QUASI-LINEAR ANALYSIS OF WATER FLOW IN THE
UNSATURATED ZONE AT YUCCA MOUNTAIN, NEVADA, USA

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ABSTRACT Philip's method of quasi-linear approximation, applied
to the fractured welded tuffs of Yucca Mountain, Nevada, USA,
yields simple relations describing groundwater movement in the
unsaturated zone. These relations suggest that water flux through the
Topopah Spring welded tuff unit, in which a proposed high-level
radioactive waste repository would be built, may be fixed at a value
close to the saturated hydraulic conductivity of the unit's porous
matrix by a capillary barrier at the unit's upper contact. Quasi-linear
methods may also be useful for predicting whether free water will
enter tunnels excavated in the tuff.

INTRODUCTION

Yucca Mountain, Nevada, USA, is being studied to determine whether it is a
suitable location for disposal of high-level radioactive wastes. The wastes would
be placed above the water table in fractured, densely welded tuff.

The flow of water in the disposal zone is of great importance for determining
whether the proposed waste repository would be safe (Sinnock & Lin, 1987). The
main mechanism by which radioactivity might be transported away from the
repository is dissolution in moving ground water. The amount of water coming in
contact with the waste packages would also be of great importance in determining
the rates of waste package corrosion and waste dissolution.

The tuffs at Yucca Mountain were emplaced by a series of volcanic eruptions,
creating a layered system. For our purposes, the layers are distinguished best by
their hydrogeologic properties (Scott et al., 1983). The principal distinction is
between welded ash-flow tuffs, which are pervasively fractured, and nonwelded
tuffs, which are coarser-grained but have far fewer fractures. A layer of
non-welded tuff may include material from both ash falls and ash flows, and the
boundary between welded and non-welded units may lie within a single ash flow.

A generalized east-west cross-section of Yucca Mountain, showing the welded
and non-welded layers, is shown in Figure 1. Most of the mountain is capped by
the Tiva Canyon welded tuff unit. Beneath this is the Paintbrush non-welded unit
which is underlain by the Topopah Spring welded unit, in which the repository
would be located.

The eastward-dipping Paintbrush non-welded tuff unit may play a major role in
water movement. Almost a decade ago (Scott et al., 1983; Montazer & Wilson,
1984), it was proposed that water may be diverted to the east as a result of the
contrast in capillary properties between the non-welded tuff and the adjoining
welded units. Numerical modeling studies (Rulon et al., 1986) have tended to
support this hypothesis.

In this communication, we apply Philip's (1969) quasi-linear approximation to
analyze unsaturated flow in the layered system at Yucca Mountain. First, the
quasi-linear approximation is extended to describe the distinct properties of the
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fractures and the matrix pores in the welded tuff. Next, a recent analysis of flow in sloping beds with contrasting pore sizes (Ross, 1990) is used to characterize flow at Yucca Mountain. Finally, quasi-linear methods are used to discuss whether free water will enter tunnels in the tuff.

QUASI-LINEAR DESCRIPTION OF TUFF UNITS

The quasi-linear approximation describes the unsaturated flow characteristics of any material by two constants, the saturated hydraulic conductivity $K$ and the sorptive number $\alpha$. At any pressure potential $\psi \leq 0$, the hydraulic conductivity is given by $K e^{\alpha \psi}$.

The non-welded tuff units at Yucca Mountain may be described in this way, but the welded units must be described as dual-porosity media. To do this, we take the hydraulic conductivity of the welded units at any value of pressure potential to be the sum of the matrix conductivity and the fracture conductivity at that potential. To apply the quasi-linear approximation, we assume that the fracture apertures in the welded units are much larger than the pores in the nonwelded units, and that these in turn are much larger than the matrix pores in the welded units. These assumptions give the following relationships among the constants for the different media:

$$\alpha_f \gg \alpha_p \gg \alpha_m$$ (1)

$$K_f \gg K_p \gg K_m$$ (2)

where the subscripts $f$, $p$, and $m$ represent, respectively, the fractures in the welded tuff, the pores in the nonwelded tuff, and the matrix pores in the welded tuff. Curves showing hydraulic conductivity as a function of pressure for the two kinds of tuff, given these assumptions, are shown schematically in Figure 2.
CAPILLARY BARRIERS

Capillary barriers are formed where a fine-grained material overlies a coarser-grained material along a sloping contact. If the system is dry enough that capillary forces prevent water from entering the coarser material, the water instead flows downdip through the finer material. Capillary barriers have been studied as artificial landfill covers (Frind et al., 1977; Johnson et al., 1983; Billiote et al., 1988) and as factors in natural hillslope hydrology (Miyazaki, 1988). It has previously been proposed (Scott et al., 1983; Montazer & Wilson, 1984) that capillary barriers may exist at Yucca Mountain.

