Effects of Collisional Zonal Flow Damping on Turbulent Transport

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(August 10, 1999)

Results from 3D global gyrokinetic particle simulations of ion temperature gradient driven microturbulence in a toroidal plasma show that the ion thermal transport level in the interior region exhibits significant dependence on the ion-ion collision frequency even in regimes where the instabilities are collisionless. This is identified as arising from the Coulomb collisional damping of turbulence-generated zonal flows.

52.25.Fi, 52.35.Ra, 52.65.Tt

Understanding the physical mechanism responsible for the turbulent transport observed in magnetized plasmas is crucial for developing techniques to improve confinement. In particular, ion thermal transport in the core region of a tokamak plasma is believed to arise from electrostatic pressure-gradient driven microinstabilities [1]. In most previous studies, ion-ion collisions have been assumed to have little or no effect on the microinstabilities most likely to be responsible for the ion thermal transport, such as ion-temperature-gradient (ITG) modes. This is because the temperature in present day major tokamak core plasmas is so high that the ion-ion collision frequency is much smaller than the characteristic frequency of the ITG mode (e.g., linear growth rate or nonlinear decorrelation rate, which is of the order of the ion diamagnetic frequency). Consequently, most theory based ion thermal diffusivities do not contain explicit dependence on the ion-ion collisionality [2,3].

Current investigations indicate that ion-ion collisions can enhance turbulent transport via Coulomb collisional damping of turbulence-generated $\mathbf{E}\times\mathbf{B}$ shear flows. These zonal flows [4], which are linearly stable $k_{||}=0$ modes, are nonlinearly driven by the flux-surface-averaged, radially local current modulations and are mainly in the poloidal direction for high aspect ratio devices. The shear decorrelation [5,6] by these small scale flows results in the reduction of turbulence and transport. Since the turbulence is regulated by zonal flows, the turbulent transport can depend on ion-ion collisions which damp poloidal flows through the "neoclassical" effects.

In this letter, we report gyrokinetic particle simulation [7] results which show that the ion thermal transport from electrostatic ITG turbulence depends on ion-ion collisions for representative tokamak core plasma parameters using the global gyrokinetic toroidal code (GTC) [8]. The collisionality-dependence of the turbulent transport comes from the neoclassical damping of zonal flows. The fluctuations and transport exhibit bursting behavior with a period corresponding to the collisional damping time of poloidal flows. These results are contrary to the usual assumption that core ion transport is "collisionless". The fact that the change of the ion heat conductivity χ_i with collision frequency cannot be attributed to the change in the linear growth rate γ or mode spectrum $k_\perp$ places considerable limitations on the applicability of most of the existing transport models that are based on an oversimplified $\gamma/k_\perp^2$ type mixing length rule.

In the experiments, despite the difficulty in isolating collisional effect by varying the collisionality while keeping other dimensionless parameters constant, there is some evidence of collisional effects on the core turbulent transport. A recent transport scaling study on DIII-D core plasmas [9] showed that one-fluid (including both ions and electrons) thermal diffusivity strongly depends on collisionality $\chi_{eff}\propto\nu^{0.49}$ in the H-mode, and is almost independent of collisionality in the L-mode. Moreover, transport analysis in C-Mod H-mode plasmas reported [10] approximately a linear dependence of χ_{eff} on ν^*. Our simulation results are consistent with this observation that the collisional dependence of transport is much more pronounced in the enhanced confinement regime where turbulence is expected to be weaker than that of typical L-mode plasmas. Finally, the bursting behavior of density fluctuations has been observed [11] in TFTR core plasmas before the transition to the enhanced reverse shear regime. The bursting period ($\sim 3ms$) was close to the collisional flow damping time calculated from measured plasma parameters.

This study employed a general geometry global gyrokinetic toroidal code (GTC) [8] using magnetic coordinates. Particles are advanced with Hamiltonian guiding center equations of motion, and the perturbed potential is calculated by a non-spectral gyrokinetic Poisson solver. The code is scalable and portable on massively parallel computers which enable us to routinely perform nonlinear simulations with realistic plasma parameters. Extensive convergence studies and nonlinear benchmarks against earlier analytic and computational models have been carried out for electrostatic toroidal ITG turbulence in the absence of self-generated zonal flows [8].

