; then, from the flux surfaces plotted in different toroidal planes, the last closed magnetic flux surface is defined as the largest flux surface that is closed and contains no stochastic regions. The result is fitted and given in terms of the components of the representation in Eqs (2) and (3).

We note that since all of the physics of the configuration for VMEC relies on the boundary, slight changes in the boundary or its representation (i.e. different fits) may give different equilibrium and stability results. A good example can be found in Ref. [24], in which the equilibrium quantities obtained from the VMEC code were carefully studied by changing the number of modes used to represent the boundary.

We use the method described in Ref. [24] to obtain good fits to the boundaries of the different ATF configurations. It is necessary to include around 30 toroidal and poloidal modes to accurately reproduce the vacuum magnetic field quantities. For the equilibrium calculations, the use of 31 and 61 radial grid points produced converged equilibrium quantities, and it was not necessary to consider finer grids or to extrapolate to zero mesh size. In studying the convergence of the Mercier criterion with the size of the radial grid, we did vary the grid size. Although we obtained stability results for flux-conserving ATF equilibria, which are more unstable [25], here we present only the results for zero current equilibria.

We define the peak beta as $\beta_0 = 2\mu_0 P_0 / B_0^2$, where B_0 is the magnitude of the magnetic field at the magnetic axis for the value of beta considered and P_0 is the plasma pressure at the magnetic axis, and the average beta, $\langle\beta\rangle$, as the ratio of the volume-averaged pressure to $B_0^2 / 2\mu_0$. The toroidal axis shift, normalized to the average minor radius a , of the configuration under study is

$$\Delta_T(\beta) = \frac{R_{00}(\beta) - R_{00}(0)}{a}$$

We define the equilibrium beta limit as that satisfying $\Delta_T(\beta) = 50\%$.

We note that the equilibria obtained using the RSTEQ code [26] were obtained by first averaging the 3-D vacuum fields, then using the 2-D average vacuum to obtain the finite beta 2-D equilibria.

3.2. Transformation from VMEC to Boozer coordinates

In order to perform the stability analysis, we numerically transform the equilibrium quantities from the VMEC coordinate system (Θ, ϕ) , which has non-straight magnetic field lines, to the Boozer coordinate system (θ, ζ) [27], which has straight magnetic field lines. We choose the flux coordinate s to be the same in both systems. The transformation of the angles is given by [11]:

$$\Theta = \theta + \tilde{\theta}, \quad \phi = \zeta + \tilde{\zeta}$$

where

$$\tilde{\theta} = \frac{-\chi'\phi + I\lambda}{-\Phi'I + \chi'J}, \quad \tilde{\zeta} = \frac{-\Phi'\phi + J\lambda}{-\Phi'I + \chi'J}$$

The quantities ϕ and λ are obtained in the VMEC coordinate system from the covariant and contravariant components of $\vec{B}$,

$$B^\phi = \frac{\Phi' + (\partial\lambda/\partial\Theta)}{\sqrt{g}}$$

$$B^\Theta = \frac{\chi' - (\partial\lambda/\partial\phi)}{\sqrt{g}}$$

$$B_\phi = -I + \frac{\partial\phi}{\partial\zeta}$$

$$B_\Theta = J + \frac{\partial\phi}{\partial\Theta}$$

Here the prime (') indicates the derivative with respect to the flux label s , i.e. $\partial/\partial s$, χ is the poloidal magnetic flux divided by 2π , and J and I are respectively the toroidal and poloidal net currents divided by 2π .

The flux surfaces are expressed in Boozer coordinates as

$$R = \sum_{m,n} R_{mn}(s) \cos(m\theta - n\zeta)$$

$$Z = \sum_{m,n} Z_{mn}(s) \sin(m\theta - n\zeta)$$

Likewise, the geometric toroidal angle is expressed as

$$\phi = \zeta + \sum_{m,n} \phi_{mn}(s) \sin(m\theta - n\zeta)$$

Here the array of mode numbers m and n in Boozer coordinates is obviously different from that in the VMEC coordinate system. To illustrate this point, we present in Fig. 3 the magnitude of the spectral quantities of R and Z in the Boozer coordinate system at the radial position $\rho = 1$ (i.e. at the boundary), for a configuration with $I_B = +0.23$, $\beta_0 = 9.6\%$, and a pressure profile $P \propto [1 - (\chi/\chi_b)]^2$, where χ_b is the value of χ at the boundary. With the selection and number of modes in the Boozer coordinate system shown in Fig. 3, it is possible to reconstruct the VMEC flux surfaces. The area of the small circles is related to the magnitude of the modes by

$$\text{Area}(Z_{mn}) = 1.0 + 0.2 \log \left[\frac{Z_{mn}}{(Z_{mn})_{\text{MAX}}} \right]$$

SPECTRAL QUANTITIES FOR THE BOUNDARY (Boozer coordinates)

Fig. 3. Spectral quantities for R (R_{mn}) and Z (Z_{mn}). $I_B = +0.23$ and $\beta_0 = 9.6\%$.