At a capillary barrier, the volume of water moving laterally increases in the downdip direction as additional infiltration is diverted by the barrier. Sufficiently far downdip, the laterally moving water wets the contact to the point that an amount of water equal to the infiltration flows downward through the coarse soil. The lateral flow at such a point represents the diversion capacity of the capillary barrier because this flow will not increase farther downdip. If the width (measured in the direction of dip) of the system is large enough that total infiltration exceeds the diversion capacity, the downdip portion of the barrier will not be effective. The diversion capacity Q can be calculated exactly in the quasi-linear approximation for single-porosity materials (Ross, 1990) as

\[ Q = \frac{K_u \tan \phi}{\alpha_u} \left[ \frac{q}{K_i} \frac{\alpha_u}{\alpha_l} - \frac{q}{K_u} \right] \]

(3)

where \( \phi \) is the dip angle of the contact, q is the downward flux from above, and the subscripts u and l refer to the fine upper layer and the coarse lower layer. If \( \alpha_u >> \alpha_l \) and \( K_u >> q \), Ross (1990) shows that this reduces to

\[ Q \approx \frac{K_u \tan \phi}{\alpha_u} \]

(4)

and the width L of the effective portion of the capillary barrier is
\[ L = \frac{K_u \tan \phi}{q \alpha_u} \]  \hspace{1cm} (5)

The possible existence of capillary barriers at Yucca Mountain has been suggested in two different situations: (1) at the lower contacts of welded units, where the barrier effect would be due to the finer texture of the welded matrix compared to the nonwelded matrix, and (2) where the Paintbrush nonwelded unit overlies the Topopah Spring welded unit, as a result of the contrast between the pores in the nonwelded unit and the fractures in the welded unit.

Where a welded unit overlies a nonwelded unit, operation of a capillary barrier requires that the fractures remain dry. The material properties to be inserted in formulas (4) and (5) are therefore those of the matrix. We may choose the following values for the Topopah Spring matrix, biased somewhat in the direction of overestimating the effectiveness of a capillary barrier, based on discussion by Montazer & Wilson (1984), Klavetter & Peters (1968), and U.S. Dept. of Energy (1988):

\[ K_m = 1 \text{ mm/yr} \]
\[ q = 0.1 \text{ mm/yr} \]
\[ \tan \phi = 0.1 \]
\[ \alpha_m^{-1} = 10 \text{ m} \]

Substitution gives us

\[ Q_{\text{max}} < 10^{-3} \text{ m}^2/\text{yr.} \text{ (i.e., } 10^{-3} \text{ m}^3/\text{yr per meter along the strike of the beds)} \]

\[ L < 10 \text{ m} \]

If \( q \) takes a value significantly larger than 0.1 mm/yr, as may very well be the case in the Topopah Spring unit and even more likely is true in the Tiva Canyon, the barrier will be even less effective.

We must conclude that capillary barrier effects where welded units overlie nonwelded units have little importance at Yucca Mountain.

Now we address the situation where the Paintbrush nonwelded unit overlies the Topopah Spring. As a fractured porous medium, the Topopah Spring has properties more complicated than allowed by the quasi-linear treatment. We will first simplify the problem by ignoring the unit's porous matrix and treating the fractures as an equivalent porous medium. In this approximation, the capillary barrier formulas cited above can be applied directly. Then we will examine the effect of the porous matrix.

Treated as an effective porous medium, the underlying Topopah Spring fractures will create a capillary barrier. Formulas (1) and (2) can then be applied to the Paintbrush nonwelded unit, which has \( K_p = 3 \text{ to } 30 \text{ m/yr.} \) (Montazer & Wilson, 1984; Peters et al., 1984; U.S. Dept. of Energy, 1988). We can roughly estimate \( \alpha^{-1} = 5 \text{ m} \) (from curve in Peters et al., 1984). The diversion capacity will be \( Q_{\text{max}} = 15 \text{ to } 150 \text{ m}^2/\text{yr.} \) For a conservatively high infiltration rate of 3 mm/yr, the width of the zone of effectiveness will be

\[ \frac{K_p \tan \phi}{q \alpha} = 500 \text{ to } 5000 \text{ m} \]

As \( q \) is probably less than 3 mm/yr, it is likely that this capillary barrier is effective all the way across Yucca Mountain.

Now we recall that the underlying welded unit is a fractured porous medium. The capillary barrier cannot prevent entry of water into the matrix pores of the
Topopah Spring; indeed, the pressure potential at the contact will be sufficient to saturate them, because they are smaller than the Paintbrush pores. Consequently, the capillary barrier will act to fix the percolation through the Topopah Spring at a value equal to its matrix saturated hydraulic conductivity, $K_m$.

