THE ROLE OF MULTIPLE BARRIERS IN ASSURING
WASTE PACKAGE RELIABILITY

Yucca Mountain Project

by

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Introduction

Yucca Mountain in southwestern Nevada is being studied as a potential repository site for the permanent storage of high-level nuclear waste. Because of the large amounts of hazardous waste to be stored, there is a great deal of concern for ensuring that dangerous releases of radioactivity to the accessible environment are unlikely. Regulators have set performance standards that the potential repository must meet in order to obtain regulatory approval. Nuclear Regulatory Commission (NRC) regulations state that containment of radioactivity must be "substantially complete" for the first 1000 years after closure of the facility. Thereafter, the acceptable annual limit on releases is 1/100,000 of each radionuclide remaining in the inventory after 1000 years.

To demonstrate that the potential facility is in compliance with the regulations, which may be interpreted as involving probabilities, it is necessary to obtain some understanding of the probability distribution of the cumulative quantity of releases by certain time points. This paper will discuss the probability distribution of waste container lifetimes and how the understanding of this distribution will play a role in finding the distribution of the release quantities over time. It will also be shown how containers with multiple layers may be much more effective than single-layer containers in isolating the waste for long periods of time. In particular, it will be shown that, for reasonable assumptions about the process of barrier failure, the reliability of a multiple-barrier container can be achieved and demonstrated much more readily than a container consisting of a single barrier. The discussion will focus primarily on the requirement of substantially complete containment for the first 1000 years; a more complete analysis, addressing the release rates after 1000 years, must be done eventually, but it is outside the scope of this report.

The Role of Container Lifetimes

Obviously, if the nuclear waste is stored in containers, none of it can reach the outside environment unless some of the containers fail. Although scenarios involving volcanic activity or large earthquakes in the vicinity of the potential repository have been studied, it is generally believed that the most likely failure mode will be the gradual corroding and degradation of container walls. Even when the containers corrode all the way through, the radionuclides will not be considered to have reached the accessible environment; ground water or magma must somehow carry the radionuclides a designated distance away from the potential repository.

These processes all involve some variability: some containers will get wetter than others; equally wet containers may corrode at different rates; contaminated groundwater flow out of the potential repository will be greater at some locations than at others, and greater at some *times* than at others. The ultimate goal is to determine the distributions of all random

variables of interest, to determine analogous functions that characterize the heterogeneity of deterministic processes, then to determine how the interaction of all of these processes influences the answer to the fundamental question of whether the potential repository meets the regulatory criteria.

After the initial emplacement of waste in containers, the decay heat from the radionuclides is expected to keep temperatures in the immediate vicinity above the boiling point of water; this condition is expected to persist for hundreds of years. As the containers cool, the amount of water in the immediate area will affect the time at which water begins to reach the waste packages. Research has suggested that the amount of water flowing across a horizontal cross-sectional area follows a lognormal distribution¹. It should be noted that this quantity is not a random variable; it is a deterministic variable whose heterogeneity is described by the lognormal distribution. On the other hand, there may be uncertainty about the values of the *parameters* of the lognormal distribution; these uncertainties should be resolvable by further site characterization work.

Once water begins to reach the waste packages, the lifetime of the container becomes the random variable of interest. An extremely desirable condition would be that the container lifetimes have a distribution that provides sufficient assurance of the regulatory criteria being met, without reliance on any other components of the Engineered Barrier System. Chesnut² has shown that this desirable condition is unlikely, given a container that acts as a single barrier whose lifetime is exponentially distributed. If multiple barriers exist, however, each barrier can have a much shorter expected lifetime than would be required of a single barrier.

Even if the container lifetime distribution fails to meet the above condition, the potential repository might still meet the regulatory requirements. First of all, a long time may pass before water reaches the containers. Secondly, even after the containers fail, it may take an extremely long time for the radionuclides to reach the accessible environment. This can result from two different, but related, factors: obviously, it may take a long time for the contaminated water to travel from the failed container to the accessible environment, but it may also take a long time for any significant amounts of radionuclides to be dislodged from their protective cladding inside the container and dissolved into the water.