Here, $(Z_{mn})_{\text{MAX}}$ is the magnitude of the largest Fourier component of Z at the radial position under consideration. The transformation of coordinates is carried out flux surface by flux surface; the reconstruction of the flux surfaces corresponding to the case in Fig. 3 is shown in Fig. 4. This selection and number of modes (≈ 300) are used for all flux surfaces, values of beta, and configurations studied in this work. We performed calculations with a large number of modes in Boozer coordinates, even for low values of beta, to ensure that the equilibria were accurately transformed for all beta values considered.

4. BALLOONING MODE EQUATION AND MERCIER STABILITY CRITERION FOR 3-D GEOMETRIES

The stability of a plasma configuration to radially localized perturbations is determined by solving the ballooning mode equation. The solutions of this equation can be very extended in the ballooning variable ζ . Solutions which are almost constant along field lines are called Mercier modes; solutions which can be localized to a much narrower width, e.g. a few times 2π , are usually called ballooning modes. In other words, Mercier modes and ballooning modes are actually a single mode, and the only difference between them is their extent in ballooning space (ζ).

Thus, Mercier modes are actually limiting solutions of the ballooning mode equation. Limiting cases can almost always be treated more easily than a general problem, and this is true in solving the ballooning mode equation. The Mercier modes can be obtained as a special case, and an analytic criterion for their stability, called the Mercier criterion [28, 29] can be obtained.

The Mercier criterion is a necessary but not a sufficient condition for stability. In determining the stability properties of a plasma configuration, it is possible first to test the Mercier criterion to determine instability to radially localized modes. If a plasma is

Fig. 4. Reconstruction of the flux surfaces corresponding to the case treated in Fig. 3.

unstable according to the Mercier criterion, then it is not necessary to solve the ballooning mode equation. However, if the plasma is stable to Mercier modes, then the ballooning mode equation must be solved in the stable Mercier region of the parameter space to determine whether the configuration is stable to ballooning modes, which may have very different widths in the ballooning variable ζ . Of course, it is necessary only to solve the ballooning mode equation to determine the stability boundaries to high- n perturbations of a configuration.

Our procedure has been to test the Mercier criterion first and then to solve the ballooning mode equation. We use the term 'ballooning modes' to refer to unstable modes not using the Mercier criterion. This allows us to make a clear distinction between Mercier instabilities and ballooning instabilities. We considered only symmetric modes in this study. For most of the cases studied, these modes were localized on the outside of the flux surfaces (bad curvature region). The ballooning mode equation is described in Section 4.1 and the Mercier criterion in Section 4.2.

4.1. Ballooning mode equation

With the definitions given in Appendix A, the ballooning mode equation can be written as [9,11,30,31]

$$\frac{\partial}{\partial \zeta} \alpha \frac{\partial}{\partial \zeta} \xi + D\xi = \left(\frac{P(\sqrt{g})^2}{\Phi' A^2} \frac{2}{\beta_0} \right) \gamma^2 \alpha \xi \quad (4)$$

where

$$\alpha = \alpha(s, \zeta) = \frac{1}{g^{ss} \sqrt{g}} \left[1 + \left(\frac{\Phi' \zeta' g^{ss}}{B} \zeta + \frac{-I g_{\theta\theta} - J g_{\theta\zeta}}{B \sqrt{g}} \right)^2 \right]$$

D is the driving term,

$$D = D(s, \zeta) = - \left[\frac{P' \zeta'}{\Phi' (J \chi' - I \Phi')} \right] \left(-I \frac{\partial \sqrt{g}}{\partial \theta} - J \frac{\partial \sqrt{g}}{\partial \zeta} \right) \zeta$$

$$+\left(\frac{1}{\Phi'}\right)^2\left[\frac{\sqrt{g}P'^2}{B^2}-P'\frac{\partial\sqrt{g}}{\partial s}+\frac{P'}{B^2}(J\chi''-I\Phi'')+\left(\frac{P'}{B}\right)^2(\sqrt{g}-V')+P'\sqrt{g}B_s\bar{B}\cdot\nabla\frac{\sqrt{g}}{J\chi'-I\Phi'}\right]$$

A is the aspect ratio, and the growth rate γ is normalized to inverse Alfvén times. The radial derivative of the volume enclosed by a flux surface V' is obtained as a surface average of the Jacobian $\sqrt{g}$,

$$V' = \iint \sqrt{g} d\theta d\zeta$$

This quantity, which is also directly proportional to the integral of $(d\ell/B)$ along the field line, gives the magnetic well.

To solve the ballooning equation, we use an explicit initial value scheme, evolving the following equations in time:

$$\partial_t \Psi = \alpha \partial_\zeta \xi$$

$$\partial_t \xi = \frac{P}{b} D + \frac{1}{b} \partial_\zeta \Psi$$

$$\partial_t P = \xi$$

where

$$b = \frac{2}{\beta_0} \frac{P(\sqrt{g})^2}{\Phi' A^2} \alpha$$

The boundary condition used to find the solution is

$$\xi_{\zeta \rightarrow \pm\infty} \approx 0, \quad \left(\frac{\partial \xi}{\partial \zeta}\right)_{\zeta \rightarrow \pm\infty} \approx 0$$

Evolving these three first order equations in time is equivalent to solving the ballooning mode equation, Eq. (4), with the appropriate boundary conditions, i.e. eigenfunction equal to zero at $\zeta \rightarrow \pm\infty$.

