NODAL ANALYSIS OF TWO-PHASE INSTABILITIES

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Nodal models having moving nodal boundaries have been developed for the analysis of two-phase flow instabilities in a boiling channel. The first model, which was based on a Galerkin method for the discretization, has been found to be accurate in the prediction of the onset of instabilities as well as the frequency of oscillations. This model however, had some problems with the prediction of chaotic phenomena and did not allow for flow reversal in the channel. A second nodal model, based on a finite difference approach, has been found to perform better for the prediction of non-linear response and it also allows for flow reversal. Both models are numerically more efficient than the existing fixed grid models for instabilities analysis.

INTRODUCTION

The phenomena known as density-wave instability may occur in phase change systems, such as Boiling Water nuclear Reactors (BWRs). Our current understanding of density-wave oscillations is fairly good for linear phenomena (eg, the onset of instabilities) but is not very advanced for non-linear phenomena [1]. In particular, limit cycle and chaotic instability modes are not well understood current and advanced generation BWRs (eg, the SBWR). Moreover, the proposed new SBWR operates in natural circulation and is thus inherently more prone to density-wave instabilities.

Previous works have dealt with the modeling and analysis of the dynamics of a boiling channel. Achard et al. [2] used a distributed parameter homogeneous equilibrium model (HEM)

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to perform linear stability and dynamic bifurcation analysis. Clausse et al [3] presented a nodal lumped parameter HEM model having moving nodal boundaries, and by numerically integrating the nodal equations found limit cycles and chaotic oscillations [3]. The effect of slip on the results for linear stability analysis and the transient response of a heated channel have been previously investigated [4,5], and it was found that the inclusion of slip improves the agreement of the calculations with experimental results. Moreover, the proper modeling of subcooled boiling was also found to be important [6].

In this topical report a new nodal lumped parameter drift-flux model for a boiling channel is presented. The results from a linear stability analysis, bifurcation analysis and time related domain evaluations are given and compared with data.

GENERAL DESCRIPTION OF THE PROBLEM

In deriving the system of ordinary differential equations which represent the lumped parameter model of a boiling channel, the following assumptions have been made:

- The system pressure is constant
- The power is uniform in the axial direction
- Both phases are incompressible
- Subcooled boiling is included via a profile-fit model [7]
- The phasic slip between the phases is quantified using a drift-flux model.
- Viscous dissipation, kinetic energy, potential energy and flow work are neglected in the energy equation.
- The inlet temperature is constant.
- The heated channel is vertical.

The one-dimensional drift-flux continuity, momentum and energy conservation equations for the two-phase mixture are [7]:

Mass

$$\frac{\partial \langle \bar{\rho} \rangle}{\partial t} + \frac{\partial G}{\partial z} = 0 \quad (1.a)$$

where

$$\langle \bar{\rho} \rangle = \rho_g \langle \alpha \rangle + \rho_f (1 - \langle \alpha \rangle)$$

Momentum

$$-\frac{\partial p}{\partial z} - g \langle \bar{\rho} \rangle - \frac{\tau_w P_f}{A_{x-s}} - \frac{\partial}{\partial z} \left[\frac{(\rho_f - \langle \bar{\rho} \rangle) \rho_f \rho_g}{(\langle \bar{\rho} \rangle - \rho_g) \langle \bar{\rho} \rangle} (V'_{gj})^2 \right] = \frac{\partial G}{\partial t} + \frac{\partial}{\partial z} \left(\frac{G^2}{\langle \bar{\rho} \rangle} \right) \quad (1.b)$$

where, $V'_{gj} = (C_o - 1) \langle j \rangle + V_{gj}$

Energy

$$\frac{q'' P_H}{A_{x-s}} - \frac{\partial}{\partial z} \left[\frac{\langle \alpha \rangle \rho_f \rho_g}{\langle \bar{\rho} \rangle} V'_{gj} h_{fg} \right] = \frac{\partial}{\partial t} (\langle \bar{\rho} \rangle \bar{h}) + \frac{\partial}{\partial z} (\langle \bar{\rho} \rangle u_m \bar{h}) \quad (1.c)$$

where,

$$\bar{h} = \frac{\rho_g \langle \alpha \rangle h_g + \rho_f (1 - \langle \alpha \rangle) h_f}{\langle \bar{\rho} \rangle}$$

and using the Zuber-Findlay void-quality model [8],

$$\langle \alpha \rangle = \frac{\langle x \rangle}{C_o \left[\langle x \rangle + \frac{\rho_g}{\rho_f} (1 - \langle x \rangle) \right] + \frac{\rho_g V_{gj}}{G}} \quad (1.d)$$