Zonal flows are generated by the Reynolds stress [12] and can be considered as a nonlinear modulational instability [13] associated with the inverse cascade of the turbulence energy [14]. This scenario of modulational instability is observed in the present gyrokinetic simulations of toroidal ITG modes. A stationary turbulence spectrum is first obtained while suppressing zonal flows. Then zonal flows are self-consistently included. Exponential growth of zonal flows is observed until the flow shear is so high that the turbulence is suppressed. The incoherent emission of turbulence [15] provides an initial seed for the growth of the nonlinear instability.

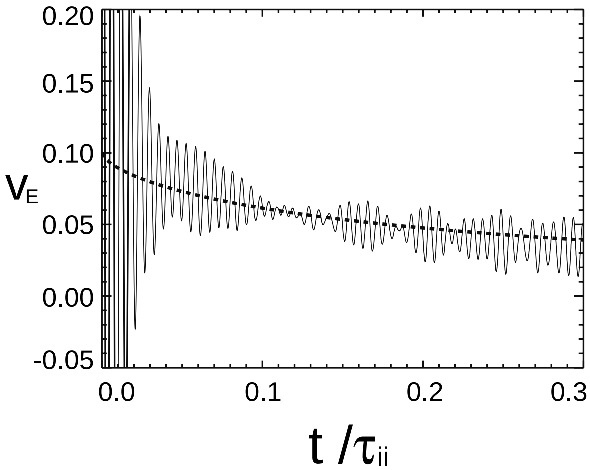

FIG. 1. Gyrokinetic simulation of linear damping of a test poloidal flow (normalized by initial value) showing magnetic pumping, GAM oscillation and collisional damping (solid curve). Dotted line is theoretical prediction of collisional damping with an initial value of the residual level measured in a collisionless simulation.

In toroidal geometry, an initial source of zonal flows is damped by the collisionless transit time magnetic pumping effects followed by a slowly damped geodesic acoustic mode (GAM) [16] oscillation. Rosenbluth and Hinton [15] have pointed out the existence of a residual flow which survives this linear collisionless damping process in the banana regime. This undamped component of poloidal flows plays a key role in determining the turbulent transport level in nonlinear simulations, thus its long time behavior must be treated accurately. This residual flow is eventually damped out by ion-ion collisions and possibly, by nonlinear effects. In the present work, the GTC linear simulation of the collisional flow damping is benchmarked with an analytical calculation [17]. In this simulation, we solve a toroidal gyrokinetic equation [18,19] with a flux-surface-averaged source which introduces an initial perturbation of the poloidal flow. Without collisions, an asymptotic residual of this flow develops and its level measured from the simulation agrees with the theoretical prediction. When ion-ion collisions are included, this residual flow is slowly damped. The collisional decay of the flow perturbation in the gyrokinetic simulation, as shown in Fig. 1, agrees [8] well with the analytical calculation [17] (see below for plasma parameters). The analytical theory only deals with time scales longer than the ion bounce period, therefore it does not describe the transient processes associated with magnetic pumping and GAM oscillations. Most of the collisional damping occurs through the friction force between trapped and passing particles [17]. The effective damping time measured from the simulation is very close to the theoretical prediction of $\tau_d = 1.5\epsilon\tau_{ii}$, where $\epsilon \equiv r/R_0$ is the local inverse aspect ratio and τ_{ii} is the ion-ion collision time. A like-species Lorentz operator is used in this simulation for comparison with the theory. In the following nonlinear simulations, we use a momentum and energy conserving Fokker-Planck operator which has been rigorously benchmarked for neoclassical transport [20].