The left-hand side of Eq. (4) has two competing terms that determine the stability of an MHD plasma: the bending term, which stabilizes the plasma, and the curvature term

(having both the normal and the geodesic curvature), which drives the instability. The curvature term is destabilizing when the direction of the magnetic field line curvature and the direction of the plasma pressure gradient are the same. The instability comes from the balance of these two terms along the field line.

4.2. Mercier criterion

The Mercier criterion is easy to use because it is an algebraic recipe which involves only the evaluation of flux surface averages of equilibrium quantities. It can easily be obtained from Eq. (4) by proposing the solution

$$\xi = Z^\delta + \tilde{f}_1(\zeta)Z^{\delta-1} + \tilde{f}_2(\zeta)Z^{\delta-2} + \tilde{f}_3(\zeta)Z^{\delta-3} + \dots$$

in the asymptotic limit $\zeta \rightarrow \infty$ and performing the standard two-space-scale analysis of the differential equation in the variables ζ and $Z = \zeta$. The criterion for stability can be written for closed flux surfaces as [17]

$$D_M = D_S + D_W + D_I + D_G > 0 \quad (5)$$

where

$$D_S = \frac{(\chi''\Phi')^2}{4} \quad (6)$$

$$D_W = P'V'' \left\langle \frac{B^2 \sqrt{g}}{g^{ss}} \right\rangle - P'^2 \left\langle \frac{\sqrt{g}}{B^2} \right\rangle \left\langle \frac{B^2 \sqrt{g}}{g^{ss}} \right\rangle \quad (7)$$

$$D_I = \left\langle \frac{B^2 \sqrt{g}}{g^{ss}} \left(\chi''J' - \chi''\Phi' \frac{\tilde{J} \cdot \tilde{B}}{B^2} \right) \right\rangle \quad (8)$$

$$D_G = - \left\langle \frac{(\tilde{J} \cdot \tilde{B})^2}{B^2} \frac{\sqrt{g}}{g^{ss}} \right\rangle \left\langle \frac{B^2 \sqrt{g}}{g^{ss}} \right\rangle + \left\langle \frac{\tilde{J} \cdot \tilde{B} \sqrt{g}}{g^{ss}} \right\rangle^2 \quad (9)$$

The angle brackets indicate integration over angular quantities; explicitly,

$$\langle (X) \rangle = \iint d\theta d\zeta (X)$$

The magnetic shear contribution D_S is always stabilizing. Its maximum contribution for the ATF device is near the boundary.

In the magnetic well contribution D_W , the first term (from the vacuum magnetic well and the toroidal shift effects) is very important because it determines the largest positive or most stabilizing contribution to the criterion (close to the magnetic axis). The second term in D_W is always negative and comes from the diamagnetic contribution of the plasma; it decreases the influence of the stabilizing magnetic well.

The first term in the net current contribution D_I vanishes for a stellarator (i.e. for zero net toroidal current equilibria). The contribution from the geodesic component of the curvature D_G enters through the Pfirsch-Schlüter current $\vec{J} \bullet \vec{B}$ [29], and it is easy to show, using the Schwartz inequality, that it is always negative. This contribution is very important for 3-D configurations.

All of the contributions play important roles in determining stability for a given configuration. Generally, D_M is determined by a delicate balance between the stabilizing and destabilizing terms, which change with beta depending on the configuration, the pressure profile and the mode of operation. For zero current equilibria, the rotational transform profile changes as beta increases. The magnetic well becomes broader in radius and deeper, the magnetic hill becomes narrower but more destabilizing, D_G effects increase as the Pfirsch-Schlüter current increases, and the magnetic shear becomes less effective for stabilization.

This criterion, applied to tokamaks with low beta and large aspect ratio, reduces to the well-known criterion of Shafranov and Yurchenko [32],

$$\frac{1}{4} \left(\frac{1}{q} \frac{dq}{dr} \right)^2 + \frac{2}{rB_0^2} \frac{dP}{dr} (1 - q^2) > 0$$

for stability.

5. ATF STABILITY RESULTS

A linear MHD stable plasma is stable to all possible solutions of the linear MHD equations. In other words, such a plasma is stable to modes with finite n , as well as to the radially localized perturbations which are solutions of the ballooning mode equation. The prevailing concept of second stability implies that as an equilibrium parameter is changed, there emerges a region that is stable to all possible MHD perturbations. Therefore, to verify the existence of a second stable region, studies of stability to low- n , intermediate- n , and high- n perturbations must be carried out.

If high- n (radially localized) modes prohibit the existence of a second stable region, then studies of finite- n perturbations are not necessary. However, stability to finite- n modes in the stable regions of localized perturbations must be studied to determine the beta limit to MHD perturbations. We have analysed stability to ideal radially localized modes by evaluating configurations with the Mercier criterion before solving the ballooning mode equation for each flux surface. These studies have examined only stability to high- n perturbations.

5.1. Comparison of 2-D and 3-D equilibrium calculations

An early set of calculations of Mercier stability for ATF used the stellarator expansion approach and examined 2-D equilibria obtained with the RSTEQ code [26]. These calculations involved only two terms in Eq. (5), the magnetic well and shear contributions. The net current and the geodesic curvature contributions cancelled out.

