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LDRD Project Progress Report — Broadly Tunable, Solid-State Coherent Light Sources

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**LDRD Project Progress Report -
Broadly Tunable, Solid-State Coherent Light Sources**

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Abstract

Our program of nanosecond optical parametric oscillator (OPO) development has concentrated on improving the spectral purity and tunability as well as the beam quality of OPO's and other tunable sources based on frequency mixing in nonlinear crystals. To facilitate quick evaluation of new OPO and frequency-mixing configurations and to provide insight to OPO operation, we developed a computer model of seeded OPO operation that includes pump depletion, birefringent walkoff, diffraction, temporal and spatial beam profiles, and cavity optics. It predicts conversion efficiency, beam quality, spectra, and time behavior. Our model development was complemented by laboratory OPO studies that served to validate the model and also to highlight the practical problems associated with limitations of optical coating and nonlinear crystal growth technologies. We present here a set of papers that compare model calculation and laboratory characterization of a seeded, nanosecond OPO, that apply the model to illustrate distortions of spectra and beam quality in frequency mixing, and that point out the peculiarities of two-crystal frequency mixing.

Broadly Tunable, Solid-State Coherent Light Sources

A. V. Smith, W. J. Alford, T. D. Raymond

This project is aimed at improving our understanding of the performance of pulsed optical parametric oscillators (OPOs) as sources for broadly tunable coherent light. The experimental portion of the work concentrated on the performance of an injection-seeded potassium titanyl phosphate ring (KTP) OPO. The theoretical portion of the work concentrated on the development of advanced two-dimensional (2-D), transient response computer models to predict OPO performance with the goal of improving their performance. Collectively, this work represents the first quantitative comparison of carefully characterized experimental data with results from 2-D, transient computer models.

We conducted a series of very well-characterized OPO and optical parametric amplifier (OPA) experiments to serve as a benchmark for our computer simulations. We measured the effective nonlinear coefficients of potassium titanyl phosphate (KTP) and lithium triborate (LBO) to a precision of about $\pm 10\%$. These measurements are essential to a quantitative predictive capability using the computer codes. We also constructed an injection-seeded, KTP ring OPO that is pumped with the spatially filtered second harmonic beam from a Q-switched Nd:YAG laser. We measured the output energy versus input energy, temporally-integrated spatial profiles and spatially-averaged temporal profiles of the signal, incident pump, and depleted pump beams. We also studied the effects of phase velocity mismatch in the crystal upon the frequency of the OPO. This data represents the most careful study to date of the performance of a simple OPO and can be used to test the accuracy of computer models.

We developed a numerical simulation of OPO and OPA operation. We believe this is the first model to incorporate beam diffraction, beam depletion, and birefringent beam walkoff in a single model. Consequently, it is the first that can model a large class of OPOs and OPAs with any degree of realism. With this tool we can investigate the behavior of these devices in unprecedented detail,

which has led to the prediction of frequency shifts due to phase velocity mismatch within the crystal, and to predictions of the frequency spectra, spatial mode patterns, and propagation effects that were not previously reported. In these initial applications of the model, most of these observations were of a qualitative nature. We are now at the point of quantitative comparisons with our experiments, and preliminary comparisons indicate that the model will match experiments well. This benchmarking will provide confidence in the model to use it as a design tool for future, more cleverly designed OPOs and OPAs.

Pyroshock Simulation for Satellite Components Using a Tunable Resonant Fixture

N. T. Davie, V. I. Bateman

Satellite and other aerospace components are often subjected to pyroshock environments due to explosively actuated hardware. Laboratory simulation of this environment is a necessary part of the flight qualification process for these components. This investigation focused on the development of a technique to provide laboratory simulation of pyroshock on large test items such as satellite components. The simulation technique utilizes a fixture that is excited into resonance by a mechanical impact. A component attached to this fixture is subjected to the simulated pyroshock. The main objective in this study was to overcome technical obstacles that had previously limited testing of these large components.

Pyroshock is a mechanical shock caused by the detonation of explosively actuated hardware common to spacecraft and other aerospace structures. This environment is characterized by a high-G, high-frequency acceleration transient. It is desirable to qualify flight hardware to this environment by means of component-level laboratory simulations. Previously existing techniques for testing small components used a resonant fixture that was excited by a mechanical impact. The present research focused on making operational improvements to these techniques and overcoming technical obstacles associated with the testing of

large items such as satellite components. Previous research resulted in a test apparatus that greatly improved operational efficiency and the ability to simulate a wide variety of pyroshock test requirements by means of a tunable fixture. The FY 93 research focused on increasing the size limit of test items.

We aimed initial efforts at simply increasing the size of the existing test apparatus. Analyses revealed that this approach was impractical and would not allow attainment of our component size goals.

A new approach was conceived in which the axial resonance of a long bar could be controlled with appropriate clamping of a large mass at various positions on the bar. The concept was to make an apparatus that approximated a perfectly fixed condition at any selected point along the bar. In theory, the axial response of the free end of the bar would have a first-mode frequency that is inversely proportional to the free length. This desired result was confirmed with a very small-scale test apparatus and finite element analyses. Unfortunately the results could not be duplicated at the larger scale, apparently due to the inability to adequately simulate the fixed condition on the bar.

Information gathered during this process led to a large axially resonant bar fixture capable of testing large satellite components. This bar had a first-mode frequency that could be changed (lowered) by adding mass to the impact end of the bar. This large apparatus was fabricated and experimentally evaluated. Measured data proved the tunability concept for this large apparatus that had a test platform capable of mounting up to a 22"x22" component.

Development of a Space-Qualified Short Wavelength Fourier Transform Spectrometer

R. E. Abbink

The intent of this study is to develop an instrument design and implement data processing methods suitable for a space-based, short wavelength imaging Fourier transform spectrometer (SWIFTS). The goal of this program is to demonstrate basic techniques suitable for a long lifetime

Automatic Planning and Programming for Robotic Construction of Planetary/Lunar Structures

P. T. Boissiere

There are many remote applications that require the dexterous manipulation of tools and materials in the field. These tasks include the assembly and maintenance of lunar and planetary structures. Traditional remote manual field operations have, unfortunately, proven to have very low productivity when compared with unencumbered human operators. Recent advances in the integration of sensors and computing into the control of remotely operated equipment show great promise for reducing the cost of remote systems while providing faster and safer remote systems.

Sandia developed the REMote TeleRobotic Vehicle for Intelligent Retrieval (RETRVIR), which integrates model-based, sensor-directed robotic manipulation with a remotely operated vehicle. RETRVIR delivers highly dexterous, sensor-controlled manipulation to remote sites for characterization and retrieval of materials. In particular, current development activities included the assembly of a remote instrument platform utilizing interconnecting structural components. The high strength manipulation capability of RETRVIR gives this remote system the capability to address many realistic applications, which up to this point have been unfeasible for remote systems.

Advanced assembly and excavation capabilities were developed and extended to address the task of remote assembly using RETRVIR. Specifically, autonomous

digging routines were improved that can prepare the remote site for the assembly of a structure. Assembly utilities were improved, which allows the operator to identify the location of the components of an instrument platform to the robotic system and then supervise the autonomous assembly of the structure. RETRVIR was showcased on CNN's *Science and Technology Week*.

Computer-assisted operations allow shared operator/computer control to provide semiautomated manipulation at remote sites as well as reducing operator stress through the use of supervisory control structures and graphical displays that review the operator's commands based on model and sensory information. Also, the graphic displays allow the operator to preview all operations prior to execution to allow the operator to ensure safe operation.

Broadly Tunable, Solid-State Coherent Light Sources

W. J. Alford, T. D. Raymond, A. V. Smith

The objective of this project is to develop improved solid-state laser sources with wide tunability. Our approach was based on the parallel development of numerical models and laboratory characterization of optical parametric oscillators (OPOs) with the goal of improving their beam quality and spectral purity. This parallel approach has produced a laboratory-validated computer model of nanosecond OPOs that should be a valuable tool for designing improved oscillators.

OPOs are laser-pumped devices that convert the pump light to two longer

wavelength coherent beams of light. They are attractive because they are all solid-state, efficient, and can be wavelength-tuned over a broad range. They have generally produced relatively poor quality beams and broad linewidths. It was our goal to improve these qualities so they can be applied to a wider range of uses. Our approach was to construct a detailed numerical model of nanosecond OPOs with the hope that it would both provide insight into the cause of these deficiencies and serve as a useful design tool for improving OPO characteristics. In parallel, we carefully characterized the performance of a laboratory OPO to validate our model and gain experience in solving practical problems of OPO development not addressed in a computer model. This approach was successful. We demonstrated close agreement between our model and laboratory OPOs in every aspect of performance that we studied. We are now using the model to predict the performance of new oscillator designs in our search for better beam quality and spectral purity. We also used both the model and our laboratory experience to construct an OPO tunable from 250 to 400 nm. We designed and may construct OPOs for trace gas detection in the 200-to-400-nm range and for remote sensing in the 3000-to-5000-nm range. Further, our computer codes are in demand by other researchers in nonlinear optics and by designers of OPOs. One of the stated objectives of this project was to design a widely tunable, grating-tuned, single-frequency, Littman cavity OPO. Although this task was not completed, we will continue to develop this device with other funding. If successful, this will be the first such device, and it would certainly be valuable in a wide variety of applications.

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LDRD Project Progress Report

Case number: 3519090000
Project title: Broadly tunable, solid-state coherent light sources
Project manager: Gerald N. Hays
Principal investigators: W. J. Alford, T. D. Raymond, and A. V. Smith

Abstract: We report our progress toward our objective of developing improved solid-state laser sources with wide tunability. Our approach has been based on the parallel development of numerical models and laboratory characterization of optical parametric oscillators with the goal of improving their beam quality and spectral purity. This parallel approach has produced a laboratory validated computer model of nanosecond optical parametric oscillators that should be a valuable tool for designing improved oscillators.

Summary: Optical parametric oscillators (OPO's) are laser-pumped devices that convert the pump light to two longer wavelength coherent beams of light. They are attractive because they are all solid state, efficient, and can be wavelength tuned over a broad range. They have generally produced relatively poor quality beams and broad linewidths. It has been our goal to improve these qualities so they can be applied to a wider range of uses. Our approach has been to construct a detailed numerical model of nanosecond OPO's with the hope that it would both provide insight into the cause of these deficiencies and serve as a useful design tool for improving OPO characteristics. In parallel, we have carefully characterized the performance of a laboratory OPO in order to validate our model and gain experience in solving practical problems of OPO development not addressed in a computer model. This approach has been successful. We have demonstrated close agreement between our model and laboratory OPO's in every aspect of performance that we have studied. We are now using the model to predict the performance of new oscillator designs in our search for better beam quality and spectral purity. We have also used both the model and our laboratory experience to construct an OPO tunable from 250 to 400 nm. We have also designed and may construct OPO's for trace gas detection in the 200 to 400 nm range and for remote sensing in the 3000 to 5000 nm range. Further, our computer codes are in demand by other researchers in nonlinear optics and by designers of OPO's. One of the stated objectives of this program was to design a widely tunable, grating tuned, single frequency, Littman cavity OPO. Although this task was not completed, we will continue to develop this device with other funding. If successful, this will be the first such device and it would certainly be valuable in a wide variety of applications.