For an idealized particle of a given radionuclide, the above discussion implies that the time required for this particle to reach the accessible environment is a random variable that can be thought of as a sum of four separate components: (1) the elapsed time before water reaches the waste package containing the particle, (2) the time it takes the water to cause the container to fail, (3) the time it takes for the particle to be dislodged from its cladding matrix, and (4) the time it takes the dislodged particle to travel the specified distance from the container to the accessible environment. These four components are almost certainly not independent, but because of their

variability, a low value of one of them could still be offset by a high value of another. The exact distributions of these four variables and the interactions among them are outside the scope of this paper, which will focus for the remainder on the time it takes for a container to fail. It should also be mentioned that knowing the results for a single particle of a single radionuclide does not constitute a final answer; the final answer must consider all particles of all radionuclides in all containers.

Failure Time Distributions for a Single Barrier

In this section, we will discuss the specific process of container failure, a process that is not necessarily well-defined. On the basis of the preceding discussion, we can say that a container fails when the process of radionuclides being dislodged from their cladding can begin. Whether this happens when a pit first penetrates all the way through the container wall, or whether the pit must have a certain diameter, or whether some other definition of failure is assumed does not matter for the sake of this discussion.

We will initially assume a container wall that consists of a single layer. The longer a waste package has been exposed to water, the more likely it is to have failed. The mathematical relationship between time and the probability of failure is quantified by a probability density function $f(t)$. This function represents the instantaneous rate of change in the probability that a failure has already taken place. The same mathematical relationship could be quantified by $F(t)$, the cumulative distribution function (CDF), where

$$F(t) = \int_0^t f(u) du.$$

The CDF evaluated at time t gives the probability that failure will happen no later than time t . Note that neither $f(t)$ nor $F(t)$ is defined for negative t . For $f(t)$ to be a legitimate probability density function, it must be the case that

$$\int_0^{\infty} f(u) du = 1$$

For the CDF that describes the container failure process, the hope is that $F(t)$ remains small even at large values of t .

Chesnut² has suggested that for a container consisting of a single barrier, the appropriate form of the probability density function is exponential (the rationale for this judgment is discussed later in this paper). Hence

$$f(t) = \lambda e^{-\lambda t}, \quad t \geq 0$$

for a suitable value of the parameter λ . The CDF for an exponential random variable is

$$F(t) = 1 - e^{-\lambda t}, \quad t \geq 0.$$

To determine a value of λ such that the CDF is 0.01 for $t = 1,000$, we solve

$$0.01 = 1 - e^{-1000\lambda}$$

which implies a parameter value of approximately 0.00001. Since the mean of an exponential distribution equals the reciprocal of the parameter value, this implies that the mean time to failure of a container must be approximately one hundred thousand years. If the containers are to last a thousand years with probability 0.999, the mean time to failure must be approximately 999,500 — essentially, one million years. Requiring a thousand-year survival probability of 0.9999 forces the mean time to failure to be almost ten million years.

Chesnut² argues that extremely robust waste packages may not be testable, as failures in such packages would be so rare that they would almost certainly not be observed even in 100 years of testing.

Multiple Barriers

It is reasonable to assume that a container wall may actually consist of more than one barrier. Thus a designer can increase the reliability of a container either by increasing the barrier reliability, or by adding barriers. If the cases discussed in the previous section were changed to have two barriers each, the changes in reliability requirements would be dramatic. If a survival probability of 0.9999 is required, the mean time to barrier failure for a waste package with two barriers is on the order of 70,000 years instead of ten million. This is a substantial difference, but it is still perhaps beyond current capabilities. If a survival probability of only 0.9 is required, the barriers' mean time to failure drops to 1,880 years. The following table gives the required mean time to barrier failure for different combinations of container survival probability, and number of barriers.