With validity checks completed, the GTC was next applied to full torus nonlinear simulations of ITG turbulence. These simulations used representative parameters of DIII-D tokamak H-mode core plasmas which have a peak ion temperature gradient at $r = 0.5a$ with the following local parameters: $R_0/L_T = 6.9$, $R_0/L_n = 2.2$, $q = 1.4$, $\hat{s} \equiv (r/q)(dq/dr) = 0.78$, $T_e/T_i = 1$, $\epsilon \equiv r/R_0 = 0.18$, and $\nu^* = 0.045$. Here R_0 is the major radius, L_T and L_n are the temperature and density gradient scale lengths, respectively, T_i and T_e are the ion and electron temperature, and $\nu^* \equiv \epsilon^{-3/2}\nu_{ii}qR_0/v_i$ with $v_i = \sqrt{T_i/m}$ the ion thermal speed. The size of the plasma column is $a = 160\rho_i$ where ρ_i is the thermal ion gyroradius measured at $r = 0.5a$. The simplified physics model included: a parabolic q profile, a pressure profile of $exp\{-[(r-0.5a)/0.3a]^6\}$, a circular cross section, no impurities, and an adiabatic electron response [21] with $\delta n_e/n_0 = e(\phi - \langle\phi\rangle)/T_e$, where $\langle\cdots\rangle$ represents the flux surface average. These global simulations used fixed boundary conditions with $\phi = 0$ enforced at $r < 0.1a$ and $r > 0.9a$. In each nonlinear simulation, we calculated 10000 time steps of the trajectories of 84 million gyrocenter centers interacting with the self-consistent turbulent field, which was discretized by 21 million grid points in a 3-dimensional configuration (128x640x256 grid points in radial, poloidal and toroidal direction, respectively). ITG modes are unstable with these plasma parameters and have a linear threshold of $R_0/L_T = 4$ [22]. The parameter of R_0/L_T represents the strength of the turbulence drive. We scan R_0/L_T in the present studies to assess sensitivity of the collisional effects on the proximity to ITG marginality.

For a strong ITG drive of $R_0/L_T = 6.9$, both turbulence and zonal flows saturate and reach steady state in collisionless simulations. These nonlinear simulations produce fluctuating $\mathbf{E}\times\mathbf{B}$ zonal flows with radial scales varying from a fraction of the system size to small radial scales comparable to those of the turbulence. The frequency spectrum of zonal flows consists of both high frequency GAM component and a relatively broad low frequency component. The turbulence eddies are broken mostly through random shearing by the low frequency component of zonal flows [23]. This results predominantly in the reduction of the radial correlation length and subsequently the turbulence [8]. By comparing to the simulation with zonal flows suppressed, we observed that a significant reduction of ion heat conductivity χ_i in steady state occurs when zonal flows are retained, consistent with earlier local gyrofluid [21,24] and gyrokinetic [25] simulations in a flux-tube geometry. Clearly, zonal flow shear induced decorrelation is the dominant turbulence saturation mechanism. On the other hand, the fact that zonal flows saturate even without collisional damping indicates that strong turbulence can also damp zonal flows possibly through nonlinear effects. When the realistic ion-ion collisions with $\nu^* = 0.045$ are included in the simulation, the steady state ion heat conductivity χ_i is increased by about one half from the collisionless value. The saturated zonal flow level decreases due to collisions. Meanwhile, the change in linear growth rate is negligible. Furthermore, when zonal flows are not included in the simulations, collisions have little effects on the turbulence transport. We therefore conclude that the enhancement of transport by collisions in the presence of zonal flows is through the neoclassical damping of zonal flows.

When turbulence drive R/L_T is reduced, nonlinear flow damping becomes insignificant. The collisionless system can undergo a transition to a flow-dominated state where zonal flows generated from the initial growth of turbulence completely suppress the turbulence and transport [13,22]. In this paper, we characterize this regime as the weak turbulent regime. Collisionless nonlinear flux-tube gyrokinetic simulations of the same plasma parameters have found [22] that, for $R/L_T = 4$ to 6, no transport is produced even though the system is linearly unstable to ITG modes. However, collisions can remove this nonlinear upshift of critical gradient by slowly damping zonal flows. The ion thermal transport level will then strongly depend on the collision frequency. Since most of the plasma volume of the representative DIII-D discharge has $R/L_T < 6$, it should be emphasized that the physics discussed here is relevant to many tokamak experiments.