Refereed publications resulting from the work:

T. D. Raymond, W. J. Alford, A. V. Smith, and M. S. Bowers, "Frequency shifts in injection seeded optical parametric oscillators with phase mismatch," to be published in Optics Letters.

A. V. Smith and M. S. Bowers, "Phase distortions in sum- and difference-frequency mixing in crystals," to be published in J. Opt. Soc. Am. B.

R. D. Mead, D. D. Lowenthal, T. D. Raymond, W. J. Alford, and A. V. Smith, "Optical parametric oscillator-based laser source continuously tunable from 250 to 400 nm," OSA Proceedings on Advanced Solid-state Lasers **20**, 381-384 (1994).

M. S. Bowers and A. V. Smith, "Optical parametric oscillator modeling with diffraction, depletion, and double refraction," OSA Proceedings on Advanced Solid-state Lasers **20**, 471-475 (1994).

W. J. Alford, T. D. Raymond, and A. V. Smith, "Characterization of a ring optical parametric oscillator," OSA Proceedings on Advanced Solid-state Lasers **20**, 476-479 (1994).

All other reports and publications resulting from the work:

P. J. Hargis, M. S. Johnson, B. Michie, G. N. Hays, G. C. Tisone, B. Clark, C. Wakefield-Reyes, J. S. Wagner, M. W. Trahan, W. E. Nelson, T. D. Raymond, W. J. Alford, A. V. Smith, S. E. Dains, T. L. Downey, R. A. Mills, H. H. Eyer, J. R. Turner, C. W. McDaniel, J. G. Taylor, and S. M. Gentry, "Multispectral UV fluorescence laser remote sensing system," CALIOPE program 1994 interim technical review Proceedings vol II, DOE 1994.

A. V. Smith and M. S. Bowers, "The role of back conversion in pulsed optical parametric oscillators," Optical Soc. Am. annual meeting, Oct. 2-7, 1994 Dallas TX.

W. J. Alford and T. D. Raymond, "Detailed spatial and temporal measurements of a nanosecond optical parametric oscillator," Optical Soc. Am. annual meeting, Oct. 2-7, 1994 Dallas TX.

T. D. Raymond, W. J. Alford, and A. V. Smith, "Frequency shifts in seeded optical parametric oscillators with phase mismatch," Optical Soc. Am. annual meeting, Oct. 3-8, 1993 Toronto, Canada.

M. S. Bowers, A. V. Smith, W. J. Alford, T. D. Raymond, "Numerical modeling of pulsed optical parametric oscillators," Optical Soc. Am. annual meeting, Oct. 3-8, 1993 Toronto, Canada.

P. J. Hargis, G. C. Tisone, J. S. Wagner, M. W. Trahan, A. V. Smith, T. D. Raymond, W. J. Alford, G. N. Hays, A. B. Wegner, J. E. M. Goldsmith, M. Lapp, and S. E. Bisson, "Multispectral-ultraviolet-fluorescence-lidar detection of volatile compounds in the atmosphere," Opt. Soc. Am. annual meeting, Oct. 3-8, 1993 Toronto, Canada.

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Phase correction in two-crystal optical parametric oscillators

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ABSTRACT

The effect of the pump, signal, and idler wave phases on three-wave nonlinear parametric mixing is investigated in a series of single-pass-gain experiments. Measurements are made with two angle-tuned KTP crystals in a 532 nm pumped, walkoff-compensated, optical parametric amplifier that is seeded by an 800 nm cw diode laser. In one of the measurements the second crystal is orientated to have its effective nonlinearity d_{eff} of opposite sign to that of the first crystal, so that all mixing that occurred in the first crystal is cancelled by the second when the phase mismatch $\Delta k_{\text{crystal 1}} = \Delta k_{\text{crystal 2}} = 0$. Efficient two-crystal amplification is subsequently restored by selecting the correct phase relationship for the three waves entering the crystal by inserting a dispersive plate between the crystals. The experimental results are explained in a straightforward manner with diagrams involving the three input wave polarizations. These results demonstrate that walkoff-compensated geometries require phase correction to achieve efficient mixing in the second crystal whenever the nonlinear interaction involves two extraordinary waves (e-waves). One practical application of this work may be lower oscillation thresholds and enhanced performance in walkoff-compensated optical parametric oscillators which use two e-waves.

Keywords: phase correction, walkoff-compensation, optical parametric oscillators

1 INTRODUCTION

High quality nonlinear crystals and pump lasers with sufficiently good spatial and temporal properties have made optical parametric oscillators (OPO's) and optical parametric amplifiers (OPA's) attractive sources of coherent light. These devices use the second order susceptibility of a nonlinear optical crystal to convert a pump photon into signal and idler photons such that $\omega_p = \omega_s + \omega_i$ (where p, s, and i denote pump, signal, and idler, respectively). Although the overall performance of optical parametric devices is improving, practical application of these devices requires further study of certain technical problems. This paper investigates the fundamental problem of the phase difference $\Delta\phi = \phi_p - \phi_s - \phi_i$ in three-wave nonlinear mixing, and demonstrates that understanding the effects of $\Delta\phi$ is critical to efficient operation of optical parametric devices that use two crystals.

Efficient conversion of pump photons into signal and idler photons is usually accomplished with phase matching in birefringent nonlinear crystals. When the phase velocity mismatch

$$\Delta k = k_p - k_s - k_i \quad (1)$$

is angle-critical and double refraction is observed, enhanced gain and output beams that experience little or no displacement with angle tuning can be achieved by using two identical crystals in a walkoff-compensated

geometry. With this geometry, the direction of birefringent walkoff of an e-wave in the first crystal is reversed in the second crystal. Walkoff compensation is not a new idea,¹ yet some misunderstanding persists regarding the use of walkoff compensation with different types of nonlinear interactions. This geometry is straightforward to implement only when the nonlinear interaction involves a single e-wave. With two e-waves, mixing in the second crystal will completely cancel mixing in the first crystal when the phase velocity mismatch Δk has been adjusted to zero for each crystal. With a series of single-pass-gain measurements and a diagrammatic method to explain the experimental results, we demonstrate that walkoff-compensation can always be employed regardless of the form of the nonlinear interaction by controlling the phases of the waves as they enter the second crystal.

Optical parametric devices that utilize a single crystal usually have just two of the three interacting waves incident on the crystal. When this is the case [e.g., sum frequency generation (SFG), or difference frequency generation (DFG)], the phases of the two input waves are unimportant since $\Delta\phi$ will assume the correct value to maximize generation of the third wave. This situation prevails in almost all OPA's and in most OPO's, where the cavity usually resonates on either the signal or idler wave, but rarely on two of the waves simultaneously. However, if the pump, signal, and idler waves are all incident on a nonlinear crystal, $\Delta\phi$ determines the flow of energy among the three waves and affects the efficiency of parametric mixing.

In an optical parametric device that uses two crystals (e.g., for walkoff compensation) with two waves incident on the first crystal, the initial $\Delta\phi$ remains unimportant as it has no effect on the mixing process in the first crystal. However, three waves enter the second crystal, so that $\Delta\phi$ again determines the flow of energy between the three waves. We will show later that if $d_{\text{eff}}^{(2)} = d_{\text{eff}}^{(1)}$, then $\Delta\phi$ remains unchanged, but $d_{\text{eff}}^{(2)} = -d_{\text{eff}}^{(1)}$ is equivalent to changing $\Delta\phi$ by π .

Figure 1: A steady state (i.e., cw) calculation of signal (—) and idler (---) intensities demonstrating the effects of the phase difference $\Delta\phi = \phi_p - \phi_s - \phi_i$ on single-pass gain in two KTP crystals. Pump and signal wavelengths are 532 nm and 800 nm, respectively. The undepleted pump intensity of $\sim 10^8$ W/cm² is not shown.

Fig. 1 emphasizes the effects of $\Delta\phi$ in the second crystal for two-crystal gain. A one dimensional calculation (i.e., plane waves, no walkoff) with $\Delta k = 0$ shows the evolution of the signal wave intensity I_s (—) and the idler wave intensity I_i (---) plotted against distance in two crystals of length $\ell = 10$ mm. With a pump and signal wave incident on the first crystal, I_s increases in intensity, while I_i begins with zero intensity and grows along

with I_s . At the entrance to the second crystal, I_s and I_i are shown to follow two different paths, depending on the sign of $d_{\text{eff}}^{(2)}$ relative to $d_{\text{eff}}^{(1)}$, or equivalently, on the phase difference $\Delta\phi$. For the paths that continue to increase in intensity, $\Delta\phi$ at the entrance to crystal 2 was set to be the same as at the exit of crystal 1. For the paths that decrease in intensity $\Delta\phi$ was changed from its previous value of $-\pi/2$ to $+\pi/2$ as the waves entered crystal 2. For this "worst case" where $\Delta\phi$ changes by π , mixing in crystal 2 returned I_s to its initial value, and returned I_i to zero.

Since $\Delta\phi$ cannot be neglected with three input waves, how does one ensure that the waves will have the correct $\Delta\phi$ when incident on the second crystal? As we'll show in Sec. 2, optimum mixing can always be obtained if $\Delta\phi$ is corrected by a phase plate (or other dispersive medium) placed between the two crystals. Use of a phase plate is straightforward, and has been previously employed, for example, to measure the phase difference between second harmonics generated in two KTP crystals.² Recent measurements suggest that if $\Delta\phi$ did have the wrong value, correcting $\Delta\phi$ would be worth the effort.⁷ An increase in two-crystal second-harmonic generation of > 3 times that for a single crystal of length 2ℓ has been reported with two walkoff-compensated KTP crystals, each of length ℓ .³

2 Understanding crystal orientation in two-crystal walkoff-compensated optical parametric devices

In Sec. 1 we stated that reversal of the sign of d_{eff} in the second crystal is equivalent to a change in $\Delta\phi$ of π . This can be deduced from the expression for the nonlinear dielectric polarization

$$\mathbf{P}_i^{(2)} = \chi_{ijk}^{(2)} \cdot \mathbf{E}_j \mathbf{E}_k. \quad (2)$$

In Eq. 2, $\mathbf{P}_i^{(2)}$, $\mathbf{E}_j$, and $\mathbf{E}_k$ are vectors and the nonlinear susceptibility coefficient $\chi_{ijk}^{(2)} = 2d_{ijk}$ is a tensor. (d_{eff} involves individual components of d_{ijk} and depends on the type of nonlinear interaction and the crystal symmetry.) The sign of $\mathbf{P}_i^{(2)}$ can be inverted by changing the sign of either $\mathbf{E}_j$ or $\mathbf{E}_k$ (adding π to $\Delta\phi$), or by changing the sign of $\chi^{(2)}$ (reversing the sign of d_{eff}). Since nonlinear frequency conversion depends on the phase (i.e., the sign) of $\mathbf{P}_i^{(2)}$ through the Maxwell wave equation,^{4,5} the equivalence of a π phase shift in $\Delta\phi$ and sign reversal of d_{eff} is straightforward to understand.

