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**Analytical and Experimental
Investigation of the Dispersion
Process During Rapid Transients
for the Aluminum-Based Nuclear
Fuel Plates**

V. Georgevich
R. P. Taleyarkhan
S. H. Kim
T. Fuketa
K. Soyama
K. Ishijima

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DISPERSION PROCESS DURING RAPID TRANSIENTS
FOR THE ALUMINUM-BASED NUCLEAR FUEL PLATES**

V. Georgevich, R. P. Taleyarkhan, S. H. Kim
ETD, Oak Ridge National Laboratory.

and

T. Fuketa, K. Soyama, K. Ishijima
Japan Atomic Energy Research Institute, Japan

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Prepared by the

OAK RIDGE NATIONAL LABORATORY

Oak Ridge, Tennessee 37831

managed by

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LIST OF ACRONYMS

ANS - Advanced Neutron Source
FCI - Fuel Coolant Interaction
HFIR - High Flux Isotope Reactor
JMTR - Japan Materials Test Reactor
NSRR - Nuclear Safety Research Reactor

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ABSTRACT

A thermally induced fuel-plate dispersion model was developed to analyze for dispersive potential and determine the onset of fuel plate dispersion for aluminum-based research and test reactor fuels. The effect of rapid energy deposition in a fuel plate was simulated. Several data types for aluminum-based fuels tested in the Nuclear Safety Research Reactor (NSRR) facility in Japan and in the Transient Reactor Test (TREAT) facility in Idaho, U.S.A., were reviewed. Analyses of experiments show that the onset of fuel dispersion is clearly linked to a sharp rise in the predicted strain rate, which further coincides with the onset of aluminum vaporization. Analysis also shows that aluminum oxidation and exothermal chemical reaction between the fuel and aluminum can significantly affect: the energy deposition characteristics (and, therefore dispersion onset connected with aluminum vaporization, and the onset of aluminum vaporization.

1.0 INTRODUCTION

Containment loads during hypothetical severe accidents of research and test reactor systems using uranium-aluminum (U-Al) based fuel can be significantly impacted by energetic fuel coolant interaction (FCI) events. To analyze energetics of FCI events the molten material breakup or dispersion characteristics need to be evaluated. Material type plays an important role in determining the phenomenological process that will disperse the fuel. Under rapid heating, aluminum-based fuels disperse at temperatures that are significantly above the melting temperature of the fuel plate. Dispersion of aluminum-based fuel is proposed to be a function of volumetric strain rate. The concept of using strain rate as the measure of dispersive potential originates from the requirement that the fuel has to be accelerated to a certain velocity in order for it to fragment into smaller pieces. The strain rate concept is used extensively in determining the spallation^[1] characteristics for high strain rate high deformation problems.

To determine the mechanism by which acceleration occurs, several separate mechanisms for strain rate were implemented in the model. These mechanisms include the strain rates caused by thermal expansion in solid state, expansion due to melting-induced phase change, thermal expansion in liquid state, expansion due to the presence of gases in the plate (present at fabrication or caused by fission product gases), and expansion due to vaporization-induced phase change. Since material strength properties are not well characterized, the modeling procedure used to determine the strain rates consists of using the concept of predicting the potential for generating strain rates. To illustrate the concept of using the potential for generating strain rates, the analogy of the conversion of potential to kinetic energy can be applied. Using the potential energy analogy, and by determining the potential for achieving a certain strain rate, the model can then be used to predict the onset of dispersion as long as material properties do not interfere with the evaluation in the range of interest. Results from analysis described in subsequent sections and associated analyses shows the assumption is justified.

2. STRAIN RATE MODEL

The strain rate model uses a local strain rate and integrates it to a global value. The peak time dependent global strain rate value is compared for different energy depositions. The next section describes how to determine the local strain and the following section describes the integration process used to determine the global strain rate.

2.1 LOCAL STRAIN RATE

To evaluate the dispersion potential, the maximum possible velocity or strain rate must be determined during the transient. To understand and rank the primary mechanisms leading to the onset of fuel-aluminum mixture dispersion, a one-dimensional (1-D) modeling framework centered on the prediction of material strain rates was developed. Strain is defined as

$$\epsilon = \frac{\Delta L}{L}, \quad (1)$$

where ϵ is the strain, ΔL is the change in length, and L is the original length. A dimensionless approach was taken to eliminate the effect of fuel-plate thickness on the results. Differentiation with respect to time results in the strain rate given by Eq. (2).

$$\dot{\epsilon} = \frac{d}{dt} \left(\frac{\Delta L}{L} \right), \quad (2)$$

Strain rate for the physical process described earlier can be expressed by

$$\dot{\epsilon} = (1-f)\{\dot{\epsilon}_{th} + \dot{\epsilon}_m + \dot{\epsilon}_v\} + f\dot{\epsilon}_{gas}, \quad (3)$$

where f is the fraction of original volume composed of vapor or gas, ϵ_{th} is the thermally induced strain rate, ϵ_m is the melting-induced strain rate, ϵ_v is the vaporizing induced strain rate, ϵ_{gas} is the strain rate induced by the expansion of gases. The volumetric change rate due to the thermal expansion of material at a particular location is given by

$$\begin{aligned} \dot{\epsilon}_{th}(z,t) &= \frac{d}{dt} \left(\frac{\Delta L}{L} \right) = \frac{d}{dt} (\beta \{T(z,t) - T_o\}) \\ &= \beta \frac{dT(z,t)}{dt}, \end{aligned} \quad (4)$$

where β is the thermal expansion coefficient, $T(z,t)$ is the temperature, and T_o is the initial temperature. When a material such as aluminum is undergoing a phase change, the density change will cause a change in volume. If material is melting at a certain rate, then the strain rate is

$$\dot{\epsilon}_m(z,t) = \frac{\rho_s - \rho_f}{\rho_f} \frac{dx_m(z,t)}{dt}, \quad (5)$$

where x_m is the melt quality (fraction) and ρ_s and ρ_f are the solid and liquid densities, respectively. Similarly, for vaporization phase change the strain rate is

$$\dot{\epsilon}_v(z,t) = \frac{\rho_f - \rho_v}{\rho_v} \frac{dx_v(z,t)}{dt}, \quad (6)$$

where x_v is vapor quality (fraction) and ρ_v is the vapor density. As gases and fission products heat up, they expand. The strain rate (the rate of change of volume) due to the gas expansion (assuming ideal gas behavior) is given by

$$\dot{\epsilon}_{\text{gas}}(z,t) = \frac{1}{T(z,t)} \frac{dT(z,t)}{dt}. \quad (7)$$

From Eqs. (5-7) the strain rate can be seen to depend on the change in temperature. To determine the rate of change in temperature as a function of time, position, and energy deposition rate, thermal energy transfer under various regimes needs to be evaluated. Section IV discusses the method used to determine the transient temperature profile. Evaluation of the local strain rate caused by several different mechanisms was previously explained. How the local strain rates interact with each other is outlined in the next section.

2.2 GLOBAL STRAIN RATE

Local strain rate acting at the center of the fuel plate will not affect the plate identically as the same magnitude strain rate acting closer to the plate surface. The expansion at the center will cause movement of the material starting from the center to the surface whereas, the expansion close to the surface will cause the expansion from that location to the surface. These are illustrated by the Figs 1. & 2. below. The first example will cause the entire plate to expand with velocity v whereas the second example will cause the last three nodes to expand with velocity v .

Fig. 1. Illustration of the effect of localized expansion.

If both nodes are expanding with velocity v then the first through the fourth node will expand with velocity v and the fifth through the seventh will expand with velocity $2v$. This example illustrates the spatial effect of expansion. This has significant implications in predicting behavior of plates that have steep temperature gradients.

Mathematically the expression is justified by determining the effective impulse that the plate is experiencing. Local impulse (I) imparted to the plate at position x is given by the mass of material after point x that is affected by expansion at point x multiplied by the expansion velocity.

$$I(\eta) = m(L - \eta)v(\eta) = v(\eta) \int_{\eta}^L \rho(x) A dx \quad (8)$$

Overall impulse, ($\bar{I}$) defined as by the integral of the impulse over the entire plate normalized by the plate thickness.

Fig. 2. Illustration of the effect of multiple expansion sites.

$$\bar{I} = \frac{\int_0^L I(\eta) d\eta}{\int_0^L d\eta} = \frac{\int_0^L v(\eta) \left\{ \int_{\eta}^L \rho(x) A dx \right\} d\eta}{\int_0^L d\eta} \quad (9)$$

Simplifying Eq. (9) results in

$$\bar{I} = \bar{v}m_t = \bar{v} \int_0^L \rho(x) A d\eta = \frac{\int_0^L v(\eta) \left\{ \int_\eta^L \rho(x) A dx \right\} d\eta}{\int_0^L d\eta}, \quad (10)$$

where, average velocity is given by

$$\bar{v} = \frac{\int_0^L v(\eta) \left\{ \int_\eta^L \rho(x) dx \right\} d\eta}{L \int_0^L \rho(x) d\eta}. \quad (11)$$

If strain is defined as the change in original length, then the strain rate is given by the rate of change of length divided by the original length

$$\bar{\dot{\epsilon}} = \frac{\int_0^L \dot{\epsilon}(\eta) \left\{ \int_\eta^L \rho(x) dx \right\} d\eta}{L \int_0^L \rho(x) d\eta}. \quad (12)$$

Therefore, the local strain rate can be integrated into a global value strain rate for the entire plate using Eq. 12. Evaluating a local strain rate, spatial transient temperature profile needs to be determined. Section 4 describes the procedure used to determine the temperature profile.

3. THERMAL ENERGY TRANSFER MODEL

Determining the global strain rate requires knowledge of spatial transient temperature behavior of the plate. To determine the transient temperature profile energy deposition rates need to be available along with the material properties for each material that composes the fuel plate. Using the conservation of energy equation the transient temperature profile can be determined. Method of solving the energy conservation equation is outlined next.

3.1 CONSERVATION OF ENERGY EQUATIONS

The conservation of energy equation that determines the 1-D transient temperature profile in a fuel plate is given by

$$k \frac{d^2 T(z,t)}{dz^2} + q''' = \rho c_p \frac{dT(z,t)}{dt}, \quad (13)$$

where $T(z,t)$ is the position and time-dependent temperature, c_p is the specific heat capacity, q''' is the volumetric heat generation source, and k is the material thermal conductivity. In a finite difference approximation the conservation equation for the i^{th} node is given by:

$$\frac{T_{i-1}}{\Delta z^2} - \left(\frac{2}{\Delta z^2} + \frac{\alpha_i}{\Delta t} \right) T_i + \frac{T_{i+1}}{\Delta z^2} = -\frac{q_i'''}{k} - \alpha_i \frac{T_i^*}{\Delta t}, \quad (14)$$

where α_i is the thermal diffusivity for the i^{th} node, T_i is the temperature for node i during the current time step, T_i^* is the temperature for node i during a previous time step, Δz is the mesh size, and Δt is time step. If any node is undergoing a phase change from solid to liquid, Eq. (10) determines the extent of change:

$$k \frac{d^2 T}{dz^2} + q''' = \rho h_{sf} \frac{dx_s}{dt}, \quad (15)$$

where x_s is the melting quality of the solid, and h_{sf} is the latent heat of fusion. In a finite difference form Eq. (10) is given by

$$x_{si} = x_{si}^* + \frac{\Delta t}{\rho h_{sf}} \left\{ k \left(\frac{T_{i+1}^* + T_{i-1}^* - 2T_i^*}{\Delta z^2} \right) + q_i''' \right\}. \quad (16)$$

Equation (16) is the conservation equation involving a material vaporization process, and Eq. (17) is its finite difference approximation.

$$k \frac{d^2 T}{dz^2} + q''' = \rho h_{fg} \frac{dx_f}{dt}, \quad (17)$$

where x_f is the mass quality of the liquid, and h_{fg} is the latent heat of vaporization.

$$x_{fi} = x_{fi}^* + \frac{\Delta t}{\rho h_{fg}} \left\{ k \left(\frac{T_{i+1}^* + T_{i-1}^* - 2T_i^*}{\Delta z^2} \right) + q_i''' \right\}. \quad (18)$$

The qualities x_s and x_f are defined over a range from zero to one. When the value of x_s is zero, the node is fully solid, and when x_s is equal to one, the node is liquid. Similarly, when material changes phases from liquid to vapor, x_f tracks the quality in an analogous manner.

An implicit method determines the transient temperature solution to Eq. (8). When melting or vaporization temperature of node i is reached, the temperature in the main matrix for node i is set to a constant melting temperature until the node fully melts or vaporizes. During the phase change process, energy equation for i^{th} node is given by Eqs. (17) or (19). These equations consider the energy deposited in the node and the energy flowing into or out of the node to determine the new quality. To solve the conservation equations, fuel plate boundary conditions have to be defined. The left boundary corresponds to the center line of the fuel (symmetry boundary condition) and the right boundary corresponds to the edge (convective/radiative boundary condition). The boundary conditions are given by Eqs. (20) and (21), respectively:

$$\frac{dT}{dz} = 0, \text{ and} \quad (19)$$

$$k \frac{dT(L,t)}{dz} = h(T(L,t) - T_{\text{inf}}), \quad (20)$$

where h is the heat transfer coefficient for the plate surface and T_{inf} is the bulk coolant temperature. Section 4.2 discusses the heat transfer coefficient modeling approach in more detail.

Results from this modeling method have been compared with thermal simulations using HEATING 7 code¹⁴. The spatial temperature predictions were always within 4% of each other.

3.2 PLATE SURFACE HEAT TRANSFER COEFFICIENT

Energy that is transferred from the surface of a heated fuel plate to the coolant is a function of temperature difference between the fuel plate surface and the medium that surrounds the fuel plate. Depending on the temperature of the fuel plate and the amount of energy that is generated in the plate, the heat transfer between the plate and the fluid can fall into several regimes. One regime corresponds to convective heat transfer from the surface to the liquid. The second region corresponds to the nucleate boiling regime or subcooled boiling, and the third region corresponds to the film boiling regime. Heat transfer characteristics in each of the states can be predicted using established steady state heat transfer correlations if the temperature of the plate surface does not change rapidly. If the surface changes rapidly, then the steady state heat transfer from the surface to the fluid may not be appropriate. Heat transfer during rapid transients can be significantly enhanced.

The mechanism by which the heat transfer is enhanced has to do with the physical characteristics of bubble formation process. Physical characteristics of bubble formation tend to change the liquid boundary layer condition. These bubble formation characteristics tend to change with the frequency of the energy generation.¹² For very short periods (high frequencies) heat transfer is significantly enhanced by the production of a larger number of bubbles that are smaller in size. The heat transfer enhancement can be up to 10 times larger than the heat transfer measured at steady state conditions^{13,14}.

The question that needs to be answered consists of determining the effective fraction of energy that the plate rejects by the transient boiling mechanism vs the amount of energy rejected

while in the film boiling stage. Analysis shows that the energy rejection mechanism for lower energy deposition where temperature of the plate does not enter film boiling regime is dominated by the transient boiling phenomena. If the energy deposition is large, the effect of the transient boiling heat transfer is not sufficient to justify a complex heat transfer model because the film boiling is reached quickly and the film boiling regime dominates the heat rejection mode.

For, large energy depositions such as in the cases where dispersion occurs, the film boiling condition is reached rapidly (viz., during the first 10% time duration of the pulse). Therefore, the heat transfer coefficient is not significantly affected by the transient nature of nucleate boiling heat transfer as much as during the lower energy deposition tests.^[2]

4. SOURCES OF ENERGY

There are several sources of energy that a fuel plate can experience during a pulse type transient. One source of energy is caused by fission heating of energy where uranium atoms release energy during fissioning process. The second source of energy is caused by chemical reaction. The chemical energy can be released by two processes. The first process is caused by the energy released when fuel particles react exothermically with the aluminum. The second process is caused by aluminum reaction with oxygen in water to form aluminum oxide. The following subsections describe the treatment of each of the energy release processes.

4.1 FISSION ENERGY

The fission energy during a pulse type tests is deposited into the test specimen (fuel) by neutron interaction with the uranium-235. The energy release follows the neutron flux level during the transient given by:

$$P(t) = \gamma \Sigma_f \phi(t). \quad (21)$$

where

$P(t)$ is the time dependent energy density generation rate (W/m^3)

γ - energy released per fission (J)

ϕ - neutron flux ($\text{neutrons/cm}^2/\text{s}$)

Σ_f - fission cross section ($1/\text{cm}$)

Determining the energy deposition is more involved than simply taking the neutron flux and integrating it over time. The complexity is caused by the inability to accurately measure the neutron flux incident on the plate. A method used to estimate the cumulative time integrated incident flux consists of identifying the activity of fission products in the irradiated fuel plate. Based on the activity of certain isotopes, total number of fission are back calculated. This activity represents the cumulative neutron-uranium interactions. Neutron flux from every nuclear reactor has a prompt and delayed neutron spectrum. Since the component of energy that is released immediately during the pulse is of interest, prompt and delayed neutron components need to be known for each transient. If the delayed neutrons are not subtracted, the amount of energy predicted by using the measured fission product activity can be overestimated. This detail of predicting the amount of energy deposited can be found in a paper presented by Ohnishi et. al ¹¹.

Each transient has its own specific ratio of prompt to delayed neutron fractions that has to be determined for each test. This procedure also depends on the type of test specimen. Once the calculation is performed to determine the cumulative amount of prompt energy that is deposited in the fuel plate, the temporal neutron flux distribution is then used as a shape function to distribute the energy.

The method in the evaluations is described in the following Sects. 5.1.1 and 5.1.2 for NSRR tests and TREAT tests respectively.

4.1.1 NSRR Energy Depositions Transients

As mentioned previously, examination of the irradiated fuel leads to determination of the amount of energy deposited in the fuel plates. The temporal shape of the prompt neutron flux is obtained from the measurement taken at the experimental facility. The time vs neutron flux data points from the experiment was then interpolated and normalized such that the integral of the neutron flux multiplied by the energy release per fission corresponded to the total number of prompt fissions within the plate.

$$\dot{Q}(t) = E \frac{\phi(t)}{\int_0^{\tau} \phi(t) dt} \quad 0 < t < \tau. \quad (22)$$

Where:

Q = energy release rate

ϕ = neutron flux

E = Energy released during the transient. (J)

τ = time span of the transient

Two types of fuel plate samples irradiated the NSRR facility. One was the JMTR fuel and the second was ANS reactor fuel. The tested fuel was U_3Si_2 aluminum based fuel. The two types of fuel samples had the same overall dimensions. Therefore, the difference between the samples was the fuel meat density, fuel meat thickness, and cladding thickness. Figure 3 represents the JMTR fuel plate and Fig. 4 represents the dimensions of the ANS fuel plate.

Fig. 3. Illustration of the JMTR fuel plates irradiated in the NSRR.

Table 1 shows the number of tests and the corresponding test conditions for the JMTR test.

Table 1. JMTR tests in NSRR facility

Test ID	Energy Deposited into JMTR Mini-Fuel Plate (kJ)	Notes
508-51	7.74	plate melted
508-52	9.17	plate melted
508-53	10.81	plate melted
508-54	12.25	Melted & Relocated
508-55	13.71	Melted & Relocated
508-56	16.05	Fuel Dispersion

Fig. 4. ANS fuel plate dimensions

Table 2 shows the test and corresponding energy deposited.

Table 2. ANS fuel tests in NSSR facility

Test ID	Energy Deposited (KJ)	Notes
518-1	3.3	Inadequate homogeneity - plate developed cracks
518-2	5.2	Inadequate homogeneity - some melting
518-3	3.5	No failure
518-4	2.5	No failure
518-5	5.66	Melting occurred

Summary of the test results will be outlined in the results section of this report.

4.1.2 TREAT Energy Deposition Transients

TREAT series of experiments were not as easy to simulate because the only information about the pulse width was given in terms of initial reactor period. The reactor period was used to determine the width of the pulse by estimating the pulse width at half maximum. Exponents in the Eq. (23) were determined for each transient

$$q(t) = q_0 K(t_0) t^n (e^{-\alpha})^m \quad 0 < t < t_0, \quad (23)$$

where q_0 is the prompt fission energy deposited during the transient, t_0 is the duration of the pulse, and the normalization factor $K(t_0)$ is given by:

$$K(t_0) = \frac{1}{\int_0^{t_0} t^n (e^{-\alpha})^m dt}, \quad (24)$$

It is important to note that the Eq. (23) only approximates the temporal pulse shape. Numerical simulation of the tests performed for the ANS and JMTR fuel plate tests have revealed that for short transients, the thermal response is very sensitive to the shape of the transient. For longer transients the thermal response is not as sensitive.

The HFIR and SL-1 reactor fuel plate samples for the TREAT tests were dimensionally much smaller than the fuel tested in the NSRR reactor. Fig. 5 shows the dimension of the SL-1 fuel plate sample. The SL-1 fuel plate samples were also uncladded fuel plate samples. Fig. 6 shows the dimensions of the HFIR fuel plates used in the tests. The plates were again smaller in size than the ANS and JMTR plates but were cladded. Tables 3 and 4 display the information for different tests and the associated energy deposited in the fuel sample along with the pulse width for SL-1 and HFIR fuel plates respectively.

Fig. 5. Dimensions of the SL-1 fuel plate tested in treat facility.

Table 3. SL-1 fuel tests in TREAT facility

Test ID	Energy Deposited into unclad SL-1 Reactor Fuel Plates				Notes
	KJ/g of fuel	KJ in the fuel plate	Reactor Period (ms)	Pulse Width (ms)	
CEN 114	1.21	3.150	112	925	melted and bulged retaining original shape
CEN 97	1.67	4.35	79	671	melted into one globule
CEN 121	1.98	5.148	79	671	Melted into one semi-circular globule
CEN 122	2.23	5.38	79	671	Melted into one spherical globule
CEN 123	2.23	5.29	79	671	Melted into one spherical globule
CEN 115	2.43	6.39	77	650	Melted and fragmented
CEN 116	2.8	7.25	62	530	Melted and fragmented
CEN 98	3.1	7.9	51	433	Melted and dispersed particles welded to inner surfaces of autoclave
CEN 137	3.69	9.2	54	460	Melted and fragmented
CEN 99	4.29	10.76	50	425	Melted and dispersed particles welded to inner surfaces of autoclave
CEN 138	5.78	15.68	41	330	Melted and fragmented

Equations (23 and 24) were used for the HFIR. Table 4 shows the list of HFIR fuel plate tests performing in the TREAT facility.

Fig.6 Dimensions of the HFIR fuel plate tested in the treat facility.

Table 4. General description of tests performed on the HFIR fuel in the TREAT facility.

Test ID	Energy Deposited into HFIR Reactor Fuel Plates				Notes
	Initial temperature of the fuel & coolant (K)	KJ in the fuel plate	Reactor Period (ms)	Approximate Pulse Width (ms)	
CEN 198	303	0.721	156	1291	
CEN 200	303	1.823	150	1252	
CEN 199	303	2.272	112	925	
CEN 201	303	2.306	10.3	85	
CEN 213	303	2.409	10.3	85	
CEN 204	303	3.521	112.1	925	Fuel Dispersion
CEN 205	303	5.798	59	570	
CEN 210H	393	1.921	116	959	
CEN 211H	393	2.993	62.8	518	
CEN 212H	393	4.874	65.8	543	
CEN 206H	558	2.260	109	900	
CEN 207H	558	3.592	107	883	

4.2 EXOTHERMIC ENERGY RELEASE FROM FUEL-AL REACTION

Several research reactors use cermet-type fuels like U_3O_8 -Al or U_3Si_2 -Al mixtures. Such mixtures undergo a thermite reaction once the aluminum melts. The reaction rate varies with the temperature of the mixture.^[3,4] The reaction of U_3Si_2 fuels with aluminum in the meat is exothermic and releases anywhere between 0.3 and 0.35 MJ/kg of U_3Si_2 , depending on the volume fraction of the fuel in the meat. The reaction begins at a very slow rate when aluminum reaches its melting temperature at 900 K and increases rapidly as the temperature increases. When fuel particles melt, the reaction rate is the fastest. The reaction of U_3O_8 fuels with aluminum in the meat is also exothermic and releases 0.75 MJ/kg of U_3O_8 . The range of temperature when the energy release occurs for the U_3O_8 -Al reaction is not the same as the temperature range for the U_3Si_2 -Al reaction. U_3O_8 releases energy in temperature bands. This effect will be described in Sect. 6.2.2. The difference in the energy release is primarily caused by the material properties of the fuel and aluminum. U_3Si_2 tends to behave as inter-metallic compound whereas U_3O_8 acts as a ceramic material embedded in the aluminum matrix.

The main problem with modeling the reaction rate process is that the rate of these reactions as a function of rate and magnitude of temperature changes are not well known; therefore, implementation of the energy deposition over a wrong time scale could significantly affect the predicted results.

The chemical reaction between fuel particle and aluminum is a diffusion controlled process that is highly temperature dependent. Section of the fuel meat is illustrated by Fig. 6. It shows that the U_3Si_2 fuel particles are dispersed in an aluminum filler material. U_3O_8 fuel is similarly dispersed in aluminum filler. When both aluminum and U_3Si_2 are in the solid state the diffusion between the two components is minimal. When aluminum melts, the reaction takes place over a period of many minutes³ (this is a very large time scale compared to the fission pulse width that is approximately 10–80 ms for NSRR fuel tests and 100 ms to 1 second in duration for TREAT facility tests). It is also known that the reaction between the fuel particle and the aluminum in the meat is very rapid when U_3Si_2 particles begin to melt. U_3O_8 fuel exhibits the first exothermal reaction at the melting temperature of the aluminum. One could expect an extremely rapid and complete reaction to occur if the temperature exceeds the melting temperature of the fuel particles assuming sufficient aluminum is available. This rate would be governed by the diffusion of two liquids.

In general, diffusion of a material in a multi-component system is a complex process. There are four separate diffusion effects that can cause migration of material from one location to another and cause chemical reaction to take place. These diffusion mechanisms are given by Eq. (21).

$$j_i = j_i^x + j_i^p + j_i^g + j_i^T, \quad (25)$$

where j_i is the total net mass flux of component i , and superscripts x , p , g , and T represent the components of diffusion by concentration gradient, diffusion by pressure gradient, diffusion induced by gravity, and diffusion caused by a temperature gradient.

$$j_i^x = \frac{c^2}{\rho RT} \sum_{j=1}^n M_i M_j D_{ij} \left[x_j \sum_{\substack{k=1 \\ k \neq j}}^n \left(\frac{\partial \bar{G}_j}{\partial x_k} \right)_{T,p,x_s} \nabla x_k \right], \quad (26)$$

$$j_i^p = -\frac{c^2}{\rho RT} \sum_{j=1}^n M_i M_j D_{ij} [x_j M_j (\frac{\bar{V}_j}{M_j} - \frac{1}{\rho}) \nabla p] , \quad (27)$$

$$j_i^g = -\frac{c}{\rho RT} \sum_{j=1}^n M_i M_j D_{ij} [x_j M_j (g_j - \sum_{k=1}^n \frac{\rho_k}{\rho} g_k)] , \quad (28)$$

$$j_i^T = -D_i^T \nabla \ln T , \quad (29)$$

where G , V , M , ρ , and c are partial molar free enthalpy (Gibbs free energy), volume, molar mass, universal gas constant, and speed of sound respectively.

To determine the reaction rate, i.e. energy release rate, the diffusion process has to be determined. This is a complex process and due to the limited data available to model the process, the method used to approximate the reaction rate did not use Eqs. (25–29). The method used to determine the reaction rate is empirical and it is outlined in the following section.

4.2.1 Empirical Method of Determining Reaction Rates

Reaction rate is expressed in terms of f , the total fraction of fuel particle that reacts with the aluminum, multiplied by the energy released by unit mass of the fuel given by Eq. 30.

$$\dot{Q} = f'(t)E , \quad (30)$$

where Q is the energy release rate per unit mass of the fuel particle (W/kg), E is the energy released per unit mass (J/kg), and $f'(t)$ is the rate of change of reacted fraction (1/s). Therefore, rate of change of reacted fraction of the fuel particle needs to be determined. To determine an empirical form of the equation, conditions need to be imposed on the equation. These conditions are:

- 1) At time equal to infinity the particle fully reacts under all conditions. Therefore, reacted fraction is equal to 1 and the rate of change of reacted fraction is equal to 0.

$$f(t \rightarrow \infty) = 1 \quad (31)$$

$$f'(t \rightarrow \infty) = 0 \quad (32)$$

- 2) At time equal to zero, the reacted fraction is equal to zero and rate of change of reacted fraction is given by Eqs. (33 and 34).

$$f(0) = 0 , \quad (33)$$

$$f'(0) = C\{T(t=0)\} , \quad (34)$$

where C is the initial fractional rate of change of reacted fuel. This parameter is temperature dependent.

- 3) As reaction progresses the diffusion through the newly formed layer will retard the reaction

leading to slower reaction rates. Therefore, the presence of reacted material is assumed to increase the resistance to the diffusion and will retard the reaction rate.

$$f'(t) \propto \{C_1 - f(t)\} \quad (35)$$

Combining Eqns. (31), (32), and (35) results in C_1 being equal to 1. This also satisfies the initial condition given by Eqns. (33) and (34).

$$f'(t) \propto \{1 - f(t)\} \quad (36)$$

Removing a proportionality and replacing it with a reaction rate constant, results in Eq. (37).

$$f'(t) = \lambda(t)\{1 - f(t)\} \quad (37)$$

Therefore, the final equation is given by Eq. (37), where λ is the effective rate coefficient (1/s). This model obeys the set of conditions such as: reacted fuel fraction at infinite time is equal to 1 and that the rate of change of the fuel fraction at that time is equal to zero. Depending on the behavior of the effective diffusion coefficient, the reacted fraction can be large or small during a pulse type transient. To model this type of reaction an empirical model was developed and a set of constants were determined using existing data.

The effective rate coefficient is modeled empirically. This approach consists of limiting the rate coefficient to very low values for temperatures below the melting temperature of the aluminum and limiting the peak value to the maximum expected rate coefficient when both fuel particles and aluminum are molten. First, a dimensionless temperature θ was defined as

$$\theta(t) = \frac{T(t) - T_{m1}}{T_{m2} - T_{m1}} \quad (38)$$

where T_{m1} and T_{m2} are melting temperature of the aluminum and melting temperature of the U_3Si_2 fuel particles respectively. The dimensionless form was chosen so that the argument is negative when temperature is below the melting temperature of the aluminum and positive form when the temperature is above the aluminum melting temperature. Since the reaction rate is small when aluminum is solid and large when both fuel and aluminum are molten, the large scale change was best modeled by some type of exponential function. Negative arguments of the exponential function are small and positive. Positive arguments of the exponential are large. It is also believed that the reaction rate will be stable and small when both fuel and aluminum are solid and large and stable when both materials are liquid. Therefore, an S - type temperature dependent curve should be imposed. This can be done using a hyperbolic tangent function. Illustrating the effect of these equations, for a given temperature in Fig. 7, the dimensionless temperature for the dominant reaction phase of the fuel is given by Fig. 8.

Fig. 7. Temperature profile used to illustrate the modeling approach.

Fig. 8. Dimensionless temperature calculated using Eq. 38.

Fig. 9. Hyperbolic tangent of the dimensionless temperature multiplied by a constant $c3 = 3$.

The equation for reaction rate is :

$$\lambda(\theta) = c_1 \text{Exp}[c_2 \tanh[c_3 \theta]] \quad (39)$$

where the constants c_1 , c_2 , and c_3 are determined and presented in the next section.

To illustrate the effect of imposing Eq. (39) on the given dimensionless temperature profile, the constants were chosen to be: 10^{-4} , 15, and 3. With these constants, Fig. 10 is obtained.