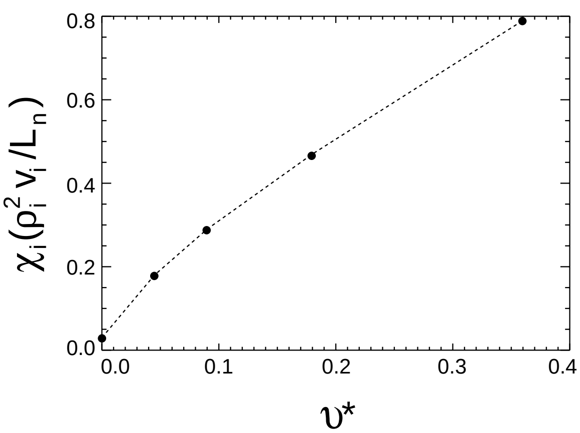

FIG. 2. Ion heat conductivity in nonlinear gyrokinetic simulations with $R/L_T = 5.3$ vs. the ion-ion collision frequency.

We now reduce the turbulence drive to $R/L_T = 5.3$ in our nonlinear simulations. ITG modes remain unstable with linear growth rate $\gamma = 0.072 v_i/L_n$ and frequency $\omega_r = -0.29 v_i/L_n$ at $k_\theta \rho_i = 0.35$. We scan the ion-ion collision frequency upward from experimental value to facilitate studies of the long term collisional effects on the zonal flow dynamics. Again, the collisional effect on linear growth rate of ITG is less than 1% for all the collision frequencies. In the collisionless simulation, the saturated heat flux decreases to insignificant level due to shearing effects of zonal flows which reach a steady state level. When collisions are introduced, zonal flows are damped and a finite transport level is obtained. As we increase collision frequency without changing other parameters, we observe the increase of time-averaged χ_i without a tendency toward a saturation for the whole scan range of collision frequency, which is up to 8 times the experimental value. Fig. 2 shows that the ion heat conductivity sensitively depends on the collision frequency in this weak turbulence regime. This collisionality-dependence of the turbulent transport implies that an accurate treatment of the linear poloidal flow damping [15,17] in nonlinear simulation codes is essential in predicting transport level.

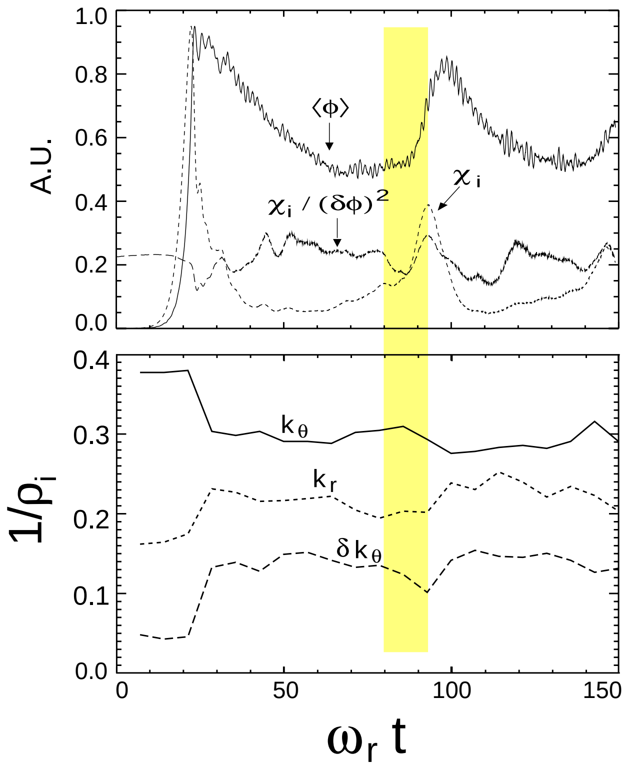

FIG. 3. Time history in nonlinear gyrokinetic simulations for zonal flow amplitude $\langle\phi\rangle$, ion heat conductivity χ_i, turbulence intensity $(\delta\phi)^2$, root mean square values of radial k_r and poloidal k_θ potential spectrum, and width of poloidal spectrum δk_θ.