Since $\chi^{(2)}$ is a tensor, d_{eff} has a well defined direction in space with respect to the crystal lattice. The absolute sign of d_{eff} , which depends on the orientation of a given nonlinear crystal, is difficult to determine. Fortunately, the absolute sign of d_{eff} is also of little importance. What is important in any two-crystal optical parametric device are the relative signs of the d_{eff} 's in the two crystals. What we want to understand then is the following fundamental problem: For which nonlinear interactions (i.e., *ooe*, *eeo*, *ooo*, etc., where *o* and *e* denote o- and e-waves for the signal, idler, and pump) is it possible to find a correct orientation for the second crystal in a walkoff-compensated geometry? This question can be answered by a purely mathematical approach through the equations of nonlinear optics.^{4,5} Alternatively, it can be answered with simple diagrams involving the input wave polarizations as shown in Fig. 2. Both approaches are equally rigorous (i.e., there are no approximations used), and lead to the same interesting result: The relative sign of d_{eff} does not depend on the particular symmetry of the crystal, but only on the type of nonlinear interaction involved.

The diagrammatic method in Fig. 2 addresses the second-crystal-orientation problem by indicating a sign for d_{eff} while observing the phase relationship of three input waves. In Fig. 2(a), a nonlinear crystal is depicted by a box with an arrow drawn on top of it. The arrow indicates the direction of walkoff θ_{WO} , but tells us nothing about the crystal symmetry or actual sign of d_{eff} . The diagrams only serve to determine if there is a correct way to orient the second crystal to obtain the most efficient two-crystal mixing. It is worth emphasizing that every crystal orientation in Fig. 2 will phase match, regardless of the signs of θ_{WO} and d_{eff} .

Figure 2: Diagrams involving the input wave polarizations that are used to understand crystal orientation in two-crystal walkoff-compensated optical parametric devices. Mixing with one e-wave is used as an example.

At the entrance to the crystal in Fig. 2(a), three polarization vectors are drawn depicting the *ooo* interaction, appropriate for the KTP crystals used in our experiments. We now establish a convention: When the e-wave points out of the page $d_{\text{eff.}} = d_o$, and when the e-wave points into the page $d_{\text{eff.}} = -d_o$. To use the diagrams to determine if there is a correct orientation for the second crystal, we simply rotate the crystal and the input waves 180° about three orthogonal axes in space. The correct orientation results when $d_{\text{eff.}} = d_o$ and $\theta_{\text{wo}} = -\theta_o$.

In Fig. 2(b), the crystal has been rotated about a horizontal axis (dashed line) that intersects the sides of the crystal. The two o-waves have been inverted, which is equivalent to two o-waves pointing up, and the e-wave is pointing out of the page so that $d_{\text{eff.}} = d_o$. We see from the walkoff arrow on the crystal that $\theta_{\text{wo}} = -\theta_o$, so we have already found the correct orientation. In Fig. 2(c), the crystal has been rotated about a vertical axis. The walkoff direction has not changed, so this orientation is of no interest. In Fig. 2(d), the crystal has been rotated about a horizontal axis passing through the entrance and exit faces of the crystal. As in Fig. 2(c), the e-wave is pointing into the page indicating $d_{\text{eff.}} = -d_o$.

The methodology outlined above can be applied to any nonlinear interaction. With the type-I *ooe* interaction, the results are identical to the type-II *ooo* interaction in Fig. 2. For a type-II *oeo* interaction, one finds that there is no correct orientation for the second crystal. While it is possible to orient the second crystal to compensate for walkoff, the sign of $d_{\text{eff.}}$ in the second crystal will always be opposite to $d_{\text{eff.}}$ in the first crystal. Repeated application of the model leads to two valuable rules for all possible forms of the nonlinear interaction. With one e-wave, there is always one correct orientation for the second crystal; with two e-waves there is *never* a correct orientation for the second crystal. When using two crystals cut for a two e-wave interaction in a walkoff-compensated geometry, one must adjust the phase difference $\Delta\phi$ to maximize the gain.

3 EXPERIMENT

A diagram of the apparatus used for the single-pass-gain experiments is shown in Fig. 3. The measurements were performed with an OPA consisting of two length $\ell = 10$ mm KTP crystals with $5 \text{ mm} \times 5 \text{ mm}$ faces mounted in a walkoff-compensated geometry. The crystals were separated by ~ 5 mm of air. Both crystals had cut angles of $\theta = 58^\circ$ with $\Phi = 0$ (type-II, *eo* interaction, propagation in the XZ plane). The crystals were rotated by stepping motors, through a reduction gear assembly, with external angle resolution of $\sim 78.5 \mu\text{rad}$ and internal angle resolution $\sim 43.5 \mu\text{rad}$. The angular resolution was more than sufficient as this cut of KTP at a signal wavelength of 800 nm has an acceptance angle of 0.93 mrad-cm. The walkoff angle is 49 mrad, which results in < 0.5 mm displacement of the extraordinary signal wave for $\ell = 10$ mm. The OPA was pumped by the second harmonic of an injection-seeded, spatially-filtered Nd:YAG laser with typical pulse energies of 12 mJ and pulse lengths of $\gtrsim 7$ ns FWHM. A 30 mW seed beam was provided by a spatially-filtered, single-longitudinal-mode 800 nm cw diode laser. The pump and signal beam diameters were 2.5 mm and 1.5 mm, respectively, at their $1/e^2$ points. With the low peak pump fluence, $\lesssim 0.25 \text{ J/cm}^2$, and single-crystal single-pass-gain of typically $\lesssim 6$, there was no observable pump depletion.

To collect data, the pump and signal beams were each carefully collimated and then overlapped with the help of a camera. The crystals were then placed in the beams and rotated to positions where the signal gain was high (not necessarily $\Delta k_1, \Delta k_2 \approx 0$, depending on the orientation of the second crystal), and a $525 \mu\text{m}$ diameter aperture was placed downstream from the crystals in the signal beam. The small aperture was then positioned to sample that portion of the signal beam spatial profile where the amplified signal was maximum. The $\Delta k_1, \Delta k_2$ "gain surfaces" were then recorded by first individually locating $\Delta k_1 = 0$ and $\Delta k_2 = 0$ as accurately as possible, and then rotating each crystal a known number of steps away from $\Delta k = 0$ to its starting point. A computer controlled data acquisition system then recorded the input signal level, the peak amplified signal, and the incident and transmitted peak pump intensities while rotating the crystals on a $\Delta k_1, \Delta k_2$ grid. The grids consisted of 40×40 or 40×50 points, depending on the orientation of the second crystal, with 1 or 2 stepping motor steps between each point. Three laser shots were averaged for the data recorded at each $\Delta k_1, \Delta k_2$ grid point. This procedure was repeated three times as the second crystal was placed first in the correct orientation, then in the d_{eff} -reversed incorrect orientation, and finally in the d_{eff} -reversed incorrect orientation with a phase compensation plate inserted between the crystals. The phase plate used in these experiments consisted of an uncoated $100 \mu\text{m}$ thick optically flat window of BK7 glass.

As shown in Fig. 3, great care was taken to ensure that the 800 nm signal alone reached the signal detector by first separating the pump and signal beams with a dichroic beam splitter. The pump, idler, and any residual Nd:YAG fundamental were then rejected by a combination of an RT830 glass filter, followed by high reflectors for 532 nm and 1064 nm.

Although the experimental results presented in this paper are primarily qualitative, the experiment was configured to make absolute measurements of d_{eff} for single crystals. This was accomplished by using a highly collimated, spatially and temporally smooth pump beam with a known intensity distribution (i.e., the central spot of an Airy pattern), and recording the peak intensities of the pump and amplified signal pulses with fast-samplers with gate widths of 200 ps. Both pump and signal detectors had a bandwidth of ~ 1 GHz. Any time jitter of the 200 ps gates with respect to the pump and signal pulses was effectively eliminated by triggering the fast-samplers with a constant-fraction-discriminator that was triggered by detecting the pump pulse with a 200 ps rise time photodiode. The detector that monitored the transmitted pump pulse energy was calibrated against a thermal detector.

With the $525 \mu\text{m}$ diameter aperture in the signal beam, the gain of the signal $I_s(2\ell)/I_s(0)$ was measured only for the small central portion of the interacting beams where the pump beam and the signal gain were nearly uniform. The signal beam walkoff of < 0.5 mm could have only a negligible effect on the measurement due to the comparatively large 2.5 mm pump beam diameter. With a 200 ps gate and the small sample area, it was possible to accurately determine the peak pump intensity. The absolute value of d_{eff} could then be extracted

Figure 3: Diagram of the experimental apparatus used to measure two-crystal single-pass signal gain on a $\Delta k_1 \ell / \pi$, $\Delta k_2 \ell / \pi$ grid with and without a phase compensation plate.

(albeit *approximately* for two-crystals) from the peak signal gain and peak pump intensity. When $\Delta k = 0$, the gain for one crystal is given by⁶

$$I_s(\ell)/I_s(0) = \cosh^2 \Gamma \ell \quad (3)$$

where

$$\Gamma^2 = \frac{8\pi^2 d_{\text{eff}}^2 I_p}{\epsilon_0 \lambda_s \lambda_i n_p n_s n_i c}. \quad (4)$$

With two crystals, $\ell \rightarrow 2\ell$ in Eq.3.

When either $|\Delta k_1|$ or $|\Delta k_2|$ is nonzero, the simple expression in Eq. 3 cannot be used to reliably extract d_{eff} for the two crystals. Instead, analytic expressions in Byer and Herbst⁶ for nonzero Δk can be extended to describe two-crystal gain with no pump depletion. Fields $E_s^{(1)}$ and $E_i^{(1)}$ from the first crystal are inserted in equations for $E_s^{(2)}$ and $E_i^{(2)}$ for the second crystal while including a linear phase shift $\Delta k_1 \ell_1$ from the first crystal and an arbitrary phase shift θ , while keeping track of $d_{\text{eff}}^{(1)}$ and $d_{\text{eff}}^{(2)}$ and their respective signs if necessary.⁷

Our method was intended for single crystal measurements of d_{eff} .⁷ Nonetheless, what we call “effective d_{eff} ,” where d_{eff} is obtained by treating two crystals as a single crystal with $\Delta k = 0$, is still a useful quantity for comparing our two-crystal gain measurements. The accuracy of effective d_{eff} is limited however, as phase shifts from AR coatings on the crystals and from air between the crystals, as well as any reflection losses from exiting crystal 1 and entering crystal 2, are all ignored.