Fig. 10. Illustration of general shape of the reaction rate parameter.

Combination of values of c_1 and c_2 will determine the maximum and minimum reaction rates since the value of hyperbolic tangent is between -1 and 1. Constant c_3 affects the slope of the hyperbolic tangent curve, affecting the sensitivity of the reaction coefficient to temperature. The next section shows the results of a parametric study relating diffusion parameter constants to the final reacted fraction.

4.2.2 Effective Diffusion Coefficient for U_3Si_2

There is a limited number of data points available for use with this model. Constant temperature data points at the melting point of aluminum are available along with the data from the several other transient tests. Data given in Table 5 are reported data given in terms of reacted fraction at a particular temperature and the time required to complete the reaction. The following procedure of determining the reaction rate constant assumes the rate is constant at a given temperature and that it varies with temperature. To obtain dimensionless temperature, the melting temperatures of the aluminum and particles are taken to be $T_{melt1} = 870$ K and $T_{melt2} = 1973$ K, respectively. If the reaction is given by

$$g'(t) = \lambda(T)(1 - g(t)), \quad (40)$$

for a constant temperature, the fraction g is given by

$$g(t) = 1 - e^{-\lambda(T)t}, \quad (41)$$

$$\lambda[T] = \frac{-\ln[1 - g[t_r]]}{t_r}. \quad (42)$$

Table 5. Estimated reaction rate parameters for U₃Si₂ fuel

Temperature (K)	Dimensionless Temperature	Reacted Fraction	Reaction time (s)	Reaction rate constant λ
300	- 0.45	0.9	8.6e6	2.6e-7
900	0	0.9	600.0	0.00384
1400	0.384	0.6	1.0	0.916
1973	1	0.9	5.0e-3	460

Plotting the data on a log based plot as shown in Fig. 11, and by an iterative process the coefficients c_1 , c_2 , and c_3 in Eq. (39) are determined.

$$\lambda(\theta) = c_1 \text{Exp}[c_2 \tanh(c_3 \theta)] \quad (43)$$

Fig. 12 illustrates the effect of this modeling method on the total predicted energy for the 508 - 54 in NSRR reactor. More detailed evaluations can be found in appendix A where all the plots for each test show the fission and chemical energy deposited in the fuel sample. As it can be seen in Fig.12 the energy contribution from the chemical reaction tends to be a significant fraction of the fission energy that is deposited in the fuel sample.

Fig. 11. Comparison of data points to the calculated reaction rate parameter.

Fig. 12. Components of power density deposited in the fuel plate during a transient test.

4.2.3 Effective Diffusion Coefficient for U_3O_8

The major difference in which U_3O_8 and U_3Si_2 react with the aluminum lies in the temperatures at which the reaction takes place and the amount of energy released during the process. The energy released per unit mass of the U_3O_8 aluminum mixture is shown in Fig. 13 as a function of fuel aluminum composition.

Fig. 13. Energy release as a function of U_3O_8 - aluminum mass fraction

The temperatures at which the reactions take place are over several temperature ranges. First exotherm occurs at temperature close to the melting temperature of the aluminum, approximately 870 K. The second exothermal reaction occurs close to 1220 K. The last exothermal reaction is observed to occur between the 1400 - 1500 K temperature range. The onset temperature seems to be affected by the partial conversion of U_3O_8 to U_4O_9 . For this we obtained,

$$\lambda(T) = c_1 \text{Exp}[c_2 \tanh[c_3 \theta_1(T)]] + c_1 \text{Exp}[c_2 \tanh[c_3 \theta_2(T)]] , \quad (44)$$

$$\theta_1(T) = \frac{T - T_{m1}}{T_{m2} - T_{m1}} , \quad (45)$$

$$\theta_2(T) = \frac{T - T_{m3}}{T_{m4} - T_{m3}} , \quad (46)$$

and T_{m1} , T_{m2} , T_{m3} , and T_{m4} are ranges of temperatures where the chemical reaction occurs.

Taking the value of constants c_1 , c_2 , and c_3 in Eq. (44) to be $4.0 \cdot 10^{-4}$, 14.0, and 1.1. and plotting Eq. 44 on Fig. 14 one can see that there is a similarity between the U_3Si_2 and U_3O_8 .

Fig. 14. Reaction rate parameter calculated for U_3O_8 .

Fig.15 Calculated power deposition density into HFIR fuel plate test caused by fissioning and chemical reaction process during a TREAT test.

This is the same as for the fuel given by Fig. 12, Fig. 15 shows the calculated total power density deposited in the fuel plates during TREAT experiment CEN 211H. As it can be seen from the plot the fission power density is smaller than the predicted chemical energy. Chemical power is larger than the fission although it is much narrower. Despite the modeling uncertainty, this is a significant finding and underscores the importance and relevance of fuel - aluminum reactions. Two peaks are observed because the chemical reaction releases energy over two temperature ranges: one at 1200 K and the other at 1500 K.

4.3 ENERGY RELEASE FROM ALUMINUM-WATER REACTION

The chemical oxidation phenomenon between aluminum and steam is very complex. The energy from its reaction was not included in the temperature predictions. The energy associated with this reaction is approximately 18 MJ/kg. To estimate the error associated with neglect of the reactions, some physical examination of the phenomenological events occurring during the reaction needs to be performed. Two mechanisms by which aluminum oxidizes are vapor phase burning and steam diffusion through the solid/liquid aluminum oxide layer.^[5] Fig - 16 illustrates the two effects.

When aluminum is at temperatures where aluminum vapor pressure is appreciable, evaporated aluminum chemically reacts with steam and burns away from the aluminum surface. For this reaction to sustain itself, there has to be steady supply of aluminum vapor and steam that feeds into the reaction. For that process to occur, energy from the reaction needs to be transferred to the aluminum surface. Part of this energy is then used for changing the phase of the aluminum from liquid to vapor.^[5] The energy from the oxidation reaction is close to 18 MJ/kg; however, the latent heat of vaporization of aluminum is 10.8 MJ/kg. To sustain the reaction, 60% of the energy needs to be transferred to the aluminum surface. If an aluminum oxide layer is present on the surface, significant evaporation of aluminum will not occur until the aluminum oxide allows the aluminum vapor to pass through the oxide. This occurs when the temperature of the aluminum oxide reaches 2330 K (the melting temperature of the aluminum oxide).

The second phenomenon involves diffusive oxidation. When aluminum is in a cold solid state, a thin oxide film builds up rapidly. This oxide forms a protective layer that prevents further diffusion of steam or oxygen, and the reaction ends. The diffusion process is governed by the diffusivity of steam in the oxide layer. As the temperature increases, the diffusivity increases, allowing more steam to penetrate through the oxide layer. As the temperature is increased to the

melting temperature of the oxide layer, the diffusion coefficient changes rapidly and allows steam to penetrate at a significantly higher rate than when the oxide layer is in the solid state. When aluminum oxide melts, significant chemical oxidation can take place. This oxidation continues until all the aluminum oxidizes or when the temperature of the aluminum oxide falls below solidification temperature. Solidification occurs when heat transfer from the surface exceeds the heat generation rate.

Fig. 16. Illustration of diffusive oxidation and vapor phase burning

Under certain conditions aluminum ignition can occur at subcooled aluminum oxide temperatures.[5,6] This phenomenon seems to occur when the oxide layer is in a subcooled metastable liquid state and the aluminum surface-area-to-volume ratio is large. Due to the finite time associated with the growth of the aluminum oxide crystal, sufficient steam can diffuse and oxidize a large fraction of aluminum because of the large surface-area-to-volume ration. When the surface-area-to-volume ratio is small, the fraction of oxidized aluminum is also small. The condition that determines whether vapor phase burning or the diffusive oxidation process governs the reaction is not clear. It seems that the vapor phase burning will occur when significant aluminum vapor pressure is exerted. The diffusion-governed oxidation occurs when aluminum oxide is barely molten and diffusion of steam through the aluminum oxide layer is more likely than evaporation of aluminum. Both of these reactions clearly can become predominant at temperatures that are near or above the melting temperature of the aluminum oxide layer. Therefore, oxidation energy clearly becomes an important source when 2330 K is exceeded. As the following section shows, the available data confirms this effect. The uncertainty in the energy release rate is the main reason for not including the oxidation as the energy source in the analysis. To predict the energy release rate detailed phenomenological models need to be implemented. These models of fundamental processes are not currently available. As shown later by the predictions, most of the deviation in data originates when temperatures are around 2300 K.

5. STRAIN RATE MODELING AND ANALYSIS FRAMEWORK

Two key assumptions of the present work are noteworthy. These assumptions are as follows:

1. The as-developed model represents the potential for acceleration and not the actual acceleration. The dynamic interaction between expansion and retention forces through material strength properties was not modeled.
2. Because of the insufficient knowledge base, the rate characteristics of the secondary sources of energy were not taken into account. The secondary source of energy not considered is the energy release from aluminum oxidation with steam.

5.1 THE EFFECT OF NEGLECTING MATERIAL STRENGTH PROPERTIES

In general, the material cohesive forces will reduce the strain rate and cumulative strain caused by the deposition of energy into a plate. The consequences of not taking into account the effect of material strength properties should therefore be quantified. During a heating transient four separate stages can be identified where material properties change and could influence the retention forces. The four stages are (1) solid, (2) slurry, (3) liquid, and (4) vapor. Restraining force depends on the state of the material.

When energy is deposited into the plate, the fuel changes from solid into liquid. Restraint in the solid is governed by elasticity of the material. The transition from solid to liquid is fuzzy because of eutectic formation. The melting point is not a single temperature but a range of temperatures. The cermet fuels seem to form a viscous slurry. During the slurry stage, it is not known whether the fuel will behave according to Newtonian or Bingham flow mechanism. When fluid properties exhibit non Newtonian behavior, the mixture may have solid like properties for a range of imposed shear stresses and exhibit linear Newtonian behavior past a critical shear stress. Some experimental evidence suggests that the cermet fuel in the plate/ tube geometry remains stiff (solid like) up to very high temperatures that are significantly beyond the melt temperatures of the pure aluminum.^[3,4] Further deposition of energy causes full melting and collapse of the fuel plate. When in liquid form, the restraint is caused by the viscosity of the liquid. This restraint is significantly lower than the elasticity-based restraint. Further energy deposition leads to vaporization. Vapor states generally do not display any significant restraint.

The liquid and vapor stages provide very little resistance to flow when compared with the solid and slurry stages. Therefore, the only two stages where material strength effects have to be considered are solid and slurry. As will be shown in the examination of experimental data, dispersion is observed at very high temperatures. Material of the fuel exists only in liquid and vapor stages before dispersion. This means that neglecting material strength may not significantly affect the predicted strain rates during the important stages where dispersion is likely to occur.

5.2 EFFECT OF FUEL IRRADIATION ON DISPERSION

When fuel in a fuel plate fissions the uranium atom breaks up into two or more fission products. The break up the uranium atoms can cause changes in the physical properties of the fuel plate. The extent of the change directly depends on the fraction of fuel that has been converted from uranium to fission products. For U_3Si_2 fueled plates, fissioning causes minimal changes in the thermal performance of the fuel plate¹⁵. The change in thermal conductivity caused by the breakup of uranium is also affected by the swelling of the fuel particles. The swelling causes closure of preexisting voids and increases the contact between the fuel and aluminum. Specific

heat capacity of the noble gases are not significantly different from the specific heat capacity of the silicide fuel. Therefore, the rate of change of temperature during a transient energy deposition is not expected to differ significantly. Table (6) illustrates the effect of irradiation on the material properties at various irradiation levels.

Table 6. Preliminary estimate of the material properties of irradiated silicide fuel plates under ambient conditions.

Fuel type	Burn-up (%)	Thermal Conductivity (W/m/K)	Meat Density kg/m ³
ANS	0	130	3 547
ANS	50	136.5	3 547
ANS	100	143	3 547
JMTR	0	39.91	6 550
JMTR	50	65.4	6 550
JMTR	100	58	5 960

If the thermal response of irradiated fuel is very similar to the fresh fuel, the only way that the fuel can exhibit a different dispersion potential is if the volatile fission products vaporize and expand as a gas. The noble gas mass fraction for a fully irradiated fuel plate is 5.7%¹⁵. Such a small fraction at first glance may not have a significant effect however, if it converted to volume fraction in vapor state, the volume fraction is large. The vapor state density of volatile fission products such as iodine is approximately 11.27 kg/m³ and xenon is 5.88 kg/m³. This represents significant reduction in density when compared to the density of U₃Si₂ or U₃O₈ fuel with aluminum filler that has the density of several thousand kg/m³ depending on the material and loading. The density of the volatile fission products could be higher because the surface tension effects or material properties compress the gas to higher than ambient pressures. This increases the effective density. Fig. 17 shows the effect of surface tension on the bubble pressure at various temperatures and radius of bubbles.

The effective volume fraction, f_v , is determined by using the mass fraction, f_m , of the volatile fission products and multiplying the mass fraction by the fuel density, ρ_f , and effective volatile fission product density ρ_{fg} .

$$f_v = \frac{f_m \rho_f}{f_m \rho_f + (1 - f_m) \rho_{fg}}, \quad (44)$$

using this expression Fig. 18 is obtained. It shows the relationship between the mass fraction and the volume fraction.

Fig. 17. The effect of aluminum surface tension on the internal pressure of gases trapped in spherical pockets.

Fig. 18. Effective volume fraction for a given mass fraction of volatile fission products if the gas is allowed to expand to ambient pressure.

If the density of the fission product gases is higher due to the compressive effects described earlier, then the curve should shift downward. It should be noted that during slow heating of irradiated fuel, the swelling of irradiated fuel occurs when the material strength of the fuel plate decreases as the temperature approaches the melting temperature of the plate. At this temperature the material strength deteriorates significantly. The expansion model does not physically represent the reduction in material strength and expansion process caused by allowing the fission product gases to expand. Thus, the model does not perform well during this stage. To improve the model, the dynamic interaction between the bubble size, internal and external pressure, material properties have to be considered and modeled. To represent this aspect of physical phenomena models have to be developed. The current model represents a significant shortcoming for the irradiated fuel model.

Models with various gas fractions were evaluated to observe the differences in the fuel plate response. Figure 19 shows the response of the fuel plate during the JMTR test 508-53 transient with 1% effective gas volume presence.

Fig. 19. Predicted strain rate for 1% effective gas volume fraction.

Figure 20 shows the result of the 25% effective gas volume fraction has on the expansion characteristics.

Fig. 20. Predicted strain rate for 25% effective gas volume fraction.

Note that during the melting phase of the transient, the expansion rate of the gas phase is effectively zero. This causes the expansion rate during melting for the two examples of 1% and 25% gas volume fraction to be the same. When calculations for the various effective volume fractions are plotted on the peak strain rate against the effective volume fraction as given by Fig. 21 the behavior is linear. This means that the vapor dominates the expansion process if it is present in sufficient quantities.

Fig. 21. The effect of effective volume fraction on the peak strain (expansion) rate.

The significant effect that the fission product gases have on the strain rate in the irradiated fuel plate shows that the gases play an important role in determining the onset of dispersion. Because significant physical effects such as bubble dynamics interaction with the solid and liquid material, are lacking from the modeling framework, the uncertainty in the current model can not claim any significant break through in understanding of irradiated fuel dispersion. Data are required for model benchmarking, upgrading and validation

5.3 SAMPLE MODEL RESULTS

To illustrate the predictions of the model, the results from a sample transient evaluation are presented. These results compare the relative magnitudes of different effects. For this particular case, the energy is deposited over a 25 ms period into an ANS type of fuel. Figure 22 shows key parameters of interest for a sample transient. On the same figure, the cumulative expansion rate curve is given along with the individual curves representing solid/liquid expansion rate, expansion rate caused by melting, vaporization, and gas expansion along with the energy deposition curve.

For this particular case the gas fraction inside the plate was chosen to be 5% of the plate volume.

As it can be seen from the figure, the expansion in the solid state is dominated by the gas expansion. As the plate begins to melt the plate transitions from solid to liquid state. Although the energy deposition rate when the plate fully melts is higher than when the plate starts melting, the larger expansion rate is caused by the larger coefficient of expansion for the liquid than the solid. For this particular case, after the plate melts the energy deposition rate decreases. The decrease in the rate is accompanied by a decrease in the strain rate. This continues until the plate reaches the vaporization temperature. When the vaporization temperatures are reached, the change in density associated with the vaporizing aluminum causes a large expansion rate.

It should be noted that the energy deposition for this particular example consists only of energy deposited from the fissioning process. Energy contribution from the chemiocal reaction should alter the shape of the energy deposition rate curve.

Fig. 22. Illustration of various expansion modes.

6. ANALYSIS OF EXPERIMENTAL RESULTS

The experimental data for aluminum-based fuels performed in the TREAT facility in Idaho was examined. The types of plates tested were those from the High Flux Isotope Reactor (HFIR) and SL-1 reactor. Advanced Neutron Source (ANS) fuel plates are being tested in the Japan Atomic Energy Research Institute's (JEARI) NSRR facility in Japan. The NSRR results are not included as part of this work due to an incomplete data base. Some differences exist between the two facilities, producing variations in experimental conditions. One dominant difference is the duration of the neutron exposure. The TREAT facility pulse width varies between 0.3 and 1 second whereas the pulse width of the NSRR facility varies between 25 and 100 milliseconds. The amount of energy that can be deposited into a plate is a function of the neutron flux density and the duration of exposure.

Although the model has a term that accounts for the presence of fission products, the data with irradiated fuel plates is lacking. Irradiated fuel plate experiments will be performed in the NSRR facility in the future. The comparisons of the model with fission product model will be made when data become available.

6.1 SL-1 FUEL TESTS IN TREAT

SL-1 fuel plates were tested in the TREAT facility.^[8] SL-1 plate samples were U-Al alloy fuel without cladding. The dimensions of the test sample were 5.0 mm x 12.7 mm x 12.7 mm. The fuel sample was an Al-U-Ni alloy that was composed of 81% Al, 17% U, and 2% Ni. For energies deposited into the plate during the experiments, the peak computed strain rate vs. energy per unit mass of the plate was determined and plotted in Fig. 23. A comparison with the observed dispersion is also presented in the same plot. Experimental data found in the report for this particular set of experiments did not contain a break down in fragmented mass fraction. The report⁸ only reported whether fragmentation occurred or not. For this reason, the experimental data was presented in terms of being either as 0% or 100% fragmentation. The results indicate that the location where the slope of the strain rate curves changes is the location where the dispersion is likely to occur.

The experiments show that for all cases in which the energy deposition was greater than 2.3 MJ/kg, the plate dispersed. It was hoped that the prediction for dispersion would be represented by the point of inflection where the curve of peak strain rate as a function of energy begins to increase. The point of inflection where the strain rate begins to increase is 2.7 MJ/kg. This represents a discrepancy between the point of inflection and the observed point where dispersion was observed. The test number CEN 115 was not in line with the predictions. To explain the discrepancy, the aluminum oxidation phenomenon was investigated. Figure 24 shows the peak computed surface temperature and the fraction of aluminum oxidation that took place as a function of the energy deposited. The peak temperature reached by nuclear heating alone in test CEN 115 was computed to be 2300 K, which was sufficient to melt the oxide layer and allow oxidation to begin. For the CEN 115 test, the oxide melting temperature was reached during the period when nuclear heating was still taking place. Additional energy from oxidation was sufficient to raise the temperature of the fuel to the vaporization state and cause dispersion. This seems to indicate that energy from nuclear heating was supplemented by chemical oxidation to provide sufficient energy for the onset of vaporization. The chemical reaction phenomenon was not modeled; therefore, the strain rate is not representative of vaporization process.

Fig. 23. Strain rate and fragmented fraction as a function of cumulative deposited energy for SL-1 fuel plate tests in the TREAT facility

Fig. 24. Peak computed surface temperatures for different energy depositions in SL-1 fuel tests conducted in the TREAT facility.

6.2 HFIR FUEL TESTS IN TREAT

HFIR fuel plates are made with U_3O_8 particles dispersed in an aluminum matrix.[8] HFIR fuel sample dimensions were 25.4 mm x 12.7 mm x 1.27 mm. Composition by mass of the HFIR fuel is 59% Al and 41% U_3O_8 . Similar to SL-1 tests, HFIR plates were subjected to neutron bursts in the TREAT facility. The initial fuel temperatures for the HFIR tests were 303 K, 393 K, and 558 K. Since only two data points were tested with the initial temperature of 558 K, they were not plotted in Figs. 25 and 26, but were included in Fig. 27. Sensitivity to initial temperatures is shown in Fig. 5 where strain rate was compared for the two initial temperatures. Comparison of the fragmented fraction with the strain rate for different energies appears in Fig. 26.

Fig. 25. Comparison of computed peak strain rate as a function of energy for different initial temperature

The results show that the HFIR and SL-1 fuel tests exhibit reasonable similarity in response to energy deposition. Just as in test CEN 115 with the SL-1 plates, CEN 211H, and CEN 204 data fall into the category of dispersing without reaching vaporization temperature by nuclear heating alone. Figure 27 shows the peak computed temperature and measured oxidized fraction plotted against the deposited energy.

Both test points in question have significantly oxidized. The computed peak clad temperature from nuclear heating alone for both tests is above the oxide melting temperature. Therefore, just as observed with SL-1 fuel tests these two points are expected to use some of the oxidation energy to aid in reaching the vaporization state.

Fig. 26. Comparison of computed peak strain rate to experimentally measured fragmented fraction as a function of cumulative deposited energy

Fig. 27. Fraction of aluminum oxidation and peak computed temperatures for HFIR fuel tests as a function of energy deposition.

6.3 JMTR FUEL PLATE TESTS IN NSRR

This section summarizes the experimental results conducted in NSRR on JMTR fuel and analytical model simulating those tests. Six fuel plates were subjected to short duration pulse-type neutron exposure tests and are summarized in Table 7. Dimensions of the JMTR plates are given in

the section 5.1.

Table 7. JMTR tests in NSRR facility

Test ID	Energy deposited in JMTR miniplate (kJ)	Pulse base width (*)	Calculated Reacted Fuel fraction (**)	Notes
508-51	7.74	60	0.02	
508-52	9.17	45	0.14	
508-53	10.81	40	0.85	
508-54	12.25	40	1.0	Melted fuel
508-55	13.71	40	1.0	Melted fuel
508-56	16.05	30	1.0	Fuel Dispersion

(*) Pulse base width is the width of the pulse at the base as opposed to the full width at half max.

(**) Calculation based on modeling constants that were based on selected data points described in section 6.

Each test was analyzed using two energy source models. The first energy source model consists of using only the fission heat source. As discussed earlier in the report, the shape of the energy deposition by fissioning of uranium was obtained from the neutron flux curve measured by the NSRR instruments. The second energy source model uses the combined fission heat source model and the chemical reaction model. As discussed earlier in the report, the mixture of U_3Si_2 fuel particles dispersed in aluminum when subjected to elevated temperatures exothermically react with each other releasing energy. The energy release rate model described in the earlier section was implemented into the dispersion model as a part of an energy source term. The total energy deposited in the fuel was based on the neutron flux measurements and the calculated chemical energy generation term. This section of the report summarizes the results of the dispersion model with and without the chemical energy release model implemented into the dispersion model.

The total amount of chemical energy that can be released is much smaller than the fission energy deposited during a test. Addition of that energy uniformly to the fission energy does not produce significant variations in temperature. This is caused by the diffusion of energy from the meat to the cladding lowering the overall temperature of the fuel plate. Also, relatively longer time allows energy to be transferred to the coolant resulting in lower temperature and related effects. However, if this energy is weighted so that is deposited at a crucial time, the effect is significantly more pronounced.

If the peak of chemical energy release coincides with the peak of fission energy deposition, then the combined energy significantly increases the peak power deposition. This increase in power causes the temperatures to increase further, increasing the chemical reaction rate and the energy release rate. If the chemical energy is deposited over a short time span, then the effect can be multiplying. Fig. 28 shows the effect of the additional energy produced by the chemical energy.

Fig. 28. Comparison of fission and chemical energy deposited for a given fission energy deposited and their corresponding temperatures.

At high fission energy depositions, supplemental energy from the chemical reaction tends to cause higher peak temperatures. For higher fission energy depositions ($q > 10$ kJ), the peak temperature difference widens to 200 K. It should be noted that for the largest energy deposition, the peak centerline temperature without chemical energy does not reach aluminum vaporization temperatures, whereas with the chemical energy deposition the vaporization temperature is reached. Since evaluations for HFIR and SL-1 tests indicate that vaporization location is the point of increase in the strain rate, not including the chemical energy in the JMTR analysis would under predict the temperature missing the vaporization point.

Strain rate curves with and without the chemical reaction are given by Fig. 29. It can be seen that the increase in the strain rate is more dominant in the model that had a chemical energy deposition model implemented. The model without the chemical energy has a smooth increase in the strain rate, whereas the model with the chemical energy has a noticeable point of inflection. For low energy depositions, the difference in strain rates is minimal since the temperature reached by the fuel is not sufficiently high to cause the reaction to occur.

Because the energy deposition time scale for the NSRR facility is shorter than the thermal diffusion time of JMTR plates, the thermal energy does not escape from the fuel meat to the cladding during the energy deposition period. Because of this effect, large temperature gradients tend to exist for the JMTR tests. The large temperature gradients are reflected in the computed clad temperatures given by Fig. 30. As it is seen in the next section, ANS fuel plates have a higher meat thermal conductivity, allowing energy to be conducted to the clad faster and leading to lower temperature gradients.

Fig.29. Strain rate predicted by fission heating only and the combined fission heating and the fuel - aluminum chemical energy source.

Fig 30. Peak center line and clad temperature for a given fission energy deposition with chemical energy model.

The issue of the aluminum - steam reaction can also be addressed in the test 508-56. Comparison of the peak fuel centerline temperature to the peak clad temperature shows that the clad temperature for the 508-56 case never exceeds the 2300 K. This temperature seems to be the onset of aluminum steam reaction. Therefore, the oxidation is not predicted for this test which is confirmed by the data from the experiment. This interpretation leads to the conclusion that for test 508-56 the principal reason behind the dispersion is the onset vaporization of the fuel meat.

As with SL-1 and the HFIR fuel tests, the onset of dispersion seems to be caused by the vaporization of the fuel meat. Next section discusses the ANS fuel plate tests.

6.4 ANS FUEL PLATE TESTS IN NSRR

Tests performed in the NSRR Facility with the ANS fuel were not energetic enough to cause fuel dispersion. Fuel plate dimensions and energies deposited into the fuel plates are outlined in Section 5.1. Some of the fuel plates, tests 518-1 and 518-2 had large deviations in homogeneity. Hot spot induced thermal stresses caused the plates to become warped. Other more homogenous plates performed in a more expected manner. Since the experimental tests did not achieve dispersion, the next section will discuss predictions of dispersion using the current model.

7. PREDICTIONS OF ANS FUEL DISPERSION

ANS fuel plates are composed of a fuel meat section sandwiched by aluminum cladding.^[9] The fuel meat is 0.76 mm thick, and the cladding is 0.254 mm thick. Fuel meat is composed of uranium silicide particles dispersed in an aluminum matrix. The density of uranium silicide in the meat is 1400 kg/m^3 . Based on a criteria deduced from comparison of the dispersion model with the TREAT experimental data, the model was used to predict the behavior of the ANS fuel. The criteria for dispersion is the location on the peak strain rate plot where the curve changes slope. The models and results are located in appendix D. Two types of plots resulted from the model. One plot shows the variation of strain rate with deposited energy in the plate over a 25-ms period per unit mass of the plate. This plot is given by Fig. 31. The second plot, Fig. 32, shows the variation of strain rate with a the varying duration for fixed cumulative energy deposited in the plate. Both Figs. 31 and 32 show a change in the slope of the strain rate curve.

When Fig. 31 is examined the expected dispersion energy is $\sim 2.7 \text{ MJ/kg}$. This represents the total energy that is required, fission and fuel - aluminum chemical energy. This conclusion is valid as long as the aluminum oxidation chemical reaction does not occur. If a this chemical reaction occurs, then the threshold will be lower. If it is assumed that oxidation aids dispersion, with a constant 25 ms energy deposition period, the onset of oxidation will be achieved when energy deposition of 2.4 MJ/kg is exceeded.

Fig. 31. Computed strain rate and temperature variation for ANS fuel (1.4 g/cc loading) as a function of deposited energy for a fixed 25 ms pulse width.

For the second case a fixed cumulative energy of 2.15 MJ/kg was used. For very short energy depositions, most of the deposited energy is used in raising the temperature of the fuel meat, and vaporization of the meat was achieved. For longer duration's, the energy spreads, and cumulative energy cannot raise the temperature of the entire plate to the vaporization temperatures, so the plate does not disperse. In other words, as the energy deposition duration is increased, the energy has time to diffuse, which results in a lower peak temperature of the meat. If dispersion

criteria deduced earlier are applied, one can conclude that for 2.15 MJ/kg to disperse the plate the pulse width has to be shorter than 5 milliseconds. If oxidation is assumed to occur when clad temperatures exceed 2300 K, the pulse width has to be anywhere between 12 and 27 milliseconds.

Fig. 32. Peak strain rate and peak temperatures as a function of pulse width for 2.15 MJ/kg energy deposition as a function of pulse width.

Experiments conducted with ANS mini plates in the NSRR facility in Japan have not been irradiated with a neutron flux that is sufficient to cause dispersion. All of the data gathered with the ANS fuel have been in the low energy range where dispersion did not occur.[2] The energies deposited in the ANS fuel varied between approximately 0.5 to 1.1 MJ/kg of fuel meat. As it can be seen from Fig. 31, when the mass of the clad is taken into account the apparent deposited energy is even lower. It should be noted that higher energies were sufficient to melt the fuel plates, but were significantly below the predicted threshold for dispersion of 2.4 MJ/kg for a 25 milliseconds exposure.

8. GENERAL OBSERVATIONS AND DISCUSSION

The transient behavior of fuel and the effects of the fuel components can be difficult to visualize without the performance of a series of parametric evaluations. Parametric evaluations performed on the fuel plate showed that some parameters significantly affect the fuel heat-up behavior. Since heat-up behavior affects the expansion characteristics and dispersion, it is worthwhile to discuss some effects. Parameters that are controllable by design include plate thickness, fuel meat thickness, clad thickness, and meat composition. The parameter that affects the fuel behavior is the energy deposition transient. For a given energy deposition, the outcome varies significantly, depending on the input duration, as shown by the ANS fuel predictions.

The manufacturing method leaves a small volume of gases in the meat section of the fuel plate. During heat-up, the gas expands, potentially causing swelling. Meat composition affects the plate behavior by changing the meat thermal conductivity, heat capacity, and void fraction. Gases can also be generated by production of fission products during fuel burnup. This would lead to an increase in the strain rate due to the higher concentration of gas in the fuel at lower temperatures. Other effects of burnup include production of volatile isotopes. When volatiles melt and vaporize they have a more pronounced effect since the volume change associated with a phase change process is larger than gas expansion. This analysis tends to show that irradiated fuel is prone to dispersion under lower energy depositions. However, due to the lack of experimental data, this conclusion was not verified.

The additional effects that cause temperature profile variations are (1) the chemical reaction between the fuel and the aluminum that is, the extent of eutectic formed from the reaction under elevated temperatures; and (2) the melting temperature of the meat with the new eutectic. Thermal conductivity affects the transient performance of the fuel plate. Increasing the fuel loading reduces thermal conductivity in the fuel meat. Also, increasing the fuel loading reduces the specific heat capacity of the meat. The combination of these two parameters causes a more pronounced increase in temperature for a given constant volumetric energy deposition rate. The effect of meat and clad thickness on the transient behavior of the plate tends to affect the temperature distribution. For thinner meat and thicker cladding, the energy from the meat tends to diffuse out to the clad and cause the cladding to heat up. Since the specific heat of the cladding is higher than the specific heat of the meat, more energy is absorbed by the clad, thereby lowering the overall temperature. This is true if the energy is deposited over a longer period of time. If the energy deposition period is reduced, the effect can be reversed. For such cases, the fuel meat does not have sufficient time to transfer energy to the cladding. Therefore, the meat heats up faster to higher temperatures, reaching vaporization temperature with less energy.