Figure 5 shows stability boundaries for zero current operation in an inward-shifted (by 5 cm) configuration with a pressure profile $P \propto [1 - (\chi/\chi_b)]^2$. Results from the 3-D equilibrium calculations are shown in Fig. 5(a); results from the 2-D equilibrium calculations, in Fig. 5(b).

Fig. 5. Mercier unstable regions for the inward-shifted (by 5 cm) configuration with zero current and $P \propto [1 - (\chi/\chi_b)]^2$ for (a) 3-D equilibria and (b) 2-D equilibria.

Figure 6 clarifies these results by showing the effects of the contributions to the Mercier criterion for both sets of equilibria with $\langle\beta\rangle = 2\%$. Figure 6(a) shows the contributions from the 3-D calculation; Fig. 6(b), the contributions from the 2-D calculation. Figure 6(c) shows only the contributions of the shear and the magnetic well, $D_{SW} = D_S + D_W$, for the 3-D calculation.

The radial positions at which $D_M = 0$ —which determines instability—for both the 2-D and 3-D calculations are quite close, even though the behaviour of D_M is different (it is less positive for the 3-D calculation). The unstable regions close to the boundary are similar in both calculations, although some unstable regions appear for the 3-D calculation that are absent from the 2-D calculation. Clearly the geodesic curvature contributions (identified in Fig. 6(a) as 'Pfirsch-Schlüter') are important in evaluating the Mercier criterion. The agreement between Figs 6(b) and 6(c) shows that the magnetic well and magnetic shear contributions from the 2-D and 3-D calculations are similar (implying that the rotational transform profile and the toroidal shift are similar).

The importance of the comparisons in Figs 5 and 6 is that they support the simple picture that instability to Mercier modes arises in torsatrons with shallow magnetic wells and low magnetic shear. These comparisons also indicate that considering only the magnetic well and the magnetic shear can adequately represent the Mercier stability for broad pressure profiles.

However, this simplification of the Mercier criterion—while valid for large aspect ratio, planar axis configurations—is not valid for other configurations, e.g. heliac-like configurations, for which the well and the geodesic curvature dictate the stability, as studied by Varias et al. [24]. For steep pressure profiles, the geodesic curvature contribution is very important, even at low beta; 2-D and 3-D equilibria will produce different Mercier stability results, as we discuss below.

The positions of the lowest order resonances in the unstable region, shown in Fig. 5(a), indicate the most likely low- n mode instabilities. The correspondence between

Fig. 6. Contributions to Mercier criterion with $\langle \beta \rangle = 2\%$. (a) Magnetic shear, magnetic well, net currents and geodesic curvature (through the Pfirsch-Schlüter current) contributions to the Mercier criterion D_M for the 3-D calculation. (b) Magnetic shear and magnetic well contributions to the Mercier criterion D_M for the 2-D calculation. (c) Magnetic shear and magnetic well contributions from the 3-D calculation and the sum of these contributions D_{SW} .

low- n unstable modes and the Mercier stability criterion for $\ell = 2$ torsatrons is discussed in Ref. [25], in which low- n modes studied using 2-D average equilibria were found to be stable for the equilibria considered in Fig. 5. No stability threshold was given by the low- n modes for the 2-D equilibria considered in Fig. 5. In fact, in analyses using 2-D equilibria, the ideal low- n modes for both the standard configuration and the inward-shifted (by 5 cm) configuration were found to be stable for a broad range of pressure profiles; however, recent analysis of 3-D equilibria has shown that low- n modes can also be unstable [33].

5.2. Effects of shift in magnetic axis and quadrupolar magnetic fields

We studied the ATF standard configuration and configurations shifted inwards by 10 cm, 5 cm and 2.5 cm and outwards by 2.5 cm and 5 cm. We present in Fig. 7 Mercier unstable regions of zero current equilibria with pressure profile $P \propto [1 - (\chi/\chi_b)]^2$ for the standard configuration and for the inward-shifted configurations. With this broad pressure profile, the average 2-D equilibria corresponding to the standard configuration, which has an equilibrium $\langle\beta\rangle$ limit of about 4.5%, were found to be stable to low- n modes [1,3–6].

A Mercier unstable region exists for all the configurations in Fig. 1. For the standard configuration, the Mercier stability limit is $\langle\beta\rangle \approx 3\%$. Unstable regions exist near the axis for various pressure profiles and low values of beta, but they are not a serious concern because they disappear when the pressure profile is flattened near the axis. The most unstable configuration is that shifted inwards by 10 cm, because a magnetic hill exists over the entire cross section at vacuum. The results in Fig. 7 indicate that the plasma is very sensitive to changes in the magnetic dipole moment of the configuration. Because this figure shows that inward-shifted configurations are unstable to Mercier modes for $P \propto [1 - (\chi/\chi_b)]^2$, we did not study the stability of these configurations to ballooning modes.

Fig. 7. Mercier unstable regions for ATF configurations with $P \propto [1 - (\chi/\chi_b)]^2$ and zero current. (a) Configurations shifted inwards by 10 cm and 2.5 cm. (b) Configuration shifted inwards by 5 cm and standard configuration.