4 RESULTS AND DISCUSSION

The type-II KTP crystals used for these measurements were a convenient choice since correct and incorrect second-crystal orientations were available with the *eo* interaction. In addition, this cut of KTP has typical $d_{\text{eff.}} \approx 2.9 \text{ pm/V}$,⁷ so that high single-pass gain was possible. The three different two-crystal single-pass-gain measurements on $\Delta k_1 \ell / \pi$, $\Delta k_2 \ell / \pi$ grids are shown in Figs. 4, 5, and 6. Since the original number of data points on the grids was large, the data were binned to 1/2 the original density, which results in some minor smoothing by interpolation.

Fig. 4 shows the gain surface for the correct orientation of the second crystal as depicted in Fig. 2(b), plotted against an effective $d_{\text{eff.}}$ scale. For this orientation, the maximum effective $d_{\text{eff.}}$ was found to be $\sim 2.43 \text{ pm/V}$. The two KTP crystals had measured values of $d_{\text{eff.}}$ of approximately 2.9 pm/V and 2.1 pm/V ,⁷ which apparently results in the measured effective $d_{\text{eff.}}$ of 2.43 pm/V . This gain surface reveals minor deviations from "perfect symmetry," which would consist of a central peak at $\Delta k_1 = \Delta k_2 = 0$ with four equal sidelobes, two each lying slightly offset from the lines $\Delta k_1 = 0$ and $\Delta k_2 = 0$. These deviations are most likely from the unequal $d_{\text{eff.}}$'s, and from small phase shifts associated with the multi-layer AR coatings on the crystals and from the air between the crystals. Since modulations of $\sim 10\%$ could be observed on the transmitted pump and idler intensities with crystal rotation (i.e., the crystals behaved like low finesse étalons at these wavelengths), one might suspect the presence of parasitic oscillations. However, these oscillations were not present since the pump fluence in this experiment was well below the threshold for parasitic oscillations.

Fig. 5 shows the gain surface for the incorrect orientation of the second crystal as depicted in Fig. 2(d). The extent of $\Delta k_2 \ell / \pi$ is more than twice that of Fig. 4 in order to record more of this complicated surface. One striking feature of Fig. 5 is the null in the gain surface that runs diagonally across the grid near $\Delta k_1 = \Delta k_2 \approx 0$. With the mismatched $d_{\text{eff.}}$'s one might expect the null to occur some distance from $\Delta k_1 = \Delta k_2 = 0$. However this was not the case since parametric gain depends exponentially on $[\Gamma^2 - (\Delta k/2)^2]^{1/2} \ell$, so only a small difference between Δk_1 and Δk_2 was required to produce the null despite the unequal $d_{\text{eff.}}$'s. An important but perhaps less striking feature is the reduced gain available from either peak that straddles the null. For this incorrect orientation, the maximum effective $d_{\text{eff.}}$ was found to be $\sim 1.67 \text{ pm/V}$ from Eq. 3. While use of Eq. 3 assumes that $\Delta k_1 = \Delta k_2 = 0$, which is strictly an approximation, $d_{\text{eff.}} = 1.67 \text{ pm/V}$ is actually the number of interest since it indicates the maximum available gain with the incorrect orientation. A true two-crystal calculation like that described in Sec. 3, which included the nonzero Δk_1 and Δk_2 from the data returned an effective $d_{\text{eff.}} \approx 2.2$. The gain surface of Fig. 5 clearly illustrates the problem with the wrong orientation of the second crystal for a single e-wave interaction. Fig. 5 also illustrates the problem with a two e-wave interaction where there is no correct walkoff-compensated orientation for the second crystal. Not only is the gain lower than one might naively expect, but the two regions of highest gain could correspond to different oscillation frequencies in an OPO cavity. The two regions of maximum gain in Fig. 5 should be separated by two times the acceptance bandwidth of a single KTP crystal, or roughly 18 cm^{-1} (540 GHz).

If we temporarily ignore phase shifts associated with AR coatings and air between the crystals, Fig. 1 demonstrated that a perfect phase compensation plate would change $\Delta \phi$ by an odd multiple of π . In the calculation of Fig. 1, the gain was cancelled by changing $\Delta \phi$ from $-\pi/2$ to $+\pi/2$. We now want to change $\Delta \phi$ back to $-\pi/2$, $-5\pi/2$, $-9\pi/2$, etc., by shifting the phase by an odd integer multiple of π . For the pump, signal, and idler wavelengths of 532 nm , 800 nm , and 1588 nm , the uncoated $100 \mu\text{m}$ thick BK7 plate gave a phase shift of slightly less than 5π . Tilting the plate can result in some additional $\Delta \phi$, but observed changes in the gain were difficult to interpret since the input intensities to the second crystal depend on the angle of the flat, uncoated plate. AR coatings on the plate would reduce the angle dependence, but knowledge of the dispersive properties of the multilayer coatings would be essential.

Fig. 6 now shows the same incorrect crystal orientation of Fig. 5, but with the phase compensation plate inserted between the crystals. The remaining additional sidelobe structure along $\Delta k_1 = 0$, as compared to that seen in Fig. 4, confirms, as we expected, that we didn't have a perfect phase plate. Exact phase correction would

Correct Orientation

Figure 4: Single-pass signal gain as a function of $\Delta k_1 \ell / \pi$, $\Delta k_2 \ell / \pi$ for two walkoff-compensated KTP crystals, plotted on a scale of "Effective $d_{\text{eff.}}$." The second crystal has the correct orientation where $d_{\text{eff.}}^{(2)} = d_{\text{eff.}}^{(1)}$.

Incorrect Orientation

Figure 5: Single-pass signal gain as a function of $\Delta k_1 \ell / \pi$, $\Delta k_2 \ell / \pi$ for two walkoff-compensated KTP crystals, plotted on a scale of "Effective $d_{\text{eff.}}$." The second crystal has the incorrect orientation, such that the sign of $d_{\text{eff.}}^{(2)}$ is reversed with respect to the sign of $d_{\text{eff.}}^{(1)}$.

Incorrect Orientation With Phase Plate

Figure 6: Single-pass signal gain as a function of $\Delta k_1 \ell / \pi$, $\Delta k_2 \ell / \pi$ for two walkoff-compensated KTP crystals, plotted on a scale of "Effective $d_{\text{eff.}}$." The second crystal has the incorrect orientation, such that the sign of $d_{\text{eff.}}^{(2)}$ is reversed with respect to the sign of $d_{\text{eff.}}^{(1)}$. A dispersive plate inserted between the crystals changed the phase difference $\Delta\phi = \phi_p - \phi_s - \phi_i$ of the three waves by $\sim 5\pi$ before entering the $d_{\text{eff.}}$ -reversed second crystal.

have required exact knowledge of the phase shifts from the AR coatings on the crystals and from the air between the crystals. Nonetheless, a single peak near $\Delta k_1, \Delta k_2 = 0$ with nearly the same effective d_{eff} , as in Fig. 4 has been restored by simply inserting a glass plate. The maximum effective d_{eff} is now 2.27 pm/V, which is quite an improvement over the value of 1.67 pm/V measured in Fig. 5. One important result here is that gain for walkoff-compensation with two e-waves can clearly be enhanced by a very simple technique.

5 CONCLUSION

The effect of the phase relationship of three input waves on parametric mixing has been studied by measuring single-pass-gain with a two-crystal walkoff-compensated OPA. Experimental results and the use of simple diagrams demonstrate that some type of phase correction must be used to extract the maximum gain from any two-crystal parametric device that uses nonlinear crystals cut for a two e-wave interaction. Practical applications of this work could result in lower OPO oscillation thresholds and better spectral control when a two e-wave interaction is used.

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Phase distortions in sum- and difference-frequency mixing in crystals

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We show that if two waves are incident on a quadratically nonlinear crystal, with the third wave generated entirely within the crystal, a phase-velocity mismatch ($\Delta k \neq 0$) leads to intensity-dependent phase shifts of the generated wave only if there is walk-off, linear absorption, or significant diffraction of at least one of the waves as well as significant energy exchange among the waves. The result is frequency broadening and wave-front distortion of the generated wave. Although the induced phase distortions are usually quite small, they may be significant in applications that require high spectral resolution or pointing accuracy.

1. INTRODUCTION

In the limit of low mixing efficiency, three-wave mixing in a quadratically nonlinear medium has been described analytically. For lowest-order Gaussian beam profiles, Boyd and Kleinman¹ presented such a treatment, including walk-off and diffraction. In this weak mixing limit the phase profiles of the output waves are independent of the beam intensities. When energy exchange becomes significant we might expect the output phases to depend on the strength of the mixing, which is determined in part by the input intensities. In this case analytical solutions for the full problem with walk-off, diffraction, and depletion are not available. However, recent experiments and analyses of second-harmonic generation have shown²⁻⁵ that, when energy exchange is significant, a phase-velocity mismatch ($\Delta k \neq 0$) among the interacting waves does indeed lead to intensity-dependent phase shifts of the fundamental wave. In some respects these phase shifts mimic those induced by an intensity-dependent refractive index, causing self-focusing or self-defocusing.³ This raises the question whether there is a similar intensity-dependent shift in the phase of the second-harmonic wave that could lead to distortions of its wave front and, for pulsed light, to frequency chirps. Such effects could cause line-shape distortions in high-resolution spectra under some conditions. They could also cause time-dependent steering of the second-harmonic beam.

Here we consider the effects of phase-velocity mismatch in three-wave sum- and difference-frequency mixing in nonlinear crystals for pulse durations of a few nanoseconds. We consider only cases of moderate nonlinear drive such as might be encountered when one is striving for efficient frequency conversion with good output beam quality. We show that, when diffraction, walk-off, absorption, and group-velocity mismatch are insignificant, and only two waves are incident upon the crystal, the generated third wave will not acquire an

intensity-dependent phase shift, apart from possible 180-deg phase reversals. All the phase distortion introduced by phase-velocity mismatch will show up in the two waves that had nonzero input intensity. If diffraction, walk-off, or absorption becomes important, or if all three waves are incident upon the crystal, this is no longer true. The output phases of all three waves will vary with the input intensities. Combined with spatial and temporal intensity variations of the input beams, this intensity dependence produces frequency shifts and wave-front distortions for all three waves.

First we will consider plane waves and the influence of linear absorption, input amplitudes, and input phases on the output phases. Then, to illustrate the phase distortions introduced by walk-off, we will present results from a numerical model of nonlinear mixing that includes spatial and temporal beam profiles, birefringence, linear absorption, diffraction, and phase-velocity mismatch. The model also permits significant energy exchange among the three waves.