Number of Barriers	Pr(Survival at 1,000 years)			
	0.9	0.99	0.999	0.9999
1	9,491	99,499	999,527	9,999,924
2	1,880	6,732	22,025	70,345
3	907	2,293	5,248	11,603
4	573	1,215	2,333	4,315
5	411	782	1,353	2,250

Clearly, the most significant improvement results from adding the second barrier. Also, for each successive barrier, the percentage difference in required mean barrier lifetime is greater for higher survival probabilities.

Two relevant questions arise at this point: Is the assumption of an exponential failure rate valid? How do the results change if different failure distributions are assumed? The second question will be addressed in the next section. As for the first question, Chesnut gives an explanation² of why the exponential distribution may be relevant for the problem at hand. In short, the argument runs as follows: it might be expected, *a priori*, that the failure rate for containers would be relatively high initially, then constant over a long span of time, then increasing after some time horizon as containers wear out. It is hoped that any early failures can be remedied during the preclosure phase of the facility life, and that the time horizon when containers begin to wear out in significantly greater numbers is distant enough that events beyond that time are outside the scope of the regulations. In these circumstances, the containers would have a failure rate that is essentially constant over the time period of interest; this implies a failure distribution that is approximately exponential over the relevant time period.

Other Lifetime Distributions

An important model to investigate is the case of barrier lifetimes that follow the normal distribution. Unlike the exponential distribution, which is completely specified by a single parameter, a normal distribution is defined by two parameters: the mean and the standard deviation (or, equivalently, the mean and the variance). The probability density function for the normal distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where μ is the mean of the distribution, and σ is the standard deviation. This function is defined for all real numbers, so in theory it cannot be the correct form of the distribution. The function is very close to zero for values that lie several standard deviations away from the mean, however, so it could be approximately correct over the range of interest.

In general, longer mean lifetimes are to be preferred, but it is also desirable to have small standard deviations. The larger the standard deviation, the less predictable the performance of the barrier is. For a normal random variable T with mean $\mu > 0$, lowering the standard deviation also lowers the probability that T is negative. Thus, data that seem to suggest a large standard deviation may actually suggest that the normal distribution is not a good approximation of the true distribution of barrier lifetimes. One measure of whether the normal distribution gives a good approximation is the implied probability that T is negative. Specifying an upper bound on

this probability determines an upper bound on reasonable values of the standard deviation. Let $\Phi(\cdot)$ denote the standard normal CDF, i.e. the CDF of normal random variable with mean of 0 and standard deviation of 1, and let u be an upper bound on the probability that T is negative. Solving the equation

$$\Phi\left(\frac{0-\mu}{\sigma}\right)=u$$

using tabulated values for the normal cumulative distribution function Φ (see Abramowitz and Stegun³) gives a value Z^0 such that $\Phi(Z^0) = u$. This implies that $Z^0 = -\mu/\sigma$, which implies that $\sigma = -\mu/Z^0$. The following table gives several useful cases:

$\Pr\{T < 0\}$	Z^0	σ/μ
0.05	-1.645	0.6080
0.01	-2.326	0.4299
0.005	-2.576	0.3882
0.001	-3.090	0.3236
0.0005	-3.291	0.3039
0.0001	-3.719	0.2689
0.00005	-3.891	0.2570
0.00001	-4.265	0.2345

The values in the third column are sometimes referred to as the *coefficient of variation*. These values can be used to do an analysis like that done above for exponential lifetimes; results are given in the table below:

Number of Barriers	Pr(Survival at 1,000 years)			
	0.9	0.99	0.999	0.9999
1	4,531	10,337	16,450	22,632
2	1,114	1,384	1,489	1,543
3	606	697	728	744
4	410	456	471	479
5	307	335	345	349

The entries in the table are mean barrier lifetimes; the assumed coefficients of variation are 0.6080, 0.3882, 0.3039, and 0.2570 for the first through fourth columns, respectively. These assumed standard deviations are conservatively large, given the assumption of normality: half of the probability mass below 1000 years is actually below 0 years. Mathematically, this can be written as

$$\Pr\{T > 0 \mid T < 1000\} = \Pr\{T < 0 \mid T < 1000\}.$$

It is worth noting that the levels of barrier performance implied by this analysis appear achievable, at least for the multiple-barrier cases. Moreover, assuming a smaller standard deviation in each case would lower the requirements on the mean barrier lifetime.