The outward-shifted configurations offer a route to higher beta with regard to Mercier stability. The configuration shifted outwards by 2.5 cm has one unstable point at $\rho = 0.129$ for $\langle\beta\rangle = 0.42\%$, while the one shifted outwards by 5 cm has two unstable points at $\rho = 0.129$ and $\rho = 0.224$ for $\langle\beta\rangle = 0.34\%$.

Reference [3] describes a method for obtaining high beta plasmas in ATF by changing the current in the VF coils (and therefore the quadrupole field) as the plasma is heated. This method is based on the analysis of a sequence of average 2-D equilibria. The rotational transforms of these equilibria monotonically increase as a function of radius. We have analysed the stability of the corresponding 3-D equilibria. The stability results for Mercier modes and ballooning modes are plotted in Figs 8–12.

Some of these results are in excellent agreement with the results of the 2-D calculations. For high beta and suitable quadrupolar magnetic moment (Figs 11 and 12), there is no Mercier instability on the outside of the configuration. Likewise, the general picture for change in the rotational transform is similar to that described in Ref. [3], i.e. the rotational transform is always a monotonically increasing function of the radius for the appropriate value of I_B as beta increases.

However, our results reveal the existence of unstable ideal ballooning modes for configurations with strong quadrupole moments. We found that as I_B increases the ideal ballooning modes become narrower in ballooning angle, even though the rotational transform is monotonically increasing as a function of the radius. This occurs because the ballooning modes become more localized in the ballooning coordinate owing to strong magnetic shear, so that the ballooning modes must be very narrow in order to overcome the stabilizing influence of the magnetic field line curvature.

Our stability analysis for the configuration shifted outwards by 2.5 cm ($I_B = 0$) reveals that as beta increases, the rotational transform becomes double valued and the configuration becomes unstable to ballooning modes. The growth rate γ for the ballooning modes is plotted in Fig. 13 for $\langle\beta\rangle = 4\%$, indicating strong instabilities.

Fig. 8. Mercier unstable regions and rotational transform for different values of beta for $I_B = -0.13$.

Fig. 9. Stability boundaries and rotational transform for different values of beta for $I_B = 0.0$ (standard configuration).

Fig. 10. Stability boundaries and rotational transform for different values of beta for $I_B = +0.13$.

Fig. 11. Unstable regions and rotational transform for different values of beta for $I_B = +0.19$.

Fig. 12. Unstable regions and rotational transform for different values of beta for $I_B = +0.23$.

Fig. 13. Growth rates for ballooning modes for the outward-shifted (by 2.5 cm) configuration with $\langle \beta \rangle = 4\%$.

5.3. Effects of pressure profile

During the first phase of operation of ATF in 1988, experiments were carried out in a configuration that was shifted inwards by 5 cm owing to the existence of field errors. The pressure profile for these experiments was necessarily different from the pressure profiles considered during the design phase of ATF. Earlier studies of these experiments [21,34] used a pressure profile prescribed by

$$P = P_0 \tanh\left(12.0 \frac{\chi}{\chi_b} - 1.56\right) \quad (10)$$

This equation gives a fit [34] to the broadest possible electron temperature profile observed during the first phase of ATF operation [21] (the electron temperature profile could not be broadened further because of the reduction in volume due to the field error).

In Fig. 14 we present the toroidal shifts as a function of peak beta from calculations using 3-D VMEC equilibria and from calculations using the 2-D equilibria obtained using the classical stellarator expansion, both carried out with the pressure profile prescription of Refs [21,34]. The toroidal shifts disagree by less than 20%. For this pressure profile with $\beta_0 = 0.8\%$, we present in Fig. 15 the rotational transform calculated using both VMEC and RSTEQ as well as the pressure profile of Eq. (10), the position of the rational flux surface 2/5 and the region of the magnetic hill from VMEC.

In Fig. 16 we plot the different terms of the Mercier criterion corresponding to this value of beta. There is a Mercier unstable region close to the magnetic axis and another unstable region around $\rho = 0.5$. The most important unstable region is located where $-P'$ is maximum and the geodesic curvature is the most destabilizing.

In Fig. 17, we compare the Mercier stability results for the 3-D and 2-D equilibria. For this value of beta and the pressure profile of Eq. (10), the 2-D and 3-D equilibria show important differences; the minimum of D_M for the 2-D calculation was inside the region of the magnetic hill (the only destabilizing term), and the minimum of D_M for the

Fig. 14. Toroidal shifts as a function of peak beta from calculations using 3-D VMEC equilibria and calculations using the 2-D equilibria obtained with the classical stellarator expansion. The pressure profile is given in Eq. (10).

Fig. 15. Pressure profile given in Eq. (10), rotational transform, and magnetic hill regions for $\beta_0 = 0.8\%$.

Fig. 16. Mercier criterion contributions for the pressure profile given in Eq. (10), with $\beta_0 = 0.8\%$.

Fig. 17. Mercier criterion for 2-D and 3-D equilibria for the pressure profile given in Eq. (10), for $\beta_0 = 0.8\%$.