Results show that reducing the thermal conductivity of the meat section increases the strain rate. The maximum strain rate corresponds to the adiabatic heat up of the fuel meat ($k = 0$). The strain rate increase is caused by a reduced transfer of thermal energy from the meat to the cladding; thus, the temperature increases faster than it would if the thermal conductivity was higher. The buildup of temperature causes other effects to become significant, such as: the expansion of gases, or the increase in melt or vaporization rate. Lower thermal conductivity of the meat can occur if fuel is not fabricated up to specifications. Nonhomogeneities in the fuel cause high and low concentrations of particles in the meat. High concentrations of fuel (U_3Si_2) particles not only have lower thermal conductivity but also have lower specific heat capacity. Thus, increasing the concentration of fuel particles decreases both thermal conductivity and heat capacity of the meat. This causes the temperature of the meat during a transient to increase and possible strain rates to be higher than for the lower concentration case. This means that homogeneity of the fuel plates is very important and flawed production could cause premature failure of the plate.

9. SUMMARY AND CONCLUSION

To summarize, a mathematical modeling framework was developed to predict the threshold of dispersion during heat-up transients for U-Al plate-type reactor fuels. The model centers on a determination of thermally induced strain rates that considers several physical phenomena. Exact closed form solution compared with a numerical model for an arbitrary simplified problem shows good agreement. Data from TREAT facility tests with SL-1 and HFIR fuel plate samples were analyzed to determine dispersion and rapid Al-H₂O reaction thresholds. Comparison of model predictions with the TREAT experiments produced a good agreement for the onset of dispersion. The study shows that the strain rate parameter displays a sharp increase coincident with the onset of material vaporization. Model predictions against the NSRR data taken with ANS reactor test samples indicate no onset of dispersion for the energies that were deposited. The NSRR data obtained so far with ANS miniplate samples corroborate this.

Overall, this work shows that the rate of energy deposition is just as important as the cumulative energy deposited in the fuel when one is trying to determine the dispersion potential during rapid transients. Strain rate as a method of determining the fragmentation characteristics predicts correctly overall trends. In this report, an empirically-based approach has been used to model exothermic fuel-aluminum reactions. Upon the melting of aluminum oxide, exothermic energy from an aluminum-steam chemical reaction is added to the fuel plate. However, for aluminum steam reactions modeling has yet to be put in place. To improve the model, a chemical energy deposition model needs to be implemented. For evaluating the criteria that should be used to determine the onset of dispersion, the energy deposition history needs to be taken into account. For slower transients, the temperature gradient in the fuel tends to be small. For such cases, when surface temperature of the clad exceeds the aluminum oxide melting temperature, dispersion should be assumed to follow. This is due to the onset of aluminum ignition with water. For rapid heat-up rates, the temperature gradients across the plate thickness can be very large. For such cases, the meat vaporizes before the clad surface temperature reaches the melting temperature of the oxide layer. For this case, the vaporization of the meat triggers the onset of dispersion.

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APPENDIX A: MATHEMATICAL INPUT AND RESULTS FOR JMTR PLATE TEST SIMULATIONS

51

```
(*
THIS IS A REVISED DISPERSION MODEL THAT USES THE
TIME ENERGY DATA FROM THE EXPERIMENTS AND TEMPERATURE
DEPENDANT CHEMICAL REACTION RATES FUEL- ALUMINUM
This copy has chemical reaction rates included in the
model
LAST UPDATED 4-19-95
Renormalized pulse width was used to a narrower band width.
*)
Print["Date last updated =",Date[]];

Date last updated ={1995, 4, 21, 8, 39, 3}

l=0.000635;m=10;dx=l/m;n=Round[m*.4];time=0;Tinf=300;
volmeat=2*dx*n*(25*10^-3)*(70*10^-3);
Do[xx[i]=i*dx-dx/2,{i,1,m}];
Tm=900.;Tv=2700.;
Do[To[i]=300;Tc[i]=300;xm[i]=0;xv[i]=0,{i,1,m}];

k[i_]:= 39.91 /; i <= n ; (* assign k to the meat *)
k[i_]:= kcl[To[i]] /; i > n ; (* assign k to the clad *)
rho[i_]:= 6550.0 /; i <= n ; (* assign meat density *)
rho[i_]:= 2700.0 /; i > n ; (* assign clad density *)

cp[i_]:= cpcl[To[i]] /; i > n ; (* assign clad cpecific heat *)
cp[i_]:= cpf[To[i]] /; i <= n ; (* assign meat cpecific heat *)

htc=Interpolation[{{150,13232},{350,13232},{410,14770},{726,12542},
{776,11561},{1256,9924},{1458,1750},{1526,1729},{1556,800},
{5000,800}},InterpolationOrder->1];

kcl=Interpolation[{{100,161},{400,177},{500,186},{600,193},
{750,193},{850,193},{900,193},{930,193},
{1200,84},{1300,84},{1500,84},{2000,84},{5000,84}},
InterpolationOrder->1];

cpcl=Interpolation[{{100,878},{300,878},{366,942},{422,963},
{588,1034},{700,1067},{811,995},{1000,995},{2000,995},
{5000,995}},InterpolationOrder->1];

cpf=Interpolation[{{100,351},{300,351},{933,463},{1933,546},
{2500,546},{3500,546},{5000,546}},InterpolationOrder->1];

hsf[i_]:=398000*fmal /; i <= n;
hfg[i_]:=10800000*fmal /; i <= n;
```

```

hsf[i_]:=398000 /; i > n;
hfg[i_]:=10800000 /; i > n;

l51=ReadList["508.51",Number,RecordLists->True];
shape=Interpolation[l51,InterpolationOrder->1];
num=Part[Part[Position[l51,Last[l51]],1],1];
TimeBeg=Part[l51[[1]],1]; TimeEnd=Part[l51[[num]],1];
c1=Sum[0.5*(shape[Part[l51[[i]],1]]+shape[Part[l51[[i+1]],1]])*
(Part[l51[[i+1]],1]-Part[l51[[i]],1]),{i,200,500}];
qs[qtot_,time_]:=qtot*shape[time]/(volmeat*c1);
qtot=7277;
(* Functions : nrt - new reacted fraction
orf - old reacted fraction *)

Tm1=863;Tm2=1970;Td[i_]:= (To[i]-Tm1)/(Tm2-Tm1);
lamda[i_]:= (4*10^-4)*E^(14.73*Tanh[1.8*Td[i]]);
z1[i_,dt_]:=NDSolve[{RF'[t]==lamda[i]-lamda[i]*RF[t],
RF[0]==orf[i]},RF,{t,0,dt},AccuracyGoal->15];
nrf[i_,dt_]:=Part[RF[dt]/.z1[i,dt],1]
frf[i_]:=lamda[i]-lamda[i]*orf[i];
UpdateRF[dt_]:=Do[orf[i]=nrf[i,dt],{i,1,n}]
qchem[i_]:=ww*frf[i];

q[i_,time_,qtot_]:=qs[qtot,time]+qchem[i]/; i<=n; (* assign heat generation in the meat *)
q[i_,time_,qtot_]:=0 /; i>n; (* assign heat generation in the clad *)
bb[i_]:=htc[To[m]]/dx /; i==m ;
bb[i_]:=0 /; i != m ; (* value for the heat transfer out of the plate *)

k1[i_]:= (2*k[i-1]*k[i]/(k[i-1]+k[i]))/dx^2 /; i!=1
k1[i_]:=0 /; i==1
k2[i_]:=k1[i]+k3[i];
k3[i_]:= (2*k[i+1]*k[i]/(k[i+1]+k[i]))/dx^2 /; i!=m
k3[i_]:=0 /; i==m ;

phase[i_]:= solid /; Tc[i] < Tm ;
phase[i_]:= liquid /; Tc[i] < Tv && Tc[i] >= Tm && xm[i] >= 0.85;
phase[i_]:= vapor /; Tc[i] >= Tv && xv[i] > 1;
phase[i_]:= melting /; Tc[i] >= Tm && xm[i] < 0.85 ;
phase[i_]:= vaporizing /; Tc[i] >= Tv && xv[i] < 1 && xv[i] > -0.01;

Tn[i_]:=Tc[i] /; phase[i] != melting && phase[i] != vaporizing;
Tn[i_]:=Tm /; phase[i] == melting ;
Tn[i_]:=Tv /; phase[i] == vaporizing ;

vv[i_,j_,dt_]:= k1[i] /; i-1==j && phase[j]!=melting && phase[j]!=vaporizing
vv[i_,j_,dt_]:= -k2[i]-rho[i]*cp[i]/dt-bb[i] /; i==j && phase[j]!=melting &&
phase[j]!=vaporizing
vv[i_,j_,dt_]:= k3[i] /; i+1==j && phase[j]!=melting && phase[j]!=vaporizing
vv[i_,j_,dt_]:= 0 /; i != j && i-1 != j && i+1 != j ;
(*-----*)
vv[i_,j_,dt_]:=1 /; i==j && phase[j]==melting;
vv[i_,j_,dt_]:=0 /; i-1==j && phase[j]==melting;
vv[i_,j_,dt_]:=0 /; i+1==j && phase[j]==melting;
vv[i_,j_,dt_]:=1 /; i==j && phase[j]==vaporizing;

```

```

vv[i_,j_,dt_]:=0 /; i-1==j && phase[j]==vaporizing;
vv[i_,j_,dt_]:=0 /; i+1==j && phase[j]==vaporizing;

(*-----*)
f[i_,dt_,time_,qtot_]:= -q[i,time,qtot]-To[i]*rho[i]*cp[i]/dt - bb[i]*Tinf /; phase[i]!=melting &&
phase[i]!=vaporizing;
f[i_,dt_,time_,qtot_]:= Tm /; phase[i]==melting;
f[i_,dt_,time_,qtot_]:= Tv /; phase[i]==vaporizing;

Plot[shape[t],{t,.24,0.32},PlotRange->{0,20}]

```

-Graphics-

TB=.25;TE=.31;

```

RhoMeat=6500.;RhoAl=2700.;RhoU=12200.;
fval=0.5;
fvu=1-fval;
DenRat=(RhoU/RhoAl);
fmal=0.212;
fmu=1-fmal;
Echem=350000.0;
ww=Echem*5100;

```

General::spell1:

Possible spelling error: new symbol name "fval"
is similar to existing symbol "fmal".

```

time=TB;
Do[To[i]=300;Tc[i]=300;xm[i]=0;xv[i]=0;orf[i]=0;,{i,1,m}];
dtt[i_]:=10*dx*dx*rho[i]*cp[i]/(2*k[i]);
dt=Min[Table[dtt[i],{i,1,m}]];
nt=Floor[(TE-TB)/dt]-1;
Print["dt= ",dt," nt= ",nt];

```

dt= 0.000278413 nt= 214

```

Do[
s=Table[vv[i,j,dt],{j,m},{i,m}];
sinv=Inverse[s];(* invert the matrix *)
qq=Table[f[i,dt,time,qtot],{i,m}];
ans=sinv.qq;

```

```

Do[Tc[i]=ans[[i]];
If[phase[i]==melting, xm[i]+=(k1[i]*To[i-1]-k2[i]*To[i]+k3[i]*To[i+1] + q[i,time,qtot]-
bb[i]*(To[m]-Tinf))*dt/(rho[i]*hsf[i]);,0];
If[phase[i]==vaporizing,xv[i]+=(k1[i]*To[i-1]-k2[i]*To[i]+k3[i]*To[i+1] + q[i,time,qtot]-
bb[i]*(To[m]-Tinf))*dt/(rho[i]*hfg[i]);,0];
,{i,1,m}];

```

```

If[Mod[numtimesteps,5]==0 || numtimesteps >=60,
Print[numtimesteps," ",time," ",To[1]," ",xv[1]," ",phase[1]," "],

```

```
q[1,time,qtot]," ",orf[1]," ",qs[qtot,time]," ",qchem[1]]];
```

```
Do[To[i]=Tn[i];
xx1[i,numtimesteps]={time,To[i]};
xx2[i,numtimesteps]={time,xm[i]};
xx3[i,numtimesteps]={time,xv[i]};
xx4[i,numtimesteps]={time,orf[i]};
xx5[i,numtimesteps]={time,q[i,time,qtot]};
xx6[i,numtimesteps]={time,qchem[i]};
xx7[i,numtimesteps]={time,qs[qtot,time]};
```

```
,{i,1,m}];
To[i]=Tn[i];
time += dt;
UpdateRF[dt];
,{numtimesteps,1,nt}];
Print[Finished];
```

5	0.251114	300.338	0	solid	1.50726	10	1.04687	10	1.50726	10	16.8081
10	0.252506	301.042	0	solid	2.59661	10	2.36337	10	2.59661	10	16.9438
15	0.253898	301.975	0	solid	3.16824	10	3.69263	10	3.16824	10	17.1257
20	0.25529	303.442	0	solid	4.46902	10	5.04193	10	4.46902	10	17.4165
25	0.256682	305.181	0	solid	6.20982	10	6.41484	10	6.20982	10	17.769
30	0.258074	307.926	0	solid	8.89874	10	7.82498	10	8.89874	10	18.343
35	0.259466	311.78	0	solid	1.31083	10	9.29098	10	1.31083	10	19.1869
40	0.260858	316.855	0	solid	1.83252	10	1.0839	10	1.83252	10	20.3703
45	0.26225	323.668	0	solid	2.51041	10	1.2506	10	2.51041	10	22.0992
50	0.263642	332.611	0	solid	3.22541	10	1.43414	10	3.22541	10	24.6411
55	0.265034	345.655	0	solid	4.87788	10	1.64483	10	4.87788	10	28.9979
60	0.266426	363.738	0	solid	6.66247	10	1.90432	10	6.66247	10	36.6312
61	0.266705	368.158	0	solid	6.99078	10	1.9649	10	6.99078	10	38.8394
62	0.266983	372.726	0	solid	7.24521	10	2.02929	10	7.24521	10	41.2861
63	0.267262	377.351	0	solid	8.09779	10	2.09784	10	8.09779	10	43.9477
64	0.26754	382.707	0	solid	8.44391	10	2.17159	10	8.44391	10	47.2814
65	0.267818	388.182	0	solid	9.02385	10	2.25113	10	9.02385	10	50.9952
66	0.268097	394.031	0	solid	9.6641	10	2.33744	10	9.6641	10	55.3385

		11	-10	11			
67	0.268375	400.299	0	solid 1.02718	10	2.43176	10 1.02718 10 60.473
				11	-10	11	
68	0.268654	406.928	0	solid 1.09391	10	2.53549	10 1.09391 10 66.5049
				11	-10	11	
69	0.268932	413.961	0	solid 1.16569	10	2.6504	10 1.16569 10 73.6695
				11	-10	11	
70	0.269211	421.434	0	solid 1.21881	10	2.7787	10 1.21881 10 82.2602
				11	-10	11	
71	0.269489	429.114	0	solid 1.29451	10	2.92266	10 1.29451 10 92.2951
				11	-10	11	
72	0.269767	437.242	0	solid 1.39401	10	3.08557	10 1.39401 10 104.452
				11	-10	11	
73	0.270046	446.057	0	solid 1.48326	10	3.27231	10 1.48326 10 119.72
				11	-10	11	
74	0.270324	455.406	0	solid 1.57189	10	3.48866	10 1.57189 10 138.711
				11	-10	11	
75	0.270603	465.25	0	solid 1.65246	10	3.74201	10 1.65246 10 162.43
				11	-10	11	
76	0.270881	475.472	0	solid 1.76358	10	4.0414	10 1.76358 10 191.953
				11	-10	11	
77	0.271159	486.384	0	solid 1.88257	10	4.40046	10 1.88257 10 230.202
				11	-10	11	
78	0.271438	498.029	0	solid 1.99616	10	4.83806	10 1.99616 10 280.564
				11	-10	11	
79	0.271716	510.305	0	solid 2.11167	10	5.37951	10 2.11167 10 347.141
				11	-10	11	
80	0.271995	523.193	0	solid 2.22274	10	6.05985	10 2.22274 10 436.185
				11	-10	11	
81	0.272273	536.611	0	solid 2.36384	10	6.92721	10 2.36384 10 556.097
				11	-10	11	
82	0.272551	550.845	0	solid 2.47588	10	8.05589	10 2.47588 10 723.631
				11	-10	11	
83	0.27283	565.545	0	solid 2.66002	10	9.54629	10 2.66002 10 955.549
				11	-9	11	
84	0.273108	581.433	0	solid 2.79492	10	1.15728	10 2.79492 10 1299.26
				11	-9	11	
85	0.273387	597.929	0	solid 2.91188	10	1.43812	10 2.91188 10 1800.53
				11	-9	11	
86	0.273665	614.811	0	solid 3.10183	10	1.83321	10 3.10183 10 2533.08
				11	-9	11	
87	0.273944	632.804	0	solid 3.22061	10	2.40623	10 3.22061 10 3673.82
				11	-9	11	
88	0.274222	651.125	0	solid 3.38518	10	3.24974	10 3.38518 10 5408.02
				11	-9	11	
89	0.2745	670.219	0	solid 3.51628	10	4.52224	10 3.51628 10 8158.46
				11	-9	11	
90	0.274779	689.715	0	solid 3.71433	10	6.47431	10 3.71433 10 12515.4
				11	-9	11	
91	0.275057	710.26	0	solid 3.86671	10	9.56354	10 3.86671 10 19806.1
				11	-8	11	
92	0.275336	731.349	0	solid 3.99936	10	1.45509	10 3.99936 10 31975.8
				11	-8	11	
93	0.275614	752.762	0	solid 4.1405	10	2.27204	10 4.1405 10 52376.9

¹¹ ⁻⁸ ¹¹
 94 0.275892 774.573 0 solid 4.32043 10 3.63107 10 4.32043 10 87132.6
¹¹ ⁻⁸ ¹¹
 95 0.276171 797.145 0 solid 4.4593 10 5.94495 10 4.4593 10 148350.
¹¹ ⁻⁸ ¹¹
 96 0.276449 820.057 0 solid 4.6111 10 9.93321 10 4.6111 10 255700.
¹¹ ⁻⁷ ¹¹
 97 0.276728 843.425 0 solid 4.73736 10 1.69027 10 4.73735 10 446839.
¹¹ ⁻⁷ ¹¹
 98 0.277006 867.004 0 solid 4.9237 10 2.91601 10 4.9237 10 785859.
¹¹ ⁻⁷ ¹¹
 99 0.277284 891.355 0 melting 5.01914 10 5.11127 10 5.01913 10

⁶
 1.40745 10

¹¹ ⁻⁷ ¹¹ ⁶
 100 0.277563 900. 0 melting 5.18824 10 7.80997 10 5.18822 10 1.73023 10
¹¹ ⁻⁶ ¹¹ ⁶
 101 0.277841 900. 0 melting 5.24951 10 1.05087 10 5.24949 10 1.73023 10
¹¹ ⁻⁶ ¹¹ ⁶
 102 0.27812 900. 0 liquid 5.29762 10 1.32074 10 5.2976 10 1.73023 10
¹¹ ⁻⁶ ¹¹ ⁶
 103 0.278398 900. 0 liquid 5.38129 10 1.59061 10 5.38128 10 1.73023 10
¹¹ ⁻⁶ ¹¹
 104 0.278677 926.056 0 liquid 5.47843 10 2.09221 10 5.4784 10

⁶
 3.21594 10

¹¹ ⁻⁶ ¹¹
 105 0.278955 940.022 0 liquid 5.55618 10 2.79005 10 5.55613 10

⁶
 4.47409 10

¹¹ ⁻⁶ ¹¹
 106 0.279233 947.688 0 liquid 5.60487 10 3.62592 10 5.60482 10

⁶
 5.35904 10

¹¹ ⁻⁶ ¹¹
 107 0.279512 975.656 0 liquid 5.7223 10 5.23157 10 5.72219 10

⁷
 1.02943 10

¹¹ ⁻⁶ ¹¹ ⁷
 108 0.27979 999.667 0 liquid 5.76274 10 8.02022 10 5.76256 10 1.7879 10
¹¹ ¹¹
 109 0.280069 1018.51 0 liquid 5.71058 10 0.0000122935 5.7103 10

⁷
 2.73974 10

¹¹ ¹¹
 110 0.280347 1041.77 0 liquid 5.65357 10 0.0000194658 5.65311 10

7

4.59838 10
 111 0.280625 1067.8 0 liquid 5.73221 10 0.000032103 5.7314 10 8.10207 10
 112 0.280904 1093.02 0 liquid 5.7149 10 0.0000536655 5.71352 10
 1.38243 10
 113 0.281182 1115.25 0 liquid 5.67079 10 0.0000877598 5.6686 10
 2.18586 10
 114 0.281461 1137.15 0 liquid 5.57916 10 0.000140649 5.57577 10
 3.39084 10
 115 0.281739 1159.07 0 liquid 5.44696 10 0.000221674 5.44176 10
 5.19456 10
 116 0.282018 1179.53 0 liquid 5.38223 10 0.000340854 5.37459 10
 7.64058 10
 117 0.282296 1198.26 0 liquid 5.30969 10 0.000508787 5.29892 10
 1.07658 10
 118 0.282574 1214.83 0 liquid 5.18506 10 0.000734264 5.1706 10 1.44544 10
 119 0.282853 1228.72 0 liquid 5.0299 10 0.00102108 5.01151 10 1.83858 10
 120 0.283131 1241.07 0 liquid 4.95078 10 0.00137458 4.92812 10 2.26606 10
 121 0.28341 1252.39 0 liquid 4.79429 10 0.00180102 4.76695 10 2.73343 10
 122 0.283688 1261.69 0 liquid 4.67199 10 0.00229704 4.6402 10 3.17934 10
 123 0.283966 1269.66 0 liquid 4.52853 10 0.00286041 4.49242 10 3.61097 10
 124 0.284245 1276.36 0 liquid 4.35391 10 0.00348638 4.31379 10 4.01203 10
 125 0.284523 1281.53 0 liquid 4.19661 10 0.00416459 4.15314 10 4.34676 10
 126 0.284802 1285.26 0 liquid 4.08655 10 0.00488264 4.04053 10 4.60197 10
 127 0.28508 1288.2 0 liquid 3.91592 10 0.00563344 3.8678 10 4.81186 10

128	0.285358	1290.04	0	liquid	3.71918	10	0.00640516	3.66972	10	4.94583	10
			11			11			9		
129	0.285637	1290.61	0	liquid	3.57436	10	0.00718297	3.52451	10	4.98482	10
			11			11			9		
130	0.285915	1290.3	0	liquid	3.46261	10	0.00795655	3.41304	10	4.95777	10
			11			11			9		
131	0.286194	1289.35	0	liquid	3.26992	10	0.00871832	3.2211	10	4.88207	10
			11			11			9		
132	0.286472	1287.3	0	liquid	3.12774	10	0.00945599	3.08047	10	4.72772	10
			11			11			9		
133	0.286751	1284.76	0	liquid	2.97652	10	0.0101647	2.93109	10	4.54225	10
			11			11			9		
134	0.287029	1281.61	0	liquid	2.81073	10	0.0108392	2.76751	10	4.32274	10
			11			11			9		
135	0.287307	1277.65	0	liquid	2.65499	10	0.0114728	2.61438	10	4.06108	10
			11			11			9		
136	0.287586	1272.99	0	liquid	2.54647	10	0.0120613	2.50876	10	3.77168	10
			11			11			9		
137	0.287864	1268.11	0	liquid	2.40842	10	0.0126054	2.37354	10	3.48794	10
			11			11			9		
138	0.288143	1262.77	0	liquid	2.27873	10	0.0131047	2.24673	10	3.19991	10
			11			11			9		
139	0.288421	1257.03	0	liquid	2.15838	10	0.0135594	2.12924	10	2.91461	10
			11			11			9		
140	0.288699	1250.95	0	liquid	2.02849	10	0.0139708	2.00212	10	2.63704	10
			11			11			9		
141	0.288978	1244.39	0	liquid	1.90334	10	0.0143396	1.87969	10	2.36453	10
			11			11			9		
142	0.289256	1237.49	0	liquid	1.81027	10	0.0146681	1.78921	10	2.10529	10
			11			11			9		
143	0.289535	1230.72	0	liquid	1.69029	10	0.0149607	1.67153	10	1.87613	10
			11			11			9		
144	0.289813	1224.06	0	liquid	1.59184	10	0.0152217	1.57511	10	1.6727	10
			11			11			9		
145	0.290091	1217.82	0	liquid	1.50106	10	0.0154558	1.48605	10	1.50064	10
			11			11			9		
146	0.29037	1212.14	0	liquid	1.43556	10	0.0156676	1.42198	10	1.3579	10
			11			11			9		
147	0.290648	1207.21	0	liquid	1.34564	10	0.0158617	1.3332	10	1.24413	10
			11			11			9		
148	0.290927	1202.75	0	liquid	1.24714	10	0.0160409	1.23565	10	1.1488	10
			11			11			9		
149	0.291205	1198.61	0	liquid	1.17536	10	0.0162072	1.1647	10	1.06622	10
			11			11			8		
150	0.291484	1194.92	0	liquid	1.08034	10	0.0163627	1.07037	10	9.97325	10
			11			11			8		
151	0.291762	1191.4	0	liquid	1.0511	10	0.0165086	1.04174	10	9.35306	10
			10			10			8		
152	0.29204	1188.52	0	liquid	9.65338	10	0.016647	9.56465	10	8.87313	10
			10			10			8		
153	0.292319	1185.72	0	liquid	9.14914	10	0.0167785	9.06487	10	8.4266	10
			10			10			8		
154	0.292597	1183.22	0	liquid	8.59253	10	0.016904	8.51207	10	8.04575	10
			10			10			8		

155	0.292876	1180.91	0	liquid	8.24286	10	0.0170242	8.16578	10	7.70776	10
			10			10			8		
156	0.293154	1178.9	0	liquid	7.58746	10	0.01714	7.51321	10	7.42513	10
			10			10			8		
157	0.293432	1176.88	0	liquid	6.99696	10	0.0172516	6.92546	10	7.14994	10
			10			10			8		
158	0.293711	1174.85	0	liquid	6.68903	10	0.0173589	6.6202	10	6.8837	10
			10			10			8		
159	0.293989	1173.03	0	liquid	6.36619	10	0.0174627	6.29967	10	6.65186	10
			10			10			8		
160	0.294268	1171.36	0	liquid	6.00986	10	0.0175632	5.94541	10	6.44533	10
			10			10			8		
161	0.294546	1169.77	0	liquid	5.67797	10	0.0176608	5.61542	10	6.25451	10
			10			10			8		
162	0.294825	1168.25	0	liquid	5.01368	10	0.0177556	4.95291	10	6.07713	10
			10			10			8		
163	0.295103	1166.48	0	liquid	4.86446	10	0.0178472	4.80569	10	5.87715	10
			10			10			8		
164	0.295381	1164.91	0	liquid	4.40484	10	0.0179362	4.34781	10	5.70339	10
			10			10			8		
165	0.29566	1163.22	0	liquid	4.3143	10	0.0180224	4.25907	10	5.52309	10
			10			10			8		
166	0.295938	1161.73	0	liquid	3.74006	10	0.0181061	3.68638	10	5.36812	10
			10			10			8		
167	0.296217	1160.	0	liquid	3.49742	10	0.0181871	3.4455	10	5.1927	10
			10			10			8		
168	0.296495	1158.3	0	liquid	3.68003	10	0.0182655	3.62978	10	5.0257	10
			10			10			8		
169	0.296773	1156.98	0	liquid	3.41878	10	0.0183419	3.36978	10	4.89988	10
			10			10			8		
170	0.297052	1155.64	0	liquid	3.01612	10	0.0184164	2.96838	10	4.77456	10
			10			10			8		
171	0.29733	1154.13	0	liquid	2.7681	10	0.0184887	2.72173	10	4.63748	10
			10			10			8		
172	0.297609	1152.58	0	liquid	3.04493	10	0.0185589	2.99992	10	4.50076	10
			10			10			8		
173	0.297887	1151.45	0	liquid	2.60625	10	0.0186276	2.56222	10	4.40243	10
			10			10			8		
174	0.298165	1150.07	0	liquid	2.42303	10	0.0186944	2.38017	10	4.28643	10
			10			10			8		
175	0.298444	1148.68	0	liquid	2.18633	10	0.0187595	2.14461	10	4.1717	10
			10			10			8		
176	0.298722	1147.21	0	liquid	2.21629	10	0.0188227	2.17575	10	4.0541	10
			10			10			8		
177	0.299001	1145.9	0	liquid	2.16775	10	0.0188844	2.12824	10	3.95143	10
			10			10			8		
178	0.299279	1144.66	0	liquid	1.87915	10	0.0189445	1.84058	10	3.85672	10
			10			10			8		
179	0.299558	1143.28	0	liquid	1.77629	10	0.0190031	1.73876	10	3.75342	10
			10			10			8		
180	0.299836	1141.91	0	liquid	1.98229	10	0.0190601	1.94575	10	3.65367	10
			10			10			8		
181	0.300114	1140.81	0	liquid	1.70716	10	0.0191158	1.6714	10	3.5757	10
			10			10			8		

182	0.300393	1139.56	0	liquid	1.88496	10	0.0191703	1.85008	10	3.48844	10
			10			10			8		
183	0.300671	1138.54	0	liquid	1.29749	10	0.0192236	1.26331	10	3.41885	10
			10			10			8		
184	0.30095	1137.08	0	liquid	1.37367	10	0.0192754	1.34045	10	3.32131	10
			10			10			8		
185	0.301228	1135.76	0	liquid	1.53827	10	0.0193259	1.50592	10	3.23547	10
			10			10			8		
186	0.301506	1134.65	0	liquid	1.38321	10	0.0193752	1.35156	10	3.16507	10
			10			10			8		
187	0.301785	1133.46	0	liquid	1.15328	10	0.0194234	1.12237	10	3.09144	10
			10			10			8		
188	0.302063	1132.14	0	liquid	1.16067	10	0.0194704	1.13056	10	3.01079	10
			10			10			8		
189	0.302342	1130.87	0	liquid	1.14153	10	0.0195162	1.11217	10	2.93588	10
			10			10			8		
190	0.30262	1129.65	0	liquid	1.21139	10	0.0195609	1.18274	10	2.86483	10
			9			9			8		
191	0.302899	1128.53	0	liquid	8.18183	10	0.0196046	7.90168	10	2.80155	10
			9			9			8		
192	0.303177	1127.12	0	liquid	9.65876	10	0.0196471	9.38644	10	2.7232	10
			9			9			8		
193	0.303455	1125.88	0	liquid	9.10584	10	0.0196885	8.8402	10	2.65637	10
			9			9			8		
194	0.303734	1124.64	0	liquid	9.84929	10	0.0197289	9.5902	10	2.59082	10
			9			9			8		
195	0.304012	1123.5	0	liquid	6.78358	10	0.0197684	6.53037	10	2.53211	10
			9			9			8		
196	0.304291	1122.13	0	liquid	8.04901	10	0.0198068	7.80271	10	2.46306	10
			9			9			8		
197	0.304569	1120.9	0	liquid	6.64743	10	0.0198443	6.40713	10	2.40302	10
			9			9			8		
198	0.304847	1119.6	0	liquid	7.8758	10	0.0198808	7.64178	10	2.34016	10
			9			9			8		
199	0.305126	1118.43	0	liquid	9.08586	10	0.0199164	8.85732	10	2.28539	10
			9			9			8		
200	0.305404	1117.39	0	liquid	6.50053	10	0.0199513	6.27675	10	2.23785	10
			9			9			8		
201	0.305683	1116.15	0	liquid	5.55928	10	0.0199854	5.34107	10	2.18211	10
			9			9			8		
202	0.305961	1114.85	0	liquid	6.83319	10	0.0200185	6.62067	10	2.12519	10
			9			9			8		
203	0.306239	1113.69	0	liquid	7.97381	10	0.0200509	7.76627	10	2.07547	10
			9			9			8		
204	0.306518	1112.65	0	liquid	5.72272	10	0.0200826	5.51954	10	2.03183	10
			9			9			8		
205	0.306796	1111.43	0	liquid	2.36223	10	0.0201135	2.16406	10	1.98169	10
			9			9			8		
206	0.307075	1109.93	0	liquid	5.55674	10	0.0201435	5.36455	10	1.92191	10
			9			9			8		
207	0.307353	1108.74	0	liquid	5.77984	10	0.0201727	5.59227	10	1.87565	10
			9			9			8		
208	0.307632	1107.59	0	liquid	5.95047	10	0.0202013	5.76727	10	1.83194	10
			9			9			8		

```

209 0.30791 1106.48 0 liquid 3.24404 10 0.0202292 3.06501 10 1.79034 10
      9      9      8
210 0.308188 1105.14 0 liquid 3.24748 10 0.0202564 3.07332 10 1.7416 10
      9      9      8
211 0.308467 1103.82 0 liquid 3.74266 10 0.0202828 3.57318 10 1.69481 10
      9      9      8
212 0.308745 1102.56 0 liquid 4.55404 10 0.0203086 4.38891 10 1.65138 10
      9      9      8
213 0.309024 1101.39 0 liquid 5.12912 10 0.0203337 4.96792 10 1.61194 10
      9      9      8
214 0.309302 1100.29 0 liquid 5.06907 10 0.0203583 4.91153 10 1.57546 10
      9      9      8
Finished

