RADIOACTIVE NUCLEAR BEAMS AND THE EVOLUTION OF COLLECTIVITY IN NUCLEI

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ABSTRACT

The advent of Radioactive Nuclear Beams (RNBs) has opened up new nuclear territories to exploration. Recent theoretical work suggests that nuclear structure in these new regions may be unlike anything observed to date and that its study may substantially alter our concepts of shell structure, collectivity and the evolution of nuclear behavior. To face the challenge of inevitably low RNB intensities, new, more efficient signatures of structure and new experimental techniques are being developed. We discuss these topics and also summarize the current status of RNB facilities and facility planning in North America.

1. Introduction

Nuclear structure physics is entering a new era and enjoying a dramatic renaissance. This is being driven by the new experimental capabilities embodied in the use of radioactive nuclear beams (RNBs) which are opening up for us realms of heretofore unaccessible nuclei in exotic regions. These nuclei include medium mass \( N = Z \) nuclei and neutron-rich species, where there is every likelihood of discovering nuclear behavior and structure unlike anything observed to date.

Aside from extensive work on developing the facility concepts to produce RNBs in the first place, the dawning of the RNB age has engendered at least four major new thrusts in nuclear physics. Nuclear astrophysics is entering a new phase where critical nuclei and critical reactions will finally be accessible to direct study. The opportunities offered by RNBs, combined with the fact that, initially, RNB intensities will be orders of magnitude lower than the stable beam intensities we are accustomed to, has led to twin efforts to develop, on the one hand, new and much more efficient instruments and measuring techniques and, on the other, simple empirical signatures of structure based on the easiest-to-obtain data. Finally, the
possibility to access truly exotic, near-drip-line nuclei has led to a major upheaval in our theoretical understanding of the properties of these nuclei. With this has come the recognition that the traditional underpinnings of nuclear structure in a more or less standard Shell Model Hamiltonian acting on single particle states occurring in familiar shells with familiar sequences of single particle energies (s.p.e.'s) and familiar residual interactions is not, in fact, an immutable edifice. Rather, the Shell Model near stability is likely to be only a particular reflection of a richer and more general framework.

The excitement engendered by the opportunities for research with RNBs is widely recognized worldwide. Major RNB facilities (primarily of projectile fragmentation type—see Section 4) already exist in North America, Europe and Japan and most of these facilities have major upgrade initiatives approved or in the planning stages. Wholly new first-generation facilities of ISOL-type (see also Section 4) are being built, and advanced ISOL facilities are being proposed.

The growing momentum embodied in this groundswell was recently reflected in the Long Range Plan (LRP) for Nuclear Science carried out in the first half of 1995 in the United States. In this document, a very strong endorsement of RNB science was made. The LRP Interim Report¹ stated that “The scientific opportunities made available by world-class radioactive beams are extremely compelling and merit very high priority.” The first recommendation for new construction initiatives in the LRP states that “We strongly recommend the immediate upgrade of the MSU facility to provide intense beams of radioactive nuclei via fragmentation”, and that ”We strongly recommend development of a cost-effective plan for a next generation ISOL-type facility and its construction when RHIC construction is substantially complete.” This recommendation explicitly recognizes the complementary role of the two principal approaches to the production and use of RNBs.

The remainder of the present discussion will focus on three of the four thrusts cited above, namely a qualitative summary of some of the new theoretical ideas and concepts for exotic nuclei; the development of new signatures of structure (exploiting a different framework for viewing nuclear structure); and the adaptation of experimental techniques to the challenges offered by RNBs. We will also include a brief summary of the current (late 1995) status of RNB facilities in North America.

2. Structure of Exotic Nuclei

Recently, there has been a growing recognition that the exotic nuclei made accessible with RNBs may have structure and properties quite different from anything observed to date. Radical changes in shell structure, residual interactions, symmetries, collective modes and the evolution of structure are envisioned. Even the basic concepts of nuclear shape and surface and shell structure themselves are being re-evaluated. Coupling to the continuum must be taken seriously. New forms of nuclear matter(e.g., halo nuclei) have already been observed, and their study expands the horizons of our science. It is therefore useful to review a few of the concepts motivating, and resulting from, recent pioneering theoretical efforts to understand
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exotic near-drip-line nuclei.

