INVESTIGATION OF A METHOD OF COMPARISON OF METALLIC SURFACE CONTOURS BY MEANS OF STEREOSCOPIC ELECTRON MICROGRAPHS

By Allen S. Powell, Thomas P. Clark, and Milton C. Shaw

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SUMMARY

Object. - To determine the feasibility of measuring surface contours with an aerial-mapping contour finder on stereoscopic electron micrographs of replicas of metal surfaces.

Scope. - Peak-to-valley distances of metallic surfaces finished by eight different methods were measured with a surface analyzer, with a Profilometer, and by taper sectioning. The values for the surface contours thus obtained were used as guides for testing the feasibility, the scope, and the accuracy of surface-contour measurements based on stereoscopic electron micrographs of positive and negative replicas of the standard metallic surfaces. Methods of preparing and photographing the replicas were critically investigated. Stereoscopic electron micrographs of an aircraft-engine cylinder wall and semipolished surfaces have been prepared.

Summary of results. - The magnitudes of surface elevations measured by an aerial-mapping contour finder on these micrographs are of the order of one-tenth the magnitude anticipated for vertical dimensions on the basis of results of surface-analyzer, Profilometer, and taper-section determinations. Contour-finder measurements of such small magnitude are unreliable for quantitative comparison of metallic surface contours. In spite of this limitation, stereoscopic pictures are an aid to the study of the fine structure of metals beyond the resolving power of light and give a qualitative representation of the third dimension of surface contours, which cannot be obtained by light micrography.

INTRODUCTION

It has been observed that the magnitude of the coefficient of friction and the amount and rate of wear of lubricated surfaces are dependent upon the roughness of rubbing surfaces. Two theories have
been developed as to the degree of finish that should be given to rubbing surfaces. The first theory maintains that surfaces should be made as smooth as is technically feasible; the second maintains that interrupted surfaces of moderate roughness make better operation possible. In order to examine the two theories by a series of critical experiments, a method of determining the absolute shape and magnitude of the peaks and valleys that form a surface is required.

Numerous methods have been proposed for determining surface contours. A most complete treatment of common methods has been presented by Schmaltz (reference 1, pp. 28-107); other comprehensive reviews have been written by Way (reference 2) and Bikerman (reference 3). Taper sectioning (reference 4) is a suitable method for use in research with certain types of surface. Tracer instruments (references 5 and 6) are valuable for production control of surface finishes. These methods, together with all the other common ones, are limited to use with specimens for which the peak-to-valley distances are greater than 10 microinches (reference 7). The error in measurement of surfaces by commercial tracer instruments with peak-to-valley distance of 70 to 80 microinches is approximately 40 percent (reference 2).

The electron microscope has been used as a qualitative instrument in surface-contour studies with considerable success, largely because of its great resolving power. A few quantitative studies of surface contour, in which the electron microscope was used, have also been made. Measurement of the density variation of a surface-finish electron micrograph has been suggested as a means of evaluating peak-to-valley distances (references 8 and 9). A serious objection to this method is that the back of the replica film is not flat, which makes the interpretation of densitometer readings inaccurate. It is also difficult to obtain equivalent exposures with different specimens.

Another method of employing electron micrographs in surface-finish studies has been proposed by Heidenreich (reference 10). The relatively great depth of focus of the electron microscope (1 to 10 microns) enables stereoscopic electron micrographs to be made of a specimen and enables the parallax of a pair of stereoscopic electron micrographs to be obtained. A study of the application of this method to metallographic investigations and a preliminary test of its application to a mechanically finished surface have been made by Heidenreich and Matheson (reference 11). In order to determine the feasibility, the scope, and the accuracy of this method for surfaces dealt with in aircraft-engine research, the investigation herein described was conducted at the NACA Cleveland laboratory from November 1943 to June 1944.
THEORY OF STEREOSCOPIC DEPTH MEASUREMENT

Depth evaluations of surface contours may be made by measurement of parallax on a three-dimensional image. Stereoscopic images for parallax measurement may be obtained by taking two successive photographs of an object with the camera displaced laterally between pictures. The source of parallax in such a stereoscopic pair of negatives and the method of using these photographs to reconstruct the object into a three-dimensional image is shown in figure 1. In addition to this qualitative picture of the spatial position of point C above the base line AB, the elevation of point C may be computed from a knowledge of the parallax of point C with respect to point A or B on the negatives \((A_1C_1 - A_2C_2 \text{ or } B_2C_2 - B_1C_1)\), the focal length of the lens, and the locations of the camera relative to the object.