3-D calculation was closer to the maximum of the destabilizing geodesic curvature contribution (the most important destabilizing contribution) than to the maximum of the magnetic hill. The results of this analysis indicate that equilibria obtained using the pressure profile given in Eq. (10) are in fact stable to Mercier modes for $\beta_0 > 1.07\%$.

The next step in our analysis is to solve the ballooning equation for the pressure profile given in Eq. (10). The radial positions that are unstable to symmetric ballooning modes are shown in Fig. 18. For $\beta_0 < 1.07\%$, we obtained very extended modes along the field lines (Mercier-type modes), and the unstable region matched the Mercier unstable region. This indicates that the modes obtained by solving the ballooning equation are in fact the Mercier modes already obtained from the Mercier criterion. Beyond the small stable window, the modes became narrower in angle (true ballooning). As noted in Section 4, we use the term "ballooning modes" to refer to unstable modes not identified using the Mercier criterion. The largest growth rates were found in flux surfaces near the rational flux surfaces $\iota = 2/5$ and $\iota = 1/2$. There is no self-stabilization, as can be seen in Fig. 19, because the growth rate of the ballooning modes increases with beta for $\beta_0 > 1.527\%$. These results clearly show that plasmas modelled with the pressure profile given in Eq. (10), which was used in earlier studies of ATF experimental results [21,34], are not in an ideal MHD second stable region.

We note further that this pressure profile, although it is called the experimental pressure profile in Ref. [34], is not a fit to the plasma pressure profile and is not even a good fit for the electron temperature profile. Figure 1 of Ref. [21] shows that the electron temperature profile was measured in only a few (<10) radial positions, and these were not equally spaced. In addition, the experimental error was very large (about $\pm 15\%$). Thus, the pressure profile given by Eq. (10) and the actual experimental pressure profile may have been very different. As a result, any value of any plasma beta recorded during the experiment can differ from our estimate. In the absence of good measurements of

Fig. 18. Ideal Mercier and ballooning unstable regions for the pressure profile given in Eq. (10) and zero current mode of operation. There is no second stability region.

Fig. 19. Growth rates of the Mercier-ballooning modes located in the unstable regions presented in Fig. 18. There is no second stability region because the growth rate increases with beta for $\beta_0 > 1.527\%$.

equilibrium to reconstruct the pressure profile corresponding to a particular value of beta, there is no clear and direct relationship—nor should there be—between a beta limit recorded in the experiment and the beta limit obtained from the theoretical calculations using the pressure profile given in Eq. (10). Any agreement would be purely coincidental.

In the same way, there is no reason to believe that the highest value of beta obtained from calculations using the electron density profile as a model for the plasma pressure profile should match any value of beta measured in the experiment. Temperature and density profiles, if measured in a number of radial positions and without large experimental errors, could provide reasonably good pressure profiles for use in numerical calculations.

On the basis of the stability results presented here, we present a possible explanation for the experimental behaviour of the plasma with respect to the pressure profile. The experimental pressure profile must have been narrower than that given by Eq. (10) for $\beta_0 < 1.07\%$, but continuously broadening in order to provide a continuous stable MHD path up to $\beta_0 = 1.07\%$. For larger values of beta, there is a window of stability; then as the plasma beta increases even more, the plasma becomes unstable to ideal ballooning modes. For plasmas with $1.07\% < \beta_0 < 1.527\%$, the experimental pressure profiles may have been close to that given by Eq. (10). However, for $\beta_0 \geq 1.527\%$, the experimental pressure profiles must have been different from that given by Eq. (10), because stable plasmas would be possible with higher values of beta but with different pressure profiles. This may in fact have occurred, but the experimental pressure profiles cannot be reconstructed from the available data.

5.4. Comparison with other stability analyses

In Ref. [12], Cooper et al. reported results based on a simplified model for one of many possible ATF configurations. The vacuum configuration of this model is slightly

different from the ATF standard configuration, as can be seen by comparing the rotational transform in Fig. 2 with that in Ref. [12].

Our results on stability to Mercier and ballooning modes generally confirm those given in Ref. [12]. Using the pressure profile of Ref. [12], we find the stability boundaries to localized perturbations shown in Fig. 20. A second stability region does not appear. Our stability results are somewhat different from those in Ref. [12], as a result of the differences in the fixed boundary, the definition of $\langle\beta\rangle$, and the version of the VMEC code (the use of different straight field line coordinate systems is not important). We did not carry out a complete and detailed study of Cooper's model as he did.

Our boundaries were obtained by using the results of a field line following code as input to VMEC, as described in detail in Section 2. It is worth repeating that a small change in the boundary (to be used as input for VMEC) causes small changes in the equilibrium properties obtained by the VMEC code and consequently in the beta limit. However, the general scenario of stability does not change: finite beta equilibria obtained using different fits to the boundary have stability beta limits to the localized perturbations studied in this paper.

The results of Ref. [12] reiterate the well-known stability properties of finite beta zero current equilibria, namely that stability degrades as soon as the rotational transform profile becomes double valued. While ι' / ι remains positive on the outside, it can be negative close to the magnetic axis (tokamak-like), and ballooning modes could therefore be excited there. At the same time (as observed using a 3-D finite beta equilibrium) the magnetic hill becomes more dangerous on the outside, and Mercier modes as well as symmetric ballooning modes can become unstable there.