```

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Tcoefsol=2.25*10^-5;
Tcoefliq=1.17*10^-4;
(* TA - average temperature *)
ta[ts_]:=Sum[Part[xx1[node,ts],2],{node,1,m}]/m;
(* tr - Temperature time rate of change *)
tr[node_,ts_]:=Part[xx1[node,ts]-xx1[node,ts-1],2]/Part[xx1[node,ts]-xx1[node,ts-1],1];
(* tra - average rate of change of temp for the plate *)
tra[ts_]:=Sum[tr[node,ts],{node,1,m}]/m;

(* expth - expansion thermal *)
(* texprate - thermal expansion rate for the plate *)

expth[node_,ts_]:=Tcoefsol*tr[node,ts] /; Part[xx1[node,ts],2]<Tm;
expth[node_,ts_]:=Tcoefliq*tr[node,ts] /; Part[xx1[node,ts],2]>Tm && Part[xx1[node,ts],2]<Tv;
expth[node_,ts_]:=tr[node,ts]/ Part[xx1[node,ts],2]/; Part[xx1[node,ts],2]>Tv;
expth[node_,ts_]:=0 /; Part[xx1[node,ts],2]==Tm;
expth[node_,ts_]:=0 /; Part[xx1[node,ts],2]==Tv;

texprate[ts_]:=Sum[expth[node,ts],{node,1,m}]/m;
(* qm - quality of the melt *)
(* mv - quality of the vapor *)
qm[ts_]:=Sum[Part[xx2[node,ts],2],{node,1,m}]/m;
qv[ts_]:=Sum[Part[xx3[node,ts],2],{node,1,m}]/m;

vfaln[i_]:=fval /; i<=n ;
vfaln[i_]:=1 /; i>n ;

(* Fractional rate of change of volume due to various factors *)
(* ger - Gas Expansion Rate *)
(* VMR - Volumetric Melting Rate *)
(* VVR - Volumetric Vaporization Rate *)
(* VOFR - Volumetric Oxide Formation Rate *)
(* QMR - Quality Melt Rate *)
(* QVR - Quality Vaporization Rate *)
rhosolid=2700; rholiquid=2500; rhovapor=1.0 ;
DencRatLV=(1/rhovapor^(0.3)-1/rholiquid^(0.3))/(1/rholiquid^(0.3));
DencRatSL=(1/rholiquid^(0.3)-1/rhosolid^(0.3))/(1/rhosolid^(0.3));
qmr[node_,ts_]:=Part[xx2[node,ts]-xx2[node,ts-1],2]/Part[xx2[node,ts]-xx2[node,ts-1],1];
qvr[node_,ts_]:=Part[xx3[node,ts]-xx3[node,ts-1],2]/Part[xx3[node,ts]-xx3[node,ts-1],1];
vmr[ts_]:=Sum[DencRatSL*qmr[node,ts]*vfaln[node],{node,1,m}]/m;
vvr[ts_]:=Sum[DencRatLV*qvr[node,ts]*vfaln[node],{node,1,m}]/m;

```

```

ger[ts_]:=Sum[ tr[node,ts]/ Part[xx1[node,ts],2] ,{node,1,n}]/m;
ff=0.01;
TotalExpansionRate[ts_]:= (1-ff)*(vmr[ts]+vvr[ts]+texprate[ts]) + ff*ger[ts]
IntegralTotExp[ts_]:=Sum[TotalExpansionRate[tt]*dt,{tt,2,ts}]
ListPlot[Table[{Part[xx1[1,ts],1],TotalExpansionRate[ts]},{ts,2,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)","Fractional Exp rate (1/s)"}]

ter=1;
(* ter total expansion *)
Do[ter*=(1+TotalExpansionRate[tt]*dt);
qww[tt]={Part[xx1[1,tt],1],ter-1 }
,{tt,2,nt}]
ListPlot[Table[qww[tt],{tt,2,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)","Expanded fraction"}]
Do[eff[ts]={Part[xx1[1,ts],1],ta[ts],TotalExpansionRate[ts],Part[qww[ts],2]},{ts,2,nt}];
ListPlot[Table[{Part[xx1[1,ts],1],ta[ts]},{ts,2,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)","average temp (K)"}]

```

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```

ListPlot[Table[{Part[xx1[1,i],1],Part[xx1[1,i],2]},{i,1,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)"," temp (K)"}]

ListPlot[Table[{Part[xx1[m,i],1],Part[xx1[m,i],2]},{i,1,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)"," temp (K)"}]

```

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```
ListPlot[Table[{Part[xx4[1,i],1],Part[xx4[1,i],2]},{i,1,nt}],
  PlotJoined->True,PlotRange->All,
  AxesLabel->{"time (s)"," fraction of fuel reacted"}];
```

```
szs1=ListPlot[Table[{Part[xx5[1,i],1],Part[xx5[1,i],2]},{i,1,nt}],
  PlotJoined->True,PlotRange->All,
  AxesLabel->{"time (s)"," Q fission (W/m^3)"}];
szs2=ListPlot[Table[{Part[xx6[1,i],1],Part[xx6[1,i],2]},{i,1,nt}],
  PlotJoined->True,PlotRange->All,
  AxesLabel->{"time (s)"," Q chemical (W/m^3)"}];
szs3=ListPlot[Table[{Part[xx7[1,i],1],Part[xx7[1,i],2]},{i,1,nt}],
  PlotJoined->True,PlotRange->All,
  AxesLabel->{"time (s)"," Q total (W/m^3)"}];
Show[szs1,szs2,szs3]
```

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```
*****
52
*****
```

```
(*
  THIS IS A REVISED DISPERSION MODEL THAT USES THE
  TIME ENERGY DATA FROM THE EXPERIMENTS AND TEMPERATURE
  DEPENDANT CHEMICAL REACTION RATES FUEL- ALUMINUM
  This copy has chemical reaction rates included in the
  model
  LAST UPDATED 4-19-95
  Renormalized pulse width was used to a narrower band width.
*)
Print["Date last updated =",Date[]];

Date last updated ={1995, 5, 1, 16, 44, 46}

l=0.000635;m=10;dx=l/m;n=Round[m*.4];time=0;Tinf=300;
volmeat=2*dx*n*(25*10^-3)*(70*10^-3);
Do[xx[i]=i*dx-dx/2,{i,1,m}];Tm=900.;Tv=2700.;
Do[To[i]=300;Tc[i]=300;xm[i]=0;xv[i]=0,{i,1,m}];

k[i_]:=39.91 /; i <= n ; (* assign k to the meat *)
k[i_]:=kcl[To[i]] /; i > n ; (* assign k to the clad *)
```



```

rho[i_]:= 6550.0 /; i <= n ; (* assign meat density *)
rho[i_]:= 2700.0 /; i > n ; (* assign clad density *)

cp[i_]:= cpcl[To[i]] /; i > n ; (* assign clad cpecific heat *)
cp[i_]:= cpf[To[i]] /; i <= n ; (* assign meat cpecific heat *)

htc=Interpolation[{{150,13232},{350,13232},{410,14770},{726,12542},
{776,11561},{1256,9924},{1458,1750},{1526,1729},{1556,800},
{5000,800}},InterpolationOrder->1];

kcl=Interpolation[{{100,161},{400,177},{500,186},{600,193},
{750,193},{850,193},{900,193},{930,193},
{1200,84},{1300,84},{1500,84},{2000,84},{5000,84}},
InterpolationOrder->1];

cpcl=Interpolation[{{100,878},{300,878},{366,942},{422,963},
{588,1034},{700,1067},{811,995},{1000,995},{2000,995},
{5000,995}},InterpolationOrder->1];

cpf=Interpolation[{{100,351},{300,351},{933,463},{1933,546},
{2500,546},{3500,546},{5000,546}},InterpolationOrder->1];

hsf[i_]:=398000*fmal /; i <= n;
hfg[i_]:=10800000*fmal /; i <= n;
hsf[i_]:=398000 /; i > n;
hfg[i_]:=10800000 /; i > n;

l51=ReadList["508.52",Number,RecordLists->True];
shape=Interpolation[l51,InterpolationOrder->1];
num=Part[Part[Position[l51,Last[l51]],1],1];
TimeBeg=Part[l51[[1]],1]; TimeEnd=Part[l51[[num]],1];
c1=Sum[0.5*(shape[Part[l51[[i]],1]]+shape[Part[l51[[i+1]],1]])*
(Part[l51[[i+1]],1]-Part[l51[[i]],1]),{i,200,500}];
qs[qtot_,time_]:=qtot*shape[time]/(volmeat*c1);
qtot=8685;
(* Functions : nrt - new reacted fraction
orf - old reacted fraction *)

Tm1=863;Tm2=1970;Td[i_]:= (To[i]-Tm1)/(Tm2-Tm1);
lamda[i_]:= (4*10^-4)E^(14.73*Tanh[1.8*Td[i]]);
z1[i_,dt_]:=NDSolve[{RF'[t]==lamda[i]-lamda[i]*RF[t],
RF[0]==orf[i]},RF,{t,0,dt}, AccuracyGoal->15];
nrf[i_,dt_]:=Part[RF[dt]/.z1[i,dt],1]
frr[i_]:=lamda[i]-lamda[i]*orf[i];
UpdateRF[dt_]:=Do[orf[i]=nrf[i,dt],{i,1,n}]
qchem[i_]:=ww*frr[i];

q[i_,time_,qtot_]:=qs[qtot,time]+qchem[i]/; i<=n; (* assign heat generation in the meat *)
q[i_,time_,qtot_]:=0 /; i>n; (* assign heat generation in the clad *)
bb[i_]:=htc[To[m]]/dx /; i==m ;
bb[i_]:=0 /; i != m ; (* value for the heat transfer out of the plate *)

k1[i_]:= (2*k[i-1]*k[i]/(k[i-1]+k[i]))/dx^2 /; i!=1
k1[i_]:=0 /; i==1

```

```

k2[i_]:=k1[i]+k3[i];
k3[i_]:= (2*k[i+1]*k[i]/(k[i+1]+k[i]))/dx^2 /; i!=m
k3[i_]:=0 /; i==m ;

phase[i_]:= solid /; Tc[i] < Tm ;
phase[i_]:= liquid /; Tc[i] < Tv && Tc[i] >= Tm && xm[i] >= 0.85;
phase[i_]:= vapor /; Tc[i] >= Tv && xv[i] > 1;
phase[i_]:= melting /; Tc[i] >= Tm && xm[i] < 0.85 ;
phase[i_]:= vaporizing /; Tc[i] >= Tv && xv[i] < 1 && xv[i] > -0.01;

Tn[i_]:=Tc[i] /; phase[i] != melting && phase[i] != vaporizing;
Tn[i_]:=Tm /; phase[i] == melting ;
Tn[i_]:=Tv /; phase[i] == vaporizing ;

vv[i_,j_,dt_]:= k1[i] /; i-1==j && phase[j]!=melting && phase[j]!=vaporizing
vv[i_,j_,dt_]:= -k2[i]-rho[i]*cp[i]/dt-bb[i] /; i==j && phase[j]!=melting &&
phase[j]!=vaporizing
vv[i_,j_,dt_]:= k3[i] /; i+1==j && phase[j]!=melting && phase[j]!=vaporizing
vv[i_,j_,dt_]:= 0 /; i != j && i-1 != j && i+1 != j ;
(*-----*)
vv[i_,j_,dt_]:=1 /; i==j && phase[j]==melting;
vv[i_,j_,dt_]:=0 /; i-1==j && phase[j]==melting;
vv[i_,j_,dt_]:=0 /; i+1==j && phase[j]==melting;
vv[i_,j_,dt_]:=1 /; i==j && phase[j]==vaporizing;
vv[i_,j_,dt_]:=0 /; i-1==j && phase[j]==vaporizing;
vv[i_,j_,dt_]:=0 /; i+1==j && phase[j]==vaporizing;

(*-----*)
f[i_,dt_,time_,qtot_]:= -q[i,time,qtot]-To[i]*rho[i]*cp[i]/dt - bb[i]*Tinf /; phase[i]!=melting &&
phase[i]!=vaporizing;
f[i_,dt_,time_,qtot_]:= Tm /; phase[i]==melting;
f[i_,dt_,time_,qtot_]:= Tv /; phase[i]==vaporizing;

Plot[shape[t],{t,.245,0.29},PlotRange->{0,50}]

```

-Graphics-

```

TB=.245;TE=.29;
RhoMeat=6500.;RhoAl=2700.;RhoU=12200.;
fval=0.5;
fvu=1-fval;
DenRat=(RhoU/RhoAl);
fmal=0.212;
fmu=1-fmal;
Echem=350000.0;
ww=Echem*5100;

time=TB;
Do[To[i]=300;Tc[i]=300;xm[i]=0;xv[i]=0;orf[i]=0,.,{i,1,m}];
dtt[i_]:=10*dx*dx*rho[i]*cp[i]/(2*k[i]);

```

```

dt=Min[Table[dt[i],{i,1,m}]];
nt=Floor[(TE-TB)/dt]-1;
Print["dt= ",dt," nt= ",nt];

dt= 0.000278413 nt= 160

Do[
s=Table[vv[i,j,dt],{j,m},{i,m}];
sinv=Inverse[s];(* invert the matrix *)
qq=Table[f[i,dt,time,qtot],{i,m}];
ans=sinv.qq;

Do[Tc[i]=ans[[i]];
If[phase[i]==melting, xm[i]+=(k1[i]*To[i-1]-k2[i]*To[i]+k3[i]*To[i+1] + q[i,time,qtot]-
bb[i]*(To[m]-Tinf))*dt/(rho[i]*hsf[i]);,0];
If[phase[i]==vaporizing,xv[i]+=(k1[i]*To[i-1]-k2[i]*To[i]+k3[i]*To[i+1] + q[i,time,qtot]-
bb[i]*(To[m]-Tinf))*dt/(rho[i]*hfg[i]);,0];
,{i,1,m}]];

If[Mod[numtimesteps,5]==0 || numtimesteps >=60,
Print[numtimesteps," ",time," ",To[1]," ",xv[1]," ",phase[1]," ",
q[1,time,qtot]," ",orf[1]," ",qs[qtot,time]," ",qchem[1]]];

Do[To[i]=Tn[i];
xx1[i,numtimesteps]={time,To[i]};
xx2[i,numtimesteps]={time,xm[i]};
xx3[i,numtimesteps]={time,xv[i]};
xx4[i,numtimesteps]={time,orf[i]};
xx5[i,numtimesteps]={time,q[i,time,qtot]};
xx6[i,numtimesteps]={time,qchem[i]};
xx7[i,numtimesteps]={time,qs[qtot,time]};

,{i,1,m}];
To[i]=Tn[i];
time += dt;
UpdateRF[dt];
,{numtimesteps,1,nt}];
Print[Finished];

```

	9	-11	9						
5	0.246114	302.292	0	solid	7.34496	10	1.06093	10	7.34496 10 17.188
	10	-11	10						
10	0.247506	306.166	0	solid	1.40937	10	2.43727	10	1.40937 10 17.9724
	10	-11	10						
15	0.248898	311.748	0	solid	1.91851	10	3.89368	10	1.91851 10 19.1797
	10	-11	10						
20	0.25029	319.911	0	solid	2.66216	10	5.47598	10	2.66216 10 21.1253
	10	-11	10						
25	0.251682	331.343	0	solid	3.69164	10	7.25861	10	3.69164 10 24.2605
	10	-11	10						
30	0.253074	348.019	0	solid	6.21422	10	9.38632	10	6.21422 10 29.8818
	10	-10	10						
35	0.254466	373.04	0	solid	8.95488	10	1.22053	10	8.95488 10 41.4612
	11	-10	11						

40 0.255858 408.432 0 solid 1.296 10 1.65288 10 1.296 10 67.9674
 11 -10 11
 45 0.25725 458.085 0 solid 1.87624 10 2.48562 10 1.87624 10 144.758
 11 -10 11
 50 0.258642 526.403 0 solid 2.59358 10 4.80081 10 2.59358 10 462.057
 11 -9 11
 55 0.260034 617.306 0 solid 3.5179 10 1.59661 10 3.5179 10 2665.82
 11 -8 11
 60 0.261426 734.536 0 solid 4.65523 10 1.34035 10 4.65523 10 34397.8
 11 -8 11
 61 0.261705 760.794 0 solid 4.89122 10 2.32503 10 4.89122 10 63130.8
 11 -8 11
 62 0.261983 788.002 0 solid 5.15806 10 4.18913 10 5.15805 10 119514.
 11 -8 11
 63 0.262262 816.4 0 solid 5.39198 10 7.84461 10 5.39198 10 234365.
 11 -7 11
 64 0.26254 845.608 0 solid 5.61143 10 1.51878 10 5.61143 10 470795.
 11 -7 11
 65 0.262818 875.449 0 melting 5.81675 10 3.01925 10 5.81674 10 962004.
 11 -7 11 6
 66 0.263097 900. 0 melting 5.99562 10 5.71796 10 5.9956 10 1.73023 10
 11 -7 11 6
 67 0.263375 900. 0 melting 6.26754 10 8.41666 10 6.26752 10 1.73023 10
 11 -6 11 6
 68 0.263654 900. 0 liquid 6.47999 10 1.11154 10 6.47997 10 1.73023 10
 11 -6 11 6
 69 0.263932 900. 0 liquid 6.65224 10 1.38141 10 6.65223 10 1.73023 10
 11 -6 11 6
 70 0.264211 932.21 0 liquid 6.77413 10 1.96168 10 6.77409 10 3.72035 10
 11 -6 11 6
 71 0.264489 949.461 0 liquid 6.99205 10 2.83311 10 6.99199 10 5.587 10
 11 -6 11 7
 72 0.264767 987.888 0 liquid 7.10984 10 4.96247 10 7.1097 10 1.3652 10
 11 -6 11
 73 0.265046 1019.53 0 liquid 7.18236 10 9.33412 10 7.18208 10

7
 2.80281 10

 11 11
 74 0.265324 1044.23 0 liquid 7.29197 10 0.0000169055 7.29149 10

7
 4.85422 10

 11 11
 75 0.265603 1075.51 0 liquid 7.41234 10 0.0000318076 7.41138 10

7
 9.55416 10

 11 11
 76 0.265881 1111.54 0 liquid 7.39574 10 0.0000634205 7.39372 10

8
 2.02678 10

 11 11 8

77	0.266159	1144.93	0	liquid	7.41384	10	0.000125046	7.40989	10	3.95089	10
					11	11	8				
78	0.266438	1174.49	0	liquid	7.40882	10	0.000233542	7.40186	10	6.9557	10
					11	11	9				
79	0.266716	1203.92	0	liquid	7.39678	10	0.00041945	7.38486	10	1.1918	10
					11	11	9				
80	0.266995	1234.19	0	liquid	7.33027	10	0.0007344	7.31008	10	2.01893	10
					11	11	9				
81	0.267273	1263.18	0	liquid	7.22371	10	0.00124301	7.19111	10	3.26007	10
					11	11	9				
82	0.267551	1289.43	0	liquid	7.1766	10	0.00201094	7.12739	10	4.92156	10
					11	11	9				
83	0.26783	1313.0	0	liquid	7.0483	10	0.00310271	6.97834	10	6.99586	10
					11	11	9				
84	0.268108	1334.74	0	liquid	6.9133	10	0.00459071	6.81797	10	9.53293	10
					11	11	10				
85	0.268387	1354.36	0	liquid	6.77286	10	0.00653413	6.64838	10	1.24477	10
					11	11	10				
86	0.268665	1371.41	0	liquid	6.61835	10	0.00896119	6.46293	10	1.55417	10
					11	11	10				
87	0.268944	1386.27	0	liquid	6.45257	10	0.0118846	6.26542	10	1.87152	10
					11	11	10				
88	0.269222	1399.18	0	liquid	6.27702	10	0.0153008	6.05838	10	2.18648	10
					11	11	10				
89	0.2695	1410.12	0	liquid	6.0482	10	0.0191816	5.79988	10	2.48318	10
					11	11	10				
90	0.269779	1418.62	0	liquid	5.86683	10	0.0234509	5.5937	10	2.73122	10
					11	11	10				
91	0.270057	1425.43	0	liquid	5.69626	10	0.0280477	5.40224	10	2.94019	10
					11	11	10				
92	0.270336	1430.97	0	liquid	5.47292	10	0.0329187	5.16141	10	3.11512	10
					11	11	10				
93	0.270614	1434.89	0	liquid	5.23713	10	0.0379828	4.9133	10	3.23826	10
					11	11	10				
94	0.270892	1437.08	0	liquid	5.03018	10	0.0431448	4.70012	10	3.30061	10
					11	11	10				
95	0.271171	1438.14	0	liquid	4.76455	10	0.0483392	4.43242	10	3.32127	10
					11	11	10				
96	0.271449	1437.77	0	liquid	4.54383	10	0.0534847	4.21483	10	3.29	10
					11	11	10				
97	0.271728	1436.37	0	liquid	4.31035	10	0.0585235	3.98816	10	3.22188	10
					11	11	10				
98	0.272006	1433.84	0	liquid	4.06659	10	0.0633962	3.755	10	3.11599	10
					11	11	10				
99	0.272284	1430.16	0	liquid	3.85693	10	0.0680478	3.55945	10	2.97483	10
					11	11	10				
100	0.272563	1425.66	0	liquid	3.62451	10	0.072446	3.34319	10	2.81316	10
					11	11	10				
101	0.272841	1420.13	0	liquid	3.41118	10	0.0765546	3.14835	10	2.62832	10
					11	11	10				
102	0.27312	1413.7	0	liquid	3.19704	10	0.0803503	2.95419	10	2.42854	10
					11	11	10				
103	0.273398	1406.47	0	liquid	2.98677	10	0.0838211	2.76466	10	2.22105	10
					11	11	10				

104	0.273677	1398.76	0	liquid	2.8122	10	0.0869727	2.61048	10	2.01713	10
			11			11			10		
105	0.273955	1391.17	0	liquid	2.66426	10	0.0898349	2.48105	10	1.83218	10
			11			11			10		
106	0.274233	1384.14	0	liquid	2.45891	10	0.0924493	2.29154	10	1.67372	10
			11			11			10		
107	0.274512	1377.29	0	liquid	2.28149	10	0.0948399	2.12842	10	1.53071	10
			11			11			10		
108	0.27479	1370.86	0	liquid	2.14862	10	0.0970357	2.00802	10	1.40605	10
			11			11			10		
109	0.275069	1365.18	0	liquid	1.97001	10	0.09907	1.83973	10	1.30281	10
			11			11			10		
110	0.275347	1359.74	0	liquid	1.84727	10	0.100959	1.72626	10	1.21013	10
			11			11			10		
111	0.275625	1354.89	0	liquid	1.71678	10	0.102727	1.60355	10	1.13224	10
			11			11			10		
112	0.275904	1350.44	0	liquid	1.59528	10	0.104389	1.48883	10	1.06447	10
			11			11			10		
113	0.276182	1346.34	0	liquid	1.48239	10	0.105958	1.38189	10	1.00502	10
			11			11			9		
114	0.276461	1342.54	0	liquid	1.40645	10	0.107445	1.31121	10	9.52417	10
			11			11			9		
115	0.276739	1339.23	0	liquid	1.31267	10	0.108863	1.22182	10	9.08511	10
			11			11			9		
116	0.277018	1336.17	0	liquid	1.18842	10	0.11022	1.10149	10	8.69317	10
			11			11			9		
117	0.277296	1332.99	0	liquid	1.12277	10	0.111516	1.03974	10	8.30282	10
			11			10			9		
118	0.277574	1330.1	0	liquid	1.05488	10	0.112758	9.75263	10	7.96137	10
			10			10			9		
119	0.277853	1327.42	0	liquid	9.82356	10	0.113953	9.05817	10	7.65396	10
			10			10			9		
120	0.278131	1324.82	0	liquid	9.03961	10	0.115103	8.30294	10	7.36667	10
			10			10			9		
121	0.27841	1322.2	0	liquid	8.63317	10	0.116209	7.92446	10	7.08715	10
			10			10			9		
122	0.278688	1319.84	0	liquid	7.66188	10	0.117277	6.97771	10	6.8417	10
			10			10			9		
123	0.278966	1317.19	0	liquid	7.23797	10	0.118303	6.58024	10	6.57732	10
			10			10			9		
124	0.279245	1314.7	0	liquid	6.87706	10	0.119292	6.24344	10	6.33614	10
			10			10			9		
125	0.279523	1312.37	0	liquid	6.44411	10	0.120247	5.83232	10	6.11791	10
			10			10			9		
126	0.279802	1310.12	0	liquid	6.26223	10	0.121169	5.67098	10	5.91248	10
			10			10			9		
127	0.28008	1308.11	0	liquid	5.64921	10	0.122064	5.07578	10	5.7343	10
			10			10			9		
128	0.280358	1305.95	0	liquid	5.27741	10	0.12293	4.72256	10	5.54853	10
			10			10			9		
129	0.280637	1303.82	0	liquid	4.92206	10	0.123768	4.38499	10	5.37069	10
			10			10			9		
130	0.280915	1301.72	0	liquid	4.73856	10	0.12458	4.21858	10	5.19978	10
			10			10			9		

131	0.281194	1299.76	0	liquid	4.4252	10	0.125367	3.92071	10	5.04496	10
						10			9		
132	0.281472	1297.82	0	liquid	4.03201	10	0.126131	3.54246	10	4.89542	10
						10			9		
133	0.281751	1295.81	0	liquid	3.51879	10	0.126871	3.04431	10	4.74479	10
						10			9		
134	0.282029	1293.61	0	liquid	3.35598	10	0.127587	2.89745	10	4.58536	10
						10			9		
135	0.282307	1291.51	0	liquid	3.41068	10	0.128279	2.96691	10	4.43775	10
						10			9		
136	0.282586	1289.69	0	liquid	3.01789	10	0.128952	2.58664	10	4.31243	10
						10			9		
137	0.282864	1287.74	0	liquid	2.60813	10	0.129604	2.18992	10	4.18212	10
						10			9		
138	0.283143	1285.64	0	liquid	2.67071	10	0.130236	2.26612	10	4.04589	10
						10			9		
139	0.283421	1283.78	0	liquid	2.57378	10	0.130849	2.18094	10	3.92844	10
						10			9		
140	0.283699	1282.01	0	liquid	1.85547	10	0.131445	1.47352	10	3.81948	10
						10			9		
141	0.283978	1279.79	0	liquid	2.11494	10	0.13202	1.74622	10	3.68729	10
						10			9		
142	0.284256	1277.96	0	liquid	1.9072	10	0.132579	1.54913	10	3.58074	10
						10			9		
143	0.284535	1276.09	0	liquid	1.92723	10	0.133121	1.57969	10	3.47536	10
						10			9		
144	0.284813	1274.39	0	liquid	1.81002	10	0.133648	1.47192	10	3.38104	10
						10			9		
145	0.285091	1272.7	0	liquid	1.20734	10	0.134162	8.78304	10	3.29034	10
						10			9		
146	0.28537	1270.62	0	liquid	1.57699	10	0.134658	1.25881	10	3.18173	10
						10			9		
147	0.285648	1268.98	0	liquid	1.30172	10	0.135141	9.91942	10	3.09778	10
						10			9		
148	0.285927	1267.2	0	liquid	1.52943	10	0.135611	1.22848	10	3.00958	10
						10			9		
149	0.286205	1265.72	0	liquid	1.26334	10	0.136069	9.69592	10	2.93745	10
						10			9		
150	0.286484	1264.09	0	liquid	1.36424	10	0.136516	1.07821	10	2.86036	10
						10			9		
151	0.286762	1262.64	0	liquid	1.22462	10	0.136951	9.45356	10	2.79263	10
						9			9		
152	0.28704	1261.13	0	liquid	9.01229	10	0.137376	6.28802	10	2.72427	10
						10			9		
153	0.287319	1259.42	0	liquid	1.00364	10	0.137789	7.38793	10	2.64848	10
						9			9		
154	0.287597	1257.87	0	liquid	9.53533	10	0.138192	6.9541	10	2.58123	10
						10			9		
155	0.287876	1256.33	0	liquid	1.19238	10	0.138585	9.40745	10	2.51639	10
						10			9		
156	0.288154	1255.06	0	liquid	1.01432	10	0.138969	7.67959	10	2.46358	10
						10			9		
157	0.288432	1253.69	0	liquid	1.01788	10	0.139345	7.77113	10	2.40763	10
						10			9		


```

158 0.288711 1252.36 0 liquid 1.0533 10 0.139712 8.17833 10 2.35466 10
          9          9          9
159 0.288989 1251.1 0 liquid 8.42375 10 0.140072 6.11836 10 2.30539 10
          9          9          9
160 0.289268 1249.69 0 liquid 8.53329 10 0.140423 6.28163 10 2.25166 10
Finished