It is convenient to rewrite these equations in non-dimensional form as:

$$\frac{\partial \langle \bar{\rho} \rangle^+}{\partial t^+} + \frac{\partial (\langle \bar{\rho} \rangle^+ u_m^+)}{\partial z^+} = 0 \quad (2.a)$$

$$\begin{aligned} -Eu \frac{\partial p^+}{\partial z^+} - \frac{1}{Fr} \langle \bar{\rho} \rangle^+ - \Lambda \langle \bar{\rho} \rangle^+ u_m^{+2} - \left[\sum_{i=1}^N K_i \delta(z - z_i) \right] \frac{\langle \bar{\rho} \rangle^+ u_m^{+2}}{2} \\ - \frac{v_f}{v_g} \frac{\partial}{\partial z^+} \left[\left(\frac{1 - \langle \bar{\rho} \rangle^+}{\langle \bar{\rho} \rangle^+ - \frac{\rho_g}{\rho_f}} \right) \frac{1}{\langle \bar{\rho} \rangle^+} (V_{gj}^+)^2 \right] = \frac{\partial [\langle \bar{\rho} \rangle^+ u_m^+]}{\partial t^+} + \frac{\partial [\langle \bar{\rho} \rangle^+ u_m^{+2}]}{\partial t^+} \end{aligned} \quad (2.b)$$

$$\frac{N_{sub}}{N_{pch}} Q^+ - \frac{1}{\frac{v_f}{v_{fg}} N_{pch}} \frac{\partial}{\partial z^+} \left[\frac{\langle \alpha \rangle}{\langle \bar{\rho} \rangle^+} V_{gj}^+ \right] = \frac{\partial}{\partial t^+} (\langle \bar{\rho} \rangle^+ \bar{h}^+) + \frac{\partial}{\partial z^+} (\langle \bar{\rho} \rangle^+ u_m^+ \bar{h}^+) \quad (2.c)$$

The non-dimensional state equation for the two-phase mixture using a profile-fit model for the quality in the subcooled boiling region can be written as [7]:

$$\langle \bar{\rho} \rangle^+ = 1 - \frac{\langle h \rangle^+ - h_{td}^+ \exp\left(\frac{\langle h \rangle^+}{h_{td}^+} - 1\right)}{C_0 \left[\langle h \rangle^+ - h_{td}^+ \exp\left(\frac{\langle h \rangle^+}{h_{td}^+} - 1\right) + 1 \right] + \frac{V_{gj}^+}{G^+}} \quad (2.d)$$

A definition of these non-dimensional quantities is given in the Nomenclature.

Finite element approach

The lumped parameter model was based on a Galerkin nodal approximation of the conservation equations for a boiling channel [3] in which a spatially linear shape function was used for enthalpy.

Single-Phase Region

The single-phase region extends from the inlet of the channel to the net vapor generation point, L_{NVG} . The net vapor generation point (NVG) is defined as the axial position where the onset of subcooled boiling is produced. The criterion used in the determination of the onset of subcooled boiling is given by the phenomenological correlation given by Clausse et al [6].

As noted in Figure-1, this region has been divided into $N_{1\phi}$ nodes. In each of these nodes the enthalpy increase is assumed to be linear and equal to $\frac{(h_{td} - h_i)}{N_{1\phi}}$, where h_{td} is the enthalpy of the liquid at the net vapor generation point (i.e., void departure point). The boundary between consecutive nodes is defined as the point where the enthalpy of the liquid is,

$$h_n = h_i + \frac{n}{N_{1\phi}}(h_{td} - h_i) \quad (3)$$

where [6],

$$(h_{td} - h_f) = \begin{cases} q'' D_h c_{pf} / (455 k_f) & , Pe < Pe_c \\ \min \left\{ \begin{array}{l} (h_f - h_i) \\ (154 q'' / G)^{(1-\eta)} \end{array} \right\} & , Pe > Pe_c \end{cases} \quad (4)$$

$$Pe_c = 70,000 \left(1 + 99 \frac{q''}{c_{pf} G (T_{sat} - T_i)} \right)^{-1}$$

and,

$$\eta = \frac{99}{99 + (h_f - h_i)G / q''} \quad (5)$$

It is important to note that the position of the various nodal boundaries, L_n , are a function of time.

The single-phase energy equation was integrated in space over each node, using Leibnitz's rule for the time derivatives. The resulting nodal equations are:

$$\frac{dL_n}{dt} = 2u_i + \frac{2q'' P_H}{A_{x-s}} \frac{N_{1\phi}}{(h_{td} - h_i)\rho_l} (L_n - L_{n-1}) - \frac{dL_{n-1}}{dt} \quad (6.a)$$

which in non-dimensional form becomes:

$$\frac{dL_n^+}{dt^+} = 2u_{in}^+ + 2N_{1\phi} \frac{N_{pch}}{\Delta h_{1\phi}^+} Q^+ (L_n^+ - L_{n-1}^+) - \frac{dL_{n-1}^+}{dt^+} \quad (6.b)$$

where $\Delta h_{1\phi}^+$ is the non-dimensional enthalpy change in the single-phase region.