2. EXACT PLANE-WAVE SOLUTIONS

The equations that describe the nonlinear interaction for plane waves in SI units are⁶

$$\frac{d\epsilon_s}{dz} = i \frac{d_{eff}}{c} \frac{\omega_s}{n_s} \epsilon_p \epsilon_i^* \exp(i\Delta k z) - \alpha_s \epsilon_s,$$

$$\frac{d\epsilon_i}{dz} = i \frac{d_{eff}}{c} \frac{\omega_i}{n_i} \epsilon_p \epsilon_s^* \exp(i\Delta k z) - \alpha_i \epsilon_i,$$

$$\frac{d\epsilon_p}{dz} = i \frac{d_{eff}}{c} \frac{\omega_p}{n_p} \epsilon_s \epsilon_i \exp(-i\Delta k z) - \alpha_p \epsilon_p, \quad (1)$$

where the electric field E_ω at frequency ω is given by

$$E_\omega = 1/2 \{ \epsilon_\omega \exp[-i(\omega t - kz)] + \epsilon_\omega^* \exp[i(\omega t - kz)] \},$$

(2)

the phase velocity mismatch Δk is defined by

$$\Delta k = k_p - k_s - k_i, \quad (3)$$

and

$$\omega_p = \omega_s + \omega_i. \quad (4)$$

The subscripts p , s , and i refer to the pump, signal, and idler waves, respectively, as is customary. The coefficients d_{eff} and α are the effective nonlinear mixing coefficient and the linear absorption coefficient, respectively. Following the method of solution given by Armstrong *et al.*,⁷ we write the fields as

$$\epsilon_n = \epsilon_n \exp(i\vartheta_n), \quad (5)$$

where the ϵ 's are real valued. The mixing equations become

$$\begin{aligned} \frac{d\epsilon_s}{dz} &= -\epsilon_p \epsilon_i \frac{d_{\text{eff}}}{c} \frac{\omega_s}{n_s} \sin \theta - \alpha_s \epsilon_s, \\ \frac{d\epsilon_i}{dz} &= -\epsilon_p \epsilon_s \frac{d_{\text{eff}}}{c} \frac{\omega_i}{n_i} \sin \theta - \alpha_i \epsilon_i, \\ \frac{d\epsilon_p}{dz} &= +\epsilon_s \epsilon_i \frac{d_{\text{eff}}}{c} \frac{\omega_p}{n_p} \sin \theta - \alpha_p \epsilon_p, \\ \frac{d\vartheta_s}{dz} &= \frac{d_{\text{eff}}}{c} \frac{\omega_s}{n_s} \frac{\epsilon_p \epsilon_i}{\epsilon_s} \cos \theta, \\ \frac{d\vartheta_i}{dz} &= \frac{d_{\text{eff}}}{c} \frac{\omega_i}{n_i} \frac{\epsilon_p \epsilon_s}{\epsilon_i} \cos \theta, \\ \frac{d\vartheta_p}{dz} &= \frac{d_{\text{eff}}}{c} \frac{\omega_p}{n_p} \frac{\epsilon_s \epsilon_i}{\epsilon_p} \cos \theta, \end{aligned} \quad (6)$$

where θ is defined by

$$\theta = \vartheta_p - \vartheta_i - \vartheta_s + \Delta k z. \quad (7)$$

If the linear absorptions are all zero, the solution for θ given by Armstrong *et al.*¹⁷ is

$$\cos \theta = \frac{\left(\Gamma - \frac{c \Delta k n_p \epsilon_p^2}{2 d_{\text{eff}} \omega_p} \right)}{\epsilon_s \epsilon_i \epsilon_p}, \quad (8)$$

where Γ is an integration constant.

It can be shown that, if $\epsilon_i(z=0)$ and $\alpha_i = \alpha_s + \alpha_p$,

$$\cos \theta = \frac{\Delta k c}{2 d_{\text{eff}}} \frac{n_i}{\omega_i} \frac{\epsilon_i}{\epsilon_s \epsilon_p} \quad (9)$$

is the solution for θ . Similarly, if $\epsilon_p(z=0) = 0$ and $\alpha_p = \alpha_i + \alpha_s$,

$$\cos \theta = -\frac{\Delta k c}{2 d_{\text{eff}}} \frac{n_p}{\omega_p} \frac{\epsilon_p}{\epsilon_s \epsilon_i} \quad (10)$$

is the solution. Thus, if the idler starts with zero intensity, from Eqs. (6) the equation for the idler phase becomes

$$\frac{d\vartheta_i}{dz} = +\frac{\Delta k}{2}. \quad (11)$$

This means that the phase of the idler wave is shifted relative to that of a solo idler wave by $+\Delta k z/2$ but is independent of the intensities of the three interacting waves.

We should point out that, although the idler phase is correctly given by Eq. (11), the amplitude ϵ_i can change sign if the mixing is strong enough to deplete the idler wave totally. When this occurs it results in an apparent abrupt 180° phase shift as the amplitude ϵ_i passes through zero. Because Eqs. (6) are symmetric in signal and idler, our discussion for the idler applies equally to the signal.

If the pump-wave intensity starts from zero, the equation for the pump phase becomes

$$\frac{d\vartheta_p}{dz} = -\frac{\Delta k}{2}, \quad (12)$$

and the pump phase is independent of the intensities of the three waves.

If the linear losses do not satisfy $\alpha_i = \alpha_s + \alpha_p$ in the case where $\epsilon_i(z=0) = 0$, the phase of the idler wave will vary with intensity. Similarly, if the condition $\alpha_p = \alpha_i + \alpha_s$ is violated for $\epsilon_p(z=0) = 0$ the pump phase will be intensity dependent.

In contrast to the intensity independence of the output phase that we just demonstrated for a wave that starts with zero intensity, if Δk is nonzero and the wave starts with nonzero intensity its input phase cannot be adjusted to make its output phase independent of the input intensities. We show this for the idler wave by combining Eq. (8) with the expression for $d\vartheta_i/dz$ in Eqs. (6) to get

$$\begin{aligned} \frac{d\vartheta_i}{dz} &= \frac{d_{\text{eff}} \omega_i}{c n_i \epsilon_i^2} \left\{ \epsilon_i(0) \epsilon_s(0) \epsilon_p(0) \cos \theta(0) \right. \\ &\quad \left. + \frac{\Delta k c n_p}{2 d_{\text{eff}} \omega_p} [\epsilon_p^2(0) - \epsilon_p^2] \right\}. \end{aligned} \quad (13)$$

Applying the Manley-Rowe relation

$$\frac{n_i}{\omega_i} [\epsilon_i^2 - \epsilon_i^2(0)] = \frac{n_p}{\omega_p} [\epsilon_p^2(0) - \epsilon_p^2] \quad (14)$$

yields, for Eq. (13),

$$\begin{aligned} \frac{d\vartheta_i}{dz} &= \frac{d_{\text{eff}} \omega_i}{c n_i \epsilon_i^2} \left[\epsilon_s(0) \epsilon_i(0) \epsilon_p(0) \cos \theta(0) \right. \\ &\quad \left. - \frac{\Delta k c n_i}{2 d_{\text{eff}} \omega_i} \epsilon_i^2(0) \right] + \frac{\Delta k}{2}. \end{aligned} \quad (15)$$

The idler output phase will be independent of the idler intensity only if the first term on the right-hand side in Eq. (15) is also independent of the idler intensity. This requires that the quantity in brackets be zero, i.e., that

$$\cos \theta(0) = \frac{\Delta k c n_i}{2 d_{\text{eff}} \omega_i} \frac{\epsilon_i(0)}{\epsilon_s(0) \epsilon_p(0)}. \quad (16)$$

For any set of input intensities the idler input phase could be adjusted to meet this condition. However, this input phase would depend on the input intensities, violating the goal of intensity independence. Thus there is no single choice of input idler phase that allows the output idler phase to be intensity independent. If all three waves have nonzero incident intensity, all three output phases vary with input intensity.

To summarize, we have shown that, for plane waves with no linear absorption, if one wave enters the crystal

with zero intensity its output phase will be independent of the intensities of the input waves and will depend only on $\Delta k L$, where L is the crystal length. The output phases of the other two waves will be intensity dependent if $\Delta k \neq 0$. If there is linear absorption the phase of the wave that started with zero intensity will also become intensity-dependent for $\Delta k \neq 0$, unless a special condition of the absorption coefficients is met ($\alpha_i = \alpha_s + \alpha_p$ or $\alpha_p = \alpha_i + \alpha_s$). If the third wave enters the crystal with nonzero intensity, its input phase cannot be adjusted to make its output phase intensity independent if $\Delta k \neq 0$.

Clearly, when $\Delta k \neq 0$, the intensity independence of the output phase of one wave is satisfied only in the special circumstance that the wave starts with zero intensity. It is also necessary that the balance of amplitudes and phases of the three interacting waves as they progress through the crystal not be altered from that of plane waves interacting without loss (or meeting the special loss criterion). These results were derived for plane waves. In the remainder of this paper we consider waves that are spatially and temporally nonuniform. In that case, effects that upset the special balance required for an intensity-independent output phase include different rates of diffractive spreading for the three waves, group-velocity mismatch among the waves, and walk-off among the three waves. If any of these effects is strong enough that the plane-wave approximation is invalid, and if $\Delta k \neq 0$, we expect the phases of all three waves at the crystal output face to depend on the input intensities. The phases will vary in space and time, leading to frequency chirps and wavefront distortions for all three waves. Furthermore, even if Δk is nominally zero, we expect walk-off to lead to intensity-dependent output phases for all three waves if the beams have diameters small enough that diffractive phase shifts are significant. The explanation is that the diffractive phase slippage mimics nonzero Δk . Alternatively, one can argue that small-diameter beams consist of a sum of plane waves with a range of transverse k -vector components, so Δk is not truly zero for nonlinear interactions among many contributions of the plane-wave components.

3. NUMERICAL MODELING WITH NONUNIFORM SPATIAL AND TEMPORAL PROFILES

Our discussion of output phases for nonuniform beams was based on plane-wave interactions and was by necessity qualitative in nature because analytic solutions of the mixing equations that include all the effects important for nonuniform beams are not available. To achieve quantitative results we must resort to numerical modeling when we include the effects of nonzero Δk in the presence of birefringence and large energy exchange among the three mixing waves.