```

```

Tcoefsol=2.25*10^-5;
Tcoefliq=1.17*10^-4;
(* TA - average temperature *)
ta[ts_]:=Sum[Part[xx1[node,ts],2],{node,1,m}]/m;
(* tr - Temperature time rate of change *)
tr[node_,ts_]:=Part[xx1[node,ts]-xx1[node,ts-1],2]/Part[xx1[node,ts]-xx1[node,ts-1],1];
(* tra - average rate of change of temp for the plate *)
tra[ts_]:=Sum[tr[node,ts],{node,1,m}]/m;

(* expth - expansion thermal *)
(* texprate - thermal expansion rate for the plate *)

expth[node_,ts_]:=Tcoefsol*tr[node,ts] /; Part[xx1[node,ts],2]<Tm;
expth[node_,ts_]:=Tcoefliq*tr[node,ts] /; Part[xx1[node,ts],2]>Tm && Part[xx1[node,ts],2]<Tv;
expth[node_,ts_]:=tr[node,ts]/ Part[xx1[node,ts],2]/; Part[xx1[node,ts],2]>Tv;
expth[node_,ts_]:=0 /; Part[xx1[node,ts],2]==Tm;
expth[node_,ts_]:=0 /; Part[xx1[node,ts],2]==Tv;

texprate[ts_]:=Sum[expth[node,ts],{node,1,m}]/m;
(* qm - quality of the melt *)
(* mv - quality of the vapor *)
qm[ts_]:=Sum[Part[xx2[node,ts],2],{node,1,m}]/m;
qv[ts_]:=Sum[Part[xx3[node,ts],2],{node,1,m}]/m;

vfaln[i_]:=fval /; i<=n ;
vfaln[i_]:=1 /; i>n ;

(* Fractional rate of change of volume due to various factors *)
(* ger - Gas Expansion Rate *)
(* VMR - Volumetric Melting Rate *)
(* VVR - Volumetric Vaporization Rate *)
(* VOFR - Volumetric Oxide Formation Rate *)
(* QMR - Quality Melt Rate *)
(* QVR - Quality Vaporization Rate *)
rhosolid=2700; rholiquid=2500; rhovapor=1.0 ;
DencRatLV=(1/rhovapor^(0.3)-1/rholiquid^(0.3))/(1/rholiquid^(0.3));
DencRatSL=(1/rholiquid^(0.3)-1/rhosolid^(0.3))/(1/rhosolid^(0.3));
qmr[node_,ts_]:=Part[xx2[node,ts]-xx2[node,ts-1],2]/Part[xx2[node,ts]-xx2[node,ts-1],1];
qvr[node_,ts_]:=Part[xx3[node,ts]-xx3[node,ts-1],2]/Part[xx3[node,ts]-xx3[node,ts-1],1];
vmr[ts_]:=Sum[DencRatSL*qmr[node,ts]*vfaln[node],{node,1,m}]/m;
vvr[ts_]:=Sum[DencRatLV*qvr[node,ts]*vfaln[node],{node,1,m}]/m;
ger[ts_]:=Sum[ tr[node,ts]/ Part[xx1[node,ts],2] ,{node,1,n}]/m;
ff=0.01;
TotalExpansionRate[ts_]:= (1-ff)*(vmr[ts]+vvr[ts]+texprate[ts]) + ff*ger[ts]
IntegralTotExp[ts_]:=Sum[TotalExpansionRate[tt]*dt,{tt,2,ts}]
ListPlot[Table[{Part[xx1[1,ts],1],TotalExpansionRate[ts]},{ts,2,nt}],
PlotJoined ->True,

```



```

PlotRange->All,
AxesLabel->{"time (s)","Fractional Exp rate (1/s)"}]

ter=1;
(* ter total expansion *)
Do[ter*=(1+TotalExpansionRate[tt]*dt);
qwg[tt]={Part[xx1[1,tt],1],ter-1}
,{tt,2,nt}}
ListPlot[Table[qwg[tt],{tt,2,nt}],
PlotJoined ->True,
PlotRange->All,
AxesLabel->{"time (s)","Expanded fraction"}]
Do[eff[ts]={Part[xx1[1,ts],1],ta[ts],TotalExpansionRate[ts],Part[qwg[ts],2]},{ts,2,nt}];
ListPlot[Table[{Part[xx1[1,ts],1],ta[ts]},{ts,2,nt}],
PlotJoined ->True,
PlotRange->All,
AxesLabel->{"time (s)","average temp (K)"}]

```

-Graphics-

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```

ListPlot[Table[{Part[xx1[1,i],1],Part[xx1[1,i],2]},{i,1,nt}],
PlotJoined ->True,
PlotRange->All,
AxesLabel->{"time (s)"," temp (K)"}]

ListPlot[Table[{Part[xx1[m,i],1],Part[xx1[m,i],2]},{i,1,nt}],
PlotJoined ->True,
PlotRange->All,
AxesLabel->{"time (s)"," temp (K)"}]

```

-Graphics-

-Graphics-

```
bsb1[i_]:=Sum[Part[xx4[j,i],2],{j,1,n}]/n;
```

```
bsb1[26]
bsb2[26]
```

-11
7.47451 10

-10
2.9898 10

```
bsb1[i_]:= (Sum[Part[xx4[j,i],2],{j,1,n}])/n;  
ListPlot[Table[{Part[xx4[1,i],1],bsb1[i]}, {i,1,nt}],  
  PlotJoined ->True,PlotRange->All,  
  AxesLabel->{"time (s)"," fraction of fuel reacted"}];
```

```
szs1=ListPlot[Table[{Part[xx5[1,i],1],Part[xx5[1,i],2]}, {i,1,nt}],  
  PlotJoined ->True,PlotRange->All,  
  AxesLabel->{"time (s)"," Q fission (W/m^3)"}];  
szs2=ListPlot[Table[{Part[xx6[1,i],1],Part[xx6[1,i],2]}, {i,1,nt}],  
  PlotJoined ->True,PlotRange->All,  
  AxesLabel->{"time (s)"," Q chemical (W/m^3)"}];  
szs3=ListPlot[Table[{Part[xx7[1,i],1],Part[xx7[1,i],2]}, {i,1,nt}],  
  PlotJoined ->True,PlotRange->All,  
  AxesLabel->{"time (s)"," Q total (W/m^3)"}];  
Show[szs1,szs2,szs3]
```

-Graphics-

```
*****  
53  
*****
```

```
(*  
THIS IS A REVISED DISPERSION MODEL THAT USES THE  
TIME ENERGY DATA FROM THE EXPERIMENTS AND TEMPERATURE  
DEPENDANT CHEMICAL REACTION RATES FUEL- ALUMINUM  
This copy has chemical reaction rates included in the  
model  
LAST UPDATED 4-19-95  
Renormalized pulse width was used to a narrower band width.  
)  
Print["Date last updated =",Date[]];  
  
Date last updated ={1995, 5, 2, 9, 53, 2}
```

```

l=0.000635;m=10;dx=l/m;n=Round[m*.4];time=0;Tinf=300;
volmeat=2*dx*n*(25*10^-3)*(70*10^-3);
Do[xx[i]=i*dx-dx/2,{i,1,m}];Tm=900.;Tv=2700.;
Do[To[i]=300;Tc[i]=300;xm[i]=0;xv[i]=0,{i,1,m}];

k[i_]:=39.91 /; i <= n ; (* assign k to the meat *)
k[i_]:=kcl[To[i]] /; i > n ; (* assign k to the clad *)
rho[i_]:=6550.0 /; i <= n ; (* assign meat density *)
rho[i_]:=2700.0 /; i > n ; (* assign clad density *)

cp[i_]:=cpcl[To[i]] /; i > n ; (* assign clad cpecific heat *)
cp[i_]:=cpf[To[i]] /; i <= n ; (* assign meat cpecific heat *)

htc=Interpolation[{{150,13232},{350,13232},{410,14770},{726,12542},
{776,11561},{1256,9924},{1458,1750},{1526,1729},{1556,800},
{5000,800}},InterpolationOrder->1];

kcl=Interpolation[{{100,161},{400,177},{500,186},{600,193},
{750,193},{850,193},{900,193},{930,193},
{1200,84},{1300,84},{1500,84},{2000,84},{5000,84}},
InterpolationOrder->1];

cpcl=Interpolation[{{100,878},{300,878},{366,942},{422,963},
{588,1034},{700,1067},{811,995},{1000,995},{2000,995},
{5000,995}},InterpolationOrder->1];

cpf=Interpolation[{{100,351},{300,351},{933,463},{1933,546},
{2500,546},{3500,546},{5000,546}},InterpolationOrder->1];

hsf[i_]:=398000*fmal /; i <= n;
hfg[i_]:=10800000*fmal /; i <= n;
hsf[i_]:=398000 /; i > n;
hfg[i_]:=10800000 /; i > n;

l51=ReadList["508.53",Number,RecordLists->True];
shape=Interpolation[l51,InterpolationOrder->1];
num=Part[Part[Position[l51,Last[l51]],1],1];
TimeBeg=Part[l51[[1]],1]; TimeEnd=Part[l51[[num]],1];
c1=Sum[0.5*(shape[Part[l51[[i]],1]]+shape[Part[l51[[i+1]],1]])*
(Part[l51[[i+1]],1]-Part[l51[[i]],1]),{i,200,500}];
qs[qtot_,time_]:=qtot*shape[time]/(volmeat*c1);
qtot=9997;
(* Functions : nrt - new reacted fraction
orf - old reacted fraction *)

Tm1=863;Tm2=1970;Td[i_]:= (To[i]-Tm1)/(Tm2-Tm1);
lamda[i_]:= (4*10^-4)*E^(14.73*Tanh[1.8*Td[i]]);
z1[i_,dt_]:=NDSolve[{RF'[t]==lamda[i]-lamda[i]*RF[t],
RF[0]==orf[i]},RF,{t,0,dt}, AccuracyGoal->15];
nrf[i_,dt_]:=Part[RF[dt],z1[i,dt],1]
frr[i_]:=lamda[i]-lamda[i]*orf[i];
UpdateRF[dt_]:=Do[orf[i]=nrf[i,dt],{i,1,n}]
qchem[i_]:=ww*frr[i];

```

```

q[i_,time_,qtot_]:=qs[qtot,time]+qchem[i]/; i<=n; (* assign heat generation in the meat *)
q[i_,time_,qtot_]:=0 /; i>n; (* assign heat generation in the clad *)
bb[i_]:=htc[To[m]]/dx /; i==m ;
bb[i_]:=0 /; i != m ; (* value for the heat transfer out of the plate *)

k1[i_]:= (2*k[i-1]*k[i]/(k[i-1]+k[i]))/dx^2 /; i!=1
k1[i_]:=0 /; i==1
k2[i_]:=k1[i]+k3[i];
k3[i_]:= (2*k[i+1]*k[i]/(k[i+1]+k[i]))/dx^2 /; i!=m
k3[i_]:=0 /; i==m ;

phase[i_]:= solid /; Tc[i] < Tm ;
phase[i_]:= liquid /; Tc[i] < Tv && Tc[i] >= Tm && xm[i] >= 0.85;
phase[i_]:= vapor /; Tc[i] >= Tv && xv[i] > 1;
phase[i_]:= melting /; Tc[i] >= Tm && xm[i] < 0.85 ;
phase[i_]:= vaporizing /; Tc[i] >= Tv && xv[i] < 1 && xv[i] > -0.01;

Tn[i_]:=Tc[i] /; phase[i] != melting && phase[i] != vaporizing;
Tn[i_]:=Tm /; phase[i] == melting ;
Tn[i_]:=Tv /; phase[i] == vaporizing ;

vv[i_,j_,dt_]:= k1[i] /; i-1==j && phase[j]!=melting && phase[j]!=vaporizing
vv[i_,j_,dt_]:= -k2[i]*rho[i]*cp[i]/dt-bb[i] /; i==j && phase[j]!=melting &&
phase[j]!=vaporizing
vv[i_,j_,dt_]:= k3[i] /; i+1==j && phase[j]!=melting && phase[j]!=vaporizing
vv[i_,j_,dt_]:= 0 /; i != j && i-1 != j && i+1 != j ;
(*-----*)
vv[i_,j_,dt_]:=1 /; i==j && phase[j]==melting;
vv[i_,j_,dt_]:=0 /; i-1==j && phase[j]==melting;
vv[i_,j_,dt_]:=0 /; i+1==j && phase[j]==melting;
vv[i_,j_,dt_]:=1 /; i==j && phase[j]==vaporizing;
vv[i_,j_,dt_]:=0 /; i-1==j && phase[j]==vaporizing;
vv[i_,j_,dt_]:=0 /; i+1==j && phase[j]==vaporizing;

(*-----*)
ff[i_,dt_,time_,qtot_]:= -q[i,time,qtot]-To[i]*rho[i]*cp[i]/dt - bb[i]*Tinf /; phase[i]!=melting &&
phase[i]!=vaporizing;
ff[i_,dt_,time_,qtot_]:= Tm /; phase[i]==melting;
ff[i_,dt_,time_,qtot_]:= Tv /; phase[i]==vaporizing;

Plot[shape[t],{t,.240,0.28},PlotRange->{0,100}]
TB=0.240;TE=0.28;

```

-Graphics-

```

RhoMeat=6500.;RhoAl=2700.;RhoU=12200.;
fval=0.5;
fvu=1-fval;
DenRat=(RhoU/RhoAl);
fmal=0.212;
fmu=1-fmal;
Echem=350000.0;

```

```
ww=Echem*5100;
```

```
General::spell1:
```

```
Possible spelling error: new symbol name "fval"  
is similar to existing symbol "fmal".
```

```
time=TB;
```

```
Do[To[i]=300;Tc[i]=300;xm[i]=0;xv[i]=0;orf[i]=0, {i,1,m}];
```

```
dt[i_]:=10*dx*dx*rho[i]*cp[i]/(2*k[i]);
```

```
dt=Min[Table[dt[i], {i,1,m}]];
```

```
nt=Floor[(TE-TB)/dt]-1;
```

```
Print["dt= ",dt," nt= ",nt];
```

```
dt= 0.000278413 nt= 142
```

```
Do[
```

```
s=Table[vv[i,j,dt],{j,m},{i,m}];
```

```
sinv=Inverse[s];(* invert the matrix *)
```

```
qq=Table[f[i,dt,time,qtot],{i,m}];
```

```
ans=sinv.qq;
```

```
Do[Tc[i]=ans[[i]];
```

```
If[phase[i]==melting, xm[i]+=(k1[i]*To[i-1]-k2[i]*To[i]+k3[i]*To[i+1] + q[i,time,qtot]-  
bb[i]*(To[m]-Tinf))*dt/(rho[i]*hsf[i]);,0];
```

```
If[phase[i]==vaporizing, xv[i]+=(k1[i]*To[i-1]-k2[i]*To[i]+k3[i]*To[i+1] + q[i,time,qtot]-  
bb[i]*(To[m]-Tinf))*dt/(rho[i]*hfg[i]);,0];
```

```
,{i,1,m}];
```

```
If[Mod[numtimesteps,5]==0 || numtimesteps >=60,
```

```
Print[numtimesteps," ",time," ",To[1]," ",xv[1]," ",phase[1]," ",
```

```
q[1,time,qtot]," ",orf[1]," ",qs[qtot,time]," ",qchem[1]]];
```

```
Do[To[i]=Tn[i];
```

```
xx1[i,numtimesteps]={time,To[i]};
```

```
xx2[i,numtimesteps]={time,xm[i]};
```

```
xx3[i,numtimesteps]={time,xv[i]};
```

```
xx4[i,numtimesteps]={time,orf[i]};
```

```
xx5[i,numtimesteps]={time,q[i,time,qtot]};
```

```
xx6[i,numtimesteps]={time,qchem[i]};
```

```
xx7[i,numtimesteps]={time,qs[qtot,time]};
```

```
,{i,1,m}];
```

```
To[i]=Tn[i];
```

```
time += dt;
```

```
UpdateRF[dt];
```

```
,{numtimesteps,1,nt}];
```

```
Print[Finished];
```

```
10      -11      10  
5 0.241114 305.398 0 solid 1.35773 10 1.08612 10 1.35773 10 17.8136  
10      -11      10  
10 0.242506 311.897 0 solid 2.76168 10 2.53858 10 2.76168 10 19.2134  
10      -11      10  
15 0.243898 321.966 0 solid 2.99173 10 4.15127 10 2.99173 10 21.6514
```


77	0.261159	1581.74	0	liquid	9.07657	10	0.0840103	7.85788	10	1.21869	10
					11	11		11			
78	0.261438	1602.08	0	liquid	8.96232	10	0.105879	7.57709	10	1.38523	10
					11	11		11			
79	0.261716	1620.69	0	liquid	8.74598	10	0.130203	7.20794	10	1.53804	10
					11	11		11			
80	0.261995	1636.86	0	liquid	8.52841	10	0.156561	6.86437	10	1.66404	10
					11	11		11			
81	0.262273	1650.83	0	liquid	8.27397	10	0.184508	6.51224	10	1.76174	10
					11	11		11			
82	0.262551	1662.53	0	liquid	8.05386	10	0.213547	6.22561	10	1.82825	10
					11	11		11			
83	0.26283	1672.52	0	liquid	7.74562	10	0.243272	5.87639	10	1.86923	10
					11	11		11			
84	0.263108	1680.36	0	liquid	7.33992	10	0.273207	5.45913	10	1.88079	10
					11	11		11			
85	0.263387	1685.53	0	liquid	7.00338	10	0.302819	5.14409	10	1.85929	10
					11	11		11			
86	0.263665	1688.76	0	liquid	6.67342	10	0.331746	4.8579	10	1.81553	10
					11	11		11			
87	0.263944	1690.36	0	liquid	6.24965	10	0.35972	4.4942	10	1.75546	10
					11	11		11			
88	0.264222	1690.01	0	liquid	5.93517	10	0.386472	4.25638	10	1.67878	10
					11	11		11			
89	0.2645	1689.12	0	liquid	5.48509	10	0.41198	3.88421	10	1.60088	10
					11	11		11			
90	0.264779	1686.95	0	liquid	5.22641	10	0.43613	3.7103	10	1.51611	10
					11	11		11			
91	0.265057	1685.22	0	liquid	4.80131	10	0.459064	3.3613	10	1.44	10
					11	11		11			
92	0.265336	1682.65	0	liquid	4.47224	10	0.480745	3.11044	10	1.36181	10
					11	11		11			
93	0.265614	1679.97	0	liquid	4.25365	10	0.50124	2.96594	10	1.28772	10
					11	11		11			
94	0.265892	1677.96	0	liquid	3.92356	10	0.5207	2.70065	10	1.22291	10
					11	11		11			
95	0.266171	1675.56	0	liquid	3.65068	10	0.539141	2.49139	10	1.15929	10
					11	11		11			
96	0.266449	1673.09	0	liquid	3.45121	10	0.556619	2.35223	10	1.09898	10
					11	11		11			
97	0.266728	1670.97	0	liquid	3.19861	10	0.573225	2.15416	10	1.04445	10
					11	11		10			
98	0.267006	1668.63	0	liquid	2.92151	10	0.588988	1.92977	10	9.91743	10
					11	11		10			
99	0.267284	1665.74	0	liquid	2.83507	10	0.60391	1.89599	10	9.39076	10
					11	11		10			
100	0.267563	1663.72	0	liquid	2.58108	10	0.618117	1.68679	10	8.94282	10
					11	11		10			
101	0.267841	1661.12	0	liquid	2.45532	10	0.6316	1.60636	10	8.48959	10
					11	11		10			
102	0.26812	1658.87	0	liquid	2.28788	10	0.644429	1.47986	10	8.08023	10
					11	11		10			
103	0.268398	1656.55	0	liquid	2.07214	10	0.656637	1.30309	10	7.69051	10
					11	11		10			

104	0.268677	1653.7	0	liquid	1.84429	10	0.668219	1.11439	10	7.29902	10
					11	11			10		
105	0.268955	1650.17	0	liquid	1.75466	10	0.679167	1.06448	10	6.90184	10
					11	11			10		
106	0.269233	1647.04	0	liquid	1.72055	10	0.689547	1.06598	10	6.54577	10
					11	11			10		
107	0.269512	1644.69	0	liquid	1.64242	10	0.699442	1.01825	10	6.24171	10
					11	10			10		
108	0.26979	1642.7	0	liquid	1.56365	10	0.7089	9.6689	10	5.96764	10
					11	10			10		
109	0.270069	1641.	0	liquid	1.36454	10	0.717962	7.92733	10	5.71809	10
					11	10			10		
110	0.270347	1638.57	0	liquid	1.27082	10	0.726604	7.25324	10	5.455	10
					11	10			10		
111	0.270625	1636.22	0	liquid	1.21344	10	0.734855	6.92557	10	5.20884	10
					11	10			10		
112	0.270904	1634.21	0	liquid	1.09462	10	0.742752	5.95968	10	4.98656	10
					11	10			10		
113	0.271182	1632.	0	liquid	1.03289	10	0.750302	5.55974	10	4.76917	10
					11	10			10		
114	0.271461	1630.02	0	liquid	1.01108	10	0.757536	5.54114	10	4.56971	10
					10	10			10		
115	0.271739	1628.55	0	liquid	9.90296	10	0.764492	5.5082	10	4.39476	10
					10	10			10		
116	0.272018	1627.56	0	liquid	9.26419	10	0.771203	5.02339	10	4.24079	10
					10	10			10		
117	0.272296	1626.64	0	liquid	8.54589	10	0.777683	4.45079	10	4.0951	10
					10	10			10		
118	0.272574	1625.7	0	liquid	7.5867	10	0.78394	3.63221	10	3.9545	10
					10	10			10		
119	0.272853	1624.52	0	liquid	7.17437	10	0.789973	3.36125	10	3.81312	10
					10	10			10		
120	0.273131	1623.51	0	liquid	6.35952	10	0.795798	2.67773	10	3.68179	10
					10	10			10		
121	0.27341	1622.32	0	liquid	6.3093	10	0.801415	2.75795	10	3.55134	10
					10	10			10		
122	0.273688	1621.55	0	liquid	6.57866	10	0.806849	3.14288	10	3.43578	10
					10	10			10		
123	0.273966	1621.41	0	liquid	6.56633	10	0.81213	3.22762	10	3.33871	10
					10	10			10		
124	0.274245	1621.64	0	liquid	6.70603	10	0.817274	3.45356	10	3.25247	10
					10	10			10		
125	0.274523	1622.32	0	liquid	5.85505	10	0.822301	2.67719	10	3.17786	10
					10	10			10		
126	0.274802	1622.62	0	liquid	5.3234	10	0.827199	2.22692	10	3.09648	10
					10	10			10		
127	0.27508	1622.76	0	liquid	4.99021	10	0.831967	1.97616	10	3.01405	10
					10	10			10		
128	0.275358	1622.9	0	liquid	5.09264	10	0.836608	2.15892	10	2.93371	10
					10	10			10		
129	0.275637	1623.38	0	liquid	4.77503	10	0.841135	1.9132	10	2.86183	10
					10	10			10		
130	0.275915	1623.84	0	liquid	4.22691	10	0.84555	1.43596	10	2.79095	10
					10	10			10		


```

131 0.276194 1624.06 0 liquid 3.82697 10 0.84985 1.10949 10 2.71748 10
      10      10      10
132 0.276472 1624.18 0 liquid 4.13664 10 0.854032 1.49281 10 2.64382 10
      10      10      10
133 0.276751 1624.74 0 liquid 4.72201 10 0.858114 2.14218 10 2.57983 10
      10      10      10
134 0.277029 1625.95 0 liquid 4.02079 10 0.862114 1.49281 10 2.52798 10
      10      10      10
135 0.277307 1626.75 0 liquid 3.54986 10 0.866023 1.08012 10 2.46974 10
      10      9      10
136 0.277586 1627.32 0 liquid 3.29121 10 0.869835 8.82491 10 2.40872 10
      10      10      10
137 0.277864 1627.82 0 liquid 3.41636 10 0.873551 1.06844 10 2.34792 10
      10      10      10
138 0.278143 1628.56 0 liquid 3.38903 10 0.877178 1.09699 10 2.29204 10
      10      9      10
139 0.278421 1629.4 0 liquid 2.87104 10 0.880722 6.32374 10 2.23867 10
      10      9      10
140 0.278699 1629.94 0 liquid 2.9303 10 0.884176 7.48471 10 2.18183 10
      10      10      10
141 0.278978 1630.65 0 liquid 3.39426 10 0.887545 1.26582 10 2.12844 10
      10      10      10
142 0.279256 1631.83 0 liquid 3.47078 10 0.890843 1.38828 10 2.0825 10
Finished

```

Tcoefsol=2.25*10⁻⁵;

Tcoefliq=1.17*10⁻⁴;

(* TA - average temperature *)

ta[ts_]:=Sum[Part[xx1[node,ts],2],{node,1,m}]/m;

(* tr - Temperature time rate of change *)

tr[node_,ts_]:=Part[xx1[node,ts]-xx1[node,ts-1],2]/Part[xx1[node,ts]-xx1[node,ts-1],1];

(* tra - average rate of change of temp for the plate *)

tra[ts_]:=Sum[tr[node,ts],{node,1,m}]/m;

(* expth - expansion thermal *)

(* texprate - thermal expansion rate for the plate *)

expth[node_,ts_]:=Tcoefsol*tr[node,ts] /; Part[xx1[node,ts],2]<Tm;

expth[node_,ts_]:=Tcoefliq*tr[node,ts] /; Part[xx1[node,ts],2]>Tm && Part[xx1[node,ts],2]<Tv;

expth[node_,ts_]:=tr[node,ts]/ Part[xx1[node,ts],2]/; Part[xx1[node,ts],2]>Tv;

expth[node_,ts_]:=0 /; Part[xx1[node,ts],2]==Tm;

expth[node_,ts_]:=0 /; Part[xx1[node,ts],2]==Tv;

texprate[ts_]:=Sum[expth[node,ts],{node,1,m}]/m;

(* qm - quality of the melt *)

(* mv - quality of the vapor *)

qm[ts_]:=Sum[Part[xx2[node,ts],2],{node,1,m}]/m;

qv[ts_]:=Sum[Part[xx3[node,ts],2],{node,1,m}]/m;

vfaln[i_]:=fval /; i<=n ;

vfaln[i_]:=1 /; i>n ;

(* Fractional rate of change of volume due to various factors *)

(* ger - Gas Expansion Rate *)

```

(* VMR - Volumetric Melting Rate *)
(* VVR - Volumetric Vaporization Rate *)
(* VOFR - Volumetric Oxide Formation Rate *)
(* QMR - Quality Melt Rate *)
(* QVR - Quality Vaporization Rate *)
rhosolid=2700; rholiquid=2500; rhovapor=1.0 ;
DencRatLV=(1/rhovapor^(0.3)-1/rholiquid^(0.3))/(1/rholiquid^(0.3));
DencRatSL=(1/rholiquid^(0.3)-1/rhosolid^(0.3))/(1/rhosolid^(0.3));
qmr[node_,ts_]:=Part[xx2[node,ts]-xx2[node,ts-1],2]/Part[xx2[node,ts]-xx2[node,ts-1],1];
qvr[node_,ts_]:=Part[xx3[node,ts]-xx3[node,ts-1],2]/Part[xx3[node,ts]-xx3[node,ts-1],1];
vmr[ts_]:=Sum[DencRatSL*qmr[node,ts]*vfaln[node,{node,1,m}]/m;
vvr[ts_]:=Sum[DencRatLV*qvr[node,ts]*vfaln[node,{node,1,m}]/m;
ger[ts_]:=Sum[ tr[node,ts]/ Part[xx1[node,ts],2] ,{node,1,n}]/m;
ff=0.01;
TotalExpansionRate[ts_]:= (1-ff)*(vmr[ts]+vvr[ts]+texprate[ts]) + ff*ger[ts]
IntegralTotExp[ts_]:=Sum[TotalExpansionRate[tt]*dt,{tt,2,ts}]
ListPlot[Table[{Part[xx1[1,ts],1],TotalExpansionRate[ts]},{ts,2,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)","Fractional Exp rate (1/s)"}]

ter=1;
(* ter total expansion *)
Do[ter*=(1+TotalExpansionRate[tt]*dt);
qww[tt]={Part[xx1[1,tt],1],ter-1 }
,{tt,2,nt}]
ListPlot[Table[qww[tt],{tt,2,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)","Expanded fraction" }]
Do[eff[ts]={Part[xx1[1,ts],1],ta[ts],TotalExpansionRate[ts],Part[qww[ts],2]},{ts,2,nt}];
ListPlot[Table[{Part[xx1[1,ts],1],ta[ts]},{ts,2,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)","average temp (K)"}]

```

-Graphics-

-Graphics-

-Graphics-

```

bsb1[i_]:=Sum[Part[xx4[j,i],2],{j,1,n}]/n;
ListPlot[Table[{Part[xx4[1,i],1],bsb1[i]},{i,1,nt}],
  PlotJoined ->True,PlotRange->All,
  AxesLabel->{"time (s)"," fraction of reacted fuel"}];

ListPlot[Table[{Part[xx1[1,i],1],Part[xx1[1,i],2]},{i,1,nt}],

```

```

PlotJoined ->True,
PlotRange->All,
AxesLabel->{"time (s)"," temp (K)"}]

ListPlot[Table[{Part[xx1[m,i],1],Part[xx1[m,i],2]},{i,1,nt}],
PlotJoined ->True,
PlotRange->All,
AxesLabel->{"time (s)"," temp (K)"}]

szs1=ListPlot[Table[{Part[xx5[1,i],1],Part[xx5[1,i],2]},{i,1,nt}],
PlotJoined ->True,PlotRange->All,
AxesLabel->{"time (s)"," Q fission (W/m^3)"}];
szs2=ListPlot[Table[{Part[xx6[1,i],1],Part[xx6[1,i],2]},{i,1,nt}],
PlotJoined ->True,PlotRange->All,
AxesLabel->{"time (s)"," Q chemical (W/m^3)"}];
szs3=ListPlot[Table[{Part[xx7[1,i],1],Part[xx7[1,i],2]},{i,1,nt}],
PlotJoined ->True,PlotRange->All,
AxesLabel->{"time (s)"," Q total (W/m^3)"}];
Show[szs1,szs2,szs3]

```

-Graphics-

-Graphics-

-Graphics-

```

*****
54
*****

```

(*

THIS IS A REVISED DISPERSION MODEL THAT USES THE
TIME ENERGY DATA FROM THE EXPERIMENTS AND TEMPERATURE
DEPENDANT CHEMICAL REACTION RATES FUEL- ALUMINUM
This copy has chemical reaction rates included in the
model
LAST UPDATED 4-19-95
Renormalized pulse width was used to a narrower band width.