Because of the limited field of the electron microscope, it is not feasible to obtain a stereoscopic pair of pictures by shifting the object relative to the photographic plate. The great depth of focus of the electron microscope makes it possible to obtain stereoscopic pictures by an angular displacement of the object about an axis perpendicular to the optical axis of the microscope. A diagrammatic view of the special specimen holder that is required when stereoscopic electron micrographs are made is shown in figure 2.

The source of the parallax, which gives rise to the stereoscopic effect in a pair of electron micrographs, is shown in figure 3. The lines representing the peak-to-valley distance of a scratch on a surface at a distance OA or OK from the optical axis are AB and KL. The angle \(\sigma\) is the angle between the specimen plane and the plane normal to the electron beam and is 4° when the standard RCA stereo specimen holder is used. The parallax introduced on the photographic plate is equal to the quantity \(H''J'' - D''C''\) when the plane of the specimen is rotated through an angle equal to 2\(\sigma\). The following relation (reference 11, p. 425) between the measurable parallax on the photographic plates \(y\), the total magnification \(M\), the elevation of the point on the surface \(x\), and the stereo angle \(\sigma\) is derived in appendix A:

\[
y = 2M \sin \sigma
\]

Heidenreich and Matheson show experimental verification of equation (1) in reference 11 and give a chart for checking parallax readings and elevations for various stereo angles.
APPARATUS AND EXPERIMENTAL PROCEDURE

Replica technique for electron microscopy. Two replica techniques were employed in this investigation. The first was the polystyrene-silica method proposed by Heidenreich and Peck (references 12 and 13). For this series of experiments, pieces of sheet steel 3/4 inch by 1/2 inch by 1/16 inch were mounted in bakelite and surfaces of four different roughnesses were prepared. The finishes were obtained by polishing on 000 metallographic paper, polishing on 0000 metallographic paper, lapping with 600-mesh carborundum on a wax lap, and lapping with levigated alumina on broadcloth.

The preparation of bubble-free polystyrene replicas proved to be difficult. Scuffing of the mold when using polystyrene was also troublesome. These difficulties were eliminated by substituting methyl methacrylate molding powder for polystyrene. Molding conditions for this resin are a temperature of 135° C to 150° C and a pressure of 2000 to 3000 pounds per square inch. Clear moldings were easily obtained under these conditions. The use of methyl methacrylate required a different solvent for removing the positive silica replica from the negative plastic replica. Chloroform proved most suitable for this purpose. No other changes in the technique of Heidenreich and Peck (references 12 and 13) were necessary.

Experiments were also made with a replica technique developed by V. J. Schaefer (references 14, 15, and 16). A 0.5-percent solution, or for the rougher surfaces a 0.75-percent solution, of polyvinyl-formal resin in dioxane was used for preparing the replica films, which were mechanically stripped under water. A set of the standard specimens was used to prepare polyvinyl-formal replicas. This procedure yielded a negative replica of the surface, whereas the two-step plastic-silica method gave a positive replica. It was found necessary to clean the metal surface carefully before preparing the replica. An effective method of cleaning is to strip two or three preliminary films from the surface. The dioxane employed must be dry or difficulty will be experienced with under-water stripping. Moisture in the solution was also found to cause loss of resolving power in the replica. Dioxane used to prepare the solutions for this investigation was dried by distillation over sodium. A humid atmosphere caused the film to blush upon drying, making it impossible to strip the replica. Stereoscopic determinations of surface contours, which should be directly comparable with the tracer- and taper-section determinations, were made from stereoscopic electron micrographs of negative replicas prepared by this technique.
Preparation of standard surfaces. - In order to determine the order of magnitude of surface irregularities of commercially finished surfaces, eight different types of finish - planed, shaped, milled, ground, polished on 000 metallographic paper, lapped, polished on 0000 metallographic paper, and polished metallographically - were prepared on steel blocks 1 by 1 by 1/2 inch and the surface finishes were measured by a surface analyzer, a Profilometer, and with taper sectioning.