We have developed a time-dependent model of three-wave mixing that numerically integrates the three wave equations with the inclusion of walk-off and diffraction in the paraxial approximation. The model describes the stepwise time evolution of each beam on a two-dimensional spatial grid of intensity and phase. We assume Gaussian spatial and temporal input profiles

for the three beams. The integration of the three wave equations through the nonlinear crystal is performed by use of Fourier-transform techniques, as for the method described by Dreger and McIver.⁸ When nonuniform transverse profiles are considered and the transverse derivatives are kept in the wave equations in order to account for walk-off and diffraction, the equations that replace Eqs. (1) take the form

$$\frac{\partial \epsilon_j(x, y, z, t)}{\partial z} = \frac{i}{2k_j} \left[\frac{\partial^2 \epsilon_j(x, y, z, t)}{\partial y^2} + \frac{\partial^2 \epsilon_j(x, y, z, t)}{\partial x^2} \right] - \tan \rho \frac{\partial \epsilon_j(x, y, z, t)}{\partial x} + P_j(x, y, z, t) - \alpha_j \epsilon_j(x, y, z, t), \quad (17)$$

where j is the frequency index and ρ is the walk-off angle in the x direction. For ordinary or y -polarized light, ρ is zero. For extraordinary or x -polarized light, ρ is the walk-off angle appropriate for the crystal orientation of interest. We have ignored the small anisotropy in the diffractive term for extraordinary waves propagating in a birefringent crystal.⁸ P_j is the polarization term at frequency ω_j and is given by

$$\begin{aligned} P_s(x, y, z, t) &= i \frac{d_{\text{eff}}}{c} \frac{\omega_s}{n_s} \epsilon_p(x, y, z, t) \\ &\quad \times \epsilon_i^*(x, y, z, t) \exp(i\Delta k z), \\ P_i(x, y, z, t) &= i \frac{d_{\text{eff}}}{c} \frac{\omega_i}{n_i} \epsilon_p(x, y, z, t) \\ &\quad \times \epsilon_s^*(x, y, z, t) \exp(i\Delta k z), \\ P_p(x, y, z, t) &= i \frac{d_{\text{eff}}}{c} \frac{\omega_p}{n_p} \epsilon_i(x, y, z, t) \\ &\quad \times \epsilon_s(x, y, z, t) \exp(-i\Delta k z). \end{aligned} \quad (18)$$

Fourier transforming the electric fields and polarizations in the transverse dimension, using

$$\begin{aligned} \epsilon_j(x, y, z, t) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \epsilon_j(s_x, s_y, z, t) \\ &\quad \times \exp[i2\pi(s_x x + s_y y)] ds_x ds_y, \\ P_j(x, y, z, t) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_j(s_x, s_y, z, t) \\ &\quad \times \exp[i2\pi(s_x x + s_y y)] ds_x ds_y, \end{aligned} \quad (19)$$

and substituting these definitions of $\epsilon_j(x, y, z, t)$ and $P_j(x, y, z, t)$ into Eq. (17), we arrive at the following equation for the propagation of the individual spatial-frequency component waves:

$$\frac{\partial \epsilon_j(s_x, s_y, z, t)}{\partial z} = -i \left[\frac{2\pi^2}{k_j} (s_x^2 + s_y^2) + 2\pi s_y \tan \rho \right] \times \epsilon_j(s_x, s_y, z, t) + P_j(s_x, s_y, z, t). \quad (20)$$

This procedure results in three coupled first-order ordinary differential equations for the change in each spatial-frequency component of the fields as they propagate through the crystal. The equations are coupled through the nonlinear interaction term $P_j(s_x, s_y, z, t)$.

Fig. 1. Example fluence profiles for (a) the input fundamental wave and (b) the output second-harmonic wave. The second-harmonic wave walks off in the $+x$ direction. $\Delta k = 0.15 \text{ mm}^{-1}$.

We model the mixing process for each time step by propagating half of a z step, using a Runge-Kutte algorithm to numerically integrate the coupled ordinary differential equations. We then apply a fast-Fourier-transform algorithm to transform the resulting spatial-frequency fields $\epsilon_j(s_x, s_y, z, t)$ into fields $\epsilon_j(x, y, z, t)$ in x - y space. We insert these fields into Eqs. (18) to find $P_j(x, y, z, t)$. We then apply the fast-Fourier transform algorithm again to obtain the $P_j(s_x, s_y, z, t)$'s, which are used in Eq. (20) to propagate the second half of the z step.

The x - y spatial grid is typically 32×32 , and the integration of a single time slice through the crystal is performed in approximately 32 steps. The number of time slices is typically approximately 75. Run time on a Pentium PC is of the order of 1000 s. As described previously,⁹ we have rigorously validated the model by comparison with experiments.

We will focus our discussion on two examples. In the first, we model a situation in which birefringence is combined with nonzero Δk . In the second, birefringence is combined with significant diffraction but with $\Delta k = 0$.

Case 1: Nonzero Phase Mismatch with Birefringence

The first case is one that has been experimentally studied by Gangopadhyay *et al.*¹⁰ They frequency doubled 3.4-mJ 7-ns (FWHM) pulses of 645-nm light in a 3-cm-long KDP crystal. For this Type I doubling, the 645-nm light had ordinary polarization. The 322.5-nm light had extraordinary polarization and a walk-off, or birefringent, angle of 28 mrad. The input beam diameter (FWHM) was 0.25 mm. For this process, $d_{\text{eff}} = 0.32 \text{ pm/V}$. They measured second-harmonic conversion efficiency and phase shifts for $\Delta k = 0$ and $\Delta k = 0.15 \text{ mm}^{-1}$.

Because the input beam diameter is quite large, diffraction is insignificant for this case (the Rayleigh range¹¹ in the crystal is 33 cm). We have verified this by performing calculations with and without diffraction and obtaining identical results. We also neglect group-velocity mismatch as inconsequential. Figure 1 shows our calculated input fundamental [Fig. 1(a)] and output second-harmonic [Fig. 1(b)] fluence profiles for $\Delta k = 0.15 \text{ mm}^{-1}$. Figure 2 displays the time development of the phase of the second-harmonic light at three spatial grid points, again for $\Delta k = 0.15 \text{ mm}^{-1}$. At these three locations the phase of the second-harmonic wave clearly varies with the input intensity of the fundamental waves. If Δk is zero or if the walk-off is artificially set to zero we find that the phase of the second-harmonic light is constant in time and independent of the input fundamental intensity. These observations are consistent with our conclusions drawn from examining the plane-wave solutions above.

To characterize the wave-front distortions at an instant of time we calculate the tilt, curvature, and residual distortion of the output waves. In our notation, x is the walk-off direction and y is perpendicular to it. The tilt angle in the walk-off direction, β_x , is the first moment in the spatial-frequency domain, defined by

$$\beta_x(t) = \frac{-\lambda \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s_x |\epsilon(s_x, s_y, t)|^2 ds_x ds_y}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\epsilon(s_x, s_y, t)|^2 ds_x ds_y}, \quad (21)$$

where s_x and s_y are the transverse components of the spatial frequency and

Fig. 2. Phases of the output second-harmonic optical field for $\Delta k = 0.15 \text{ mm}^{-1}$ at three positions on the calculational grid. The y positions are zero, and the x positions are as labeled.

$$\varepsilon(s_x, s_y, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \varepsilon(x, y, t) \exp[-i2\pi(s_x x + s_y y)] dx dy. \quad (22)$$

The centroid of a beam's intensity propagates at the tilt angle β_x .

The cylindrical curvature of the wave fronts in the plane of walk-off and perpendicular to it is characterized by use of methods similar to those described by Siegman.¹² The number of waves of curvature is

$$c_x(t) = \frac{\sigma_x^2(t)}{\lambda R_x(t)}, \quad (23)$$

where

$$R_x(t) = Z_x(t) \left[1 + \frac{\sigma_{ox}^2(t)}{\lambda^2 \sigma_{sx}^2(t) Z_x^2(t)} \right]. \quad (24)$$

The z positions of the x -dimension beam waist, $Z_x(t)$, and the waist size, $\sigma_{ox}(t)$, are given by

$$Z_x(t) = \frac{A_x(t) + 2\beta_x(t)\bar{x}(t)}{2\lambda^2 \sigma_{sx}^2(t)}, \quad (25)$$

$$\sigma_{ox}^2(t) = \sigma_{ox}^2(t) - Z_x^2(t) \lambda^2 \sigma_{sx}^2(t). \quad (26)$$

$Z_x(t)$ is measured relative to the plane where $A_x(t)$ and $\bar{x}(t)$ are specified. $\bar{x}(t)$ is the x position of the intensity centroid:

$$\bar{x}(t) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x |\varepsilon(x, y, t)|^2 dx dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\varepsilon(x, y, t)|^2 dx dy}. \quad (27)$$

$A_x(t)$ is

$$A_x(t) = \frac{i\lambda}{2\pi} \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s_x \left[\varepsilon(s_x, s_y, t) \frac{\partial \varepsilon^*(s_x, s_y, t)}{\partial s_x} - \text{c.c.} \right] ds_x ds_y}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\varepsilon(s_x, s_y, t)|^2 ds_x ds_y}; \quad (28)$$

$$\sigma_x^2(t) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [x - \bar{x}(t)]^2 |\varepsilon(x, y, t)|^2 dx dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\varepsilon(x, y, t)|^2 dx dy}, \quad (29)$$

$$\sigma_{sx}^2(t) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [\lambda s_x - \beta_x(t)]^2 |\varepsilon(s_x, s_y, t)|^2 ds_x ds_y}{\lambda^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\varepsilon(s_x, s_y, t)|^2 ds_x ds_y}. \quad (30)$$

After tilt and cylindrical curvature are accounted for, the remaining distortion is characterized by two values of Siegman's¹³ M^2 . One, M_x^2 , is calculated in the walk-off plane, and the other, M_y^2 , is calculated in the plane perpendicular to walk-off. The quantity M_x^2 is defined by

$$M_x^2(t) = 4\pi \sigma_{ox}(t) \sigma_{sx}(t). \quad (31)$$

It is the product of the real-space variance and the spatial-frequency-space variance of intensity for an actual beam

normalized to that for an ideal Gaussian beam. An M^2 of 1 corresponds to a Gaussian beam with no phase distortions. Any amplitude or phase distortion makes M^2 larger than unity.

Figure 3 compares the output powers, tilts, curvatures, and M_x^2 's for $\Delta k = 0$ with those for $\Delta k = 0.15 \text{ mm}^{-1}$. Figures 3(a)–3(e) show results for $\Delta k = 0$, and Figs. 3(f)–3(j) show results for $\Delta k = 0.15 \text{ mm}^{-1}$. Figures 3(a) and 3(f) compare the power out of the crystal as a function of time for a Gaussian input pulse profile (7 ns FWHM). The calculated mixing efficiencies, defined as 322.5-nm energy out divided by 645-nm energy in, are 0.56 and 0.37 for these two cases. These are larger than the measured efficiencies¹⁰ of 0.36 and ~ 0.18 . This discrepancy is probably due to differences between the experimental and model beams. Figures 3(b) and 3(g) compare the tilts in the walk-off, or x , direction. For $\Delta k = 0$ there is no tilt of either the fundamental or the second-harmonic beam. For $\Delta k = 0.15 \text{ mm}^{-1}$ the second-harmonic wave tilts in the direction of walk-off, and the fundamental wave tilts by nearly an equal amount in the opposite direction, with pronounced intensity dependence. These tilts reverse sign if the sign of Δk is reversed. Tilts in the y direction are zero for all values of Δk .