The time derivatives of the (n-1) previous nodal boundaries can be eliminated using backward substitution to achieve an ordinary differential equation (ODE) for the new state variable, $L_n(t)$. The position of the last single-phase boundary, $L_{N_{1\phi}}^+$, defines the so-called net vapor generation point, L_{nvg}^+ .

Two-Phase Region

The two-phase region extends from the net vapor generation point, to the exit of the heated channel. As can be seen in Fig. 1, this region was divided into $N_{2\phi}$ nodes. At the boundaries of these nodes the enthalpy is given by, $h_n = h_{td} + \frac{(h_e - h_{td})}{N_{2\phi}} n$, or in non-dimensional

form, $h_n^+ = h_{td}^+ + \frac{(h_e^+ - h_{td}^+)}{N_{2\phi}} n$, where h_c^+ is the non-dimensional two-phase enthalpy at the exit of the heated channel.

The flow quality in the subcooled boiling region was approximated using Levy's profile-fit model [7]. This model relates the flow quality, x , to the thermodynamic equilibrium quality, x_e , in the form,

$$x = x_e - x_{e,d} \Psi \quad (7.a)$$

where Ψ is a function of the equilibrium quality obtained by fitting the experimental data,

$$\Psi = \exp \left(\frac{x_e}{x_{e,d}} - 1 \right) \quad (7.b)$$

Substituting the flow quality into the state equation for $\langle \bar{\rho} \rangle^+$, Eq.(2.d) is obtained.

As before, a linear enthalpy profile was assumed within each two-phase node and the continuity equation was integrated over each node. The resulting non-dimensional nodal mass conservation equation is:

$$\frac{dM_n^+}{dt^+} - \langle \rho \rangle_n^+ \frac{dL_n^+}{dt^+} + \langle \rho \rangle_{n-1}^+ \frac{dL_{n-1}^+}{dt^+} + \langle \rho \rangle_n^+ u_{m_n}^+ - \langle \rho \rangle_n^+ u_{m_{n-1}}^+ = 0 \quad (8)$$

where the non-dimensional mass in node- n is given by:

$$M_n^+ = \int_{L_{n-1}^+}^{L_n^+} \langle \rho \rangle^+ dz^+ \quad (9)$$

and $\langle \rho \rangle^+$ is given by Eq. (2.d).

We may also assume that G is constant and equal to its average value within the node.

That is,

$$G_n = \bar{G}_n = \frac{(G_n + G_{n+1})}{2} = \frac{[\langle \bar{\rho} \rangle_n u_{m_n} + \langle \bar{\rho} \rangle_{n+1} u_{m_{n+1}}]}{2} \quad (10)$$

Unfortunately, the density in Eq. (9) does not have an analytical integral. However, we can use Gauss quadratures to obtain the mass of the node in the two-phase region:

$$M_{n2\Phi}^+ = (L_{n2\Phi}^+ - L_{(n-1)2\Phi}^+) - \frac{(L_{n2\Phi}^+ - L_{(n-1)2\Phi}^+)}{2} \sum_{j=1}^4 w_j f(x_j) \quad (11)$$

where,

$$f(x_j) = \frac{\langle h \rangle^+(x_j) - h_{td}^+ \exp\left(\frac{\langle h \rangle^+(x_j)}{h_{td}^+} - 1\right)}{C_0 \left[\langle h \rangle^+(x_j) - h_{td}^+ \exp\left(\frac{\langle h \rangle^+(x_j)}{h_{td}^+} - 1\right) + 1 \right] + \frac{V_{gj}^+}{G^+}} \quad (12)$$

The coordinates, x_j , and the weights, w_j , used are the ones given by Abramowitz and Stegun [13].

Differentiating Eq. (11) with respect to time and substituting the result into Eq. (5) the ODE which characterizes mass conservation for a node in the two-phase region was obtained,

$$\left(\frac{dL_n^+}{dt^+} - \frac{dL_{n-1}^+}{dt^+} \right) \left[1 - \frac{1}{2} \sum_{j=1}^4 w_j f(x_j) \right] - (L_n^+ - L_{n-1}^+) \frac{1}{2} \sum_{j=1}^4 w_j \frac{df(x_j)}{dt^+} - \langle \bar{\rho} \rangle_n^+ \frac{dL_n^+}{dt^+} + \langle \bar{\rho} \rangle_{n-1}^+ \frac{dL_{n-1}^+}{dt^+} + \langle \bar{\rho} \rangle_n^+ u_{mn}^+ - \langle \bar{\rho} \rangle_{n-1}^+ u_{mn-1}^+ = 0 \quad (13)$$

Unheated riser

The adiabatic riser above the heated channel was divided in N_r nodes having fixed boundaries. The equations for the unheated riser were obtained by integrating the continuity equation along each node of the riser, allowing for the possibility of having subcooled liquid with voids in the riser. The ODE for node- n in the unheated riser is given by:

$$\frac{dM_n^+}{dt^+} + \langle \rho \rangle_n^+ u_{m_n}^+ - \langle \rho \rangle_n^+ u_{m_{n-1}}^+ = 0 \quad (14)$$

where the mass of the n^{th} node of the riser is given by an expression analogous to Eq.(10).