The surface analyzer and the Profilometer had tracer points of 500-micron inch radius. The surface analyzer was equipped with a meter directly reading the rms deviation from the average surface level \( h_{\text{rms}} \) and a recording oscillograph, the chart of which gave the maximum peak-to-valley distance \( h_{\text{max}} \) on the surface. Photomicrographs at a magnification of 100 diameters of the profiles of 25:1 taper sections were measured to determine another set of \( h_{\text{max}} \) values for the standard specimens. Two observers made check readings at three points on the surfaces. These three points were obtained by grinding back the taper-section profile twice in addition to the original preparation. Photomicrographs of the profile were taken for measurement after each grinding and polishing step.

Preparation of stereoscopic electron micrographs. - The stereoscopic electron micrographs were taken with a type EMB-4 RCA microscope and a standard RCA 4° angle stereo specimen holder. A 30,000-volt accelerating potential was used to prepare micrographs of negative polyvinyl-formal-resin replicas; a 55,000-volt accelerating potential was used to prepare micrographs of positive silica replicas. Under these conditions high contrast images could be obtained by controlling the thickness of the replicas. In the case of negative replicas, thickness was controlled by the concentration of the polyvinyl-formal solution and, in the case of positive replicas, by the amount of silica evaporated. Standard RCA 200-mesh specimen screens having square holes, 0.0025 inch on a side, were used for mounting the replica films.

The orientation of the stereoscopic base line is governed by the lens currents and accelerating potential of the electron beam used in taking the pictures. This base line is the intersection of the plane in which the angle \( \pm \sigma \) is turned with the plane of the photographs (that is, perpendicular to the axis of rotation) and must be found experimentally. A specimen screen was folded over along a diameter in the stereo specimen holder with the folded edge lying in the plane in which the stereo angle is generated to determine the base line. Pairs of pictures of the edge of the folded screen were prepared under the various conditions used in making the stereoscopic electron micrographs. From the corresponding negative
the proper orientation of stereoscopic pairs could be determined for mounting prints. A base line determined in this manner can deviate approximately 10° from the true value without affecting the accuracy of contour measurements (reference 11). Pictures are best mounted by fusing the images while observing them through a stereoscope. When the pictures are interchanged from left to right, the sign of the parallax changes but its magnitude does not. One arrangement of pictures usually appears to give a better stereoscopic effect than the other.

Fields close to the center of the specimen screen were chosen in order to prevent discrepancy in magnification between the two settings of the sterec specimen holder. Magnifications were determined from micrographs of a negative replica of a 30,000-line-per-inch grating and are accurate to 10 percent.

**Measurement of parallax.** - Parallax was measured by an Abrams model FC-2 aerial-mapping contour finder. Figure 4 shows the contour finder and the mechanism used to keep the instrument aligned with the stereoscopic base. The alignment mechanism is a conventional drafting machine A; B and B' are adjustable lenses having magnifications of 4 diameters; C is a fixed dot that is set over a point in the left-hand picture; D is a movable dot that is set over the corresponding point in the right-hand picture; E is a dial-type indicator graduated in 0.01-millimeter divisions to give the position of D along the base line; F is a dial-type indicator, which can be used to determine the displacement of C perpendicular to the stereoscopic base necessary to compensate for distortion in the photographs.

In order to use the contour finder, the pictures are first mounted with their stereoscopic base lines co-linear and adjusted laterally until they fuse. The instrument is then moved until C falls upon a selected control point. Then D is moved until it is superimposed on C and on the apparent position of the stereoscopic image of the control point. Indicator E is set at zero and the parallax of all other points is measured relative to the control by the same procedure. If the elevation of this control point is known, the parallax readings can be converted to absolute heights. If the control-point elevation is not known, all heights and depressions are computed relative to the selected point.

**DISCUSSION OF METHODS AND RESULTS**

**Measurements on standard surfaces for comparison with stereographic determinations.** - The results of surface-analyzer, Profilometer, and taper-section evaluations of h_max for the surface
standards used in this investigation are shown in table I. Some
typical taper sections are shown in figure 5. Values of the maximum
peak-to-valley distance in the taper-section profile could be
measured to 0.02 inch, which represents 8 microinches in the verti-
cal direction for this investigation. The poor precision of these
results is due to dependence of results on the judgement of the
observer and to the actual variation of surface roughness from point
to point on the surface. The ratio between $h_{\text{max}}$ and $h_{\text{rms}}$ was
taken as 10:3 for approximate comparison of taper-section and
tracer-instrument measurements. (See reference 2.)