```

*)
Print["Date last updated =",Date[]];

Date last updated ={1995, 5, 2, 10, 6, 51}

l=0.000635;m=10;dx=l/m;n=Round[m*.4];time=0;Tinf=300;
volmeat=2*dx*n*(25*10^-3)*(70*10^-3);
Do[xx[i]=i*dx-dx/2,{i,1,m}];Tm=900.;Tv=2700.;
Do[To[i]=300;Tc[i]=300;xm[i]=0;xv[i]=0,{i,1,m}];

k[i_]:=39.91 /; i <= n ; (* assign k to the meat *)
k[i_]:=kcl[To[i]] /; i > n ; (* assign k to the clad *)
rho[i_]:=6550.0 /; i <= n ; (* assign meat density *)
rho[i_]:=2700.0 /; i > n ; (* assign clad density *)

cp[i_]:=cpcl[To[i]] /; i > n ; (* assign clad cpecific heat *)
cp[i_]:=cpf[To[i]] /; i <= n ; (* assign meat cpecific heat *)

htc=Interpolation[{{150,13232},{350,13232},{410,14770},{726,12542},
{776,11561},{1256,9924},{1458,1750},{1526,1729},{1556,800},
{5000,800}},InterpolationOrder->1];

kcl=Interpolation[{{100,161},{400,177},{500,186},{600,193},
{750,193},{850,193},{900,193},{930,193},
{1200,84},{1300,84},{1500,84},{2000,84},{5000,84}},
InterpolationOrder->1];

cpcl=Interpolation[{{100,878},{300,878},{366,942},{422,963},
{588,1034},{700,1067},{811,995},{1000,995},{2000,995},
{5000,995}},InterpolationOrder->1];

cpf=Interpolation[{{100,351},{300,351},{933,463},{1933,546},
{2500,546},{3500,546},{5000,546}},InterpolationOrder->1];

hsf[i_]:=398000*fmal /; i <= n;
hfg[i_]:=10800000*fmal /; i <= n;
hsf[i_]:=398000 /; i > n;
hfg[i_]:=10800000 /; i > n;

l51=ReadList["508.54",Number,RecordLists->True];
shape=Interpolation[l51,InterpolationOrder->1];
num=Part[Part[Position[l51,Last[l51]],1],1];
TimeBeg=Part[l51[[1]],1]; TimeEnd=Part[l51[[num]],1];
c1=Sum[0.5*(shape[Part[l51[[i]],1]]+shape[Part[l51[[i+1]],1]])*
(Part[l51[[i+1]],1]-Part[l51[[i]],1]),{i,200,500}];
qs[qtot_,time_]:=qtot*shape[time]/(volmeat*c1);
qtot=11006;
(* Functions : nrt - new reacted fraction
orf - old reacted fraction *)

Tm1=863;Tm2=1970;Td[i_]:=(To[i]-Tm1)/(Tm2-Tm1);
lamda[i_]:=(4*10^-4)E^(14.73*Tanh[1.8*Td[i]]);
z1[i_,dt_]:=NDSolve[{RF'[t]==lamda[i]-lamda[i]*RF[t],
RF[0]==orf[i]},RF,{t,0,dt},AccuracyGoal->15];

```

```

nrf[i_,dt_]:=Part[RF[dt]/.z1[i,dt],1]
frr[i_]:=lamda[i]-lamda[i]*orf[i];
UpdateRF[dt_]:=Do[orf[i]=nrf[i,dt],{i,1,n}]
qchem[i_]:=ww*frr[i];

q[i_,time_,qtot_]:=qs[qtot,time]+qchem[i]/; i<=n; (* assign heat generation in the meat *)
q[i_,time_,qtot_]:=0 /; i>n; (* assign heat generation in the clad *)
bb[i_]:=htc[To[m]]/dx /; i==m ;
bb[i_]:=0 /; i != m ; (* value for the heat transfer out of the plate *)

k1[i_]:= (2*k[i-1]*k[i]/(k[i-1]+k[i]))/dx^2 /; i!=1
k1[i_]:=0 /; i==1
k2[i_]:=k1[i]+k3[i];
k3[i_]:= (2*k[i+1]*k[i]/(k[i+1]+k[i]))/dx^2 /; i!=m
k3[i_]:=0 /; i==m ;

phase[i_]:= solid /; Tc[i] < Tm ;
phase[i_]:= liquid /; Tc[i] < Tv && Tc[i] >= Tm && xm[i] >= 0.85;
phase[i_]:= vapor /; Tc[i] >= Tv && xv[i] > 1;
phase[i_]:= melting /; Tc[i] >= Tm && xm[i] < 0.85 ;
phase[i_]:= vaporizing /; Tc[i] >= Tv && xv[i] < 1 && xv[i] > -0.01;

Tn[i_]:=Tc[i] /; phase[i] != melting && phase[i] != vaporizing;
Tn[i_]:=Tm /; phase[i] == melting ;
Tn[i_]:=Tv /; phase[i] == vaporizing ;

vv[i_,j_,dt_]:= k1[i] /; i-1==j && phase[j]!=melting && phase[j]!=vaporizing
vv[i_,j_,dt_]:=-k2[i]-rho[i]*cp[i]/dt-bb[i] /; i==j && phase[j]!=melting &&
phase[j]!=vaporizing
vv[i_,j_,dt_]:= k3[i] /; i+1==j && phase[j]!=melting && phase[j]!=vaporizing
vv[i_,j_,dt_]:= 0 /; i != j && i-1 != j && i+1 != j ;
(*-----*)
vv[i_,j_,dt_]:=1 /; i==j && phase[j]==melting;
vv[i_,j_,dt_]:=0 /; i-1==j && phase[j]==melting;
vv[i_,j_,dt_]:=0 /; i+1==j && phase[j]==melting;
vv[i_,j_,dt_]:=1 /; i==j && phase[j]==vaporizing;
vv[i_,j_,dt_]:=0 /; i-1==j && phase[j]==vaporizing;
vv[i_,j_,dt_]:=0 /; i+1==j && phase[j]==vaporizing;

(*-----*)
f[i_,dt_,time_,qtot_]:= -q[i,time,qtot]-To[i]*rho[i]*cp[i]/dt - bb[i]*Tinf /; phase[i]!=melting &&
phase[i]!=vaporizing;
f[i_,dt_,time_,qtot_]:= Tm /; phase[i]==melting;
f[i_,dt_,time_,qtot_]:= Tv /; phase[i]==vaporizing;

TB=0.230;TE=0.27;
Plot[shape[t],{t,TB,TE},PlotRange->{0,100}]

```

-Graphics-

RhoMeat=6500.;RhoAl=2700.;RhoU=12200.;

```
fval=0.5;
fvu=1-fval;
DenRat=(RhoU/RhoAl);
fmal=0.212;
fmu=1-fmal;
Echem=350000.0;
ww=Echem*5100;
```

```
time=TB;
Do[To[i]=300;Tc[i]=300;xm[i]=0;xv[i]=0;orf[i]=0;,{i,1,m}];
dt[i_]:=10*dx*dx*rho[i]*cp[i]/(2*k[i]);
dt=Min[Table[dt[i],{i,1,m}]];
nt=Floor[(TE-TB)/dt]-1;
Print["dt= ",dt," nt= ",nt];
```

```
dt= 0.000278413 nt= 142
```

```
Do[
s=Table[vv[i,j,dt],{j,m},{i,m}];
sinv=Inverse[s];(* invert the matrix *)
qq=Table[f[i,dt,time,qtot],{i,m}];
ans=sinv.qq;
```

```
Do[Tc[i]=ans[[i]];
If[phase[i]==melting, xm[i]+=(k1[i]*To[i-1]-k2[i]*To[i]+k3[i]*To[i+1] + q[i,time,qtot]-
bb[i]*(To[m]-Tinf))*dt/(rho[i]*hsf[i]),0];
If[phase[i]==vaporizing,xv[i]+=(k1[i]*To[i-1]-k2[i]*To[i]+k3[i]*To[i+1] + q[i,time,qtot]-
bb[i]*(To[m]-Tinf))*dt/(rho[i]*hfg[i]),0];
,{i,1,m}];
```

```
If[Mod[numtimesteps,5]==0 || numtimesteps >=60,
Print[numtimesteps," ",time," ",To[1]," ",xv[1]," ",phase[1]," ",
q[1,time,qtot]," ",orf[1]," ",qs[qtot,time]," ",qchem[1]]];
```

```
Do[To[i]=Tn[i];
xx1[i,numtimesteps]={time,To[i]};
xx2[i,numtimesteps]={time,xm[i]};
xx3[i,numtimesteps]={time,xv[i]};
xx4[i,numtimesteps]={time,orf[i]};
xx5[i,numtimesteps]={time,q[i,time,qtot]};
xx6[i,numtimesteps]={time,qchem[i]};
xx7[i,numtimesteps]={time,qs[qtot,time]};
```

```
,{i,1,m}];
To[i]=Tn[i];
time += dt;
UpdateRF[dt];
,{numtimesteps,1,nt}];
Print[Finished];
```

```

          9      -11      9
5 0.231114 302.026 0 solid 5.91789 10 1.06255 10 5.91789 10 17.1356
          10      -11      10
10 0.232506 305.476 0 solid 1.18493 10 2.42411 10 1.18493 10 17.8296
```


73	0.250046	1570.6	0	liquid	1.27071	10	0.0465846	1.15443	10	1.16279	10
					12	12			11		
74	0.250324	1617.44	0	liquid	1.29556	10	0.0719483	1.13512	10	1.60433	10
					12	12			11		
75	0.250603	1662.74	0	liquid	1.30758	10	0.105038	1.09926	10	2.08318	10
					12	12			11		
76	0.250881	1705.36	0	liquid	1.32368	10	0.145698	1.06901	10	2.54674	10
					12	12			11		
77	0.251159	1746.47	0	liquid	1.34088	10	0.193406	1.04371	10	2.97171	10
					12	11			11		
78	0.251438	1786.75	0	liquid	1.33151	10	0.247259	9.98033	10	3.33482	10
					12	11			11		
79	0.251716	1824.56	0	liquid	1.31399	10	0.305566	9.5503	10	3.5896	10
					12	11			11		
80	0.251995	1859.8	0	liquid	1.2843	10	0.366426	9.11733	10	3.72571	10
					12	11			11		
81	0.252273	1892.04	0	liquid	1.24143	10	0.427885	8.67148	10	3.74278	10
					12	11			11		
82	0.252551	1920.88	0	liquid	1.19409	10	0.488148	8.28825	10	3.65263	10
					12	11			11		
83	0.25283	1946.61	0	liquid	1.12294	10	0.545815	7.7487	10	3.48072	10
					12	11			11		
84	0.253108	1967.76	0	liquid	1.05437	10	0.599679	7.30371	10	3.24003	10
					11	11			11		
85	0.253387	1984.76	0	liquid	9.75115	10	0.649031	6.79058	10	2.96056	10
					11	11			11		
86	0.253665	1997.06	0	liquid	9.03153	10	0.693472	6.3708	10	2.66073	10
					11	11			11		
87	0.253944	2005.73	0	liquid	8.30238	10	0.732998	5.93908	10	2.36329	10
					11	11			11		
88	0.254222	2011.22	0	liquid	7.56693	10	0.767818	5.48681	10	2.08012	10
					11	11			11		
89	0.2545	2013.92	0	liquid	6.95128	10	0.798264	5.13327	10	1.81801	10
					11	11			11		
90	0.254779	2015.02	0	liquid	6.25785	10	0.824775	4.67502	10	1.58283	10
					11	11			11		
91	0.255057	2013.98	0	liquid	5.73974	10	0.847755	4.36757	10	1.37217	10
					11	11			11		
92	0.255336	2012.13	0	liquid	5.17009	10	0.867647	3.98197	10	1.18811	10
					11	11			11		
93	0.255614	2008.95	0	liquid	4.70388	10	0.884828	3.67718	10	1.0267	10
					11	11			10		
94	0.255892	2005.06	0	liquid	4.29849	10	0.899659	3.41167	10	8.86821	10
					11	11			10		
95	0.256171	2000.75	0	liquid	3.89084	10	0.912465	3.12463	10	7.66206	10
					11	11			10		
96	0.256449	1995.81	0	liquid	3.54326	10	0.923521	2.88129	10	6.61971	10
					11	11			10		
97	0.256728	1990.55	0	liquid	3.226	10	0.933071	2.65369	10	5.72307	10
					11	11			10		
98	0.257006	1985.02	0	liquid	2.8879	10	0.941328	2.39266	10	4.95238	10
					11	11			10		
99	0.257284	1978.94	0	liquid	2.56059	10	0.948468	2.1319	10	4.28688	10
					11	11			10		

100	0.257563	1972.26	0	liquid	2.33248	10	0.954645	1.96126	10	3.71211	10
			11			11			10		
101	0.257841	1965.65	0	liquid	2.18883	10	0.959998	1.86677	10	3.22054	10
			11			11			10		
102	0.25812	1959.64	0	liquid	1.89642	10	0.964652	1.61614	10	2.80277	10
			11			11			10		
103	0.258398	1952.98	0	liquid	1.7852	10	0.968699	1.54125	10	2.4395	10
			11			11			10		
104	0.258677	1946.97	0	liquid	1.57328	10	0.972229	1.36025	10	2.13034	10
			11			11			10		
105	0.258955	1940.75	0	liquid	1.37188	10	0.975313	1.18564	10	1.86239	10
			11			11			10		
106	0.259233	1934.3	0	liquid	1.27075	10	0.978008	1.10775	10	1.62997	10
			11			11			10		
107	0.259512	1928.33	0	liquid	1.1943	10	0.980372	1.05123	10	1.43071	10
			11			11			10		
108	0.25979	1922.95	0	liquid	1.13998	10	0.982452	1.01401	10	1.25969	10
			10			10			10		
109	0.260069	1918.23	0	liquid	9.37108	10	0.984287	8.25841	10	1.11267	10
			10			10			9		
110	0.260347	1912.94	0	liquid	8.54347	10	0.985907	7.56082	10	9.82643	10
			10			10			9		
111	0.260625	1907.96	0	liquid	8.05485	10	0.987339	7.18507	10	8.69778	10
			10			10			9		
112	0.260904	1903.48	0	liquid	7.19909	10	0.98861	6.4271	10	7.71991	10
			10			10			9		
113	0.261182	1899.16	0	liquid	7.1947	10	0.989739	6.50836	10	6.86347	10
			10			10			9		
114	0.261461	1895.59	0	liquid	6.14894	10	0.990745	5.53671	10	6.12232	10
			10			10			9		
115	0.261739	1891.9	0	liquid	5.20878	10	0.991643	4.66228	10	5.46494	10
			10			10			9		
116	0.262018	1888.12	0	liquid	5.32405	10	0.992444	4.83584	10	4.8821	10
			10			10			9		
117	0.262296	1885.04	0	liquid	4.71817	10	0.993162	4.28068	10	4.37485	10
			10			10			9		
118	0.262574	1882.04	0	liquid	4.43382	10	0.993806	4.04135	10	3.92471	10
			10			10			9		
119	0.262853	1879.32	0	liquid	3.52377	10	0.994384	3.17107	10	3.52695	10
			10			10			9		
120	0.263131	1876.37	0	liquid	3.68237	10	0.994904	3.36539	10	3.16977	10
			10			10			9		
121	0.26341	1873.98	0	liquid	2.64927	10	0.995372	2.36366	10	2.85603	10
			10			10			9		
122	0.263688	1871.2	0	liquid	3.01236	10	0.995793	2.75516	10	2.57205	10
			10			10			9		
123	0.263966	1869.08	0	liquid	2.85457	10	0.996173	2.62228	10	2.32291	10
			10			10			9		
124	0.264245	1867.18	0	liquid	2.18572	10	0.996517	1.97566	10	2.10067	10
			10			10			9		
125	0.264523	1865.08	0	liquid	2.89844	10	0.996828	2.70848	10	1.89957	10
			10			10			9		
126	0.264802	1863.82	0	liquid	1.52186	10	0.99711	1.34952	10	1.72335	10
			10			10			9		

```

127 0.26508 1861.76 0 liquid 2.37842 10 0.997365 2.22241 10 1.56012 10
      10      10      9
128 0.265358 1860.61 0 liquid 1.47431 10 0.997597 1.33258 10 1.4173 10
      10      10      9
129 0.265637 1858.98 0 liquid 1.70544 10 0.997807 1.57683 10 1.28605 10
      10      10      9
130 0.265915 1857.74 0 liquid 1.81334 10 0.997998 1.69644 10 1.16898 10
      10      9      9
131 0.266194 1856.78 0 liquid 1.09947 10 0.998172 9.93083 10 1.0639 10
      10      10      8
132 0.266472 1855.43 0 liquid 1.57485 10 0.99833 1.47811 10 9.67365 10
      10      10      8
133 0.266751 1854.62 0 liquid 1.43663 10 0.998474 1.34848 10 8.81458 10
      10      10      8
134 0.267029 1853.84 0 liquid 1.29239 10 0.998605 1.21205 10 8.03474 10
      10      10      8
135 0.267307 1853.08 0 liquid 1.12204 10 0.998724 1.04878 10 7.32607 10
      9      9      8
136 0.267586 1852.3 0 liquid 5.27007 10 0.998834 4.60195 10 6.68118 10
      10      10      8
137 0.267864 1851.18 0 liquid 1.42095 10 0.998933 1.36008 10 6.08759 10
      10      10      8
138 0.268143 1850.86 0 liquid 1.06099 10 0.999024 1.00536 10 5.56325 10
      10      10      8
139 0.268421 1850.35 0 liquid 1.17653 10 0.999107 1.12572 10 5.08141 10
      10      10      8
140 0.268699 1850.02 0 liquid 1.75579 10 0.999182 1.70935 10 4.64463 10
      9      9      8
141 0.268978 1850.2 0 liquid 7.41146 10 0.999252 6.98615 10 4.25305 10
      9      9      8
142 0.269256 1849.64 0 liquid 8.72162 10 0.999315 8.33313 10 3.88489 10
Finished

```

```

Tcoefsol=2.25*10^-5;
Tcoefliq=1.17*10^-4;
(* TA - average temperature *)
ta[ts_]:=Sum[Part[xx1[node,ts],2],{node,1,m}]/m;
(* tr - Temperature time rate of change *)
tr[node_,ts_]:=Part[xx1[node,ts]-xx1[node,ts-1],2]/Part[xx1[node,ts]-xx1[node,ts-1],1];
(* tra - average rate of change of temp for the plate *)
tra[ts_]:=Sum[tr[node,ts],{node,1,m}]/m;

(* expth - expansion thermal *)
(* texprate - thermal expansion rate for the plate *)

expth[node_,ts_]:=Tcoefsol*tr[node,ts] /; Part[xx1[node,ts],2]<Tm;
expth[node_,ts_]:=Tcoefliq*tr[node,ts] /; Part[xx1[node,ts],2]>Tm && Part[xx1[node,ts],2]<Tv;
expth[node_,ts_]:=tr[node,ts]/ Part[xx1[node,ts],2]/; Part[xx1[node,ts],2]>Tv;
expth[node_,ts_]:=0 /; Part[xx1[node,ts],2]==Tm;
expth[node_,ts_]:=0 /; Part[xx1[node,ts],2]==Tv;

texprate[ts_]:=Sum[expth[node,ts],{node,1,m}]/m;
(* qm - quality of the melt *)
(* mv - quality of the vapor *)

```

```

qm[ts_]:=Sum[Part[xx2[node,ts],2],{node,1,m}]/m;
qv[ts_]:=Sum[Part[xx3[node,ts],2],{node,1,m}]/m;

vfa[n[i_]]:=fval /; i<=n ;
vfa[n[i_]]:=1 /; i>n ;

(* Fractional rate of change of volume due to various factors *)
(* ger - Gas Expansion Rate *)
(* VMR - Volumetric Melting Rate *)
(* VVR - Volumetric Vaporization Rate *)
(* VOFR - Volumetric Oxide Formation Rate *)
(* QMR - Quality Melt Rate *)
(* QVR - Quality Vaporization Rate *)
rhosolid=2700; rholiquid=2500; rhovapor=1.0 ;
DencRatLV=(1/rhovapor^(0.3)-1/rholiquid^(0.3))/(1/rholiquid^(0.3));
DencRatSL=(1/rholiquid^(0.3)-1/rhosolid^(0.3))/(1/rhosolid^(0.3));
qmr[node_,ts_]:=Part[xx2[node,ts]-xx2[node,ts-1],2]/Part[xx2[node,ts]-xx2[node,ts-1],1];
qvr[node_,ts_]:=Part[xx3[node,ts]-xx3[node,ts-1],2]/Part[xx3[node,ts]-xx3[node,ts-1],1];
vmr[ts_]:=Sum[DencRatSL*qmr[node,ts]*vfa[n[node]],{node,1,m}]/m;
vvr[ts_]:=Sum[DencRatLV*qvr[node,ts]*vfa[n[node]],{node,1,m}]/m;
ger[ts_]:=Sum[ tr[node,ts]/ Part[xx1[node,ts],2] ,{node,1,n}]/m;
ff=0.01;
TotalExpansionRate[ts_]:= (1-ff)*(vmr[ts]+vvr[ts]+texprate[ts]) + ff*ger[ts]
IntegralTotExp[ts_]:=Sum[TotalExpansionRate[tt]*dt,{tt,2,ts}]
ListPlot[Table[{Part[xx1[1,ts],1],TotalExpansionRate[ts]},{ts,2,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)","Fractional Exp rate (1/s)"}]

ter=1;
(* ter total expansion *)
Do[ter*=(1+TotalExpansionRate[tt]*dt);
  qwq[tt]={Part[xx1[1,tt],1],ter-1}
  ,{tt,2,nt}]
ListPlot[Table[qwq[tt],{tt,2,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)","Expanded fraction"}]
Do[eff[ts]={Part[xx1[1,ts],1],ta[ts],TotalExpansionRate[ts],Part[qwq[ts],2]},{ts,2,nt}];
ListPlot[Table[{Part[xx1[1,ts],1],ta[ts]},{ts,2,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)","average temp (K)"}]

```

-Graphics-

-Graphics-

-Graphics-

```
bsb1[i_]:=Sum[Part[xx4[j,i],2],{j,1,n}]/n;
ListPlot[Table[{Part[xx4[1,i],1],bsb1[i]},{i,1,nt}],
  PlotJoined->True,PlotRange->All,
  AxesLabel->{"time (s)"," fraction of reacted fuel"}];

ListPlot[Table[{Part[xx1[1,i],1],Part[xx1[1,i],2]},{i,1,nt}],
  PlotJoined->True,
  PlotRange->All,
  AxesLabel->{"time (s)"," temp (K)"}]

ListPlot[Table[{Part[xx1[m,i],1],Part[xx1[m,i],2]},{i,1,nt}],
  PlotJoined->True,
  PlotRange->All,
  AxesLabel->{"time (s)"," temp (K)"}]

szs1=ListPlot[Table[{Part[xx5[1,i],1],Part[xx5[1,i],2]},{i,1,nt}],
  PlotJoined->True,PlotRange->All,
  AxesLabel->{"time (s)"," Q fission (W/m^3)"}];
szs2=ListPlot[Table[{Part[xx6[1,i],1],Part[xx6[1,i],2]},{i,1,nt}],
  PlotJoined->True,PlotRange->All,
  AxesLabel->{"time (s)"," Q chemical (W/m^3)"}];
szs3=ListPlot[Table[{Part[xx7[1,i],1],Part[xx7[1,i],2]},{i,1,nt}],
  PlotJoined->True,PlotRange->All,
  AxesLabel->{"time (s)"," Q total (W/m^3)"}];
Show[szs1,szs2,szs3]
```

-Graphics-

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-Graphics-

```
*****
. 55
*****
```

```

(*)
THIS IS A REVISED DISPERSION MODEL THAT USES THE
TIME ENERGY DATA FROM THE EXPERIMENTS AND TEMPERATURE
DEPENDANT CHEMICAL REACTION RATES FUEL- ALUMINUM
This copy has chemical reaction rates included in the
model
LAST UPDATED 4-19-95
Renormalized pulse width was used to a narrower band width.
*)
Print["Date last updated =",Date[]];

Date last updated ={ 1995, 5, 2, 10, 6, 51}

l=0.000635;m=10;dx=l/m;n=Round[m*.4];time=0;Tinf=300;
volmeat=2*dx*n*(25*10^-3)*(70*10^-3);
Do[xx[i]=i*dx-dx/2,{i,1,m}];Tm=900.;Tv=2700.;
Do[To[i]=300;Tc[i]=300;xm[i]=0;xv[i]=0,{i,1,m}];

k[i_]:= 39.91 /; i <= n ; (* assign k to the meat *)
k[i_]:= kcl[To[i]] /; i > n ; (* assign k to the clad *)
rho[i_]:= 6550.0 /; i <= n ; (* assign meat density *)
rho[i_]:= 2700.0 /; i > n ; (* assign clad density *)

cp[i_]:= cpcl[To[i]] /; i > n ; (* assign clad cpecific heat *)
cp[i_]:= cpf[To[i]] /; i <= n ; (* assign meat cpecific heat *)

htc=Interpolation[{{150,13232},{350,13232},{410,14770},{726,12542},
{776,11561},{1256,9924},{1458,1750},{1526,1729},{1556,800},
{5000,800}},InterpolationOrder->1];

kcl=Interpolation[{{100,161},{400,177},{500,186},{600,193},
{750,193},{850,193},{900,193},{930,193},
{1200,84},{1300,84},{1500,84},{2000,84},{5000,84}},
InterpolationOrder->1];

cpcl=Interpolation[{{100,878},{300,878},{366,942},{422,963},
{588,1034},{700,1067},{811,995},{1000,995},{2000,995},
{5000,995}},InterpolationOrder->1];

cpf=Interpolation[{{100,351},{300,351},{933,463},{1933,546},
{2500,546},{3500,546},{5000,546}},InterpolationOrder->1];

hsf[i_]:=398000*fmal /; i <= n;
hfg[i_]:=10800000*fmal /; i <= n;
hsf[i_]:=398000 /; i > n;
hfg[i_]:=10800000 /; i > n;

l51=ReadList["508.55",Number,RecordLists->True];
shape=Interpolation[l51,InterpolationOrder->1];
num=Part[Part[Position[l51,Last[l51]],1],1];
TimeBeg=Part[l51[[1]],1]; TimeEnd=Part[l51[[num]],1];
c1=Sum[0.5*(shape[Part[l51[[i]],1]]+shape[Part[l51[[i+1]],1]])*
(Part[l51[[i+1]],1]-Part[l51[[i]],1]),{i,200,500}];
qs[qtot_,time_]:=qtot*shape[time]/(volmeat*c1);

```

```

qtot=12218;
(* Functions : nrt - new reacted fraction
               orf - old reacted fraction *)

Tm1=863;Tm2=1970;Td[i_]:= (To[i]-Tm1)/(Tm2-Tm1);
lamda[i_]:= (4*10^-4)*E^(14.73*Tanh[1.8*Td[i]]);
z1[i_,dt_]:=NDSolve[{RF'[t]==lamda[i]-lamda[i]*RF[t],
                    RF[0]==orf[i]},RF,{t,0,dt},AccuracyGoal->15];
nrf[i_,dt_]:=Part[RF[dt]/z1[i,dt],1]
frr[i_]:=lamda[i]-lamda[i]*orf[i];
UpdateRF[dt_]:=Do[orf[i]=nrf[i,dt],{i,1,n}]
qchem[i_]:=ww*frr[i];

q[i_,time_,qtot_]:=qs[qtot,time]+qchem[i]; i<=n; (* assign heat generation in the meat *)
q[i_,time_,qtot_]:=0 /; i>n; (* assign heat generation in the clad *)
bb[i_]:=htc[To[m]]/dx /; i==m ;
bb[i_]:=0 /; i != m ; (* value for the heat transfer out of the plate *)

k1[i_]:= (2*k[i-1]*k[i]/(k[i-1]+k[i]))/dx^2 /; i!=1
k1[i_]:=0 /; i==1
k2[i_]:=k1[i]+k3[i];
k3[i_]:= (2*k[i+1]*k[i]/(k[i+1]+k[i]))/dx^2 /; i!=m
k3[i_]:=0 /; i==m ;

phase[i_]:= solid /; Tc[i] < Tm ;
phase[i_]:= liquid /; Tc[i] < Tv && Tc[i] >= Tm && xm[i] >= 0.85;
phase[i_]:= vapor /; Tc[i] >= Tv && xv[i] > 1;
phase[i_]:= melting /; Tc[i] >= Tm && xm[i] < 0.85 ;
phase[i_]:= vaporizing /; Tc[i] >= Tv && xv[i] < 1 && xv[i] > -0.01;

Tn[i_]:=Tc[i] /; phase[i] != melting && phase[i] != vaporizing;
Tn[i_]:=Tm /; phase[i] == melting ;
Tn[i_]:=Tv /; phase[i] == vaporizing ;

vv[i_,j_,dt_]:= k1[i] /; i-1==j && phase[j] != melting && phase[j] != vaporizing
vv[i_,j_,dt_]:= -k2[i]-rho[i]*cp[i]/dt-bb[i] /; i==j && phase[j] != melting &&
phase[j] != vaporizing
vv[i_,j_,dt_]:= k3[i] /; i+1==j && phase[j] != melting && phase[j] != vaporizing
vv[i_,j_,dt_]:= 0 /; i != j && i-1 != j && i+1 != j ;
(*-----*)
vv[i_,j_,dt_]:=1 /; i==j && phase[j] == melting;
vv[i_,j_,dt_]:=0 /; i-1==j && phase[j] == melting;
vv[i_,j_,dt_]:=0 /; i+1==j && phase[j] == melting;
vv[i_,j_,dt_]:=1 /; i==j && phase[j] == vaporizing;
vv[i_,j_,dt_]:=0 /; i-1==j && phase[j] == vaporizing;
vv[i_,j_,dt_]:=0 /; i+1==j && phase[j] == vaporizing;

(*-----*)
f[i_,dt_,time_,qtot_]:= -q[i,time,qtot]-To[i]*rho[i]*cp[i]/dt - bb[i]*Tinf /; phase[i] != melting &&
phase[i] != vaporizing;
f[i_,dt_,time_,qtot_]:= Tm /; phase[i] == melting;
f[i_,dt_,time_,qtot_]:= Tv /; phase[i] == vaporizing;

TB=0.22;TE=0.26;

```

```
Plot[shape[t],{t,TB,TE},PlotRange->{0,100}]
```

-Graphics-

```
RhoMeat=6500.;RhoAl=2700.;RhoU=12200.;
fval=0.5;
fvu=1-fval;
DenRat=(RhoU/RhoAl);
fmal=0.212;
fmu=1-fmal;
Echem=350000.0;
ww=Echem*5100;
```

```
time=TB;
Do[To[i]=300;Tc[i]=300;xm[i]=0;xv[i]=0;orf[i]=0, {i,1,m}];
dtt[i_]:=10*dx*dx*rho[i]*cp[i]/(2*k[i]);
dt=Min[Table[dtt[i],{i,1,m}]];
nt=Floor[(TE-TB)/dt]-1;
Print["dt= ",dt," nt= ",nt];
```

```
dt= 0.000278413 nt= 142
```

```
Do[
s=Table[vv[i,j,dt],{j,m},{i,m}];
sinv=Inverse[s];(* invert the matrix *)
qq=Table[f[i,dt,time,qtot],{i,m}];
ans=sinv.qq;
```

```
Do[Tc[i]=ans[[i]];
If[phase[i]==melting, xm[i]+=(k1[i]*To[i-1]-k2[i]*To[i]+k3[i]*To[i+1] + q[i,time,qtot]-
bb[i]*(To[m]-Tinf))*dt/(rho[i]*hsf[i]);,0];
If[phase[i]==vaporizing,xv[i]+=(k1[i]*To[i-1]-k2[i]*To[i]+k3[i]*To[i+1] + q[i,time,qtot]-
bb[i]*(To[m]-Tinf))*dt/(rho[i]*hfg[i]);,0];
,{i,1,m}];
```

```
If[Mod[numtimesteps,5]==0 || numtimesteps >=60,
Print[numtimesteps," ",time," ",To[1]," ",xv[1]," ",phase[1]," ",
q[1,time,qtot]," ",orf[1]," ",qs[qtot,time]," ",qchem[1]]];
```

```
Do[To[i]=Tn[i];
xx1[i,numtimesteps]={time,To[i]};
xx2[i,numtimesteps]={time,xm[i]};
xx3[i,numtimesteps]={time,xv[i]};
xx4[i,numtimesteps]={time,orf[i]};
xx5[i,numtimesteps]={time,q[i,time,qtot]};
xx6[i,numtimesteps]={time,qchem[i]};
xx7[i,numtimesteps]={time,qs[qtot,time]};
```