Another important model to investigate is that in which the barrier lifetimes are assumed to follow the Weibull distribution. The Weibull distribution is one of three functional forms that have been derived to model the distributions of *extreme values*, i.e. the largest or smallest outcomes, in random samples. It is generally applicable for distributions that are bounded below by some value ϵ and that follow certain other regularity conditions (see Benjamin and Cornell⁴). If a large sample is taken from such a distribution, the probability that the smallest observation in the sample is less than z is approximately

$$1 - \exp\left[-\left(\frac{z - \epsilon}{u - \epsilon}\right)^k\right] \quad z \geq \epsilon$$

for suitably chosen values of u and k . Obviously, when ϵ equals zero, this expression simplifies greatly. The ϵ represents a linear shift in the z coordinate. We will use $x = z - \epsilon$ and $u' = u - \epsilon$. This distributional form has been used to describe the distribution of breaking strengths of materials, the breakdown of capacitors, and many other phenomena⁵.

There is reason to believe that the Weibull distribution should accurately describe the distribution of barrier failure times. For any barrier there are likely to be a large number of locations on the barrier surface where corrosion is acting on the barrier and could eventually lead to failure. If failure of the container happens as soon as one of these locations experiences failure, and if the corrosion processes at the various locations are independent and satisfy certain regularity conditions, the barrier lifetime should follow a Weibull distribution.

Like the normal, the Weibull distribution has two parameters. Unlike the normal, the Weibull parameters do not have a straightforward interpretation. The probability density function of the Weibull can be written as

$$f(x) = abx^{b-1}e^{-ax^b}$$

for $x \geq 0$, for suitably chosen parameters a and b . ($a = (u')^{-k}$ and $b = k$ defined above). The mean of a Weibull distribution is

$$\mu = \alpha^{\beta} \Gamma(\beta+1),$$

where $\alpha = 1/a$, $\beta = 1/b$, and $\Gamma(\cdot)$ is Euler's gamma function; the standard deviation is

$$\sigma = \alpha^\beta \sqrt{\Gamma(2\beta+1) - \Gamma^2(\beta+1)}.$$

When $b = 1$, the Weibull distribution is the same as the exponential. This is the case when the mean and standard deviation are equal. When the standard deviation is greater than the mean, the probability density function is asymptotic to the vertical axis. When the standard deviation is less than the mean, the density function is peaked. Illustrative cases are plotted in the graph below:

As with the normal distribution, illustrative cases for the Weibull can be chosen by specifying a mean barrier lifetime and a standard deviation that is a given multiple of the mean. Unlike the cases of exponential or normally distributed barrier lifetimes, the Weibull distribution is complicated by the fact that the distribution of a sum of Weibull random variables is not well understood. Results can be obtained by simulation, however, and this was the method used to derive the results in the following table (lifetimes are rounded to the nearest hundred years, and the standard deviation is assumed to be 80% of the mean):

Number of Barriers	Pr(Survival at 1,000 years)			
	0.9	0.99	0.999	0.9999
1	5,600	36,000	225,000	1,403,100
2	1,400	3,800	9,800	24,600
3	700	1,500	2,900	5,500
4	500	900	1,400	2,300
5	350	600	900	1,300

These cases illustrate the fact that improvements in total reliability can be achieved either by increasing the expected barrier lifetime or by adding barriers. A comparison of these results with those from the exponential case illustrates how requirements on mean barrier life can be eased if the variance in the lifetime distribution is reduced (the exponential distribution is actually a Weibull distribution with the mean equal to the standard deviation).