Surface-analyzer and Profilometer measurements give lower
values for maximum peak-to-valley distance than taper sections. As
appendix B shows, the relatively large tracer-point radius is presum-
ably the source of this error. The circuit converting tracer-point
motion to $h_{\text{rms}}$ values apparently further lowers the results, as
shown by comparison of $h_{\text{rms}}$ values from the meter and from the
oscillograph chart of the surface analyzer. It seems probable that
taper-section readings of $h_{\text{max}}$ are also low owing to the tearing
away of nickel from the points of the profile; for example, in fig-
ure 5 the bluntness and raggedness of the points of the milled-
surface taper section indicate the pulling away of nickel during
preparation of this section. Nickel in valleys is much less likely
to break off than metal in the peaks because it is supported to a
greater extent. In spite of these considerations, surface-contour
determinations by taper sectioning are thought to be more accurate
than determinations made with the tracer instruments (reference 4).
The peak-to-valley distances measured on taper-section profiles are
probably of the correct order of magnitude.

Measurement of Stereoscopic Electron Micrographs

Scope of the plastic-silica replica method. - A number of fac-
tors limit the field of usefulness of stereoscopic electron micro-
graphs in surface-finish measurements. About the roughest surface
that has been reproduced in silica was one finished on 0000 metal-
lographic paper. Table I shows that such a surface has a peak-to-
valley distance of approximately 55 microinches (13,750 Å). During
the present investigation, efforts failed to produce silica replicas
of surfaces rougher than those finished by polishing with 600-mesh
carbideum. The deep scratches in rougher surfaces caused the
silica films to break up as they came off the polystyrene or methyl
methacrylate. When silica films were made strong enough to hold
together during removal from the plastic, they were too thick to
transmit the electron beam. Only a narrow range of thickness of
silica replica seems suitable for reproduction of polished surfaces. Control of the silica evaporation is not precise enough to permit attainment of proper replica thickness except by numerous repetitions. Heidenreich and Matheson (reference 11) obtained replicas from surfaces with elevations up to 120 microinches when etched metallographic specimens with an irregular lay were used. In the case of mechanically finished specimens, the regular lay of the surface finish caused the silica replica to tear along the sharp edges of the parallel finish marks to such an extent that the whole replica easily ruptured. Etched surfaces, however, have only short edges of the type found in commercial surface finishes, the tendency to tear is less, and the silica replica is less likely to disintegrate.

Silica replicas can be used at the highest magnification available in the RCA electron microscope. Occasional replicas attain the resolving power of the microscope, which is at least 50 Å, although a day-to-day average replica resolution of only 150 Å can be maintained. Measurements on an enlargement of figure 6 show that points closer than 150 Å apart can be clearly distinguished as separate. The specimen was deep-etched Inconel, the structure of which is almost entirely beyond the limit of resolution and depth of focus of the light microscope. Qualitative studies of such deep-etched or submicroscopic structures have been the main applications of the electron microscope in the examination of metal surfaces.

Scope of the polyvinyl-formal-resin replica. - Polyvinyl-formal replicas may be made of rough surfaces because they are strong enough to withstand more abrupt surface variations than silica replicas. Negative replicas were taken from surfaces as rough as those formed during the average grinding process. Edges and corners in these replicas are not sharply defined because of a building up the thickness of the plastic film at such discontinuities of the surface as a result of the surface tension of the polyvinyl-formal solution. Figures 7 and 8 were made from negative replicas and illustrate the resolution obtainable with this type of replica in its present state of development. Stereoscopic electron micrographs of polyvinyl-formal replicas showed some saucering of the resin film at the center of specimen screen holes, indicating a slight distortion of the replica as a result of mounting.

Measurement of Surface Contours

Equation (1) shows that parallax is a function of magnification and stereo-holder angle. If 12-percent precision in parallax measurement with the contour finder is sought, when a peak-to-valley height of 10 microinches (2.5 × 10⁻⁴ mm) is being measured from
stereoscopic electron micrographs made with a 4° angle stereo specimen holder, it is evident from equation (1) that the magnification of the stereoscopic pair measured must be

\[
\frac{10 (0.02)}{(2.5 \times 10^{-4})(2)(0.0698)} = 5740 \times
\]

or greater. The factor 0.02 millimeter is the precision of the contour finder, which was verified by the tests reported in appendix C. Stereoscopic electron micrographs at a magnification of 2500 diameters should be measurable with a precision of 20 to 25 percent if 10-microinch elevation differences are present. The 0.02-millimeter-diameter floating dots of the contour finder are responsible for some loss of precision. In order to be accurately measured, a point or line must be resolved to a dimension less than the diameter of the dot. As a result the contour finder can be applied only to clear, sharp pictures. The stereoscopic electron micrographs available did not meet these conditions.