```
,{i,1,m}];
To[i]=Tn[i];
time += dt;
```

```
UpdateRF[dt];
,{numtimesteps,1,nt}];
Print[Finished];
```

```

      9      -11      9
5 0.221114 301.819 0 solid 4.29077 10 1.06104 10 4.29077 10 17.0951
      9      -11      9
10 0.222506 303.913 0 solid 7.84088 10 2.41333 10 7.84088 10 17.5112
      9      -11      9
15 0.223898 306.916 0 solid 2.59439 10 3.80868 10 2.59439 10 18.1293
      10      -11      10
20 0.22529 309.887 0 solid 1.81309 10 5.24095 10 1.81309 10 18.7668
      10      -11      10
25 0.226682 315.302 0 solid 2.67245 10 6.76203 10 2.67245 10 19.9992
      10      -11      10
30 0.228074 326.563 0 solid 4.02536 10 8.45262 10 4.02536 10 22.8866
      10      -10      10
35 0.229466 345.204 0 solid 7.05044 10 1.04922 10 7.05044 10 28.8331
      11      -10      11
40 0.230858 375.866 0 solid 1.08443 10 1.33318 10 1.08443 10 43.072
      11      -10      11
45 0.23225 428.016 0 solid 1.92453 10 1.85072 10 1.92453 10 90.7794
      11      -10      11
50 0.233642 516.126 0 solid 3.34848 10 3.56816 10 3.34848 10 384.617
      11      -9      11
55 0.235034 658.053 0 solid 5.5005 10 2.31702 10 5.5005 10 6272.28
      11      -7      11
60 0.236426 870.314 0 melting 8.53527 10 1.98249 10 8.53526 10 850699.
      11      -7      11      6
61 0.236705 900. 0 melting 9.13968 10 4.6812 10 9.13966 10 1.73023 10
      11      -7      11      6
62 0.236983 900. 0 liquid 9.73741 10 7.37991 10 9.7374 10 1.73023 10
      12      -6      12      6
63 0.237262 900. 0 liquid 1.03834 10 1.00786 10 1.03834 10 1.73023 10
      12      -6      12      6
64 0.23754 950.276 0 liquid 1.11336 10 1.89611 10 1.11335 10 5.69485 10
      12      -6      12
65 0.237818 1023.86 0 liquid 1.17866 10 6.71434 10 1.17863 10

      7
3.08912 10

      12      12
66 0.238097 1083.06 0 liquid 1.24394 10 0.0000242057 1.24383 10

      8
1.12142 10

      12      12
67 0.238375 1150.82 0 liquid 1.28947 10 0.0000933306 1.28903 10

      8
4.43167 10

      12      12      9
68 0.238654 1220.59 0 liquid 1.34575 10 0.000342758 1.34416 10 1.59896 10
      12      12      9
```


69	0.238932	1288.8	0	liquid	1.40265	10	0.00110399	1.39777	10	4.87869	10
					12	12	10				
70	0.239211	1359.72	0	liquid	1.43447	10	0.00319847	1.42106	10	1.34143	10
					12	12	10				
71	0.239489	1431.99	0	liquid	1.47775	10	0.0082518	1.44544	10	3.23163	10
					12	12	10				
72	0.239767	1503.68	0	liquid	1.54228	10	0.0187607	1.47526	10	6.70176	10
					12	12	11				
73	0.240046	1574.34	0	liquid	1.57892	10	0.037798	1.45806	10	1.20863	10
					12	12	11				
74	0.240324	1643.04	0	liquid	1.66242	10	0.0681433	1.47097	10	1.91453	10
					12	12	11				
75	0.240603	1712.68	0	liquid	1.71949	10	0.112148	1.44413	10	2.75361	10
					12	12	11				
76	0.240881	1780.97	0	liquid	1.79381	10	0.170004	1.43523	10	3.58576	10
					12	12	11				
77	0.241159	1849.54	0	liquid	1.83426	10	0.240274	1.40337	10	4.30894	10
					12	12	11				
78	0.241438	1915.99	0	liquid	1.82939	10	0.319229	1.35045	10	4.78942	10
					12	12	11				
79	0.241716	1978.39	0	liquid	1.79634	10	0.401967	1.2995	10	4.96838	10
					12	12	11				
80	0.241995	2036.48	0	liquid	1.73525	10	0.483903	1.24768	10	4.87566	10
					12	12	11				
81	0.242273	2089.45	0	liquid	1.6494	10	0.561359	1.19208	10	4.5732	10
					12	12	11				
82	0.242551	2136.51	0	liquid	1.54117	10	0.631873	1.12753	10	4.13634	10
					12	12	11				
83	0.24283	2176.39	0	liquid	1.42975	10	0.694116	1.06651	10	3.63244	10
					12	12	11				
84	0.243108	2209.1	0	liquid	1.31362	10	0.747769	1.00173	10	3.11889	10
					12	11	11				
85	0.243387	2234.5	0	liquid	1.18717	10	0.793174	9.23992	10	2.63176	10
					12	11	11				
86	0.243665	2252.04	0	liquid	1.08862	10	0.831031	8.69608	10	2.19011	10
					11	11	11				
87	0.243944	2264.1	0	liquid	9.90194	10	0.862298	8.09538	10	1.80657	10
					11	11	11				
88	0.244222	2271.	0	liquid	8.95707	10	0.887935	7.47691	10	1.48016	10
					11	11	11				
89	0.2445	2273.5	0	liquid	8.11829	10	0.908843	6.91141	10	1.20689	10
					11	11	10				
90	0.244779	2272.76	0	liquid	7.2669	10	0.92584	6.28574	10	9.81162	10
					11	11	10				
91	0.245057	2268.89	0	liquid	6.55682	10	0.939621	5.76096	10	7.95863	10
					11	11	10				
92	0.245336	2263.06	0	liquid	5.90417	10	0.950784	5.25914	10	6.45028	10
					11	11	10				
93	0.245614	2255.64	0	liquid	5.31559	10	0.959823	4.79292	10	5.22672	10
					11	11	10				
94	0.245892	2246.99	0	liquid	4.79878	10	0.967143	4.37511	10	4.23666	10
					11	11	10				
95	0.246171	2237.5	0	liquid	4.35828	10	0.973076	4.01458	10	3.43706	10
					11	11	10				

96	0.246449	2227.58	0	liquid	3.89103	10	0.97789	3.61182	10	2.79213	10
					11	11			10		
97	0.246728	2216.85	0	liquid	3.4711	10	0.981801	3.24403	10	2.27067	10
					11	11			10		
98	0.247006	2205.53	0	liquid	3.13352	10	0.984982	2.9486	10	1.84923	10
					11	11			10		
99	0.247284	2194.12	0	liquid	2.90964	10	0.987574	2.75874	10	1.50906	10
					11	11			10		
100	0.247563	2183.35	0	liquid	2.55357	10	0.989693	2.43007	10	1.23499	10
					11	11			10		
101	0.247841	2172.08	0	liquid	2.21978	10	0.991427	2.11855	10	1.01227	10
					11	11			9		
102	0.24812	2160.36	0	liquid	1.99683	10	0.99285	1.91372	10	8.31155	10
					11	11			9		
103	0.248398	2148.95	0	liquid	1.93865	10	0.994019	1.87022	10	6.84281	10
					11	11			9		
104	0.248677	2139.	0	liquid	1.62797	10	0.994984	1.5714	10	5.65675	10
					11	11			9		
105	0.248955	2128.45	0	liquid	1.51777	10	0.995782	1.47094	10	4.68267	10
					11	11			9		
106	0.249233	2118.73	0	liquid	1.30656	10	0.996444	1.26767	10	3.88933	10
					11	11			9		
107	0.249512	2108.98	0	liquid	1.2875	10	0.996994	1.25513	10	3.23711	10
					11	11			9		
108	0.24979	2100.57	0	liquid	1.09775	10	0.997453	1.0707	10	2.70512	10
					10	10			9		
109	0.250069	2092.07	0	liquid	9.52039	10	0.997838	9.29392	10	2.26466	10
					10	10			9		
110	0.250347	2083.74	0	liquid	8.8159	10	0.998159	8.62589	10	1.90012	10
					10	10			9		
111	0.250625	2076.09	0	liquid	7.37935	10	0.99843	7.21945	10	1.59898	10
					10	10			9		
112	0.250904	2068.46	0	liquid	6.27832	10	0.998658	6.14351	10	1.34807	10
					10	10			9		
113	0.251182	2061.05	0	liquid	5.44129	10	0.998851	5.32739	10	1.139	10
					10	10			8		
114	0.251461	2053.99	0	liquid	5.09665	10	0.999013	5.00019	10	9.64644	10
					10	10			8		
115	0.251739	2047.6	0	liquid	5.35035	10	0.999152	5.26841	10	8.19341	10
					10	10			8		
116	0.252018	2042.26	0	liquid	4.98067	10	0.999269	4.91084	10	6.98314	10
					10	10			8		
117	0.252296	2037.41	0	liquid	4.54967	10	0.99937	4.49002	10	5.9651	10
					10	10			8		
118	0.252574	2032.94	0	liquid	4.38057	10	0.999456	4.32951	10	5.10546	10
					10	10			8		
119	0.252853	2028.99	0	liquid	3.34227	10	0.999529	3.29848	10	4.37902	10
					10	10			8		
120	0.253131	2024.82	0	liquid	3.06238	10	0.999592	3.0248	10	3.75862	10
					10	10			8		
121	0.25341	2020.99	0	liquid	3.08997	10	0.999647	3.05765	10	3.23181	10
					10	10			8		
122	0.253688	2017.69	0	liquid	2.72187	10	0.999694	2.69402	10	2.78451	10
					10	10			8		

```

123 0.253966 2014.57 0 liquid 2.40288 10 0.999734 2.37886 10 2.40217 10
      10      10      8
124 0.254245 2011.62 0 liquid 2.3656 10 0.999769 2.34485 10 2.07488 10
      10      10      8
125 0.254523 2009.04 0 liquid 2.08581 10 0.999799 2.06786 10 1.79492 10
      10      10      8
126 0.254802 2006.6 0 liquid 2.03633 10 0.999825 2.02079 10 1.55432 10
      10      10      8
127 0.25508 2004.45 0 liquid 1.65021 10 0.999847 1.63673 10 1.34767 10
      10      10      8
128 0.255358 2002.3 0 liquid 1.07708 10 0.999867 1.06538 10 1.16924 10
      10      10      8
129 0.255637 1999.99 0 liquid 1.3544 10 0.999884 1.34425 10 1.01474 10
      10      10      7
130 0.255915 1998.15 0 liquid 1.84537 10 0.999899 1.83655 10 8.82089 10
      10      10      7
131 0.256194 1996.92 0 liquid 2.17832 10 0.999911 2.17064 10 7.68122 10
      10      10      7
132 0.256472 1996.15 0 liquid 1.50081 10 0.999923 1.49411 10 6.69731 10
      10      10      7
133 0.256751 1995.03 0 liquid 1.04951 10 0.999932 1.04367 10 5.83674 10
      10      10      7
134 0.257029 1993.72 0 liquid 1.21892 10 0.999941 1.21383 10 5.08657 10
      9      9      7
135 0.257307 1992.7 0 liquid 9.66032 10 0.999948 9.61595 10 4.4371 10
      9      9      7
136 0.257586 1991.62 0 liquid 4.06927 10 0.999955 4.03056 10 3.87134 10
      9      9      7
137 0.257864 1990.24 0 liquid 1.67389 10 0.99996 1.64012 10 3.37681 10
      9      9      7
138 0.258143 1988.81 0 liquid 9.87643 10 0.999965 9.84697 10 2.94637 10
      9      9      7
139 0.258421 1988.15 0 liquid 4.03368 10 0.99997 4.00793 10 2.57589 10
      9      9      7
140 0.258699 1987.14 0 liquid 9.6182 10 0.999973 9.59569 10 2.25087 10
      10      10      7
141 0.258978 1986.66 0 liquid 1.12192 10 0.999977 1.11995 10 1.96961 10
      9      9      7
142 0.259256 1986.38 0 liquid 7.7479 10 0.99998 7.73066 10 1.72447 10
Finished

```

```

Tcoefsol=2.25*10^-5;
Tcoefliq=1.17*10^-4;
(* TA - average temperature *)
ta[ts_]:=Sum[Part[xx1[node,ts],2],{node,1,m}]/m;
(* tr - Temperature time rate of change *)
tr[node_,ts_]:=Part[xx1[node,ts]-xx1[node,ts-1],2]/Part[xx1[node,ts]-xx1[node,ts-1],1];
(* tra - average rate of change of temp for the plate *)
tra[ts_]:=Sum[tr[node,ts],{node,1,m}]/m;

(* expth - expansion thermal *)
(* texprate - thermal expansion rate for the plate *)

expth[node_,ts_]:=Tcoefsol*tr[node,ts] /; Part[xx1[node,ts],2]<Tm;

```

```

exph[node_,ts_]:=Tcoefliq*tr[node,ts] /; Part[xx1[node,ts],2]>Tm && Part[xx1[node,ts],2]<Tv;
exph[node_,ts_]:=tr[node,ts]/ Part[xx1[node,ts],2]/; Part[xx1[node,ts],2]>Tv;
exph[node_,ts_]:=0 /; Part[xx1[node,ts],2]==Tm;
exph[node_,ts_]:=0 /; Part[xx1[node,ts],2]==Tv;

texprate[ts_]:=Sum[exph[node,ts],{node,1,m}]/m;
(* qm - quality of the melt *)
(* mv - quality of the vapor *)
qm[ts_]:=Sum[Part[xx2[node,ts],2],{node,1,m}]/m;
qv[ts_]:=Sum[Part[xx3[node,ts],2],{node,1,m}]/m;

vfaln[i_]:=fval /; i<=n ;
vfaln[i_]:=1 /; i>n ;

(* Fractional rate of change of volume due to various factors *)
(* ger - Gas Expansion Rate *)
(* VMR - Volumetric Melting Rate *)
(* VVR - Volumetric Vaporization Rate *)
(* VOFR - Volumetric Oxide Formation Rate *)
(* QMR - Quality Melt Rate *)
(* QVR - Quality Vaporization Rate *)
rhosolid=2700; rholiquid=2500; rhovapor=1.0 ;
DencRatLV=(1/rhovapor^(0.3)-1/rholiquid^(0.3))/(1/rholiquid^(0.3));
DencRatSL=(1/rholiquid^(0.3)-1/rhosolid^(0.3))/(1/rhosolid^(0.3));
qmr[node_,ts_]:=Part[xx2[node,ts]-xx2[node,ts-1],2]/Part[xx2[node,ts]-xx2[node,ts-1],1];
qvr[node_,ts_]:=Part[xx3[node,ts]-xx3[node,ts-1],2]/Part[xx3[node,ts]-xx3[node,ts-1],1];
vmr[ts_]:=Sum[DencRatSL*qmr[node,ts]*vfaln[node],{node,1,m}]/m;
vvr[ts_]:=Sum[DencRatLV*qvr[node,ts]*vfaln[node],{node,1,m}]/m;
ger[ts_]:=Sum[ tr[node,ts]/ Part[xx1[node,ts],2] ,{node,1,n}]/m;
ff=0.01;
TotalExpansionRate[ts_]:= (1-ff)*(vmr[ts]+vvr[ts]+texprate[ts]) + ff*ger[ts]
IntegralTotExp[ts_]:=Sum[TotalExpansionRate[tt]*dt,{tt,2,ts}]
ListPlot[Table[{Part[xx1[1,ts],1],TotalExpansionRate[ts]},{ts,2,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)","Fractional Exp rate (1/s)"}]

ter=1;
(* ter total expansion *)
Do[ter*=(1+TotalExpansionRate[tt]*dt);
qww[tt]={Part[xx1[1,tt],1],ter-1} ,
,{tt,2,nt}]
ListPlot[Table[qww[tt],{tt,2,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)","Expanded fraction"}]
Do[eff[ts]={Part[xx1[1,ts],1],ta[ts],TotalExpansionRate[ts],Part[qww[ts],2]},{ts,2,nt}];
ListPlot[Table[{Part[xx1[1,ts],1],ta[ts]},{ts,2,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)","average temp (K)"}]

```

-Graphics-

-Graphics-

-Graphics-

```
bsb1[i_]:=Sum[Part[xx4[j,i],2],{j,1,n}]/n;
ListPlot[Table[{Part[xx4[1,i],1],bsb1[i]},{i,1,nt}],
  PlotJoined ->True,PlotRange->All,
  AxesLabel->{"time (s)"," fraction of reacted fuel"}];

ListPlot[Table[{Part[xx1[1,i],1],Part[xx1[1,i],2]},{i,1,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)"," temp (K)"}]

ListPlot[Table[{Part[xx1[m,i],1],Part[xx1[m,i],2]},{i,1,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)"," temp (K)"}]

szs1=ListPlot[Table[{Part[xx5[1,i],1],Part[xx5[1,i],2]},{i,1,nt}],
  PlotJoined ->True,PlotRange->All,
  AxesLabel->{"time (s)"," Q fission (W/m^3)"}];
szs2=ListPlot[Table[{Part[xx6[1,i],1],Part[xx6[1,i],2]},{i,1,nt}],
  PlotJoined ->True,PlotRange->All,
  AxesLabel->{"time (s)"," Q chemical (W/m^3)"}];
szs3=ListPlot[Table[{Part[xx7[1,i],1],Part[xx7[1,i],2]},{i,1,nt}],
  PlotJoined ->True,PlotRange->All,
  AxesLabel->{"time (s)"," Q total (W/m^3)"}];
Show[szs1,szs2,szs3]
```

-Graphics-

-Graphics-

-Graphics-

56

(*

THIS IS A REVISED DISPERSION MODEL THAT USES THE
TIME ENERGY DATA FROM THE EXPERIMENTS AND TEMPERATURE
DEPENDANT CHEMICAL REACTION RATES FUEL- ALUMINUM

This copy has chemical reaction rates included in the
model

LAST UPDATED 4-19-95

Renormalized pulse width was used to a narrower band width.

*)

Print["Date last updated =",Date[]];

Date last updated ={1995, 4, 21, 8, 39, 3}

l=0.000635;m=10;dx=l/m;n=Round[m*.4];time=0;Tinf=300;
timespan=35*10⁻³;volmeat=2*dx*n*(25*10⁻³)*(70*10⁻³);
Do[xx[i]=i*dx-dx/2,{i,1,m}];qtot=14200;Tm=900.;Tv=2700.;
Do[To[i]=300;Tc[i]=300;xm[i]=0;xv[i]=0,{i,1,m}];

k[i_]:=39.91 /; i <= n ; (* assign k to the meat *)
k[i_]:=kcl[To[i]] /; i > n ; (* assign k to the clad *)
rho[i_]:=6550.0 /; i <= n ; (* assign meat density *)
rho[i_]:=2700.0 /; i > n ; (* assign clad density *)

cp[i_]:=cpcl[To[i]] /; i > n ; (* assign clad cpecific heat *)
cp[i_]:=cpf[To[i]] /; i <= n ; (* assign meat cpecific heat *)

htc=Interpolation[{{150,13232},{350,13232},{410,14770},{726,12542},
{776,11561},{1256,9924},{1458,1750},{1526,1729},{1556,800},
{5000,800}},InterpolationOrder->1];

kcl=Interpolation[{{100,161},{400,177},{500,186},{600,193},
{750,193},{850,193},{900,193},{930,193},
{1200,84},{1300,84},{1500,84},{2000,84},{5000,84}},
InterpolationOrder->1];

cpcl=Interpolation[{{100,878},{300,878},{366,942},{422,963},
{588,1034},{700,1067},{811,995},{1000,995},{2000,995},
{5000,995}},InterpolationOrder->1];

cpf=Interpolation[{{100,351},{300,351},{933,463},{1933,546},
{2500,546},{3500,546},{5000,546}},InterpolationOrder->1];

hsf[i_]:=398000*fmal /; i <= n;
hfg[i_]:=10800000*fmal /; i <= n;
hsf[i_]:=398000 /; i > n;
hfg[i_]:=10800000 /; i > n;

```

151=ReadList["508.56",Number,RecordLists->True];
shape=Interpolation[151,InterpolationOrder->1];
num=Part[Part[Position[151,Last[151]],1],1];
TimeBeg=Part[151[[1]],1];
TimeEnd=Part[151[[num]],1];
c1=Sum[0.5*(shape[Part[151[[i]],1]]+shape[Part[151[[i+1]],1]])*
(Part[151[[i+1]],1]-Part[151[[i]],1]),{i,200,500}];
qs[qtot_,time_]:=qtot*shape[time]/(volmeat*c1);

(* Functions : nrt - new reacted fraction
orf - old reacted fraction *)

Tm1=863;Tm2=1970;Td[i_]:=(To[i]-Tm1)/(Tm2-Tm1);
lamda[i_]:=(4*10^-4)E^(14.73*Tanh[1.8*Td[i]]);
z1[i_,dt_]:=NDSolve[{RF'[t]==lamda[i]-lamda[i]*RF[t],
RF[0]==orf[i]},RF,{t,0,dt},AccuracyGoal->15];
nrf[i_,dt_]:=Part[RF[dt]/.z1[i,dt],1]
frf[i_]:=lamda[i]-lamda[i]*orf[i];
UpdateRF[dt_]:=Do[orf[i]=nrf[i,dt],{i,1,n}]
qchem[i_]:=ww*frf[i];

q[i_,time_,qtot_]:=qs[qtot,time]+qchem[i]; i<=n; (* assign heat generation in the meat *)
q[i_,time_,qtot_]:=0 /; i>n; (* assign heat generation in the clad *)
bb[i_]:=htc[To[m]]/dx /; i==m ;
bb[i_]:=0 /; i != m ; (* value for the heat transfer out of the plate *)

k1[i_]:=(2*k[i-1]*k[i]/(k[i-1]+k[i]))/dx^2 /; i!=1
k1[i_]:=0 /; i==1
k2[i_]:=k1[i]+k3[i];
k3[i_]:=(2*k[i+1]*k[i]/(k[i+1]+k[i]))/dx^2 /; i!=m
k3[i_]:=0 /; i==m ;

phase[i_]:=solid /; Tc[i] < Tm ;
phase[i_]:=liquid /; Tc[i] < Tv && Tc[i] >= Tm && xm[i] >= 0.85;
phase[i_]:=vapor /; Tc[i] >= Tv && xv[i] > 1;
phase[i_]:=melting /; Tc[i] >= Tm && xm[i] < 0.85 ;
phase[i_]:=vaporizing /; Tc[i] >= Tv && xv[i] < 1 && xv[i] > -0.01;

Tn[i_]:=Tc[i] /; phase[i] != melting && phase[i] != vaporizing;
Tn[i_]:=Tm /; phase[i] == melting ;
Tn[i_]:=Tv /; phase[i] == vaporizing ;

vv[i_,j_,dt_]:=k1[i] /; i-1==j && phase[j]!=melting && phase[j]!=vaporizing
vv[i_,j_,dt_]:=-k2[i]-rho[i]*cp[i]/dt-bb[i] /; i==j && phase[j]!=melting &&
phase[j]!=vaporizing
vv[i_,j_,dt_]:=k3[i] /; i+1==j && phase[j]!=melting && phase[j]!=vaporizing
vv[i_,j_,dt_]:=0 /; i != j && i-1 != j && i+1 != j ;
(*-----*)
vv[i_,j_,dt_]:=1 /; i==j && phase[j]==melting;
vv[i_,j_,dt_]:=0 /; i-1==j && phase[j]==melting;
vv[i_,j_,dt_]:=0 /; i+1==j && phase[j]==melting;
vv[i_,j_,dt_]:=1 /; i==j && phase[j]==vaporizing;
vv[i_,j_,dt_]:=0 /; i-1==j && phase[j]==vaporizing;
vv[i_,j_,dt_]:=0 /; i+1==j && phase[j]==vaporizing;

```

```
(*-----*)
ff[i_,dt_,time_,qtot_]:= -q[i,time,qtot]-To[i]*rho[i]*cp[i]/dt - bb[i]*Tinf /; phase[i]!=melting &&
phase[i]!=vaporizing;
ff[i_,dt_,time_,qtot_]:= Tm /; phase[i]==melting;
ff[i_,dt_,time_,qtot_]:= Tv /; phase[i]==vaporizing;
```

```
qtot=14200;
RhoMeat=6500.;RhoAl=2700.;RhoU=12200.;
fval=0.5;
fvu=1-fval;
DenRat=(RhoU/RhoAl);
fmal=0.212;
fmu=1-fmal;
Echem=350000.0;
ww=Echem*5100
```

```
9
1.785 10
```

```
time=0.22;
Do[To[i]=300;Tc[i]=300;xm[i]=0;xv[i]=0;orf[i]=0, {i,1,m}];
dtt[i_]:=2*dx*dx*rho[i]*cp[i]/(2*k[i]);
dt=Min[Table[dtt[i], {i,1,m}]];
nt=Floor[(0.03)/dt]-1;
Print["dt= ",dt," nt= ",nt];
```

```
dt= 0.0000556826 nt= 537
```

```
Do[
s=Table[vv[i,j,dt], {j,m}, {i,m}];
sinv=Inverse[s];(* invert the matrix *)
qq=Table[ff[i,dt,time,qtot], {i,m}];
ans=sinv.qq;
```

```
Do[Tc[i]=ans[[i]];
If[phase[i]==melting, xm[i]+=(k1[i]*To[i-1]-k2[i]*To[i]+k3[i]*To[i+1] + q[i,time,qtot]-
bb[i]*(To[m]-Tinf))*dt/(rho[i]*hsf[i]);,0];
If[phase[i]==vaporizing,xv[i]+=(k1[i]*To[i-1]-k2[i]*To[i]+k3[i]*To[i+1] + q[i,time,qtot]-
bb[i]*(To[m]-Tinf))*dt/(rho[i]*hfg[i]);,0];
, {i,1,m}];
```

```
If[Mod[numtimesteps,5]==0 || numtimesteps >=60,
Print[numtimesteps," ",time," ",To[1]," ",xv[1]," ",phase[1]," ",
q[1,time,qtot]," ",orf[1]," ",qs[qtot,time]," ",qchem[1]]];
```

```
Do[To[i]=Tn[i];
xx1[i,numtimesteps]={time,To[i]};
xx2[i,numtimesteps]={time,xm[i]};
xx3[i,numtimesteps]={time,xv[i]};
xx4[i,numtimesteps]={time,orf[i]};
xx5[i,numtimesteps]={time,q[i,time,qtot]};
xx6[i,numtimesteps]={time,qchem[i]};
xx7[i,numtimesteps]={time,qs[qtot,time]};
```