Fig. 20. Stability boundaries and rotational transforms for the pressure profile given in Ref. [12].

6. DISCUSSION

For the pressure profiles of Refs. [12], and [21], and the case $P \propto [1 - (\chi/\chi_b)]^2$, we find no second stable region in the 3-D ATF configurations studied. We analysed in detail the stability of ATF equilibria with a narrow pressure profile, used in Refs. [21,34] to study experimental results, and found that for low values of beta there is an unstable Mercier region. As beta increases, the plasma becomes stable, but as beta is increased further, the plasma becomes unstable to ballooning modes. Our results on stability to Mercier and ballooning modes generally confirm those given in Ref. [12], despite of the various pressure profiles used, and the study of plasma configurations with strong quadrupolar fields.

All of the currentless configurations studied, including both outward-shifted configurations and configurations with strong quadrupole fields, are unstable (close to the boundary) to Mercier modes or symmetric ballooning modes for $\langle\beta\rangle \geq 3\%$ for all pressure profiles considered. We found a stability picture similar to that described in Ref. [12]. Because symmetric ballooning modes were unstable, we did not look for nonsymmetric modes.

All configurations of ATF are consistent with regard to stability, i.e. if one configuration is unstable to Mercier and ballooning modes, then so are the others (as are model configurations of ATF)—although with different stability beta limits [13]. ATF configurations with strong quadrupole fields and monotonically increasing rotational transform are generally unstable to ballooning modes; therefore, there is no room for flexibility in these configurations with respect to high beta MHD stability. On the basis of our results, the highest beta stable plasma should be obtained experimentally in a configuration with weak quadrupolar magnetic field (i.e. close to the standard configuration).

There are some areas of agreement between 3-D and 2-D equilibrium and Mercier stability results for ATF. The agreement is better for broader pressure profiles. The equilibrium picture for ATF given by a 2-D analysis is close to that given by the 3-D VMEC code (see also Ref. [35]).

Most of the calculations used for stability analysis in the design of ATF were based on the stellarator expansion. For Mercier stability, the agreement between these calculations and the 3-D calculations is good for broad pressure profiles. With regard to ideal ballooning modes there is complete disagreement. This disagreement can be understood by remembering that the conclusion in Ref. [8] regarding the nonexistence of ballooning modes is based on the assumption that the flux surfaces are circular. As seen in Fig. 1, for equilibria with a strong quadrupolar field the flux surfaces are not circular even in vacuum. Figure 4 also shows that finite beta equilibria for configurations with quadrupolar fields have flux surfaces that do not resemble circles at all. (As an aside we report that with the VMEC code it was possible to find finite beta ATF equilibria for different pressure profiles and strong quadrupolar fields, whereas the RSTEQ code failed to do so.) It can be also seen in Ref. [3] that the flux surfaces for the 2-D ATF configurations are not circular in general. Thus, the assumption under which ballooning modes were believed not to exist in stellarators was unfounded.

Stability in ATF requires a magnetic well close to the origin and high magnetic shear close to the boundary. In stable configurations, these two quantities overcome the unstable contributions from the geodesic curvature and the magnetic hill (as in any other torsatron). For low beta, stability can be obtained if the pressure profile is flattened close to the axis and the largest values of the pressure gradient are localized close to the edge. However, for Mercier stability and for larger values of the pressure, high pressure close to the boundary is not allowed because the magnetic hill increases significantly, thus causing instabilities. It is therefore better to decrease the slope of the pressure close to the boundary and rely on self-stabilization close to the magnetic axis. The optimized pressure

profile for the standard configuration remains to be found. However, we presume that it is not far from that given in Ref. [12]. We estimate that the stability beta limit will be around $\langle\beta\rangle \approx 3\%$.

Outward-shifted configurations are stable to Mercier modes because they have a very deep magnetic well. Likewise, vacuum configurations with no magnetic well exist, and the stability beta limit for these can be very low ($\beta \approx 0$). Each configuration has its own stability beta limit, as can be seen in Figs 7–13 and 20. For the outward-shifted configurations the stability beta limit is set by ballooning modes. We did not attempt to find the largest stable beta for the ATF torsatron, so we did not carry out any optimization studies of the pressure profile for stability to ideal ballooning modes for different ATF configurations. If non-MHD effects (e.g. finite Larmor radius) are important and stabilizing, then it may be possible to obtain larger stability beta limits. These effects may be more important close to the magnetic axis and if the growth rates of the ballooning modes are small (as they are for resistive ballooning modes).

The discussion can be summarized in two points:

1. Stability studies of localized perturbations using only the Mercier criterion are incomplete, because MHD stability is determined by ballooning modes at finite beta.
2. Stability studies for finite beta, finite aspect ratio stellarator plasmas using 2-D models like the one using the stellarator expansion do not agree in general with the results using the full 3-D geometry. Even the results from the Mercier criterion are different.

7. CONCLUSIONS

Our main results are as follows.

1. We have obtained results similar to those of Ref. [12]; our more elaborate calculations support the conclusions in that work.