The simulation time history for the case of $\nu^* = 0.09$ is shown in Fig. 3. The ITG instabilities evolve from a linear phase of exponential growth to a nonlinear stage in which zonal flows are generated. When the effective shearing rate [23], or root mean square shearing rate [13] of zonal flows exceeds the ambient turbulence decorrelation rate which can be approximated by the ITG linear growth rate [24], the ITG turbulence and associated transport are significantly reduced. Zonal flows are then slowly damped by the ion-ion collisions and become weaker. When the effective shearing rate is below the growth rate, the ITG turbulence grows again and drives zonal flows. These turbulence-zonal flows interactions modulated by collisions result in a cyclic, bursting behavior of fluctuations, transport and zonal flows. From a scan of the collision frequency, we observe that the bursting period is of the order of the collisional damping time of the zonal flows, τ_d. The oscillation of zonal flows lags behind that of fluctuation by a phase of 90^o, an indication of the causal relation between the turbulence and zonal flows. The root mean square of radial and poloidal spectra and the width of the poloidal spectrum of the potential fluctuation are also plotted in Fig. 3. During the turbulence growth (shaded band), streamer-like eddies with radial elongation and narrow poloidal spectrum emerges, resembling linear eigenmode structure. This is a favorable condition for zonal flows generation [26]. The subsequent zonal flow generation and turbulence shear decorrelation lead to a much more isotropic and broader turbulence spectra [8]. The observed flow-induced broadening of the fluctuation spectra is in qualitative agreement with the nonlinear theoretical predictions [13,23]. The streamer-zonal flow interaction may have implication on the relevance of self-organized-criticality (SOC) paradigm to tokamak transport [27].

The richness of the temporal dynamics as shown in Fig. 3 presents an opportunity to further explore the fundamental physics underlying the turbulent transport. The turbulence intensity $(\delta\phi)^2$ and ion heat conductivity χ_i are in phase during burstings. The ratio $\chi_i/(\delta\phi)^2$ stays remarkably close to a constant in time despite an order of magnitude variations in both χ_i and $(\delta\phi)^2$. This observation of the weak turbulence scaling of $\chi_i \propto (\delta\phi)^2$ in nonlinear simulations with zonal flows will provide useful insight for the development of nonlinear theory. We note that the standard weak turbulence theory and early gyrokinetic simulations of similar scaling in a stronger drive regime [28] did not consider effects of zonal flows.

In a weak turbulence system, turbulence-zonal flow-turbulence interaction is the dominant triad [13] and the zonal flows are damped by collisions. The ITG modes are saturated by the zonal flow shear decorrelation. On the other hand, the zonal flows saturate when the generation rate of the modulational instability is balanced by the collisional damping rate. This analysis of the model weak turbulence system [13] yields $(\delta\phi)^2 \propto \nu_{ii}$ in steady state. Since $\chi_i \propto (\delta\phi)^2$ is expected when turbulence drive is weak, one would expect $\chi_i \propto \nu_{ii}$, which is qualitatively consistent with the results from our first-principle gyrokinetic simulations. As we increase collision frequency $\nu_{ii} \rightarrow \infty$, while keeping the linear growth rate finite, one would expect to recover the usual strong turbulence scaling. Transition from weak to strong turbulence regime, however, can be complicated by the collisional detrapping of the resonant particles that are trapped by the waves [29].

In summary, linear collisional damping of spontaneously generated $\mathbf{E} \times \mathbf{B}$ zonal flows in gyrokinetic particle simulations agrees well with recent analytic calculations. Nonlinear simulations show that the turbulent transport is proportional to turbulence intensity and increases with ion-ion collision frequency. The collisionality-dependence of turbulent transport comes from the neoclassical damping of zonal flows which regulate the turbulence. Bursting behavior of density fluctuations with a period close to collisional damping time of flows has also been observed in the weak turbulence regime.

We acknowledge useful conversations with L. Chen, F. L. Hinton, M. N. Rosenbluth, and M. A. Beer. Work supported by DoE Contract No. DE-AC02-76CH03073 (PPPL) and DoE Grant No. 88ER53275 (UCSD), and in part by the Numerical Tokamak Turbulence Project.

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