2.1. Proton Rich Nuclei—the $N = Z$ Line

On the proton rich side, for example, the heavy $N = Z$ nuclei from $^{80}$Zr to $^{100}$Sn will become accessible to serious study. $^{80}$Zr is known to be deformed. (In itself, this is an interesting phenomena in that $N, Z$ values of 40 are, in some nuclei, magic numbers.) $^{100}$Sn is expected to be the heaviest bound self-conjugate nucleus and is thought to be itself doubly magic. In $N = Z$ nuclei the $T = 0$ residual interaction dominates. As can be easily understood$^2$ from Wigner’s supermultiplet theory, this dominance is essentially a singularity: even at $N = Z \pm 2$, the $T = 1$ interaction has already assumed its traditional advantage. Therefore, $N = Z$ nuclei form a unique “trajectory” through the nuclear chart. The $^{80}$Zr to $^{100}$Sn region may therefore constitute an isolated spherical-deformed transition region, which is effectively a 1-dimensional slice through the $N/Z$ plane, and the only new shape transition we will ever be able to study that is controlled by the $T = 0$ interaction. A slice perpendicular to the $N = Z$ line could show the most rapid and dramatic phase transitions yet observed. Of course, if $^{100}$Sn turns out to be deformed, the consequences are even greater and strike at the heart of our understanding of the underlying shell structure of nuclei. There are many other features of interest in $N \sim Z$ nuclei. For example, with a strong $T = 0$ residual interaction, the coupling and alignment of high $j$ orbits due to the Coriolis force is likely to be quite different than in other nuclei. Backbending and band crossings may be delayed.

2.2. Neutron Rich Nuclei—New Manifestations of Structure and Collectivity

The neutron rich side of stability, which we focus on in this paper, is even more interesting. In part this is due to the longer “lever arm” from the valley of stability, and hence the greater uncertainty in extrapolation, but, in greater measure, it is due to the wealth of exotic phenomena expected near the neutron drip line. The rationale for this is simple. Near the edges of stability on this side, the outer realms of nuclei are comprised, essentially, of an extended, diffuse, low-density region of nearly-pure neutron matter. As suggested in recent studies$^3$, such a diffuse density distribution is unlikely to be able to support a shell model potential with sharp contours. Hence, even the traditional form of the Shell Model Hamiltonian itself, long cherished as the underlying basis for nuclear structure, may be radically altered. Its Woods-Saxon form may go over into a more rounded harmonic oscillator shape. In the language of the Nilsson model, this corresponds to the vanishing of the “$l^2$” term.
Fig. 1. Single particle energies in the standard Shell Model (middle), in a scenario with no $l \cdot s$ term (left), and with no $l^2$ term (right).

The consequences of this are hard to overestimate. Consider the single particle levels that would result in such a scenario. These are illustrated on the right in Fig. 1. The effects of the no-$l^2$ scenario are to drastically raise the energy of the unique parity orbits which return to their parent shells. Hence, the magic numbers change. Naturally, high spin phenomena and octupole correlations will also be dramatically altered since they depend in essential ways on the location of these special orbits. But, more profound changes occur. The traditional order (see Fig. 1, center) of the normal parity orbits, namely monotonically decreasing in $j$ by $\Delta j = -1$ as their energy increases is completely upset. Instead, one encounters a "nested" pattern of $j$ orbits with the highest $j$ orbits surrounding the middle $j$, which in turn enclose still lower $j$ orbits, and so on. Moreover, the $j$ spin sequence is now $\Delta l = \Delta j = -2$ throughout (recall that $1/2 \rightarrow 3/2$ is also $\Delta j = 2$). It can easily be imagined that the manifestations of collectivity, and especially its evolution with $N$ and $Z$, could be significantly different than near stability since the quadrupole force specifically favors the coupling of orbits with $\Delta l = \Delta j = 2$. Figure 1 also illustrates an alternate possibility, equally intriguing, namely the absence of the spin orbit interaction. There are suggestions that this could characterize some neutron rich nuclei or perhaps only the proton orbits inside the diffuse neutron skin.

The effects on structure near the neutron drip line go beyond such scenarios,
However. The weak binding of the outermost neutrons and the proximity of quasi-bound levels means that coupling to the continuum must be considered. Moreover, as the uppermost orbits in a shell merge into the continuum (perhaps as quasi-bound levels) the remaining bound levels no longer constitute a complete sequence of \( j \) values from some \( j_{\text{max}} \) down to \( j = 1/2 \). In the example in Fig. 1 (right), for example, the first orbit into the continuum would be \( j = 7/2 \). The distorted single \( j \)-shell sequences, and the merging into and coupling with the continuum, could easily change the symmetries [such as SU(3) or pseudo-SU(3)] associated with the fermionic states.

Beyond this, residual interactions will also be different. The pairing interaction near the drip line, where wave functions are greatly extended, could become significantly stronger. The neutron excess, leading to a difference in proton and neutron orbits, will alter the p-n interaction as well.

Finally, geometrical concepts of the nucleus are unlikely to survive intact. Just as the diffuse outer zones of neutron rich nuclei may not support sharply defined shell model potentials, they may not support strongly defined surfaces. For example, the outer boundaries of such nuclei may be, in effect, spherical. Yet the well-defined inner core may be deformed and might even rotate or vibrate relative to the neutrons. Exotic isovector collective modes might occur.


Traditionally, nuclear structure is viewed in a "vertical" sense in terms of the excitations in a specific nucleus or in a small region of nuclei. Virtually all models and experiments are designed to look at nuclear structure in this way. Yet, the present situation in nuclear physics offers an alternative approach.