Parallel Failures

It is expected that most containers will get eventually wet, corrode, and, finally, fail. Although some containers will not get wet during the time period of interest, these containers will still be subjected to the stresses associated with radiation and decay heat from the radionuclides. Unlike the corrosion process, which would probably attack multiple barriers in sequence, radiation and heat effects would act concurrently on all barriers of a multi-layered container. The time until these stresses cause a container failure is a random variable equal to the longest of the individual barrier lifetimes; hence the probability of a container failure by a certain time would be the probability that each barrier fails in that length of time or less. Let $F(t)$ be the probability that a barrier fails by time t . The probability that a container fails by time t is $[F(t)]^n$, where n is the number of barriers. As was done above for barriers failing sequentially, various assumptions can be made about the distribution of barrier lifetimes, and the required mean barrier life can be calculated in each case.

For the case of exponentially distributed lifetimes,

$$F(t) = 1 - e^{-\lambda t}, \quad t \geq 0,$$

so

$$[F(t)]^n = [1 - e^{-\lambda t}]^n.$$

The probability that the container survives for the first thousand years is

$$1 - [F(1000)]^n = 1 - [1 - e^{-1000\lambda}]^n.$$

This equation can be used to find the value of λ that gives a specified survival probability at 1000 years. Results are given in the table below:

Number of Barriers	Pr(Survival at 1,000 years)			
	0.9	0.99	0.999	0.9999
1	9,491	99,499	999,500	9,999,500
2	2,631	9,491	31,120	99,499
3	1,603	4,121	9,491	21,040
4	1,210	2,631	5,107	9,491
5	1,003	1,970	3,457	5,795

For the case of barrier lifetimes that are assumed to be normally distributed, numerical approximations to the inverse of the CDF for the normal distribution (see Abramowitz and Stegun³) can be used to determine the mean barrier lifetimes that lead to given probabilities of survival at a thousand years. Using the notation in the earlier discussion of normally-distributed failure times, the probability that a container with n identical barriers failing in parallel is completely failed by time t is

$$[\Phi((t-\mu)/\sigma)]^n$$

where μ is the mean time to barrier failure, and σ is the standard deviation of the barrier lifetime. Let $\sigma = c\mu$, where c is the coefficient of variation, as defined earlier. Then to determine a value of μ such that the probability of container survival at 1000 years equals p , we solve the equation

$$(t-\mu)/c\mu = \Phi^{-1}((1-p)^{1/n})$$

or

$$\mu = t / (1 + c\Phi^{-1}((1-p)^{1/n}))$$

Results of calculations of mean lifetimes are presented below (the assumptions about the coefficients of variation are the same as in the previous section):

Number of Barriers	Pr(Survival at 1,000 years)			
	0.9	0.99	0.999	0.9999
1	4,531	10,337	16,450	22,632
2	1,410	1,990	2,297	2,487
3	1,058	1,440	1,638	1,761
4	913	1,228	1,390	1,491
5	831	1,111	1,256	1,346

For barrier lifetimes that follow the Weibull distribution, the calculations of the required mean lifetimes are straightforward. The CDF of the Weibull distribution can be written as

$$F(x) = 1 - e^{-ax^b}$$

Suppose that a container has n identical barriers failing in parallel; to determine a mean barrier lifetime such that the probability of survival at 1000 years equals p , we must first determine values of the parameters a and b such that

$$[1 - e^{-a1000^b}]^n = 1 - p.$$

There is not a unique a and b that satisfy this condition; as in previous cases, however, we can specify a value of the coefficient of variation. From the earlier discussion of the Weibull distribution, it can be seen that

$$\sigma/\mu = \frac{\sqrt{\Gamma(2\beta+1) - \Gamma^2(\beta+1)}}{\Gamma(\beta+1)},$$

where $\beta = 1/b$; hence, specifying a value of the coefficient of variation specifies a value of the parameter b . Constraining the failure probability then determines a value for the parameter a ; the values of a and b can then be used to determine the mean lifetime, using the formula

$$\mu = a\beta\Gamma(\beta+1)$$

presented earlier. Results of calculations of mean lifetimes are shown below (this example uses $(\sigma/\mu) = 0.8$ as in the sequential Weibull example earlier):