The cylindrical curvatures shown in Figs. 3(c) and 3(h) indicate that there is no curvature for $\Delta k = 0$, whereas there are intensity-dependent curvatures for $\Delta k = 0.15 \text{ mm}^{-1}$. At the peak of the pulse the fundamental beam is slightly diverging at the crystal exit face, and the second-harmonic beam is slightly converging.

The calculated values of M_x^2 are plotted in Figs. 3(d) and 3(i). For low input intensities near the beginning and the end of the pulse, M_x^2 for the fundamental wave is unity, as it must be for the input Gaussian transverse

profile. The second-harmonic wave's M_x^2 is greater than 1 because of the walk-off-induced elongation of its beam profile. Notice that the values of M_x^2 increase with intensity in each case and that the increase is more pronounced for $\Delta k \neq 0$.

The time-varying tilts, curvatures, and residual distortions just discussed are the consequence of intensity-dependent output phases similar to those displayed in Fig. 2. Another consequence of this time variation must be frequency chirps and shifts. These could be measured in two ways. One is to heterodyne the output beam with a frequency-shifted reference beam as Gangopadhyay *et al.*¹⁰ did. This can reveal phase shifts much smaller than 1 rad over any selected part of the beam. By its nature, this technique measures $[\int \varepsilon_{\text{ref}}^* \varepsilon(t) dA + \text{c.c.}]$, that is, the electric field weighted by the reference field and integrated over an area. In our modeling we simulate this measurement by summing the output electric field over the spatial grid at each time step and calculate the phase of this summed field as a function of time.

Fig. 3. Numerical results for frequency doubling a 7-ns (FWHM Gaussian), 0.25-mm-diameter (FWHM lowest-order Gaussian) pulse of 645-nm light in a 3-cm-long KDP crystal. The conditions are described for Case 1 in the text. Results for (a)–(e) $\Delta k = 0$, (f)–(j) $\Delta k = 0.15 \text{ mm}^{-1}$. In (a) and (f) the solid curve is one half of the 645-nm power and the dashed curve is the full 322.5-nm power. The tilt, curvature, and M^2 characterize the wave-front distortions in the walk-off direction. The heterodyne phases of the depleted fundamental and generated second harmonic are at the exit face of the crystal.

An alternative method of measuring the frequency spectrum is to use a square-law-detection apparatus such as a spectrometer mated to an intensity monitor. We simulate this measurement by calculating the Fourier time transform of the field at each spatial grid point and summing the transforms weighted by the pulse energy at each grid point. As we will see, these two measurement methods can produce quite different results.

Using the latter method with $\Delta k = 0$, we find that the fundamental and the second-harmonic spectra are virtually identical to Fourier transforms of the power profiles shown in Fig. 3(a). For $\Delta = 0.15 \text{ mm}^{-1}$ the time variation of the phase shown in Fig. 2 suggests there will be frequency chirps and broadening of the spectra relative to the power profile transforms. Indeed, comparing the square-law spectra with these transforms reveals that there is some broadening, but it adds less than 2 MHz to the width (FWHM) of the fundamental and second-harmonic spectra. We attribute the broadening of the second-harmonic spectrum to a small blue shift on the leading edge of the pulse followed by an equal red shift on the trailing edge, as suggested by Fig. 2.

Figures 3(e) and 3(j) display the heterodyne phases. As Fig. 3(e) shows, there are no phase shifts for $\Delta k = 0$. For $\Delta k = 0.15 \text{ mm}^{-1}$ the heterodyne phase of the fundamental decreases with increasing intensity, whereas the phase of the second-harmonic wave shifts in the opposite direction. The maximum apparent second-harmonic and fundamental frequency shifts are ~ 30 and ~ 15 MHz, respectively.

Gangopadhyay *et al.*¹⁰ measured heterodyne phases of the second-harmonic wave as a function of time for $\Delta k = 0$ and $\Delta k = 0.15 \text{ mm}^{-1}$. For $\Delta k = 0.15 \text{ mm}^{-1}$ they found that the second harmonic appeared to be red shifted by ~ 20 MHz on the leading edge of the pulse and blue shifted by ~ 10 MHz on the falling edge compared with those in the $\Delta k = 0$ case. Unfortunately, the sign of Δk was not recorded, and the error bars for the measurement are nearly as large as the shifts, so detailed comparison of their measurement with our calculation is impossible.

The calculated and measured heterodyne phases might be interpreted as a 30-MHz red shift followed by an equal blue shift for the second-harmonic light and a 15-MHz blue shift followed by an equal red shift for the fundamental light. However, these conclusions would be at odds with the square-law spectra. For the second-harmonic wave the frequency shifts have the opposite sign from those deduced from examination of the phases of individual spatial-grid points shown in Fig. 2, which we used to calculate the square-law spectra. In addition, the frequency shifts deduced from heterodyne phases are much larger than those seen in the square-law spectra.

The cause of this apparent paradox is the combination of beam tilt with a shift in the centroid of the beams in the walk-off dimension. Because we assume that the detector is very close to the output face of the crystal, tilt alone will not cause the calculated behavior of the heterodyne phases. It will merely weaken the heterodyne signal by producing fringes on the detector face where the wave interferes with the tilt-free reference wave. In our case there is only a fraction of a fringe, so this should not be an important effect. However, the existence of a tilt toward the walk-off direction for the second-harmonic

wave implies that the phase of this wave must increase on scanning from $-x$ to $+x$. As Fig. 3(g) shows, the tilt of the second harmonic is relatively constant throughout the pulse, so the second-harmonic phase fronts are nearly stationary, apart from higher-order distortions. However, the centroid of the second-harmonic beam scans from $-x$ to $+x$ as its intensity increases. This movement of the centroid is due to fundamental wave depletion. As Fig. 1 shows, the second-harmonic output beam is quite elongated in the walk-off direction because the maximum walk-off of 0.84 mm is large compared with the input beam diameter of 0.25 mm. The part of the beam displaced the most was generated near the input face of the crystal, whereas the part displaced the least was generated near the exit face. For low input intensities the fundamental wave is not significantly depleted, so the second-harmonic wave's intensity is quite uniform over its elongated profile. Its centroid is at $x = 0.42 \text{ mm}$, i.e., shifted in the walk-off direction by half the product of the walk-off angle and the crystal length. As the input fundamental intensity increases, it generates second-harmonic light efficiently near the input face of the crystal but becomes depleted part way through the crystal, thus generating less second-harmonic output near the exit face. Consequently the second-harmonic output beam is brightest on the walk-off side. Its centroid shifts in the walk-off direction as the fundamental intensity increases. Because the walk-off direction is also the direction of increased phase owing to the tilt of the second-harmonic wave, and because the heterodyne phase is weighted by the field amplitude, this combination of nearly constant tilt and shifting centroid explains the positive second-harmonic-wave heterodyne phase shift with increasing intensity. This effect is smaller for the fundamental wave because its walk-off angle is zero so its centroid shifts much less.

The conclusion is that the heterodyne measurement of the second-harmonic wave is not necessarily a measure of the spectral shifts relevant for spectrometry. It is predominantly a measure of the tilt and the centroid shift of the wave. Indeed, the sign of the chirp measured by the heterodyne method for the second-harmonic wave is much larger than and opposite in direction to that of the chirp that would be seen by a collection of atoms. Clearly, this conclusion is based on the choice of a reference beam with zero tilt. Tilting the reference beam by $200 \mu\text{rad}$ to align it with the second-harmonic beam would dramatically alter the expected heterodyne phase. In addition, we calculated the heterodyne phases assuming infinite plane-wave reference beams, and the phases would be different if we used a reference beam matched more closely in size to the actual beams. However, the conclusion remains that its sensitivity to tilt makes it difficult to relate the heterodyne phase measurement to square-law-detector spectral measurements.

Case 2: Diffraction with Birefringence

This second example of numerical modeling illustrates the effect of diffraction combined with birefringence. As discussed above, we expect diffractive phase slippage to mimic phase-velocity mismatch and produce phase distortions of the generated wave even in the absence of a mismatch. We model the same system as in Case 1, except that here the crystal is shortened to 2 mm and

Fig. 4. Numerical results for frequency doubling a 7-ns (FWHM), 14- μ m waist-diameter (FWHM) pulse of 645-nm light in a 2-mm-long KDP crystal with $\Delta k = 0$. The conditions are described for Case 2 in the text. In (a) the solid curve is one half of the 645-nm power and the dashed curve is the full 322.5-nm power. The tilt, curvature, and M^2 characterize the wave-front distortions in the walk-off direction. The heterodyne phases of the depleted fundamental and generated second harmonic are at the exit face of the crystal.

the fundamental beam is focused to a waist of 14 μ m (FWHM of intensity) at the center of the crystal (Rayleigh range, 1 mm). This gives a conversion efficiency similar to that for $\Delta k = 0$ in Case 1 and also has the same beam-diameter-to-walk-off ratio. Figure 4 summarizes the re-

sults at the exit face of the crystal. The second-harmonic wave tilts toward the walk-off direction, and the fundamental wave tilts in the opposite direction. The tilts are much larger than for Case 1 but are still much less than the diffractive spread of the beams. Figure 4(c) shows that the fundamental wave at the crystal exit face is diverging early and late in the pulse because it is focused at the center of the crystal. The second-harmonic wave is substantially less divergent. Both beams diverge less at the peak of the pulse, and the second-harmonic wave has almost no net curvature. Figure 4(d) shows residual distortions or M^2 's comparable with those of Case 1, but here the fundamental wave is distorted more than the second harmonic. The heterodyne phase shifts are quite small, as Fig. 4(e) shows, and the square-law spectra are noticeably different from the transform of the pulse envelopes only in the far wings of the fundamental spectrum.

4. CONCLUSIONS

We have shown that in the plane-wave approximation a wave starting from zero initial intensity in sum- or difference-frequency mixing in a quadratically nonlinear medium with nonzero Δk will suffer no intensity-dependent phase shifts. This lack of sensitivity to intensity is a special situation, however, and any upset of the balance of intensity or phase among the mixing waves is expected to introduce intensity-sensitive phase shifts. We illustrated this by numerically modeling a case in which the walk-off of one wave was comparable with the beam diameters, violating the plane-wave approximation. We showed that the resulting intensity-dependent phase shifts of the second-harmonic light produced frequency chirps for pulsed light, and also wave-front tilts, focusing, and other distortions. We also showed that heterodyne techniques are not a reliable means of measuring the spectral content of beams leaving a nonlinear mixing crystal. In a second example we showed that diffraction produces similar effects. Other situations that violate the plane-wave approximation and so can be expected to lead to beam distortions include mixing with crossed beams when the crossing angle is large enough that the beams separate by an amount comparable with their diameters, group-velocity mismatches large enough to displace the pulses by an amount comparable with their durations, and linear absorption of any wave (except in the special cases mentioned). The resulting distortions are generally small, but they may be significant when beam pointing or frequency stability is critical. The phase distortions may also become significant in optical parametric oscillators, for which phase distortions can accumulate over many transversals of the optical cavity.