Over the years, a tremendous body of nuclear structure data has been built up through countless experiments and a number of different types of structure have been identified such as nuclei near closed shells, vibrational nuclei, rotational nuclei, and many varieties of intermediate forms. The vast reservoir of data now offers a unique opportunity to explore nuclear structure in a horizontal or evolutionary way.

This can be done by studying correlations of collective observables either with external quantities or with other collective observables. From such studies a growing realization has emerged that the seemingly-complex evolution of structure across the nuclear chart can in fact be viewed very simply.

Fascinating as the concepts of structure for exotic nuclei discussed above may be, they will be of little use if we cannot identify new manifestations of structure from the limited data that will be obtained with RNBs. Of course, in many cases, detailed spectroscopy will be necessary, including nucleon transfer reactions and \( \gamma \)-ray spectroscopy. However, frequently, insights into exotic nuclei can be provided by exploiting these correlations to provide new signatures of structures based on the easiest-to-obtain data. Of course, correlations such as these may or may not apply in new regions. If they do, they provide an evolutionary paradigm. If they do not,
their breakdown will itself signal the radical kinds of structural and evolutionary changes we have been discussing. In this section, we will discuss a few of these signatures. Others have also been developed, and work is continuing on new ones.

The $N_pN_n$ scheme\textsuperscript{4} (recently reviewed in ref. 5), in which collective observables are plotted against the product of the number of valence protons and the number of valence neutrons, has been a valuable method to disclose the underlying simplicity that is normally hidden in the complexity of nuclear data. The scheme is based, of course, on the assumption that the evolution of collectivity is largely controlled by the residual valence $p-n$ interaction and that the integrated $p-n$ interaction strength, averaged over all occupied valence orbits, approximately scales as $N_pN_n$. It is fitting to show some recent results of the $N_pN_n$ scheme (and the related $P$-factor\textsuperscript{6}) here since it was at the 1984 School in Poiana Brasov that the concept was first introduced.

A fascinating aspect of the $N_pN_n$ scheme is that $N_pN_n$ values for many nuclei far from stability are actually less than for known nuclei closer to stability. For example, $N_pN_n$ for $^{166}$Dy is 288 while $N_pN_n$ values for $^{146}$Xe, $^{150}$Ba and $^{160}$Sm are only 40, 72 and 192, respectively. Thus the properties of many exotic nuclei can be predicted in the $N_pN_n$ scheme by interpolation rather than extrapolation.

We illustrate the idea in Figs. 2 and 3. Figure 2 shows normal and $N_pN_n$ plots for $E(2^+_1)$ in the $A = 150$ region. The enormous simplification in the $N_pN_n$ scheme is typical. Note that, in using the scheme, it is, of course, necessary to know the nearest magic numbers in order to properly count $N_p$ and $N_n$ and to take account of important spherical subshell gaps. Indeed, we shall see that the $N_pN_n$ scheme can be used to give information on the underlying shell structure. For now, we just note that, in Figs. 2, 3, $Z = 64$ is taken as a subshell gap for $N < 90$ but not for $N \geq 90$.

Using the $N_pN_n$ plots in Fig. 2, it is trivial to make predictions for $E(2^+_1)$ in unknown nuclei simply by reading off the curve in Fig. 2b at the appropriate $N_pN_n$ value. Such predictions are shown in Figs. 2a,3. The spread of the data in Fig. 2b suggests that these predictions are accurate to about $\pm 15\%$. The reader is encouraged to attempt to use the systematics of Fig. 2a to make similar predictions. The uncertainties will often be much larger and the results sometimes simply wrong. To illustrate this, Fig. 2a shows (as dashed curves) the $N_pN_n$ predictions taken from the right side. Clearly, the predictions from Fig. 2b are hardly what one would have predicted if one had only traditional types of systematics like those shown in Fig. 2a. $^{142}$Xe provides a nice example where the $N = 88$ and $Z = 54$ trendlines in Fig. 2a suggest quite different extrapolations. When the idea of Fig. 2b was first proposed, $^{142}$Xe was unknown. Subsequently, it has been studied\textsuperscript{7} with the result $E(2^+_1) = 0.287$ MeV compared to the $N_pN_n$ prediction of 0.300(40) MeV.
Fig. 2. Correlation of $E(2^+_1)$ with $N$ and with $N_pN_n$ in the $A \sim 150$ region. On the left, the dashed lines are the predictions given by the $N_pN_n$ scheme of Fig. 2b. They are not the most obvious extrapolations one would make if guided only by the trends in Fig. 2a itself.

Fig. 3. Predictions of the $N_pN_n$ scheme for $E(2^+_1)$ (in MeV) in the $A \sim 150$ region. Based on Fig. 2b.
Despite the obvious utility of $N_pN_n$, its actual numerical value is not too meaningful due to different shell sizes. A useful quantity whose absolute value conveys a more precise physical idea is the $P$ - factor, defined as

$$P = \frac{N_pN_n}{N_p + N_n}$$

$P$ is a “normalized” $N_pN_n$ giving the number of $p-n$ interactions per valence nucleon. More significantly, it is the number of valence $p-n$ interactions divided by the number of pairing interactions and is therefore related to the ratio of the integrated strengths of these interactions. Since typical $p-n$ matrix elements are $\sim 0.2$ MeV, and the pairing interaction strength is $\sim 1$ MeV, $P \sim 5$ corresponds to the point at which the $p-n$ interaction strength begins to dominate the pairing strength. It is hardly surprising that nuclei become deformed for $P$ values near 5.