Parallax measurements were made on figure 7, which is a stereoscopic electron micrograph at a magnification of 2500 diameters of a netative replica taken from a surface polished on 00C metallographic paper, and on figure 8, which is a stereoscopic electron micrograph at a magnification of 5000 diameters of a replica from an aircraft-cylinder wall. The results are presented in the following table:

<table>
<thead>
<tr>
<th>Surface</th>
<th>Relative elevation (micron.)</th>
<th>Stereographic peak-to-valley distance (h_max)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>Steel polished with 000 paper (fig. 7):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observer 1</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Observer 2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Honed aircraft-cylinder wall (fig. 8):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observer 1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Observer 2</td>
<td>0</td>
<td>-8</td>
</tr>
</tbody>
</table>
Lack of reproducibility of results by different observers arose from inability to measure exactly the same point owing to low resolving power of stereoscopic electron micrograph negatives, which made enlargements greater than a magnification of 2 diameters show graininess under the contour finder, and to the inherent inability of the replicas to reproduce sharp corners and edges. This characteristic of the replicas made it useless to work with negatives taken at magnifications above 2500 diameters. A typical example of the precision obtained may be taken from the individual measurements of point F (fig. 8) on the honed cylinder-wall stereoscopic electron micrograph. The elevations of point F were 3.3, 8.8, 8.8, and -3.3 microinches (average, 4.4 microin.). The maximum deviation from the mean was 7.7 microinches or 175 percent. It is evident that the results are not quantitatively reliable.

The principal cause of poor precision expressed on a percentage basis was that the magnitude of the parallax readings obtained was close to the limit of the parallax measuring method. This magnitude was only a tenth of that expected on the basis of measurements of surface roughness by other methods. The $h_{\text{max}}$ value for a taper section of a surface polished on 000 metallographic paper is about 93 microinches according to table I. From the stereoscopic electron micrographs (fig. 7), the $h_{\text{max}}$ value is found to be 10 to 25 microinches. The $h_{\text{max}}$ value of the honed cylinder-wall specimen (fig. 8) was computed to be 16.7 microinches ($10/3 \times h_{\text{rms}}$ meter reading, 5 microin.). No taper section of this specimen was prepared but the maximum peak-to-valley distance on its surface must be in the range from 50 to 150 microinches on the basis of taper-section results for the finer surfaces shown in table I. Parallax measurements on figure 8 give the $h_{\text{max}}$ value as 20 to 40 microinches. Heidenreich and Matheson (reference 11) have reported measurements on a polished steel surface, the finish of which was obtained on a felt wheel using magnesium oxide as the abrasive in water. This polished-steel surface corresponds to the metallographically polished surface in table I, which has an $h_{\text{max}}$ value of approximately 40 microinches. The greatest elevation difference measured by Heidenreich and Matheson corresponds to 2 microinches. These results suggest a discrepancy between the electron-optical conditions assumed in deriving equation (1) and the conditions that actually exist during the preparation of stereoscopic electron micrographs.
Possible Sources of Low Stereographic $h_{\text{max}}$ Values

Because of the poor definition in the micrographs taken in this investigation, the possibility that low $h_{\text{max}}$ values are entirely due to an unsatisfactory replica method must be given the greatest weight. Several other factors are worthy of consideration. Equation (1) shows that the parallax found on a pair of stereoscopic electron micrographs is a function of magnification, stereo angle, and surface elevation. For given elevations and a fixed stereo angle, reference 11 had shown experimentally that the relation between $y$ and $M$ is linear. This relation was tested only for oxide replicas from deep-etched aluminum and therefore does not entirely exclude the possibility of a nonlinear relation between $y$ and $M$ if other types of replica were considered. It is more likely, however, that the cause for low $h_{\text{max}}$ values should be sought in some other factor. One possibility is that the variation of $y$ with $\psi$ may not be sinusoidal. Heidenreich and Matheson have not reported a test of this variable. The angle between beam and specimen can be different from the angle between specimen and microscope axis if the beam is not parallel to the axis. The derivation of equation (1) assumes that angle $\psi$ is the angle between the specimen plane and the plane normal to the electron beam. Another possible source of low $h_{\text{max}}$ values is an electron-optical effect in the lens system reducing the parallax appearing in the final images. Without calibration for known elevations in the range up to $h_{\text{max}}$ values of 100 microinches, contour determinations from stereoscopic electron micrographs cannot be considered reliable. No method of preparing such elevation standards is now known.