Closure

The system of equations has $N_{1\phi} + N_{2\phi}$ equations and $N_{1\phi} + N_{2\phi} + 1$ unknowns. An additional independent equation is therefore necessary to achieve closure for the case of interest here, parallel channels. This equation comes from the boundary condition which specifies the pressure drop across the heated channel. In order to introduce this boundary conditions, the non-dimensional momentum equation, Eq. (2.b), was integrated along the channel. The following ODE was obtained.

$$\frac{dW^+}{dt^+} = Eu - \Delta p_a^+ - \Delta p_f^+ - \Delta p_L^+ - \Delta p_g^+ \quad (15)$$

where,

$$W^+ = \int_0^{L_{NVG}^+} \frac{\partial u_{in}^+}{\partial t^+} dz^+ + \int_{L_{NVG}^+}^1 \frac{\partial G^+}{\partial t^+} dz^+ \quad (16a)$$

$$\Delta p_a^+ = \langle \rho \rangle^+ u_{n_e}^{+2} - u_i^{+2} + \frac{\rho_g}{\rho_f} \left(\frac{1 - \langle \rho \rangle_e^+}{\langle \rho \rangle_e^+ - \frac{\rho_g}{\rho_f}} \right) \frac{1}{\langle \rho \rangle_e^+} (V_{gj}^+)^2 \quad (16b)$$

$$\Delta p_f^+ = \Lambda \left[\int_0^{L_{NVG}^+} u_{in}^{+2} dz^+ + \int_{L_{NVG}^+}^1 \frac{G^{+2}}{\langle \bar{\rho} \rangle^+} dz^+ \right] \quad (16c)$$

$$\Delta p_L = K_i u_{in}^{+2} + K_e \frac{G_e^{+2}}{\langle \rho \rangle_e^+} \quad (16d)$$

$$\Delta p_g^+ = \frac{1}{Fr} \left\{ \left(1 - L_{NVG}^+ \right) + \sum_{i=1}^{N_{2\phi}} M_{i_{2\phi}}^+ + \frac{b N_{sub}}{2 N_{1\phi}} \left[\sum_{i=1}^{N_{1\phi}} \left\{ (2n-1) (L_n^+ - L_{n-1}^+) \right\} L_{NVG}^+ \right] \right\} \quad (16e)$$

In a closed loop, additional ordinary differential equations are written for the downcomer region. In that case the sum of the pressure drop in the heated channel, the adiabatic riser and the downcomer is equal to zero. Otherwise, the pressure drop across the channel must be specified (i.e., the parallel channel case).

Finite difference approach

An alternative technique has been used for the transformation of the original conservation PDEs into a set of non-linear ODEs. As in the previous section, we will obtain a set of ODEs where the dependent variables are the positions of the boundaries of the nodes. The criterion for the nodalization of the heated channel remains the same.

Single-phase region

As before, we begin with the single-phase energy equation, Eq. (5). If we follow the position inside the channel where the liquid reaches an enthalpy equal to $h_n = h_i + n \Delta h_{1\Phi}$. The enthalpy at this position L_n does not change with time, therefore,

$$\frac{\partial h^+}{\partial t^+} = 0 = \frac{dh_n^+}{dt^+} + \frac{dL_n^+}{dt^+} \frac{\partial h_n^+}{\partial t^+} \quad (17)$$

From this expression, we can solve for the partial derivative of the enthalpy with respect to time at L_n and substitute in the energy equation:

$$\frac{dL_n^+}{dt^+} = u_i^+ + Q^+ \frac{N_{pch}}{N_{sub}} N_{1\Phi} [\gamma(L_n^+ - L_{n-1}^+) + (1-\gamma)(L_{n+1}^+ - L_n^+)] \quad (18)$$

where the coefficient γ will determine the scheme for the space derivative of the enthalpy.