![Graph](image)

**Fig. 4.** Comparison of empirical $\epsilon/\Delta$ and $P$ values. Ref. 8.

There is an elegant way of demonstrating that $P$ senses the competition between $p-n$ and pairing interactions. An empirical measure of this competition, the ratio $\epsilon/\Delta$, where $\epsilon$ is the usual quadrupole deformation and $\Delta$ is the average pairing gap [$\Delta = (\Delta_p + \Delta_n)/2$], can be empirically extracted from $B(E2)$ values and odd-even mass differences. A nearly global comparison of $\epsilon/\Delta$ values with $P$ is shown in Fig. 4. The match is excellent, even in fine details.

The significance of this is worth stressing: it is possible to account for the evolution of collectivity and the competition between deformation and spherical-driving forces without resorting to complex microscopic calculations or to multi-parameter phenomenological schemes. Instead, reproduction of the behavior of
virtually all nuclei from \( Z = 40 - 100 \) is obtained merely by multiplying two numbers and dividing by their sum.

### 3.1. \( B(E2) \) values and magic numbers

We mentioned above that the \( N_pN_n \) scheme can be useful in identifying magic numbers and hence in elucidating the underlying shell structure. We give two illustrations of this.

**Fig. 5.** \( B(E2) \) values as a function of \( N_pN_n \) for the \( A \sim 100 \) region under three assumptions concerning magic numbers and shell gaps. (See text.)

The basic idea is that \( B(E2 : 2^+ \to 0^+) \) values (hereafter "\( B(E2) \) values") behave simply in \( N_pN_n \) plots. Of course, a previso for this is an accurate counting of \( N_pN_n \). If shell structure is altered from that defined by the traditional magic numbers, or is unknown, the use of incorrect \( N_p \) and \( N_n \) values can generate large deviations from simple patterns. Figure 5 illustrates this for the \( A \sim 100 \) region. On the left, standard magic numbers are used, namely \( Z = 40, 50 \) for \( 50 < N < 60, Z = 28, 50 \) for \( N \geq 60 \). A compact, indeed linear, correlation is achieved. If the well known subshell at \( Z = 40 \) for \( N < 60 \) were *not* assumed, however, the correlation becomes messy (Fig. 5, middle). Suppose further that \( N = 50 \) is not taken as a magic number. Then the plot on the right of Fig. 5 is obtained. Clearly, the compact correlation for \( B(E2) \) values against \( N_pN_n \) is destroyed. Even in advance of specific level scheme data on \(^{106}\text{Sn}\), this certainly suggests that the \( Z = 50 \) proton shell closure is intact, at least for nuclei within a few proton or neutron numbers of 50.

A second example concerns \(^{32}\text{Mg}\). Although it has \( N = 20 \), it is a quasi-deformed nucleus. This suggests that \( N = 20 \) is not magic. We can test this notion using the \( B(E2) \) value. If \( N = 20 \) were magic, then \( N_n \) would be zero as would \( N_pN_n \), giving a near-vanishing \( B(E2) \) value. If \( N = 20 \) is not magic, then the nearest closed neutron shell is \( N = 28 \), and \( N_pN_n = 4 \times 8 = 32 \). This \( N_pN_n \) value gives
$B(E2: 2^+_1 \rightarrow 0^+_1)_{N_pN_n} \sim 0.04e^2b^2$ in good agreement with the measured value of $0.045(8)e^2b^2$.

3.2. Hexadecapole Deformations and $B(E2)$ Values

Finally, we show one more illustration of the use of the $N_pN_n$ scheme. In this case, it (or, equally well, the $P$ factor) provides semi-quantitative information on subtle aspects of nuclear shape, in particular on hexadecapole deformations ($\beta_4$).

![Graphs and Diagrams]

Fig. 6. (Top) $B(E2)$ values against $N_pN_n$. (Bottom) $B(E2)_Q$ values [i.e., $B(E2)$ values corrected for $\beta_4$—see text] against $N_pN_n$. Based on ref. 10. Crosses denote nuclei where $\beta_2, \beta_4$ values from ref. 11 were used to correct $B(E2)$ values: in all other cases, empirical $\beta_2, \beta_4$ values were used.