Number of Barriers	Pr{Survival at 1,000 years}			
	0.9	0.99	0.999	0.9999
1	5,560	35,979	225,048	1,403,120
2	2,006	5,560	14,286	35,979
3	1,353	2,866	5,560	10,467
4	1,082	2,006	3,398	5,560
5	932	1,594	2,492	3,757

Conclusions

The cases analyzed in this report demonstrate the general principle that overall container reliability can be increased by increasing the number of container barriers, as well as increasing the reliability of each barrier.

This general principle holds true regardless of the barrier lifetime distribution and regardless of whether barriers fail in sequence or in parallel. The series and parallel cases examined above give bounds on the required lifetimes of the barriers: a multi-barrier container with identical barriers cannot fail any faster than if all its barriers fail in parallel, nor can it fail any slower than if each barrier does not begin degrading until its predecessor has failed. Combining the results from the series and parallel cases, we can construct, for each distribution, a table of bounds on barrier lifetimes. For the assumption of exponential barrier lifetimes, the table is:

Number of Barriers	Pr(Survival at 1,000 years)			
	0.9	0.99	0.999	0.9999
	$(\sigma/\mu) = 1$	$(\sigma/\mu) = 1$	$(\sigma/\mu) = 1$	$(\sigma/\mu) = 1$
1	9,491	99,499	999,527	9,999,924
2	1,880-2,631	6,732-9,491	22,025-31,120	70,345-99,499
3	907-1,603	2,293-4,121	5,248-9,491	11,603-21,040
4	573-1,210	1,215-2,631	2,333-5,107	4,315-9,491
5	411-1,003	782-1,970	1,353-3,457	2,250-5,795

For the assumption of normally distributed barrier lifetimes, the table is:

Number of Barriers	Pr(Survival at 1,000 years)			
	0.9	0.99	0.999	0.9999
	$(\sigma/\mu) = 0.6080$	$(\sigma/\mu) = 0.3882$	$(\sigma/\mu) = 0.3039$	$(\sigma/\mu) = 0.2570$
1	4,531	10,337	16,450	22,632
2	1,114-1,410	1,384-1,990	1,489-2,297	1,543-2,487
3	606-1,058	697-1,440	728-1,638	744-1,761
4	410-913	456-1,228	471-1,390	479-1,491
5	307-831	335-1,111	345-1,256	349-1,346

For barrier lifetimes that follow the Weibull distribution, the table is:

Number of Barriers	Pr(Survival at 1,000 years)			
	0.9	0.99	0.999	0.9999
	$(\sigma/\mu) = 0.8$	$(\sigma/\mu) = 0.8$	$(\sigma/\mu) = 0.8$	$(\sigma/\mu) = 0.8$
1	5,560	35,979	225,048	1,403,120
2	1,400-2,006	3,800-5,560	9,800-14,286	24,600-35,979
3	700-1,353	1,500-2,866	2,900-5,560	5,500-10,467
4	500-1,082	900-2,006	1,400-3,398	2,300-5,560
5	350-932	600-1,594	900-2,492	1,300-3,757

These results clearly demonstrate the substantial gains in reliability made possible by multiple barriers. This conclusion holds for numerous types of distributions and for either sequential or parallel exposure to failure-causing processes. The benefits from these gains would be reflected both in the reduced effort required to design barriers to meet the appropriate lifetime standards, as well as in the increased amenability of these barriers to testing and verification of performance.

It is also important to emphasize that the specific numerical examples shown in this report deal only with the container itself, not with the other parts of the overall system, e.g. the surrounding rock, the cladding of the waste, etc. A more complete analysis would take these components into account. Furthermore, a complete analysis must also address the long-term release phase after the initial thousand years.

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