Two-phase region

We begin with the two-phase continuity equation (1.b). To eliminate the partial derivative of the enthalpy with respect to time, we can not follow the same procedure as in the single-phase region, since the enthalpy at the boundary of the n^{th} node is now a function of time, $h_n = h_f + \Delta h_{2\Phi}$ and $\Delta h_{2\Phi} = (h_e(t) - h_f) / N_{2\Phi}$. The ratio of the change of enthalpy in the each node to the total change of enthalpy in the two-phase region is equal to:

$$\frac{\Delta h_{2\Phi}^+}{h_e^+} = \frac{h_n^+}{h_e^+} \quad (20)$$

and it does not change with time, therefore,

$$\frac{d}{dt^+} \left(\frac{h_n^+}{h_e^+} \right) = 0 = \frac{1}{h_e^+} \frac{\partial h_n^+}{\partial z^+} \frac{dL_n^+}{dt^+} - \frac{h_n^+}{h_e^{+2}} \frac{dh_e^+}{dt^+} \quad (21)$$

Solving for the time derivative of the enthalpy at L_n^+ and substituting it into the energy equation, we obtain a set of $N_{2\Phi_n}$ ODEs

$$\begin{aligned} \frac{dL_n^+}{dt^+} \left[1 + \frac{h_n^+}{c_0(1+h_n^+) + \frac{V_{gj}^+}{G_n^{+2}}} \left(1 - \frac{V_{gj}^+}{< j >_n^+} \right) \right] = & - \frac{1}{h_e^+} \frac{\partial h_e^+}{\partial z^+} \frac{dh_e^+}{dt^+} + \\ & \frac{\rho_{hn}^+ h_n^+ \frac{V_{gj}^+}{G_n^+} \frac{du_i^+}{dt^+} - Q^+ N_{pchl} \rho_{hn}^+ h_n^+ \frac{V_{gj}^+}{G_n^+} \frac{d\lambda^+}{dt^+}}{\left[c_0(1+h_n^+) + \frac{V_{gj}^+}{G_n^+} \right] \frac{\partial h_n^+}{\partial z^+}} + \\ & \left[-Q^+ N_{npchl} \rho_n^+ \left[c_0(1+h_n^+) + \frac{V_{gj}^+}{G_n^+} \right] + < j >_n^+ \rho_{hn}^{+2} \left[c_0(1+h_n^+) + \frac{V_{gj}^+}{G_n^+} \right] \frac{\partial h_n^+}{\partial z^+} \right] \end{aligned} \quad (22)$$

where the dependent variables are the positions of the boundaries of the nodes, $L_{N_{2\Phi_n}}^+$, the enthalpy at the exit of the channel, h_e^+ , and the boiling boundary, λ^+ . The space derivative of the enthalpy at the boundary of the node can be approximated as in the single-phase region.

In the ODEs for the two-phase region, the time derivative of the exit enthalpy appears explicitly. The ODE for the exit enthalpy is obtained by applying the same substitution, but taking into account that the boundary at the exit is fixed.

Closure of the model

Since the boundary condition is still given by the pressure imposed on the channel, the closure equation used is the same as the one used in the finite element approach, Eq. (15). With this approach we again obtain a system of $N_2\Phi+1$ equations and variables.

Linear Stability Analysis

A linear stability analysis for the system of ODEs for a heated channel was performed. The sensitivity of the linear stability map to the number of nodes was studied.

The resulting system of nonlinear ODEs can be written in matrix form as,

$$\dot{\underline{x}} = \underline{f}(\underline{x}) \quad (23)$$

where,

$$\underline{x} = (L_1^+, \dots, L_{N_1\Phi-1}^+, L_{NVG}^+, L_{N_1\Phi+1}^+, \dots, \rho_e^+, \rho_{r1}^+, \dots, \rho_{rN_r}^+, W^+) \quad (24)$$

The system of nonlinear ODE was linearized around their fixed points, $\underline{x}_0$, using first order perturbations, $\delta \underline{x} = \underline{x} - \underline{x}_0$. The resulting linear system is,

$$\delta \dot{\underline{x}} = \underline{J}_0 \delta \underline{x} \quad (25)$$

where the Jacobian matrix is ,

$$J_0 = \left. \frac{\partial \underline{f}}{\partial \underline{x}} \right|_{\underline{x}=\underline{x}_0} \quad (26)$$

The linear stability of the system can be determined from the eigenvalues, λ_i , of the Jacobian matrix:

$$\det[\underline{J}_0 - \lambda \underline{I}] = 0 \quad (27)$$

If all the eigenvalues (λ) have negative real parts, the system is linearly stable. If an eigenvalue is real and positive, the system has an excursive instability. If any pair of complex conjugate eigenvalues have positive real parts, the system is linearly unstable and oscillatory (i.e., an unstable phase plane spiral).

The eigenvalues of the system were calculated numerically for different numbers of axial nodes, and the linear stability boundary plotted in the $N_{pch} - N_{sub}$ phase plane. Figure 2 shows the linear stability map for 2 nodes in the single-phase region, and two nodes in the two-phase region, for both bulk boiling and subcooled boiling. As expected the inclusion of subcooled boiling improves the agreement with the data of Saha [9]. We have observed that the linear stability map changes only slightly when increasing from two to four single-phase nodes and that it does not change at all from four to six nodes. This is significant since the more nodes we have, the larger the computer time requirements in the nonlinear time domain evaluations of the model. The small number of nodes needed with this moving boundary model is an important feature, since other nodal models having fixed boundaries require on the order of 100 axial nodes to give results that compare favorably with experiments. [5]

BIFURCATION ANALYSIS

A Hopf bifurcation signals the transition of the solution between a stable fixed point and a limit cycle. The limit cycle can be either stable (supercritical bifurcation) or unstable (subcritical bifurcation) [2].