Figure 6a shows $B(E2)$ values against $N_pN_n$ for all even nuclei with $Z > 28$. Although the correlation is simpler than one against $N$ or $Z$, it seems to show several branches. To understand the origin of this phenomenology, note that the quadrupole moment

\[ Q(2^+) \sim \beta_2(1 + 0.3604\beta_2 + 0.9672\beta_4 + 0.3277\frac{\beta_4^2}{\beta_2} + \ldots) \]  

depends on $\beta_4$ and on its sign. Since $\sqrt{B(E2)} \propto Q$, the $B(E2)$ values do also. In any major shell $\beta_4$ deformations have a characteristic systematics. They are large and positive early in a shell, cross zero near mid-shell and then turn large
and negative. Thus, from eq. 2, $B(E2)$ values are increased by the effects of hexadecapole deformations early in a shell and decreased later on. This leads to the curvatures evident in Fig. 6a. If, however, the effects of $\beta_4$ deformations are removed from the $B(E2)$ values (by moving the $\beta_4$ terms to the left hand side of eq. 2), giving the values that would arise if $\beta_4$ were zero, the resulting $B(E2)$ values (which we call "$B(E2)Q$" values since they reflect the contributions from quadrupole deformations alone) are nearly straight against $N_pN_n$ as shown in Fig. 6b.

The linear trajectory means that if $B(E2)$ values are measured in new nuclei, deviations from such trajectories can be used to estimate unknown $\beta_4$ values. Figures 6c,d illustrate this by showing $B(E2)$ and $B(E2)_Q$ values for actinide nuclei. Clearly the $B(E2)_Q$ values lie on a nearly straight line trajectory. We note that, at the far right of Fig. 6d, the $B(E2)_Q$ values for $^{250,252}$Cf (obtained with theoretical $\beta_2,\beta_4$ from ref. 11) fall well below the trendline. One can estimate the required $\beta_4$ to move these two points to the line. We obtain $\beta_4 \sim -0.07$. Interestingly, theoretical calculations do not give a negative value. Thus, not only do we extract a semi-quantitative estimate of $\beta_4$, but we can address possible improvements to calculations of the heaviest elements.

3.3. $R_{4/2}$ values and underlying shell structure

In Section 2, we discussed possible scenarios for new shell structure near the neutron drip line that have recently been discussed$^3$, including the so-called "no-\(l^2\)" case. Though it is hardly likely that this scenario will be realized in the extreme limit shown in Fig. 1, it is interesting to ask if clues to such changes in shell structure could be obtained from simple observables. This has recently been discussed in ref. 12 where we proposed that the simple ratio $R_{4/2} \equiv E(4^+)/E(2^+)$ is sensitive to the $j$ and $\Delta j$ shell sequences.

Consider two like nucleons in a single-$j$ configuration, $|j^2J>$, under the influence of a short range interaction (surface $\delta$ force). Then, $R_{4/2} \sim 1.2$. Similar $R_{4/2}$ values arise for $|j^nJ>$, and for a multi-$j$ configuration of the type $|j_1^{n_1}j_2^{n_2}J.....J>$ where $n_1,n_2.....$ are even. Consider, however, a two particle system with a particle in each of two orbits, that is, $|j_1j_2J>$. If $|j_1 - j_2| \neq 2$, then the $2^+$ state is formed by a non-co-planar alignment of angular momenta. Hence, with a short range interaction, the $2^+$ state will not be lowered substantially relative to the $4^+$ state: $R_{4/2}$ remains near 1.2. However, suppose $|j_1 - j_2| = 2$. Then the $2^+$ state requires co-planar alignment of the two angular momentum vectors and hence a large overlap and interaction, and thus the $2^+$ state is strongly lowered. This gives larger $R_{4/2}$ values, which can even attain magnitudes of $\sim 1.8$ for a 2-particle system. These simple ideas may account for the existing phenomenology of $R_{4/2}$ values in magic nuclei and may provide a signature for "$\Delta j = 2$" $j$-shell structure near the neutron drip line. Figure 7 shows that, in the $s-d$ and $p-f$ shells, where, in fact, $\Delta j = 2$, empirical $R_{4/2}$ values rise even above 1.5. Only in heavy nuclei, where the $j_{\text{max}}$ state has descended to the next lower shell (where it is the unique parity orbit), and the remaining $j$-sequence is $\Delta j = 1$ (see Fig. 1, middle panel)
does $R_{4/2}$ approach $\sim 1.2$. These empirical systematics are easily reproduced by schematic calculations with a surface $\delta$-force as shown on the right (solid line) in Fig. 7. However, superposed in this figure are two calculations of $R_{4/2}$ using the $\Delta j = 2$ no-$l^2$ scenario of Fig. 1$^{13}$. Clearly, the $R_{4/2}$ values calculated with no $l^2$ term are anomalous, and such large $R_{4/2}$ values in new regions made accessible with RNBs could signal major changes in underlying $j$-structure.

![Graph showing empirical $R_{4/2}$ values for neutron magic nuclei](image)

Fig. 7. (Left) Empirical $R_{4/2}$ values for neutron magic nuclei. (Right) Calculations of $R_{4/2}$ with a $\delta$-function interaction using "normal" single particle energies and smoothly decreasing interaction strength (solid line) and for the "no-$l^2$" scenario of shell model s.p.e.'s in Fig. 1 (two upper points). Based on refs. 12,13.