2. We have considered the configurations and pressure profiles used in 2-D calculations in which second stability was found for localized modes and have determined, by analysing the corresponding 3-D equilibria, that there is no second stability region.

3. Contrary to what was believed in the past, strong positive quadrupole fields do not stabilize ATF plasmas. By analysing the 3-D equilibria, we have found that ideal ballooning modes exist in regions where ideal Mercier modes are stable.

4. The stellarator expansion approach gives good agreement with the 3-D calculation for equilibrium quantities, sometimes good agreement for Mercier analysis, and disagreement in the analysis of ballooning modes.

Recent studies of stability for a similar torsatron, the Large Helical Device (LHD) [36], consider the full 3-D geometry. In these studies, MHD stability beta limits have been found not only from ideal Mercier modes but also from ideal ballooning modes and from ideal low- n modes. Stability beta limits to ideal ballooning modes and low- n modes were not found with the stellarator expansion approach. Thus, we conclude that stellarator configurations must be considered in ideal MHD stability calculations as the 3-D configurations that they are.

As noted in Section 5, there is no reason to expect agreement of (1) a beta limit calculated using a fit to a few points of the electron temperature profile to represent the pressure profile and (2) any value of 'experimental' beta not supported by good measurements of density and temperature [21] to reconstruct the experimental pressure profile. Any agreement between values of beta obtained in these ways would be purely coincidental. By contrast, in cases where exist good measurements of tokamak experimental finite beta equilibrium profiles, the comparison of beta limits from the experiment and the theory used in this paper, not only makes sense but has resulted in an excellent agreement.

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APPENDIX A. BALLOONING MODE EQUATION

The ideal ballooning mode equation can be written as [9,11,30,31]:

$$\bar{\mathbf{B}} \cdot \nabla \left\{ \frac{1}{g^{ss}} \left[1 + \left(\frac{g^{ss}}{B} \int \frac{\sum d\ell}{B} \right)^2 \right] \right\} \bar{\mathbf{B}} \cdot \nabla \xi + 2P' \left(\frac{\kappa^s}{g^{ss}} - \frac{\kappa_\zeta}{\chi'} \int \frac{\sum d\ell}{B} \right) \xi = \Gamma^2 \frac{\rho_D}{g^{ss}} \left[1 + \left(\frac{g^{ss}}{B} \int \frac{\sum d\ell}{B} \right)^2 \right]$$

In Boozer coordinates, the perpendicular wave vector is

$$|\bar{\mathbf{k}}_\perp|^2 = |\nabla(\chi'\zeta - \Phi'\theta)|^2 = \frac{B^2}{g^{ss}} \left[1 + \left(\frac{g^{ss}}{B} \int \frac{\sum d\ell}{B} \right)^2 \right]$$

The magnetic field can be written as

$$\bar{\mathbf{B}} = (\nabla_s \times \nabla\theta)\Phi' + (\nabla_\zeta \times \nabla s)\chi'$$

where the covariant and contravariant components are

$$B_\zeta = -I$$

$$B_\theta = J$$

$$B_s = -v$$

$$B^\zeta = \frac{\Phi'}{\sqrt{g}}$$

$$B^\theta = \frac{\chi'}{\sqrt{g}}$$

$$B^s = 0$$

$$B = |\vec{B}|$$

We define the metric element

$$g^{ss} = |\nabla s|^2$$

The Jacobian of the transformation is given by

$$\sqrt{g} = \frac{J\chi' - I\Phi'}{B^2}$$

The contravariant components of the plasma current are

$$J^\zeta = \frac{J' + (\partial v / \partial \theta)}{\sqrt{g}}$$

$$J^\theta = \frac{I' - (\partial v / \partial \zeta)}{\sqrt{g}}$$

$$J^s = 0$$

The curvature of the magnetic field lines is given by

$$\bar{\kappa} = \frac{1}{B^2} \nabla \left(P + \frac{B^2}{2} \right) - \frac{\vec{B}}{2B^4} \vec{B} \cdot \nabla B^2$$

The covariant components of the curvature are

$$\kappa_\zeta = \bar{\kappa} \cdot \vec{e}_\zeta = \frac{\chi'}{2B^4 \sqrt{g}} \left(B_\theta \frac{\partial B^2}{\partial \zeta} - B_\zeta \frac{\partial B^2}{\partial \theta} \right)$$

$$\kappa_s = \bar{\kappa} \cdot \vec{e}_s = \frac{1}{B^2} \left[\frac{\partial (P + B^2/2)}{\partial s} - \frac{B_s}{2B^2} \vec{B} \cdot \nabla B^2 \right]$$

The contravariant component is obtained from the equation

$$\frac{\kappa^s}{g^{ss}} = \kappa_s - \frac{\kappa_\zeta}{g^{ss} B^\theta} (B^\zeta g^{s\theta} - B^\theta g^{s\zeta})$$

The local shear is given by

$$\Sigma = \frac{1}{g^{ss}} \left(\frac{\nabla_s \times \vec{B}}{g^{ss}} \right) \cdot \nabla \times (\nabla_s \times \vec{B})$$

In our notation, the rotational transform is

$$\mathfrak{t} = \frac{\chi'}{\Phi'}$$

P is the pressure, ρ_D the density and Γ the growth rate.

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