A Hopf bifurcation analysis has been performed on Eq(12) for a heated parallel channel. The program AUTO [10] was used for the numerical bifurcation analysis. The chosen bifurcation parameter was the phase change number, N_{pch} .

Figures 3 and 4 show the bifurcation diagram for given values of the subcooling number, N_{sub} , the friction number, Λ and the Euler number, Eu , and for $C_0=1$. and $V_{gj}=0.3$ m/s. Figure 3 presents a supercritical bifurcation in which the limit cycle is stable. In Fig. 4 the system was

found to also exhibit a subcritical bifurcation. In this case, the limit cycle is unstable. This means that if the system is perturbed beyond the amplitude of the unstable limit cycle, it diverges exponentially. This identifies a region in parameter space that is potentially very dangerous in terms of the operation of a heated boiling channel since a CHF may occur subsequent to large perturbations even though the channel is operating in the region of linear stability.

FREQUENCY DOMAIN ANALYSIS OF THE NODAL MODELS

The system of ODEs for the single-phase region is linear for both nodalizations, therefore it can be analyzed using the methods of linear control theory [14]. We may apply the Laplace transform to go from the time domain, t , to the frequency domain, $s = i\omega$. Figure 5 shows the block diagram that represents the system of ODEs for the finite element nodalization approach, where:

$$c = \frac{2u_{io}}{L_{No}} N$$

and N is the number of nodes.

The transfer function of this linear system is found to be:

$$F(s) = \frac{L_n(s)}{u_i(s)}$$

$$L_n(s) = \{L_{n-1}(s)(c-s) + 2u_i\} \frac{1}{s+c}$$

$$n = 1 \dots N$$

Figure 6 shows the corresponding block diagram for the frequency domain representation of the system of ODEs for the single-phase region obtained with the finite difference nodalization method.

The transfer function of this system is given by:

$$F(s) = \frac{L_n(s)}{u_i(s)}$$

where,

$$L_n(s) = \left\{ L_{n-1}(s)c + u_i \right\} \frac{1}{s+c}$$

$$n = 1 \dots N$$

In the calculations of the previous sections, the inlet velocity, u_i , was a variable of the problem. However, to analyze the behavior of the linear system of ODEs for the single-phase region and thus the behavior of the time evolution of the boiling boundary given by each nodalization approach, we can force the system with a given inlet velocity $u_i(t)$ and observe the response of the system.

The gain of the system is defined as the ratio between the amplitude of the output of the system, L_N , and the amplitude of the input, u_i . Figure 7 shows the gain as a function of the frequency of the forcing. It can be easily seen that the response of the system can have many anti-resonant points at different perturbation frequencies, when $N \geq 3$ and the frequency > 0.1 Hz. We believe that the minima in the gain arises from the third (and negative) term in Equation (6.a). These last terms appear as dashed lines and they represent the connection between the nodes, $-s$ in the frequency domain.

Figure 8 shows the gain as a function of the forcing frequency for the single-phase region ODEs obtained with the finite difference approach. It can be observed that the gain varies smoothly with the frequency and no minima are observed.

This observed behavior leads to the conclusion that for certain oscillation frequencies, the response of system of ordinary differential equations obtained with the finite element approach

could appear damped and even look like a stable solution. This is particularly troublesome if the model is to be used to predict chaotic behavior, since all frequencies are present in a chaotic response. The model obtained with the finite difference approach does not present this kind of behavior, and therefore it is expected to have a better behavior in the region of parameter space where chaotic oscillations could be found.

NUMERICAL ANALYSIS. COMPARISON OF THE MODELS

Both sets of ODEs for the single-phase region have been forced with a nonlinear combination of periodic functions. Figure 9 shows the shape of the oscillatory forcing inlet velocity. Figure 10 shows the comparison of a response of the finite element model for 200 nodes with the solution found using the method of characteristics [7]. It can be observed that the model has difficulty following the high frequency components of the oscillation. Figure 11 shows the comparison of the response of the finite difference model with the method of characteristics, at the same location in the channel and for the same number of nodes. It can be observed that this model exhibits a better behavior in following the non-linear forcing. Finally, Figure 12 shows the response of a fixed boundaries nodal model [5]. It is obvious that even with a large number of nodes, the fixed boundaries model cannot follow the nonlinear forcing in the inlet velocity.

FLOW REVERSAL

The finite element model has the implicit assumption of upward flow, and can therefore not be modified to account for flow reversal. The finite difference based model, however, has no assumptions made about the direction of the flow, and the modifications to include the possibility of flow reversal are straight forward.