3.4. Correlations between Collective Observables

We now discuss another kind of correlation, in this case between collective observables themselves, $E(4^+_4)$ and $E(2^+_2)$. Considering that the ratio $R_{4/2}$ varies from $\sim 1.2$ for some magic nuclei to $\sim 3.33$ for good rotors, it might seem unlikely that $E(4^+_4)$ would be simply correlated with $E(2^+_2)$. Yet it is, and the correlation is simpler, more universal, and more informative than could possibly have been expected$^{14,15}$.

We show this in Fig. 8 which includes all even-even nuclei with $Z$ between 50 and 82 regardless of $R_{4/2}$ value. The data display a tight correlation characterized by three linear segments. Each segment has a specific, physically meaningful slope, and the transitions between segments are extremely rapid. At the lowest $2^+_2$ energies, the data are well reproduced by a slope of 3.33. This segment is the rotor region and is not unexpected. The next segment, though, certainly is. All the nuclei from $E(2^+_2) \sim 150$ keV to $\sim 500$ keV lie on a linear trajectory with a least squares fitted slope of 2.00. That is

$$E(4^+_4) = 2E(2^+_2) + \epsilon_4$$

(3)

This is the equation for an anharmonic vibrator (AHV). The remarkable point
is not that the AHV can reproduce the energies of any individual nucleus, but rather that all these nuclei are fit with a constant intercept $\epsilon_4 = 0.16(1)$ MeV. Physically, in the AHV, $\epsilon_4$ is the anharmonicity and can be viewed as reflecting the phonon-phonon interaction. Clearly, from the $R_{4/2}$ values (approximate $R_{4/2}$ values are given at the top of the figure), the structure of the nuclei in the AHV region is changing continuously. Nuclei in this segment of the plot range from near rotors, to transitional nuclei, to $\gamma$-soft structures to traditional anharmonic vibrators and to near harmonic vibrators. Yet, somehow, they all display nearly the same $\epsilon_4$ value. A microscopic understanding of this has not yet been found, but IBA calculations\textsuperscript{16} do reproduce it automatically. Analysis of these calculations (see the analytic derivation in ref. 17) shows that they inherently embody a good phonon structure\textsuperscript{18} and that the near-constant anharmonicity emerges naturally for a wide variety of parameter values. Evidently, eq. 3 with constant $\epsilon_4$ must result from rather general features of shell structure and residual interactions. In passing, we note that many other features of nuclear excitations are also described well by the AHV. These include yrast $B(E2)$ values and energies extending to much higher spins, and $\gamma$- and $K = 0^+$-band energies as well\textsuperscript{19,20}.

![Fig. 8. Correlations of $E(4^+_1)$ with $E(2^+_1)$ for $Z = 50-82$.](image)

The third segment of Fig. 8 is the pre-collective regime with $R_{4/2} < 2$. Herein, again, a linear behavior is noted and the slope is unity. This value also has a simple explanation. It is not, as might first be thought, a reflection of the seniority scheme in which all yrast $| j^nJ >$ energies are constant. This is generally not the case empirically. Indeed, it is a rare occurrence, the Sn isotopes being the best case.

However, if the wave functions of the $n$-particle nucleus are constructed from those of the $(n-2)$-particle nucleus by addition of the $(n = 2)$-particle wave function
of the $0^+$ state, then the excitation energies of the $2^+_1$, $4^+_1$, ..... levels will change identically since they differ, in the $n$- and $(n-2)$-particle nuclei, by exactly the same 2-particle wave function$^{15}$. Hence $E(4^+_1) - E(2^+_1)$ will be constant. But this is equivalent to $E(4^+_1) = E(2^+_1) + \text{constant}$ throughout or, in other words, to a linear relation with slope of unity and constant intercept as observed.

We thus see from Fig. 8 that the entire, seemingly complex, evolution of structure across major shells can be subsumed into a "tri-partite" classification of rotor, AHV, and pre-collective regimes. The persistence of these linear regimes forces a squeezing of the span of $2^+_1$ energies where the transition from one structural paradigm to another occurs. Hence these transition regions become very rapid and, indeed, seem to be describable in terms of the same critical phase transitional behavior (and power law equations) that have been used to describe phase transitions in magnetic and thermodynamic systems. Thirdly, the $2^+_1$ energy takes on a new role, namely that of a kind of critical parameter, and the phase transitional behavior lies not in the behavior of a single nuclear species but in the relative behavior of nuclei as $N$ and $Z$ change. These conclusions suggest a re-examination of the applicability of phase transition concepts to nuclei, even though the structure of these nuclei is dominated by a few valence nucleons. These concepts have recently been shown$^{21}$ to apply to odd-$A$ nuclei as well.