Limitations on Stereoscopic-Contour Measurements

A primary requirement for accurate, precise evaluation of surface contours is calibration of the stereographic method as discussed in the preceding section. Magnification calibrations of the microscope are limited to an accuracy of 10 percent at present and give no indication of spatial distances. The highest magnification attainable for surface-contour measurements is restricted to that at which a peak and a valley on the surface replica still appear in the same field. Silica replicas can be used at this magnification or higher but those replicas cannot be taken from surfaces with $h_{\text{max}}$ values greater than approximately 50 microinches. Although figures 7 and 8 show surface-finish scratches as deep as they are wide, the ratio of scratch width to depth may be as high as 25:1 for some types of surface. Such scratches might be too wide to appear in a single field at the low magnification limit of the RCA electron microscope.
Improved replica techniques will have to be devised to increase the precision of the stereographic method in its application to surfaces of interest in aircraft engines.

It is not practical to employ the light microscope for determining peak-to-valley distances by the parallax method. For very smooth surfaces, there is insufficient parallax at the highest powers obtainable; for coarse surfaces, there is insufficient depth of focus to have both a peak and a valley in sharp focus simultaneously.

All the stereoscopic electron micrographs prepared showed a marked three-dimensional effect. Such three-dimensional pictures of surfaces at high magnification are a distinct advance toward gaining an insight into the character of the fine structure of a surface and its change due to wear. Improvement in replica methods would be the next logical step forward from the present work. At present, only qualitative comparison of surfaces finished by different methods can be made with certainty. The difference in performance of surfaces prepared by different methods may, of course, be attributable to variations in contour on a qualitative scale.

RESULTS

1. Quantitative measurements, which were taken during this investigation, of surface contour \( h_{\text{max}} \) from the stereoscopic electron micrographs, are uncertain to more than ±50 percent of the values measured. The stereographic \( h_{\text{max}} \) values are about 10 percent of the magnitude that would be predicted from temper-sandwich and tracer-instrument measurements. Precision in parallax measurement is lost owing to low plate resolution and blurring of the images of sharp discontinuities of surfaces, which is probably characteristic of the polyvinyl-formal-resin replica.

2. The discrepancy between stereographic values of surface roughness and values obtained by other measuring methods indicates that probably either some factor involved in the method of preparing stereoscopic electron micrographs gives erroneous parallax values or some false assumption has been made in the derivation of the equation used to convert parallax readings to elevations.
CONCLUSION

Stereoscopic electron micrographs of surfaces of the type used in aircraft engines can be prepared. These micrographs facilitate comparison among surfaces of different fine structural characteristics that are beyond the resolving power and depth of focus of the light microscope.

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APPENDIX A

DERIVATION OF PARALLAX EQUATION

Heidenreich and Matheson (reference 11) have presented a derivation for the electron-microscope parallax equation but the significance of the simplifying assumptions and the limitations of the final equations are not so evident as in the following derivation:

From the diagram in figure 3, let

\[ x = AB \text{ or } KL \]
\[ b = OA \text{ or } OK \]
\[ d = OG \]

Then

\[ OC = \frac{EF}{FG} \cdot d \]
\[ OC = \frac{(b \cos \phi + x \sin \phi)}{(b \sin \phi - x \cos \phi + d)} \cdot d \]

Likewise

\[ OD = \frac{FA}{FG} \cdot d \]
\[ OD = \frac{b \cos \phi}{b \sin \phi + d} \cdot d \]
\[ CH = \frac{PK}{FG} \cdot d \]
\[ CH = \frac{b \cos \phi}{d - b \sin \phi} \cdot d \]
\[ CJ = \frac{QL}{QG} \cdot d \]
\[ CJ = \frac{b \cos \phi - x \sin \phi}{d - b \sin \phi - x \cos \phi} \cdot d \]