The main point to consider is the behavior of the enthalpy at the inlet of the channel when the inlet velocity becomes negative. The liquid will be heated as it exits the channel, and thus the

inlet enthalpy will not be constant anymore. We need then to add a differential equation for the inlet enthalpy. This equation can be found with the same approach used to obtain the single-phase ODEs, but considering that the boundary of the node (i.e. the inlet) is fixed. This gives us the following equation:

$$\frac{dh_i^+}{dt^+} = Q^+ N_{pch} - u_i^+ / N_{1\phi} \frac{h_i^+}{L_1} \quad (28)$$

After the flow starts going upwards again, the time evolution of the inlet enthalpy is found by linear interpolation between the value of the enthalpy at the first node (outside the channel) and the value of the enthalpy at the second node (inside the channel). If more than one node leaves the channel, the linear interpolation will be between the last node outside the channel and the first one inside. When the liquid is flowing upwards at the inlet, the equation for the inlet enthalpy is simply $dh_i^+/dt=0$ and $h_i^+=-N_{sub}$.

Figure 13 shows the time evolution of the boundary of the first node with flow reversal.

SUMMARY AND CONCLUSIONS

Two drift-flux nodal models have been derived for the analysis of non-linear oscillations in boiling channels. The finite element based model has been found to be accurate for the prediction of the onset of linear instabilities when compared with experimental data. Good agreement has been found also in the Hopf bifurcation analysis when compared with the results obtained using a distributed parameter models. However, the model has shown poor response to non-linear forcing with high frequency components. Supercritical and subcritical Hopf bifurcations have been found in certain operational conditions. The ability to predict subcritical Hopf bifurcations is a major advantage due to their dangerous connotations for the safe operation of boiling systems. The frequency domain analysis shows that this models presents minima in the gain of the system for several forcing frequencies. Moreover, the finite element model does not allow for the reversal of the direction of the flow at the inlet of the channel.

The finite difference based model has been found to have better behavior in its response to non-linear forcing. This model shows a much better response than existing fixed boundaries nodal model. Also the frequency domain analysis shows that this model does not present undesirable minima in the gain. Another interesting feature of this model is that it allows for the reversal of the flow at the inlet of the channel.

We conclude that the finite element model can be used for the analysis of the onset of instability. Its ability to provide good agreement with the experiments and the distributed parameter calculations for a small number of node (5 nodes compared to 80 or more for a fixed nodal model) makes it a good tool for linear stability and Hopf bifurcation analysis. However, due to the minima in the gain vs frequency and its behavior when forced with a nonlinear inlet velocity, we do not recommend it for non linear oscillations (e.g. chaotic) analysis.

The finite difference model shows a much better response when forced with a non-linear inlet velocity. Calculations, not presented here, show good agreement with experiments for the

onset of instabilities with a slightly higher number of nodes (10 vs 5). The absence of minima in the gain vs frequency calculations and its response to nonlinear forcing makes it a better tool for nonlinear oscillations analysis. Indeed, its ability to account for flow reversal is a very advantageous feature.

NOMENCLATURE

A_{x-s}	Cross-sectional flow area	u_m	center-of-mass velocity, $G/\langle\rho\rangle$
c_{pf}	liquid heat capacity	v	specific volume
D	diameter	z	axial coordinate
Eu	Euler number	$\langle\alpha\rangle$	void fraction
f	friction factor	β	Boussinesq approximation coefficient
Fr	Froude number	Λ	nondimensional friction factor
G	mass flux	Λ_{max}	maximum Lyapunov exponent
g	gravity	ρ	density
h	enthalpy		
h_{td}	liquid enthalpy at net vapor generation (ie, void departure point)		
L	length		
M	mass		
N_{sub}	subcooling number		
N_{pch}	phase change number		
p	pressure		
P	perimeter		
q''	heat flux		
t	time		
u	axial velocity		

Subscripts

i inlet

o steady-state

 ℓ subcooled liquid

g gas

ref reference value

d void departure point

NVG net vapor generation point

Non-Dimensional Quantities

The non-dimensional quantities in the equations for the heated boiling channel are defined

as:

$$z^+ = \frac{z}{L_H}$$

$$\lambda^+ = \frac{\lambda}{L_H}$$

$$t^+ = \frac{t}{t_{\text{ref}}}$$

$$u^+ = \frac{u}{u_{\text{ref}}}$$

$$\rho^+ = \frac{\rho}{\rho_f}$$

$$p^+ = \frac{(p - p_i)}{\Delta p_{\text{tot}}}$$

$$h^+ = \frac{(h - h_f)}{h_{\text{ref}}}$$

$$Q^+ = \frac{\left(\frac{q}{A_{x-s}} P_H \right)}{\left(\frac{q}{A_p} P_H \right)}$$