4. RNB Facilities and Experiments

4.1. Status of North American RNB Activities

Worldwide interest in RNBs has grown enormously in the last five years. There are two complementary approaches to the production and use of RNBs—projectile fragmentation (PF) and the ISOL (Isotope-Separator-On-Line) technique. These are illustrated in Fig. 9. In this section we summarize these production methods and the status of North American RNB activities. The reader is referred to the recent update of the ISL (IsoSpin Laboratory) White Paper$^{22}$ and to individual facility proposals.$^{23,24,25,26}$

In PF, heavy ion projectiles impinge on a thin light target. The collisions lead to the production of a large variety of exotic species which exit the target at nearly the incident beam velocity. Downstream, the RNBs are separated, and can be focused onto a target for the study of secondary reactions, perhaps leading to even more exotic nuclei. The PF approach produces RNBs virtually immediately and at high energies, typically 50-200 MeV/A. All reaction products are available but, of course, careful separation is necessary. With beam energies well above the Coulomb barrier, the technique is best suited to reaction studies, although a wealth of exciting structure studies (e.g., of halo nuclei) have been performed at such facilities.
The ISOL technique is very nearly the inverse: light projectiles \((p,n,\text{He})\) or light heavy ions (e.g., \(^{12}\text{C}\)) bombard a heavy, thick target. The reaction products are produced at low velocity, diffuse to the surface, are ionized, and accelerated in a post-accelerator. The technique in chemically selective and inherently involves a delay—between production and ionization—which depends sensitively on diffusion, desorption and ionization rates. On one hand, this can be an advantage—a given element is selected before acceleration, but it is also a disadvantage in that some elements cannot be easily produced, and delay times for others are long enough that decay intensity losses are large. With final energies up to \(\sim 10\) MeV/A, this method is ideal for most nuclear structure and nuclear astrophysics experiments.

Most existing RNB facilities are of PF type. The principal ones worldwide are at RIKEN, GSI, MSU, and GANIL. They have been among the pioneers in developing the field of RNB physics and in revealing the exciting research opportunities it provides. RIKEN, MSU and GANIL all have proposed upgrades. That at MSU
involves the coupling of the K500 and K1200 cyclotrons, and an improved RNB separation system. Improvements in RNB intensities will often be 3 orders of magnitude. For example, $^{11}$Li beams should increase from $\sim 10^3$ p/s to $\sim 10^6$ p/s and $^{132}$Sn beams from a couple of particles per sec to $\sim 10^4$ p/s.

In the ISOL approach, an existing facility at Louvain-la-Neuve, Belgium, is aimed primarily at the study of astrophysically important reactions. In North America, two first-generation ISOL initiatives have been approved, HRIBF at Oak Ridge and ISAC-1 in Vancouver, Canada. Construction of the former is nearing completion and an experimental program focusing on nuclear astrophysics and nuclear structure should be in place in 1996. The facility utilizes the ORIC K105 cyclotron as driver and the 25 MV Tandem as re-accelerator. Initially, the focus will be on light and medium mass neutron-deficient RNBs. Initial beams of F, As and Ga are likely. The maximum RNB energies will be 20 MeV/A for $A = 10$ and 5 MeV/A for $A = 90$. There are plans for the use of actinide targets to produce a wide range of neutron-rich fission-product RNBs.

The ISAC-1 facility will use 200-500 MeV protons from TRIUMF. After ionization and separation, the exotic nuclei can be used at low energies in trapping experiments, or accelerated to the 100 keV/A range for low-energy astrophysics experiments. It is planned to build a linear post-accelerator for RNB energies up to $\sim 1.5$ MeV/A. Initially, beams will be limited to $A/q < 30$.

Several North American laboratories are interested in second generation ISL-like RNB facilities. Both the HRIBF and ISAC-1 installations could be upgraded. In the former case, an upgraded driver cyclotron and a booster linac after the Tandem are envisioned. In the latter case, further linac sections would boost the final energy. Argonne National Laboratory is also actively developing a proposal for an advanced ISOL facility using the existing ATLAS machine as post-accelerator along with its beam line and instrumentation infrastructure. The driver would be a 215 MeV linac that would produce intense beams of protons (1 mA), $d$, $^3$He and $\alpha$ particles. Light-heavy ions such as $^{12}$C would also be available. An innovative concept is to use 200 MeV deuterons from the driver to bombard a $^{238}$U target in order to produce a 0.1 pmA neutron beam that would then bombard a second (e.g., U) target to produce exotic nuclei. The use of neutrons as production beam has the intriguing advantage of much lower heat generation in the production target. The heat that is produced essentially all comes from nuclear reactions that produce exotic nuclei as well. Due to slower energy loss of the neutron beam compared to protons, quite thick targets can be used.

4.2. Low Energy Coulomb Excitation with RNBs

Of course, an extremely wide range of experimental approaches to the use of RNBs are possible. They may be used at low energies in atomic physics experiments. Many critical astrophysics experiments will rely on them. Exotic nuclei produced at RNB facilities can be studied in their own right through decay or scattering experiments. Finally, RNBs can be used to initiate secondary reactions that
can lead to even more exotic nuclei. The recent study by Dasso et al.\textsuperscript{27} highlights, for example, some of the extremely neutron-rich nuclei that can be produced with reactions of very neutron-rich RNBs on stable heavy targets. Access to the super-heavy region should also be possible. It is, of course, impossible to summarize all applications here. The reader is referred to a number of recent overviews\textsuperscript{22,23,24,25,26}.