It is important to note that the percolation flux through the Topopah Spring will be fixed by this mechanism at a value independent of the infiltration rate at the surface. If the infiltration rate increases, the capillary barrier may not extend as far downdip to the east. But the flux beneath Yucca Crest (where existing data come from, and which is near the updip boundary) will not change unless the infiltration rate reaches many cm/yr. This suggests that climate change may have little effect on the behavior of a repository in Yucca Mountain.

WATER ENTRY INTO TUNNELS

Quasi-linear analysis may also be used to analyze whether water will drip into or otherwise enter tunnels built within Yucca Mountain. A recent series of papers by Philip et al. (1989a; 1989b; Knight et al., 1989; Philip, 1989) investigates numerous problems involving water entry into cavities of various shapes in unsaturated porous media. The basic mechanism of water entry is that cavities are obstacles to downward unsaturated water flow, and thus a zone of increased pressure is created above them. If the pressure is increased above zero, water will enter the cavity.

The Philip analyses consider ordinary porous media rather than dual-porosity media. With the approximation of sharply contrasting material properties discussed above, however, Philip's results can be used to address the question of whether the pressure increase above a cavity will be sufficient to drive water out of the matrix into the fractures. The question of whether the fractures will transmit water around the cavity is left for future work.

The capillary barrier analysis presented above indicates that the matrix of the Topopah Spring unit will be nearly saturated. Near-saturation of the matrix material is possible over a wide range of pressures because $\alpha_m$ is very small; thus the pressure in the unit is not determined by the saturation of the matrix. In this situation, the question of whether water will leave the matrix reduces to the problem of flow around a cavity in an unsaturated medium whose effective hydraulic conductivity is nearly independent of pressure.

To be specific, we wish to solve the unsaturated flow equation

$$V \cdot [K_m (V \psi - \hat{z})] = 0$$

(6)

in the radial coordinates shown in Figure 3, with a no-flow boundary at the surface of a circular tunnel with a radius of $a$, and uniform potential $\psi_o$ far away from the tunnel. Because $K_m$ is uniform, (6) reduces to

$$\nabla^2 \psi = 0 \text{ for } r>a$$

(7)

and the boundary conditions are

$$\frac{\partial \psi}{\partial r} = \cos \theta \text{ at } r=a$$

(8a)

$$\psi = \psi_o \text{ as } r \to \infty$$

(8b)
The solution to this problem is

\[ \psi = \psi_0 - \frac{a^2 \cos \theta}{r} \]  \hspace{1cm} (9)

As would be expected (Philip et al., 1989a), the maximum potential is at the point (a, π), located at the apex of the tunnel. Its value is \( \psi_0 + a \). If \( a < |\psi_0| \) -- that is, if the radius of the tunnel, a, is less than the magnitude of the pre-existing pressure potential in the tuff, \( |\psi_0| \) -- water will be confined to the matrix pores. If this condition is not satisfied, water will leave the matrix and either flow through the fractures or enter the tunnel.

This implies that a necessary (but not sufficient) condition for water entry into tunnels in Yucca Mountain, given the assumptions made above, is that the magnitude of the pre-existing pressure potential be less than the tunnel radius. Measured potentials range from close to the tunnel radius (a few meters) to orders of magnitude larger (Montazer et al., 1985).

The capacity of the fractures to divert flow around cavities is a complex question deserving of study. Furthermore, the neglect of heterogeneity within a layer is a serious deficiency in the present analysis; it is easy to imagine that some local low-permeability feature might divert water into a tunnel. Consequently, it is not clear whether water entry into tunnels should be expected at Yucca Mountain. If some water entry were to be observed when tunnels are excavated, the hypothesis that water in the Topopah Spring unit is confined to the matrix would not necessarily be contradicted.

CONCLUSIONS

By approximating the Yucca Mountain unsaturated zone as a series of uniform eastward-dipping layers, and using Philip's quasi-linear approximation to solve the unsaturated flow equations, simple relations describing water flow can be obtained. These relations state that infiltrating water will be diverted laterally by a capillary barrier at the bottom of the Paintbrush nonwelded tuff unit, reducing the flux through the Topopah Spring welded unit to a value equal to saturated hydraulic conductivity of the latter unit's matrix pores. Water will not enter circular tunnels
excavated in the Topopah Spring welded unit if the magnitude of the pressure potential in the tuff exceeds the radius of the tunnel.

These conclusions will undoubtedly require modification to account for heterogeneity of rock units. Furthermore, the above discussion relates to the natural hydrologic regime of Yucca Mountain. With construction of a repository, that regime will be perturbed by removal of water through ventilation air, addition of water in drilling fluids, and redistribution of water by repository heating. Engineered features of the repository (such as backfills chosen for their capillary properties) could also affect water movement. Quasi-linear analysis may be useful to assess these phenomena.

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REFERENCES


Appendix

Information from the Reference Information Base
Used in this Report

This report contains no information from the Reference Information Base.

Candidate Information
for the
Reference Information Base

This report contains no candidate information for the Reference Information Base.

Candidate Information
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