$$t_{\text{ref}} = \frac{\rho_f (h_f - h_{i0})}{\left(\frac{q}{A_p} P_H \right)}$$

$$u_{\text{ref}} = (gL_H)^{1/2}$$

$$h_{\text{ref}} = h_{fg} v_f / v_{fg}$$

$$N_{\text{sub}} = \frac{(h_f - h_{i0}) v_{fg}}{h_{fg} v_f}$$

$$N_{\text{pch}} = \frac{q P_H u_{\text{ref}} v_{fg}}{A_{x-s} h_{fg} v_f}$$

$$\Lambda = \frac{f L_H}{2 D_H}$$

$$\text{Fr} = \frac{u_{\text{ref}}^2}{g L_H}$$

$$\text{Eu} = \frac{\Delta p_{\text{tot}}}{2 \rho_f u_{\text{ref}}^2}$$

$$b = \frac{\beta h_{fg} v_f}{C_{pf} v_{fg}}$$

REFERENCES

- [1] Lahey, R.T., Jr. and Podowski, M.Z., "On the Analysis of Instabilities in Two-Phase Flows," Multiphase Science & Technology, Vol-3, Hemisphere Press, 1987.
- [2] Achard, J.L., Drew, D.A. and Lahey, R.T., Jr., "The Analysis of Nonlinear Density-Wave Oscillations in Boiling Channels," J. of Fluid Mechanics, Vol-155, pp. 213, 1985.
- [3] Clause, A. and Lahey, R.T., Jr., "The Analysis of Periodic and Strange Attractors During Density-Wave Oscillations in Boiling Flows," J. of Chaos, Solitons and Fractals, Vol-1, No. 2, pp. 17, 1991.
- [4] Rizwan Uddin, "Linear and Nonlinear Stability Analysis of Density-Wave Oscillations in Heated Channels," PhD Thesis, Department of Nuclear Engineering, University of Illinois, Urbana-Champaign, 1987.
- [5] Pinheiro-Rosa, M and Podowski M, "Nonlinear effects in two-phase flow dynamics", ANS Student Conference, Rensselaer Polytechnic Institute, Troy, NY, 1993.
- [6] Clause, A. and Lahey R.T "The effect of flow development in subcooled boiling" Int Comm. in Heat Transfer, vol. 17 1990.
- [7] Lahey, R.T. and Moody, F.J., "The Thermal-Hydraulics of a Boiling Water Nuclear Reactor", ANS Mongraph, 1977
- [8] Zuber N. and Findlay J.A. "Average Volumetric Concentration in Two-Phase Flow Systems", J. of Heat Transfer, Vol. 87, pp. 453-468, 1965.
- [9] Saha, P. "Thermally Induced Two-Phase Flow Instabilities, Including the Effect of Thermal Non-Equilibrium Between the Phases" , Ph.D. Thesis, School of Mechanical Engineering, Georgia Institute of Technology , 1974
- [10] Doedel E., "AUTO - User Manual", California Institute of Technology, 1986.
- [11] Lahey, R.T., Jr, Chang , C.J. and Bonetto F.J., "Chaotic Phenomena in Density Wave Oscillations", *Int. Sym on New Bases for Engineering Science* --

Algorithms, Dynamics and Fractals, pp 21-64, Nov. 23-28, Okayama, Japan, 1992.

- [12] Wolf, A., Swift, J.B., Swinney, H.L. and Vastano, J.A., "Determining Lyapunov Exponents in a Time Series," Physica 16D, pp. 285, 1985.
- [13] Abramowitz M. and Stegun, I., "Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables", U.S. National Bureau of Standards, Applied Mathematics Series, 55. Washington, 1972.
- [14.] Kuo, B.C.: "Automatic Control Systems", Prentice-Hall Inc., Englewood Cliffs, New Jersey

FIGURE 1 Schematic of the boiling channel

FIGURE 2 Linear stability analyses. Effect of subcooled boiling

FIGURE 3 Supercritical bifurcation

FIGURE 4 Subcritical bifurcation

Figure 5: Block diagram for the frequency domain representation of the system of ODEs with the finite element approach

Figure 6: Block diagram for the frequency domain representation of the system of ODEs with the finite difference approach

Figure 7: Gain versus frequency-Finite element model

Figure 8: Gain versus frequency-Finite difference model

Figure 9: Time evolution of the forcing inlet velocity.

Figure 10: Time evolution of the boiling boundary. Finite element approach, 200 nodes. Comparison with the Method of characteristics.

Figure 11: Time evolution of the boiling boundary. Finite difference approach, 200 nodes. Comparison with the method of characteristics.

Figure 12: Time evolution of the boiling boundary. Fixed nodes approach, 200 nodes. Comparison with the Method of characteristics.

Figure 13: Time evolution of the boundary of the first node with flow reversal.