Here we discuss only one application, low energy Coulomb excitation (CE) in inverse kinematics since it is ideally suited to obtaining the kind of basic nuclear structure information (e.g., $E(2^+_1)$, $R_{4/2}$ and $B(E2: 2^+_1 \rightarrow 0^+_1)$ values) that provides the new signatures of structure discussed above. The use of inverse kinematics—bombardment of light targets by heavy nuclei—allows the RNBs to exit the target region in a narrow forward cone, taking their radioactive background radiation with them. With, for example, an $A \sim 150$ RNB on a light $^{12}$C target ($\sim 1$ mg/cm$^2$) all beam particles exit the target region in a cone of $\lesssim 5$ deg. The other critical element in the technique is to use low energy RNBs, of about 1.5-3.5 MeV/A, so that only the lowest $2^+$ state (and, perhaps the $4^+$) is excited. This enormously simplifies the analysis since real and virtual CE of other states can be neglected and the data reduction involves only one $E2$ matrix element. There are several methods that can be used to measure the CE yields. If the $2^+_1 \rightarrow 0^+_1$ transition is highly converted, Si PIN diode electron detectors can be used. Also, plunger techniques, taking advantage of the Doppler shifts, can be exploited. Here, we will discuss extraction of the yields from direct measurement of the de-excitation $\gamma$ rays.

Given the low beam energy, the deexcitation spectra following CE of even-even nuclei are very simple. Aside from x-rays, they only contain a single peak. Hence Doppler broadening ($\sim 5\%$ at 1.5 MeV/A) and energy resolution are not issues of concern, and NaI(Tl) detectors may be used (along with a Ge monitor detector). There are two approaches depending on the lifetime of the $2^+_1$ state. These techniques involve observation of the absolute yield or measurement of the in-flight decay curve and are applicable to lifetimes in the ps or ns regimes, respectively. In the former method, the CE target is placed in the central bore tube of a (say) 3"x3" through-well NaI(Tl) detector that has about 90\% geometrical coverage. The high efficiency of this device allows rapid accumulation of sufficient statistics so that $E(2^+_1)$ and the $B(E2)$ value can be measured in about a day even at RNB intensities of only $10^5$ p/s. Of course, with this method, one must know the RNB intensity, which can be measured, for example, with a downstream scintillator that can also serve as a trigger on the NaI detector if background from scattered upstream RNBs is large.

For larger $2^+_1$ state halflives, decay will occur in flight downstream from the target. In this case, one mounts a linear array of NaI detectors along the beam tube as illustrated in Fig.\ 10. Each detector sees a certain time window depending on its downstream distance. Their respective counting rates then directly give the lifetime via a measurement of the decay curve. It is not necessary to know the beam intensity.

In an instrument of this type that we have developed, and tested with stable Os beams ($A \sim 190$), we have used semi-annular NaI detectors surrounding a 1"
Fig. 10. Top) Schematic depiction of the CE technique discussed in the text; Bottom) Ge and NaI spectra taken with an $^{188}$Os beam at $\sim 267$ MeV incident on a $\sim 1$ mg/cm$^2$ $^{12}$C target.
diameter beam tube. This is the minimum diameter that permits all scattered RNB particles to exit the beam tube downstream without landing on its walls. (If they did there could be a buildup of unwanted radioactivity.) The cross sections are typically large (10s to 100s of mb) for heavy mass RNBs, and hence the count rates are reasonable even with weak beams. A typical set of spectra taken with a stable test beam of $^{188}$Os on a $^{12}$C target at 267 MeV is shown in the lower part of Fig. 10. The simplicity of the spectra is evident. The lower two panels show the spectra for two NaI detectors located downstream from the target at “distances” of 0.1 and 0.3 ns. The lower intensity of the 155 keV $2^+_1 \rightarrow 0^+_1$ γ-ray in the latter case is clear.

Clearly, with this simple and efficient technique, series of isotopes, extending closer and closer to the drip line, can be measured efficiently and under uniform conditions in rapid succession, allowing the evolution of structure in exotic regions to be readily mapped.

5. Summary

We have discussed aspects of nuclear structure far from stability in regions that will become accessible with RNBs. Exploiting a “horizontal” approach, we analyzed several signatures of structure that are based on the simplest data. These signatures exploit remarkably simple, often universal, correlations of collective observables, either with $N_pN_n$ (or $P$) or among themselves. An important point resulting from this work is that horizontal correlations of nuclear observables are not merely an exercise in “systematics” or “phenomenology”, in the zoological sense, but can provide real, new, and otherwise hidden, insights into nuclear behavior and the microscopic underpinnings of nuclear structure. Finally, the status of RNB facilities in North America was reviewed and a particular technique, low energy Coulomb excitation in inverse kinematics, was described.

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