The difference in parallax on the photographic plate \( y \) is equal to \( E''J'' \) minus \( D''C'' \) and

\[ y = M (HJ - DC) \]

where \( M \) is the over-all magnification.
\[ y = M \left[ (OJ - OK - (OC - OD)) \right] \]

\[ y = M d \left( \frac{b \cos \sigma - x \sin \sigma}{d - b \sin \sigma} - \frac{b \cos \sigma - x \sin \sigma}{d - b \sin \sigma} + \frac{b \cos \sigma}{d + b \sin \sigma} \right) \]

\[ y = M d \left[ \frac{(x^2 + b^2) \sin 2 \sigma - 2 x d \sin \sigma}{(d - x \cos \sigma)^2 - b^2 \sin^2 \sigma} - \frac{b^2 \sin 2 \sigma}{d^2 - b^2 \sin^2 \sigma} \right] \]

but

\[ x \cos \sigma \ll d \]
\[ b \sin \sigma \ll d \]

Therefore, to a good approximation

\[ y = M d \left( \frac{x^2 \sin 2 \sigma - 2 x d \sin \sigma}{d^2} \right) \]

but

\[ \frac{x^2}{d} \sin 2 \sigma \ll 2 x \sin \sigma \]

Therefore

\[ y = 2 x M \sin \sigma \]

It is thus seen that for all practical purposes the amount of parallax is independent of the distance of the point from the optical axis. The fact that parallax varies directly as the first power of the peak-to-valley distance greatly simplifies the interpretation of parallax measurements.
APPENDIX B

LIMITATION OF TRACER INSTRUMENTS

The limitations imposed upon the range of usefulness of tracer instruments by the finite radius of the tracer point has been recognized and discussed by several authors (references 2, 5, and 1, p. 67). The following simple computation shows the magnitude of this error for a hypothetical surface having a saw-tooth contour of wave length $L$ and peak-to-valley distance $h$ (fig. 9). For a tracer point of finite radius $r$, there will be a height $h_o$, below which the tracer may not penetrate. The percentage error will be $100 \frac{h_o}{h}$.

From figure 9

$$\frac{L}{2h} = \tan \alpha$$

$$\sin \alpha = \frac{L}{\sqrt{4h^2 + L^2}}$$

In triangle OAB

$$\sin \alpha = \frac{CB}{3A} = \frac{r}{r + h_o}$$

Therefore

$$\frac{r}{r + h_o} = \frac{L}{\sqrt{4h^2 + L^2}}$$

$$h = r\left(\frac{h}{L}\right) \left[\sqrt{\frac{L^2}{4h^2}} + 1 - 1\right]$$

Practical values of the scratch width to peak-to-valley depth ratio ($L/h$) range from 25:1 to 5:1 or less. The following table shows the minimum peak-to-valley distance that may be measured with errors of 50 and 5 percent, with a 0.0005-inch tracer radius and the values of scratch width to peak-to-valley distance ratio indicated.
### Ratio of scratch width to depth vs. Minimum height accurate to

<table>
<thead>
<tr>
<th>Ratio of scratch width to depth</th>
<th>Minimum height accurate to 50 percent (microin.)</th>
<th>Minimum height accurate to 5 percent (microin.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>3.19</td>
<td>31.9</td>
</tr>
<tr>
<td>20</td>
<td>4.98</td>
<td>49.8</td>
</tr>
<tr>
<td>15</td>
<td>8.85</td>
<td>88.5</td>
</tr>
<tr>
<td>10</td>
<td>19.80</td>
<td>198.0</td>
</tr>
<tr>
<td>5</td>
<td>77.03</td>
<td>770.3</td>
</tr>
<tr>
<td>1</td>
<td>123.61</td>
<td>1236.1</td>
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</table>
APPENDIX C

TEST OF THE AERIAL-MAPPING CONTOUR FINDER

An experiment was performed to test the accuracy and the reproducibility of the aerial-mapping contour finder. A stereoscopic system simulating that utilized in the electron microscope was set up as diagrammed in figure 10 and the following mathematical expression for the parallax occurring was derived. Let

\[ z \] spacing of bands of object

\[ \beta \] angle of specimen from picture plane

**Equation (2)** was used to calculate the parallax for several values of \( z \) and a card was made having, alternate black and white bands for these different values of \( z \). This card was pasted on a box at an angle \( \beta \) of 45°. The setup was photographed at two equivalent displacements of \( \sigma \), one on each side of the center line \( OP \) (fig. 11). The negatives from these pictures were examined with a measuring microscope having a 0.001-inch scale. Because of grain and lack of resolution, the limit of accuracy was +0.0005 inch. The absolute parallax was determined by subtracting the smaller reading from the larger reading of the width of the same band on
the two negatives. Enlargements to a magnification of 5 diameters were made of the negatives and these were examined for parallax with the contour finder. The parallax on the prints was measured in two ways to make sure that the error found was in the instrument rather than in the operator's technique. The parallax was first measured in the accepted manner by adjusting the optically fused dots to a level with the elevation to be determined and this value was compared with one found by making each dot tangent to the elevation edge in its own picture. The two methods gave the following equivalent results within the limits of experimental error:

<table>
<thead>
<tr>
<th>Values of spacing of bands of object ( z ) (mm)</th>
<th>8.92</th>
<th>13.0</th>
<th>16.9</th>
<th>20.6</th>
<th>24.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated parallax ( y ) for ( 1/10 , M )</td>
<td>0.40</td>
<td>0.59</td>
<td>0.76</td>
<td>0.835</td>
<td>1.080</td>
</tr>
<tr>
<td>Value of parallax ( y ) measured on negative</td>
<td>0.375</td>
<td>0.625</td>
<td>0.750</td>
<td>0.837</td>
<td>1.100</td>
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<tr>
<td>Average of parallax ( y ) on print using contour finder</td>
<td>0.372</td>
<td>0.552</td>
<td>0.693</td>
<td>0.852</td>
<td>1.032</td>
</tr>
<tr>
<td>Measurement of parallax ( y ) on print with microscope</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.85</td>
<td>1.00</td>
</tr>
<tr>
<td>Maximum variation of readings from average</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>Percentage error of average from actual value on negative</td>
<td>0.1</td>
<td>12.0</td>
<td>9</td>
<td>1.2</td>
<td>6</td>
</tr>
</tbody>
</table>

The table of results indicates that ±0.02 millimeter reproducibility of readings with the contour finder is possible. The
values obtained with the contour finder compare favorably with the values found by measurement of the negatives. In electron microscopic work, this accuracy is largely lost owing to diffuse replica edges and low plate resolution.

REFERENCES


<table>
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<tr>
<th>Method of measurement</th>
<th>Type of surface</th>
<th>Planed</th>
<th>Shaped</th>
<th>Milled</th>
<th>Ground</th>
<th>Polished 000 paper</th>
<th>Lapped 000 paper</th>
<th>Polished metallographically</th>
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<td>Oscillograph trace, microinches</td>
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<td></td>
<td></td>
<td>151</td>
<td>35</td>
<td>165</td>
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<tr>
<td>10/3 $h_{rms}$ meter reading, microinches</td>
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<td>350</td>
<td>187</td>
<td>90</td>
<td>60</td>
<td></td>
<td>12</td>
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<td>Profilometer:</td>
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</tr>
<tr>
<td>10/3 $h_{rms}$ meter reading, microinches</td>
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<td>400</td>
<td>213</td>
<td>87</td>
<td>60</td>
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<tr>
<td>Mean value, microinches</td>
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<td>1039</td>
<td>554</td>
<td>209</td>
<td>184</td>
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<td>93</td>
<td>63</td>
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<tr>
<td>Mean deviation, microinches</td>
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<td>54</td>
<td>52</td>
<td>52</td>
<td>52</td>
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<td>18</td>
<td>1</td>
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<tr>
<td>Percentage deviation from mean</td>
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<td>6</td>
<td>16</td>
<td>16</td>
<td></td>
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* Factor from reference 2.
Figure 1. - Illustration of the parallax in a stereoscopic pair of photographs and production of a three-dimensional image.
Figure 2. Diagrammatic view of electron-microscope stereo specimen holder.
Figure 3. - Parallax construction for stereoscopic electron micrographs.
Figure 4. - Aerial-mapping contour finder used for measuring parallax on stereographs.
Figure 5. - Typical taper sections. Horizontal, X100; vertical X2500.
Figure 6.—Deep-etched Inconel. Electron micrograph of a methyl methacrylate-silica replica. $\times 10,000$.

Figure 7.—Stereoscopic electron micrographs of a polyvinyl formal resin replica of steel polished on 000 metallographic paper. $\times 2500$.

Figure 8.—Stereoscopic electron micrographs of a polyvinyl formal resin replica of an unworn portion of Wright aircraft cylinder barrel. $\times 5000$. 
Figure 9. - Diagram of tracer point traversing a hypothetical surface.
Figure 10. - Optics of the system used to prepare stereo-graphs for testing the aerial-mapping contour finder.

Figure 11. - Stereographic pair used in testing the aerial-mapping contour finder.