```
, {i, 1, m}];
To[i] = Tn[i];
time += dt;
UpdateRF[dt];
, {numtimesteps, 1, nt}];
Print[Finished];
```

5	0.220223	301.567	0	solid	1.67333	10	2.11264	10	1.67333	10	17.0458
10	0.220501	303.847	0	solid	2.08224	10	4.81244	10	2.08224	10	17.4977
15	0.22078	305.945	0	solid	2.3301	10	7.58238	10	2.3301	10	17.9265
20	0.221058	308.303	0	solid	2.67262	10	1.04256	10	2.67262	10	18.4236
25	0.221336	310.938	0	solid	3.5025	10	1.33519	10	3.5025	10	18.9988
30	0.221615	314.252	0	solid	3.22663	10	1.63862	10	3.22663	10	19.7528
35	0.221893	317.342	0	solid	4.3287	10	1.95342	10	4.3287	10	20.4886
40	0.222172	321.156	0	solid	4.26191	10	2.28192	10	4.26191	10	21.4423
45	0.22245	324.907	0	solid	4.92083	10	2.62539	10	4.92083	10	22.4321
50	0.222728	329.249	0	solid	6.23806	10	2.98615	10	6.23806	10	23.6468
55	0.223007	334.547	0	solid	7.03586	10	3.36948	10	7.03586	10	25.2362
60	0.223285	340.307	0	solid	7.22532	10	3.78046	10	7.22532	10	27.1099
61	0.223341	341.451	0	solid	7.37053	10	3.86625	10	7.37053	10	27.5014
62	0.223397	342.613	0	solid	7.56794	10	3.9533	10	7.56794	10	27.9059
63	0.223452	343.806	0	solid	7.85813	10	4.04167	10	7.85813	10	28.3281
64	0.223508	345.05	0	solid	8.14009	10	4.13144	10	8.14009	10	28.7769
65	0.223564	346.344	0	solid	8.3375	10	4.22269	10	8.3375	10	29.2525
66	0.223619	347.668	0	solid	8.50058	10	4.31549	10	8.50058	10	29.7485
67	0.223675	349.013	0	solid	8.5993	10	4.40989	10	8.5993	10	30.2626
68	0.223731	350.363	0	solid	8.97046	10	4.50594	10	8.97046	10	30.7892
69	0.223786	351.782	0	solid	9.56273	10	4.60375	10	9.56273	10	31.3546
70	0.223842	353.323	0	solid	9.63255	10	4.70352	10	9.63255	10	31.982
71	0.223898	354.861	0	solid	9.53385	10	4.80528	10	9.53385	10	32.623

			10	-11	10				
72	0.223953	356.355	0	solid	9.43512	10	4.90904	10	9.43512 10 33.2604
			10	-11	10				
73	0.224009	357.806	0	solid	9.44994	10	5.01477	10	9.44994 10 33.8933
			11	-11	11				
74	0.224065	359.24	0	solid	1.00422	10	5.12249	10	1.00422 10 34.5327
			11	-11	11				
75	0.224121	360.794	0	solid	1.04163	10	5.23243	10	1.04163 10 35.2411
			11	-11	11				
76	0.224176	362.414	0	solid	1.04163	10	5.34472	10	1.04163 10 35.9983
			11	-11	11				
77	0.224232	364.013	0	solid	1.08684	10	5.45941	10	1.08684 10 36.7642
			11	-11	11				
78	0.224288	365.697	0	solid	1.16581	10	5.57667	10	1.16581 10 37.5914
			11	-11	11				
79	0.224343	367.544	0	solid	1.21412	10	5.69685	10	1.21412 10 38.5237
			11	-11	11				
80	0.224399	369.481	0	solid	1.2536	10	5.82016	10	1.2536 10 39.5307
			11	-11	11				
81	0.224455	371.487	0	solid	1.25436	10	5.94683	10	1.25436 10 40.6056
			11	-11	11				
82	0.22451	373.47	0	solid	1.26896	10	6.07692	10	1.26896 10 41.7016
			11	-11	11				
83	0.224566	375.461	0	solid	1.34793	10	6.21054	10	1.34793 10 42.8369
			11	-11	11				
84	0.224622	377.61	0	solid	1.40002	10	6.34812	10	1.40002 10 44.1026
			11	-11	11				
85	0.224677	379.853	0	solid	1.40989	10	6.48996	10	1.40989 10 45.4699
			11	-11	11				
86	0.224733	382.089	0	solid	1.43148	10	6.63621	10	1.43148 10 46.8827
			11	-11	11				
87	0.224789	384.347	0	solid	1.46109	10	6.78707	10	1.46109 10 48.3601
			11	-11	11				
88	0.224844	386.643	0	solid	1.53005	10	6.94279	10	1.53005 10 49.9181
			11	-11	11				
89	0.2249	389.067	0	solid	1.60888	10	7.10384	10	1.60888 10 51.6266
			11	-11	11				
90	0.224956	391.642	0	solid	1.57926	10	7.27078	10	1.57926 10 53.5152
			11	-11	11				
91	0.225011	394.115	0	solid	1.56384	10	7.44361	10	1.56384 10 55.4041
			11	-11	11				
92	0.225067	396.518	0	solid	1.60333	10	7.6224	10	1.60333 10 57.3136
			11	-11	11				
93	0.225123	398.978	0	solid	1.66707	10	7.80753	10	1.66707 10 59.3469
			11	-11	11				
94	0.225178	401.55	0	solid	1.76578	10	7.99957	10	1.76578 10 61.5619
			11	-11	11				
95	0.225234	404.315	0	solid	1.82815	10	8.19937	10	1.82815 10 64.0486
			11	-11	11				
96	0.22529	407.186	0	solid	1.86763	10	8.4076	10	1.86763 10 66.7537
			11	-11	11				
97	0.225346	410.111	0	solid	1.90712	10	8.62486	10	1.90712 10 69.6438
			11	-11	11				
98	0.225401	413.087	0	solid	1.94596	10	8.85174	10	1.94596 10 72.7324

99	0.225457	416.112	0	solid	1.95583	10	9.08893	10	1.95583	10	76.0329
					11	-11	11				
100	0.225513	419.119	0	solid	1.97239	10	9.33687	10	1.97239	10	79.4833
					11	-11	11				
101	0.225568	422.123	0	solid	2.01188	10	9.59613	10	2.01188	10	83.1081
					11	-11	11				
102	0.225624	425.174	0	solid	2.0641	10	9.86747	10	2.0641	10	86.9841
					11	-10	11				
103	0.22568	428.302	0	solid	2.1332	10	1.01519	10	2.1332	10	91.1711
					11	-10	11				
104	0.225735	431.543	0	solid	2.171	10	1.04506	10	2.171	10	95.753
					11	-10	11				
105	0.225791	434.827	0	solid	2.19074	10	1.07646	10	2.19074	10	100.661
					11	-10	11				
106	0.225847	438.11	0	solid	2.27668	10	1.10948	10	2.27668	10	105.854
					11	-10	11				
107	0.225902	441.543	0	solid	2.37455	10	1.1443	10	2.37455	10	111.609
					11	-10	11				
108	0.225958	445.151	0	solid	2.45352	10	1.18112	10	2.45352	10	118.04
					11	-10	11				
109	0.226014	448.889	0	solid	2.5252	10	1.22016	10	2.5252	10	125.146
					11	-10	11				
110	0.226069	452.74	0	solid	2.57455	10	1.26164	10	2.57455	10	132.971
					11	-10	11				
111	0.226125	456.651	0	solid	2.63725	10	1.30577	10	2.63725	10	141.484
					11	-10	11				
112	0.226181	460.65	0	solid	2.71622	10	1.35282	10	2.71622	10	150.825
					11	-10	11				
113	0.226236	464.773	0	solid	2.80812	10	1.4031	10	2.80812	10	161.182
					11	-10	11				
114	0.226292	469.046	0	solid	2.90683	10	1.45699	10	2.90683	10	172.76
					11	-10	11				
115	0.226348	473.483	0	solid	2.92924	10	1.51494	10	2.92924	10	185.768
					11	-10	11				
116	0.226404	477.91	0	solid	2.94346	10	1.57728	10	2.94346	10	199.843
					11	-10	11				
117	0.226459	482.308	0	solid	3.02243	10	1.64436	10	3.02243	10	215.006
					11	-10	11				
118	0.226515	486.82	0	solid	3.1014	10	1.7167	10	3.1014	10	231.899
					11	-10	11				
119	0.226571	491.445	0	solid	3.18037	10	1.79492	10	3.18037	10	250.748
					11	-10	11				
120	0.226626	496.18	0	solid	3.264	10	1.87971	10	3.264	10	271.815
					11	-10	11				
121	0.226682	501.034	0	solid	3.35284	10	1.97187	10	3.35284	10	295.456
					11	-10	11				
122	0.226738	506.017	0	solid	3.42835	10	2.07235	10	3.42835	10	322.098
					11	-10	11				
123	0.226793	511.098	0	solid	3.49744	10	2.18216	10	3.49744	10	352.001
					11	-10	11				
124	0.226849	516.26	0	solid	3.59258	10	2.30242	10	3.59258	10	385.531
					11	-10	11				
125	0.226905	521.559	0	solid	3.69378	10	2.43457	10	3.69378	10	423.619

	11	-10	11				
126	0.22696	527.005	0 solid 3.8221	10	2.58028	10	3.8221 10 467.096
	11	-10	11				
127	0.227016	532.656	0 solid 3.92204	10	2.74168	10	3.92204 10 517.399
	11	-10	11				
128	0.227072	538.448	0 solid 3.95166	10	2.92109	10	3.95166 10 575.127
	11	-10	11				
129	0.227127	544.224	0 solid 4.02495	10	3.12066	10	4.02495 10 639.739
	11	-10	11				
130	0.227183	550.078	0 solid 4.1434	10	3.34318	10	4.1434 10 713.331
	11	-10	11				
131	0.227239	556.106	0 solid 4.26872	10	3.59235	10	4.26872 10 798.773
	11	-10	11				
132	0.227294	562.32	0 solid 4.39704	10	3.87265	10	4.39704 10 898.536
	11	-10	11				
133	0.22735	568.722	0 solid 4.48985	10	4.18944	10	4.48985 10 1015.53
	11	-10	11				
134	0.227406	575.235	0 solid 4.5787	10	4.54866	10	4.5787 10 1151.53
	11	-10	11				
135	0.227461	581.847	0 solid 4.66753	10	4.95725	10	4.66753 10 1309.81
	11	-10	11				
136	0.227517	588.556	0 solid 4.76549	10	5.42345	10	4.76549 10 1494.47
	11	-10	11				
137	0.227573	595.378	0 solid 4.88394	10	5.95723	10	4.88394 10 1711.12
	11	-10	11				
138	0.227629	602.356	0 solid 5.0125	10	6.57105	10	5.0125 10 1967.7
	11	-10	11				
139	0.227684	609.506	0 solid 5.1507	10	7.2803	10	5.1507 10 2273.63
	11	-10	11				
140	0.22774	616.847	0 solid 5.27476	10	8.10411	10	5.27476 10 2640.87
	11	-10	11				
141	0.227796	624.346	0 solid 5.39323	10	9.06542	10	5.39323 10 3081.64
	11	-9	11				
142	0.227851	631.988	0 solid 5.53894	10	1.01921	10	5.53894 10 3611.75
	11	-9	11				
143	0.227907	639.825	0 solid 5.68209	10	1.15199	10	5.68209 10 4256.64
	11	-9	11				
144	0.227963	647.851	0 solid 5.79067	10	1.30934	10	5.79067 10 5044.07
	11	-9	11				
145	0.228018	655.989	0 solid 5.90574	10	1.49653	10	5.90573 10 6000.57
	11	-9	11				
146	0.228074	664.25	0 solid 6.03406	10	1.72014	10	6.03406 10 7168.18
	11	-9	11				
147	0.22813	672.658	0 solid 6.14135	10	1.98852	10	6.14135 10 8603.37
	11	-9	11				
148	0.228185	681.167	0 solid 6.2302	10	2.31183	10	6.2302 10 10364.4
	11	-9	11				
149	0.228241	689.737	0 solid 6.39179	10	2.70244	10	6.39179 10 12521.4
	11	-9	11				
150	0.228297	698.514	0 solid 6.57934	10	3.1772	10	6.57934 10 15219.2
	11	-9	11				
151	0.228352	707.545	0 solid 6.72974	10	3.75842	10	6.72974 10 18631.9
	11	-9	11				
152	0.228408	716.753	0 solid 6.87781	10	4.47386	10	6.87781 10 22934.9

			11	-9	11			
153	0.228464	726.129	0	solid	7.02588	10	5.35921	10 7.02588 10 28381.3
			11	-9	11			
154	0.228519	735.67	0	solid	7.18773	10	6.46052	10 7.18773 10 35304.4
			11	-9	11			
155	0.228575	745.399	0	solid	7.37529	10	7.83836	10 7.37529 10 44169.
			11	-9	11			
156	0.228631	755.363	0	solid	7.54646	10	9.57396	10 7.54646 10 55637.4
			11	-8	11			
157	0.228686	765.527	0	solid	7.7044	10	1.17732	10 7.7044 10 70501.1
			11	-8	11			
158	0.228742	775.858	0	solid	7.84739	10	1.45745	10 7.84738 10 89800.6
			11	-8	11			
159	0.228798	786.326	0	solid	7.98558	10	1.81582	10 7.98558 10 114881.
			11	-8	11			
160	0.228854	796.916	0	solid	8.14276	10	2.27609	10 8.14276 10 147546.
			11	-8	11			
161	0.228909	807.662	0	solid	8.30234	10	2.86998	10 8.30233 10 190381.
			11	-8	11			
162	0.228965	818.565	0	solid	8.47015	10	3.63977	10 8.47015 10 246770.
			11	-8	11			
163	0.229021	829.638	0	solid	8.63066	10	4.64228	10 8.63066 10 321372.
			11	-8	11			
164	0.229076	840.863	0	solid	8.77873	10	5.9533	10 8.77873 10 420269.
			11	-8	11			
165	0.229132	852.212	0	solid	8.96644	10	7.6735	10 8.96644 10 551438.
			11	-8	11			
166	0.229188	863.759	0	solid	9.1836	10	9.94166	10 9.18359 10 727098.
			11	-7	11			
167	0.229243	875.554	0	solid	9.40077	10	1.29502	10 9.40076 10 964426.
			11	-7	11			
168	0.229299	887.594	0	solid	9.61795	10	1.69631	10 9.61794 10

6
1.28643 10

			11	-7	11		
169	0.229355	899.873	0	melting	9.79635	10	2.23442 10 9.79633 10

6
1.72499 10

			11	-7	11	6	
170	0.22941	900. 0	melting	9.96668	10	2.77416 10	9.96666 10 1.73023 10
			12	-7	12	6	
171	0.229466	900. 0	melting	1.01049	10	3.3139 10	1.01049 10 1.73023 10
			12	-7	12		
172	0.229522	900. 0	melting	1.02777	10	3.85364 10	1.02777 10

6
1.73023 10

			12	-7	12	
173	0.229577	900. 0	melting	1.05048	10	4.39338 10 1.05048 10

6
1.73023 10

12 -7 12
174 0.229633 900. 0 melting 1.06908 10 4.93312 10 1.06907 10

6
1.73023 10

12 -7 12
175 0.229689 900. 0 melting 1.08487 10 5.47287 10 1.08487 10

6
1.73023 10

12 -7 12
176 0.229744 900. 0 melting 1.10224 10 6.01261 10 1.10224 10

6
1.73023 10

12 -7 12 6
177 0.2298 900. 0 liquid 1.12001 10 6.55235 10 1.12001 10 1.73023 10

12 -7 12 6
178 0.229856 900. 0 liquid 1.13877 10 7.09209 10 1.13876 10 1.73023 10

12 -7 12
179 0.229912 917.884 0 liquid 1.15773 10 7.91845 10 1.15772 10

6
2.64903 10

12 -7 12
180 0.229967 933.197 0 liquid 1.17747 10 9.10642 10 1.17747 10

6
3.80823 10

12 -6 12
181 0.230023 946.386 0 liquid 1.19762 10 1.07278 10 1.19761 10

6
5.19749 10

12 -6 12
182 0.230079 957.837 0 liquid 1.21835 10 1.28487 10 1.21834 10

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6.79889 10

12 -6 12
183 0.230134 971.915 0 liquid 1.24455 10 1.57931 10 1.24454 10

6
9.4388 10

12 -6 12 7
184 0.23019 987.48 0 liquid 1.27416 10 2.0012 10 1.27415 10 1.35246 10

12 -6 12
185 0.230246 1003.86 0 liquid 1.29327 10 2.61486 10 1.29325 10

7
1.96717 10

12 -6 12
186 0.230301 1020.4 0 liquid 1.31015 10 3.50661 10 1.31013 10

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 2.85867 10
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 187 0.230357 1036.77 0 liquid 1.3309 10 4.79102 10 1.33086 10
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 4.11737 10
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 188 0.230413 1052.85 0 liquid 1.3503 10 6.61967 10 1.35024 10
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 5.86206 10
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 189 0.230468 1068.51 0 liquid 1.36513 10 9.18584 10 1.36505 10
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 8.22625 10
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 190 0.230524 1083.61 0 liquid 1.38252 10 0.0000127254 1.38241 10
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 1.13466 10
¹² ¹² ⁸
 191 0.23058 1098.8 0 liquid 1.4033 10 0.00001759 1.40314 10 1.55942 10
¹² ¹²
 192 0.230635 1114.42 0 liquid 1.42471 10 0.0000242953 1.4245 10
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 2.14949 10
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 193 0.230691 1130.57 0 liquid 1.44651 10 0.00003358 1.44621 10
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 194 0.230747 1147.29 0 liquid 1.4584 10 0.0000464889 1.45798 10
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 195 0.230802 1164.38 0 liquid 1.46869 10 0.0000644222 1.46811 10
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 196 0.230858 1181.74 0 liquid 1.4847 10 0.0000892639 1.48391 10
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 197 0.230914 1199.41 0 liquid 1.50349 10 0.000123562 1.50239 10
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198 0.230969 1217.33 0 liquid 1.53055 10 0.000170692 1.52904 10

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199 0.231025 1235.54 0 liquid 1.55107 10 0.000235173 1.54901 10

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200 0.231081 1253.87 0 liquid 1.56366 10 0.000322672 1.56085 10

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2.80482 10

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201 0.231137 1272.09 0 liquid 1.58423 10 0.00044003 1.58047 10

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3.76187 10

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202 0.231192 1290.3 0 liquid 1.60915 10 0.000595829 1.60416 10

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4.99403 10

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203 0.231248 1308.59 0 liquid 1.63272 10 0.000800882 1.62615 10

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6.57265 10

12 12 9
204 0.231304 1327. 0 liquid 1.65625 10 0.00106852 1.64768 10 8.5785 10

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205 0.231359 1345.58 0 liquid 1.67754 10 0.00141515 1.66643 10

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1.11098 10

12 12
206 0.231415 1364.33 0 liquid 1.69761 10 0.00186053 1.68333 10

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207 0.231471 1383.25 0 liquid 1.71337 10 0.0024282 1.69518 10

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208 0.231526 1402.28 0 liquid 1.72721 10 0.00314535 1.70423 10

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209 0.231582 1421.39 0 liquid 1.73891 10 0.00404301 1.71015 10

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 210 0.231638 1440.54 0 liquid 1.7564 10 0.0051559 1.72075 10 3.56556 10
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 211 0.231693 1459.83 0 liquid 1.7774 10 0.00652395 1.73358 10
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 212 0.231749 1479.3 0 liquid 1.80419 10 0.00819229 1.75076 10
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 213 0.231805 1499.02 0 liquid 1.83337 10 0.010212 1.76869 10 6.46781 10
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 214 0.23186 1519.01 0 liquid 1.86614 10 0.012639 1.78844 10 7.77069 10
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 215 0.231916 1539.28 0 liquid 1.89544 10 0.0155344 1.80276 10
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 9.26793 10
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 216 0.231972 1559.73 0 liquid 1.91338 10 0.0189603 1.80375 10
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 1.09632 10
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 217 0.232027 1580.15 0 liquid 1.9419 10 0.022974 1.8135 10 1.28405 10
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 218 0.232083 1600.67 0 liquid 1.98136 10 0.0276367 1.83225 10
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 1.49113 10
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 219 0.232139 1621.45 0 liquid 2.01326 10 0.0330136 1.84137 10
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 1.71887 10
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 220 0.232194 1642.32 0 liquid 2.04279 10 0.0391625 1.84631 10
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 1.96484 10
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 221 0.23225 1663.23 0 liquid 2.07039 10 0.0461349 1.84769 10 2.22698 10
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 222 0.232306 1684.16 0 liquid 2.10064 10 0.0539752 1.85034 10
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 2.50297 10
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223 0.232362 1705.17 0 liquid 2.14624 10 0.0627228 1.86712 10 2.7912 10
12 12 11
224 0.232417 1726.5 0 liquid 2.18679 10 0.0724219 1.87748 10 3.09306 10
12 12 11
225 0.232473 1748.09 0 liquid 2.214 10 0.0831043 1.87353 10 3.40462 10
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226 0.232529 1769.71 0 liquid 2.25113 10 0.0947809 1.87922 10

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3.71916 10

12 12 11
227 0.232584 1791.52 0 liquid 2.2976 10 0.107459 1.89403 10 4.0357 10
12 12 11
228 0.23264 1813.67 0 liquid 2.3243 10 0.121144 1.889 10 4.35304 10
12 12 11
229 0.232696 1835.82 0 liquid 2.34236 10 0.13581 1.87616 10 4.66192 10
12 12 11
230 0.232751 1857.85 0 liquid 2.38808 10 0.151413 1.89245 10 4.95635 10
12 12 11
231 0.232807 1880.19 0 liquid 2.43128 10 0.167923 1.90723 10 5.24056 10
12 12 11
232 0.232863 1902.82 0 liquid 2.44547 10 0.185296 1.8944 10 5.5107 10
12 12 11
233 0.232918 1925.3 0 liquid 2.46699 10 0.203457 1.89133 10 5.75655 10
12 12 11
234 0.232974 1947.77 0 liquid 2.50594 10 0.222332 1.90812 10 5.97821 10
12 12 11
235 0.23303 1970.55 0 liquid 2.52276 10 0.241854 1.90486 10 6.17891 10
12 12 11
236 0.233085 1993.35 0 liquid 2.51936 10 0.261938 1.88413 10 6.35224 10
12 12 11
237 0.233141 2015.86 0 liquid 2.53021 10 0.282482 1.88089 10 6.49317 10
12 12 11
238 0.233197 2038.3 0 liquid 2.54436 10 0.303395 1.88385 10 6.6051 10
12 12 11
239 0.233252 2060.75 0 liquid 2.55107 10 0.324587 1.88216 10 6.68909 10
12 12 11
240 0.233308 2083.12 0 liquid 2.55465 10 0.345969 1.88019 10 6.74458 10
12 12 11
241 0.233364 2105.37 0 liquid 2.55546 10 0.367452 1.87822 10 6.7724 10
12 12 11
242 0.23342 2127.49 0 liquid 2.55051 10 0.388953 1.87313 10 6.7738 10
12 12 11
243 0.233475 2149.42 0 liquid 2.53726 10 0.41039 1.86227 10 6.74986 10
12 12 11
244 0.233531 2171.04 0 liquid 2.52979 10 0.431685 1.85962 10 6.70169 10
12 12 11
245 0.233587 2192.48 0 liquid 2.52682 10 0.452771 1.86357 10 6.6325 10
12 12 11
246 0.233642 2213.82 0 liquid 2.49956 10 0.47359 1.84505 10 6.54506 10
12 12 11
247 0.233698 2234.69 0 liquid 2.46327 10 0.49408 1.81939 10 6.43889 10
12 12 11
248 0.233754 2254.98 0 liquid 2.45001 10 0.514188 1.81843 10 6.3158 10

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 1.03107 10
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 291 0.236148 2700. 0.0169509 vaporizing 1.20542 10 0.940776 1.1075 10
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 292 0.236204 2700. 0.0186778 vaporizing 1.17824 10 0.943754 1.08524 10
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 293 0.236259 2700. 0.0203831 vaporizing 1.15777 10 0.946582 1.06945 10
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 294 0.236315 2700. 0.0220531 vaporizing 1.13514 10 0.949267 1.05126 10
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 295 0.236371 2700. 0.0236583 vaporizing 1.10625 10 0.951818 1.02658 10
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 296 0.236426 2700. 0.0252128 vaporizing 1.08364 10 0.954241 1.00798 10
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 297 0.236482 2700. 0.0267373 vaporizing 1.06799 10 0.956541 9.96138 10
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 298 0.236538 2700. 0.0282161 vaporizing 1.04852 10 0.958726 9.80278 10
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 299 0.236593 2700. 0.0296391 vaporizing 1.02732 10 0.960802 9.62509 10
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 300 0.236649 2700. 0.0309864 vaporizing 1.00194 10 0.962772 9.40389 10
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301 0.236705 2700. 0.0322558 vaporizing 9.77588 10 0.964644 9.1913 10
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 302 0.23676 2700. 0.0334993 vaporizing 9.68725 10 0.966422 9.13206 10
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 303 0.236816 2700. 0.0347024 vaporizing 9.55142 10 0.96811 9.02414 10
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 304 0.236872 2700. 0.0358211 vaporizing 9.29786 10 0.969714 8.79709 10
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 305 0.236928 2700. 0.036857 vaporizing 9.06515 10 0.971237 8.58957 10
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 306 0.236983 2700. 0.0378133 vaporizing 8.85368 10 0.972683 8.40201 10
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 307 0.237039 2700. 0.0387026 vaporizing 8.68478 10 0.974056 8.25582 10
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 308 0.237095 2700. 0.0395308 vaporizing 8.53489 10 0.975361 8.12749 10
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 309 0.23715 2700. 0.0402883 vaporizing 8.35935 10 0.9766 7.97244 10
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 310 0.237206 2700. 0.0409756 vaporizing 8.18932 10 0.977776 7.82187 10
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 311 0.237262 2700. 0.0416146 vaporizing 8.08201 10 0.978894 7.73303 10
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 312 0.237317 2700. 0.042194 vaporizing ¹¹7.93883 10 0.979955 7.6074 10
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 313 0.237373 2700. 0.0426833 vaporizing ¹¹7.71486 10 0.980963 7.4001 10
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 314 0.237429 2700. 0.0430955 vaporizing ¹¹7.53747 10 0.98192 7.23853 10
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 315 0.237484 2700. 0.043445 vaporizing ¹¹7.40399 10 0.982829 7.12008 10
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 316 0.23754 2700. 0.0437276 vaporizing ¹¹7.25707 10 0.983693 6.98744 10
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 317 0.237596 2700. 0.0439409 vaporizing ¹¹7.10532 10 0.984513 6.84924 10
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 318 0.237651 2700. 0.0440914 vaporizing ¹¹6.97245 10 0.985291 6.72926 10
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 2.43198 10

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 319 0.237707 2700. 0.0441793 vaporizing ¹¹6.83925 10 0.986031 6.60828 10
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 2.3097 10

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 320 0.237763 2700. 0.0441985 vaporizing ¹¹6.68944 10 0.986733 6.47008 10
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 321 0.237818 2700. 0.0441459 vaporizing ¹¹6.53368 10 0.9874 6.32535 10
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 2.08327 10

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 322 0.237874 2700. 0.0440157 vaporizing ¹¹6.36526 10 0.988034 6.16741 10
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1.97851 10
 323 0.23793 2700. 0.0438084 vaporizing 6.20266 10 0.988636 6.01476 10
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 324 0.237985 2700. 0.0435246 vaporizing 6.04514 10 0.989207 5.86669 10
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 325 0.238041 2700. 0.043185 vaporizing 5.9465 10 0.98975 5.77702 10
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 326 0.238097 2700. 0.0427995 vaporizing 5.86889 10 0.990265 5.70793 10
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 327 0.238153 2700. 0.0423502 vaporizing 5.7358 10 0.990755 5.58294 10
 1.52867 10
 328 0.238208 2700. 0.0418338 vaporizing 5.59541 10 0.99122 5.45023 10
 1.4518 10
 329 0.238264 2700. 0.0412403 vaporizing 5.43017 10 0.991661 5.29229 10
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 330 0.23832 2700. 0.0405729 vaporizing 5.27918 10 0.99208 5.14824 10
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 331 0.238375 2700. 0.0398402 vaporizing 5.15415 10 0.992479 5.02979 10
 1.24363 10
 332 0.238431 2700. 0.0390465 vaporizing 5.04042 10 0.992857 4.92231 10
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 333 0.238487 2700. 0.0381958 vaporizing 4.93578 10 0.993216 4.82361 10

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 334 0.238542 2700. 0.0372947 vaporizing 4.84642 10 0.993557 4.73989 10
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 335 0.238598 2700. 0.0363467 vaporizing 4.76209 10 0.993881 4.66091 10
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 336 0.238654 2700. 0.0353394 vaporizing 4.63997 10 0.994189 4.54388 10
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 337 0.238709 2700. 0.0342729 vaporizing 4.52001 10 0.994481 4.42875 10
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 9.12558 10
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 338 0.238765 2700. 0.033153 vaporizing 4.41671 10 0.994758 4.33005 10
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 339 0.238821 2700. 0.0319803 vaporizing 4.31364 10 0.995022 4.23133 10
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 8.23095 10
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 340 0.238876 2700. 0.0307551 vaporizing 4.21078 10 0.995272 4.13261 10
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 7.81708 10
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 341 0.238932 2700. 0.0294842 vaporizing 4.12521 10 0.99551 4.05097 10
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 342 0.238988 2700. 0.0281733 vaporizing 4.05239 10 0.995736 3.98188 10
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 343 0.239043 2700. 0.0268212 vaporizing 3.97203 10 0.99595 3.90507 10
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 344 0.239099 2700. 0.025428 vaporizing 3.88969 10 0.996154 3.82609 10

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6.35951 10
345 0.239155 2700. 0.0239837¹¹ vaporizing 3.77837¹¹ 10 0.996347 3.71797 10

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346 0.239211 2700. 0.022488¹¹ vaporizing 3.66861¹¹ 10 0.996531 3.61125 10

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5.73605 10
347 0.239266 2700. 0.0209432¹¹ vaporizing 3.567¹¹ 10 0.996705 3.51253 10

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5.44763 10
348 0.239322 2700. 0.0193535¹¹ vaporizing 3.47718¹¹ 10 0.996871 3.42544 10

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349 0.239378 2700. 0.0177258¹¹ vaporizing 3.40549¹¹ 10 0.997028 3.35636 10

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350 0.239433 2700. 0.0160657¹¹ vaporizing 3.34571¹¹ 10 0.997178 3.29905 10

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351 0.239489 2700. 0.0143778¹¹ vaporizing 3.29401¹¹ 10 0.99732 3.24969 10

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4.43187 10
352 0.239545 2700. 0.0126609¹¹ vaporizing 3.23451¹¹ 10 0.997454 3.19242 10

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4.20902 10
353 0.2396 2700. 0.0109156¹¹ vaporizing 3.17317¹¹ 10 0.997582 3.13319 10

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3.99739 10
354 0.239656 2700. 0.00914269¹¹ vaporizing 3.11193¹¹ 10 0.997704 3.07397 10

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3.79639 10
11 11

355 0.239712 2700. 0.00733923 vaporizing 3.04047 10 0.997819 3.00441 10

9
3.6055 10
Finished

\$Aborted

```
Tcoefsol=2.25*10^-5;
Tcoefliq=1.17*10^-4;
nt=350;
(* TA - average temperature *)
ta[ts_]:=Sum[Part[xx1[node,ts],2],{node,1,m}]/m;
(* tr - Temperature time rate of change *)
tr[node_,ts_]:=Part[xx1[node,ts]-xx1[node,ts-1],2]/Part[xx1[node,ts]-xx1[node,ts-1],1];
(* tra - average rate of change of temp for the plate *)
tra[ts_]:=Sum[tr[node,ts],{node,1,m}]/m;

(* expth - expansion thermal *)
(* texprate - thermal expansion rate for the plate *)

expth[node_,ts_]:=Tcoefsol*tr[node,ts] /; Part[xx1[node,ts],2]<Tm;
expth[node_,ts_]:=Tcoefliq*tr[node,ts] /; Part[xx1[node,ts],2]>Tm && Part[xx1[node,ts],2]<Tv;
expth[node_,ts_]:=tr[node,ts]/ Part[xx1[node,ts],2]/; Part[xx1[node,ts],2]>Tv;
expth[node_,ts_]:=0 /; Part[xx1[node,ts],2]==Tm;
expth[node_,ts_]:=0 /; Part[xx1[node,ts],2]==Tv;

texprate[ts_]:=Sum[expth[node,ts],{node,1,m}]/m;
(* qm - quality of the melt *)
(* mv - quality of the vapor *)
qm[ts_]:=Sum[Part[xx2[node,ts],2],{node,1,m}]/m;
qv[ts_]:=Sum[Part[xx3[node,ts],2],{node,1,m}]/m;

vfa1n[i_]:=fval /; i<=n ;
vfa1n[i_]:=1 /; i>n ;

(* Fractional rate of change of volume due to various factors *)
(* ger - Gas Expansion Rate *)
(* VMR - Volumetric Melting Rate *)
(* VVR - Volumetric Vaporization Rate *)
(* VOFR - Volumetric Oxide Formation Rate *)
(* QMR - Quality Melt Rate *)
(* QVR - Quality Vaporization Rate *)
rhosolid=2700; rholiquid=2500; rhovapor=1.0 ;
DencRatLV=(1/rhovapor^(0.3)-1/rholiquid^(0.3))/(1/rholiquid^(0.3));
DencRatSL=(1/rholiquid^(0.3)-1/rhosolid^(0.3))/(1/rhosolid^(0.3));
qmr[node_,ts_]:=Part[xx2[node,ts]-xx2[node,ts-1],2]/Part[xx2[node,ts]-xx2[node,ts-1],1];
qvr[node_,ts_]:=Part[xx3[node,ts]-xx3[node,ts-1],2]/Part[xx3[node,ts]-xx3[node,ts-1],1];
vmr[ts_]:=Sum[DencRatSL*qmr[node,ts]*vfa1n[node],{node,1,m}]/m;
vvr[ts_]:=Sum[DencRatLV*qvr[node,ts]*vfa1n[node],{node,1,m}]/m;
ger[ts_]:=Sum[ tr[node,ts]/ Part[xx1[node,ts],2] ,{node,1,n}]/m;
ff=0.01;
TotalExpansionRate[ts_]:= (1-ff)*(vmr[ts]+vvr[ts]+texprate[ts]) + ff*ger[ts]
IntegralTotExp[ts_]:=Sum[TotalExpansionRate[tt]*dt,{tt,2,ts}]
```

```

ListPlot[Table[{Part[xx1[1,ts],1],TotalExpansionRate[ts]},{ts,2,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)","Fractional Exp rate (1/s)"}]

ter=1;
(* ter total expansion *)
Do[ter*=(1+TotalExpansionRate[tt]*dt);
  qwq[tt]={Part[xx1[1,tt],1],ter-1 }
  ,{tt,2,nt}]
ListPlot[Table[qwq[tt],{tt,2,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)","Expanded fraction" }]
Do[eff[ts]={Part[xx1[1,ts],1],ta[ts],TotalExpansionRate[ts],Part[qwq[ts],2]},{ts,2,nt}];
ListPlot[Table[{Part[xx1[1,ts],1],ta[ts]},{ts,2,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)","average temp (K)"}]

```

-Graphics-

-Graphics-

-Graphics-

```

ListPlot[Table[{Part[xx1[1,i],1],Part[xx1[1,i],2]},{i,1,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)"," temp (K)"}]

```

```

ListPlot[Table[{Part[xx1[m,i],1],Part[xx1[m,i],2]},{i,1,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)"," temp (K)"}]

```

-Graphics-

-Graphics-

```

szs1=ListPlot[Table[{Part[xx4[1,i],1],Part[xx4[1,i],2]},{i,1,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)"," temp (K)"}]

```

```

szs2=ListPlot[Table[{Part[xx4[3,i],1],Part[xx4[3,i],2]},{i,1,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)"," temp (K)"}]
szs3=ListPlot[Table[{Part[xx4[n,i],1],Part[xx4[n,i],2]},{i,1,nt}],
  PlotJoined ->True,
  PlotRange->All,
  AxesLabel->{"time (s)"," temp (K)"}];
Show[szs1,szs2,szs3]

```

-Graphics-

-Graphics-

-Graphics-

```

szs1=ListPlot[Table[{Part[xx5[1,i],1],Part[xx5[1,i],2]},{i,1,nt}],
  PlotJoined ->True,PlotRange->All,
  AxesLabel->{"time (s)"," Q (W)"}];
szs2=ListPlot[Table[{Part[xx6[1,i],1],Part[xx6[1,i],2]},{i,1,nt}],
  PlotJoined ->True,PlotRange->All,
  AxesLabel->{"time (s)"," Q (W)"}];
szs3=ListPlot[Table[{Part[xx7[1,i],1],Part[xx7[1,i],2]},{i,1,nt}],
  PlotJoined ->True,PlotRange->All,
  AxesLabel->{"time (s)"," Q (W)"}];
Show[szs1,szs2,szs3]

```

-Graphics-

Internal Distribution List

1.	W. G. Craddick	8.	D. B. Simpson
2.	E. C. Fox	9-11.	R. P. Taleyarkhan
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4.	R. M. Harrington	13.	ORNL Patent Section
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- 31 K. Soda, Director, Office of Planning, Japan Atomic Energy Research Institute, 2-2, Uchisaiwai-Cho, 2-Chome, Chiyoda-ku, Tokyo 100, Japan
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