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Characterization of a Ring Optical Parametric Oscillator

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Abstract

We have studied a singly-resonant KTP ring OPO pumped by nanosecond pulses from a frequency-doubled Nd:YAG laser. We present measurements of the temporal and spatial intensity profiles of the incident pump beam and OPO output beams, including the depleted pump, as well as the output energy as a function of pump laser energy. These measurements have been carried out for both injection-seeded and unseeded operation of the OPO. The results of these measurements have been compared to the output of a computer model.

Introduction

Optical parametric oscillators (OPOs) are capable of producing widely tunable coherent light using a fixed wavelength laser as a pump source. Though OPOs were first demonstrated in the 1960's, OPOs have recently been the subject of much research and development. The two principle reasons for this renewed interest have been the development of improved pump lasers, e.g. injection seeded Nd:YAG lasers, and improved nonlinear crystals, e.g. beta-barium borate (BBO), potassium titanyl phosphate (KTP), and lithium triborate (LBO). Despite the demonstration of OPOs using a large number of nonlinear crystals and cavity configurations, little work has been devoted to carefully characterizing the output energy and the temporal and spatial intensity profiles for well defined pump and cavity conditions. Such measurements are necessary for understanding the limitations and potential of OPOs as convenient sources of tunable coherent light and to advance the state-of-the-art in computer simulations of these

devices.

Experimental Details

The experimental setup is shown in Figure 1. The 1064nm output from the oscillator of a Q-switched, injection-seeded Nd:YAG laser (Continuum NY61) is sent through a 2 m focal length lens and noncritically frequency doubled in a 2 cm LBO crystal (x-cut) placed 60 cm from the lens. The YAG laser oscillator is capable of producing 150 mJ of 1064 nm at the optimum laser Q-switch delay. However, we typically lengthen the Q-switch delay to produce <100 mJ in a beam with better beam quality than that produced at optimum delay. After separating the 532 nm light from the 1064 nm light the 532 nm light is focused onto a 400 μ m diamond pinhole to spatially filter the 532 nm beam. A collimating telescope is then used to give a 0.6 mm full width at half maximum (FWHM) 532 nm beam with an energy of up to about 15 mJ in a 8 nsec FWHM pulse. The 532 nm pump beam spatial profile is well approximated by a Gaussian intensity distribution. Pump fluences of up to 2.5 J/cm² are used to pump the OPO.

The OPO is a three-mirror (flat-flat-flat) ring configuration similar to that used by Hamilton and Bosenberg¹ consisting of two 780 nm high reflectors and a 780 nm 51% output coupler. One of the high reflectors is mounted on a piezoelectric transducer to allow fine adjustments of the cavity length. The KTP crystal ($\theta=51^\circ$, $\phi=0^\circ$) is 1cm in length with anti reflection coatings for 780 nm and 532 nm. The OPO cavity length is 6.7 cm. The resonated signal wave at 780 nm is polarized in the plane of the ring cavity and propagates through the KTP as an extraordinary

wave. The pump and idler waves are polarized out of the plane of the ring and propagate through the KTP as ordinary waves. As the 780 nm wave traverses the crystal, it experiences a walkoff of 0.52 mm in the plane of the resonator. A CW Ti:sapphire laser pumped by an Ar⁺ ion laser (Spectra Physics 2020) can be used to injection seed the OPO. Typically the seed beam contains 30-40 mW of power in a beam slightly larger than the 532 nm pump beam.

The depleted pump and signal beams exiting the OPO are monitored with CCD cameras (Cohu 4800) and a fast phototube (Hamamatsu 1328U). The cameras are part of a Big Sky Analyzer Plus beam profiler system. The OPO signal energy is measured with a pyroelectric detector (Laser Precision Rj-7200).

Results

Figure 2 shows the OPO signal energy versus 532 nm pump energy. Threshold is observed to be at a pump energy of 3.3 mJ (5.2 mJ) corresponding to a peak pump fluence of .6 J/cm² (0.95 J/cm²) for seeded (unseeded) operation. As expected, the unseeded OPO threshold is higher than for seeded operation because the unseeded OPO signal and idler waves have to build up from vacuum noise. The seeded OPO linewidth is approximately 150 MHz while the unseeded linewidth is about 2-3 cm⁻¹. The signal pulse has a duration of 6.5 nsec FWHM at a pump energy of 11 mJ. For seeded operation, the efficiency for production of signal energy (signal energy divided by the incident pump energy) peaks at 30% for a pump energy of 8 mJ. Both curves in Figure 2 show significant drop in slope efficiency with increasing pump energy. The peak conversion efficiency for signal wave production is well below the theoretical quantum limit of 68% suggesting that only a fraction of the pump beam is depleted. We believe that this is due to conversion of signal and idler photons back into pump photons (backconversion). Also shown in this figure is a computer model² prediction calculated for our conditions.

We have measured the time-integrated two-dimensional spatial profiles and the spatially-integrated time-resolved power of the incident and depleted pump beam and the signal beam. The depletion of the pump energy beam is consistent between the two measurements. Figures 3 and 4 show cross sections through the plane of the resonator of the time-integrated spatial profiles of the undepleted pump, depleted pump and the signal beam. The signal wave profile shown in Figure 3 has a Gaussian-like cross section with a 0.6 mm FWHM at modest pump energy (7.5 mJ). This beam has been

observed to be astigmatic with nearly diffraction limited divergence in the plane of the ring and greater divergence out of plane. At higher pump energies (11.7 mJ), the signal beam develops structure. This pump intensity also coincides with the development of shoulders and asymmetries in the OPO lineshape.

As shown in Fig. 4(a), the depleted pump profile at 3.1 times threshold (2.5 J/cm²) shows local depletion as high as 75% and an overall energy reduction of 50%. The depleted pump also shows an asymmetry in the walkoff (horizontal) plane, i.e. more depletion is observed on the side of the beam toward which walkoff occurs. Note also that the maximum pump depletion does not occur at the peak incident pump position; this is an indication of backconversion occurring in the center portion of the pump beam. Profiles at 2.3 times threshold (1.6 J/cm²), shown in Figure 4(b), also show significant backconversion. This spatially resolved backconversion has been observed for pump energies as low as 1.1 times threshold.

Conclusion

In conclusion, we have carefully characterized the operation of a ring OPO. Evidence of backconversion has been found in the efficiency, spatial profiles and lineshapes of the signal and the depleted pump beams. This data suggest that backconversion can limit OPO performance in the areas of efficiency, beam quality and spectral content. The experimental data compares well with the results of a computer model².

Acknowledgments

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2. M. Bowers and A.V. Smith, this proceedings.

Figure 1 The experimental layout is shown above. The Q-switched, injection-seeded Nd:YAG was frequency doubled in a LBO crystal and a spatial filter was used to clean and collimate the beam. The OPO was injected via the output coupler with a single frequency titanium-doped sapphire laser. The OPO cavity was kept in resonance with the seed frequency with a piezoelectric transducer (PZT). The signal output and the transmitted pump beams were analyzed using CCD cameras, fast photodiodes and Fabry-Perot etalons.

Figure 2 The output signal energy (a) and signal energy efficiency (b) are plotted (crosses) versus the incident pump energy for both seeded and unseeded operation. Also plotted (solid lines) are the results of the computer model. The roll off in efficiency is believed to be due to backconversion of the signal and idler into pump photons.

Figure 3 Profiles of the signal beam cut through the plane of the resonator. The signal beam walks off toward negative displacements. The small asymmetry in the 3.6x threshold case is due to this walk off.

Figure 4 Profiles of the incident and depleted pump beam, cut through the plane of the resonator, are plotted above for pump energies of 7.5 mJ (2.3x threshold) and 11.7 mJ (3.6x threshold). Also plotted are the ratios of the depleted and incident profiles. Note that the maximum depletion of the incident pump beam occurs in the wings of the profile and that little conversion occurs near the beam center. We believe this to be evidence of backconversion of the signal and idler beams to pump. The signal beam walks off toward negative displacements and that the pump depletion is greater on that side.

Sum frequency generation using an optical parametric oscillator

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Abstract

We demonstrate high photon conversion (>50%) of a tunable near infrared OPO into the ultraviolet by sum frequency mixing with the harmonics of a Nd:YAG laser.

Sum frequency generation using an optical parametric oscillator

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Introduction

A number of applications would benefit from sources of tunable coherent ultraviolet (uv) radiation in the 250 to 400 nm range. Among these are uv fluorescence laser detection and ranging (LIDAR)¹. Other applications such as photolithography could use a fixed-frequency solid-state equivalent to excimer lasers. Broad tunability in this wavelength region has generally been accessed by frequency doubling tunable visible or near infrared lasers or by frequency mixing these lasers with the harmonics of a Nd:YAG laser. Unfortunately, to cover the full wavelength range requires multiple dye changes and/or multiple nonlinear crystals because of the relatively narrow tuning range of any one laser medium. The use of optical parametric oscillators (OPO's) with their wide tuning range as a tunable source of near infrared light coupled with a single β -barium borate (BBO) crystal for sum frequency mixing with the Nd:YAG harmonics should yield a convenient alternative to cover this wavelength range.

In this paper we demonstrate that an OPO can be efficiently sum frequency mixed with the Nd:YAG harmonics to produce tunable uv radiation in the 250-400 nm range. This is significant because OPO's generally have poor beam quality which limits their nonlinear conversion in critically phase matched processes. Specifically, we have mixed the 780 nm signal beam from a potassium titanyl phosphate (KTP) OPO with the second harmonic from a Nd:YAG laser in BBO converting up to 50% of the OPO photons into 316 nm photons. We have also demonstrated the generation of narrow bandwidth 248 nm by mixing 827 nm from the OPO with the third harmonic of the Nd:YAG.

Summary

Our experimental setup for the 316-nm generation experiment is shown in Figure 1. The pump laser consists of the oscillator of a Continuum NY61 flashlamp-pumped, Q-switched, injection-seeded Nd:YAG laser. Because the oscillator was found to produce much smoother beam profiles than the oscillator/amplifier configuration we chose to use only the oscillator output and to efficiently frequency double it in an external crystal. The 1.06 μm beam diameter from the graded-reflectivity output coupled oscillator was reduced with a telescope to yield a beam approximately 2 mm in diameter.

The 1.06 μm beam was frequency doubled in an uncoated 2 cm long, Type I lithium triborate (LBO) crystal using noncritical phase matching at approximately 150°C. The 110 mJ of fundamental produced approximately 70 mJ of second harmonic. The spatial profile of the second harmonic had a relatively intense central region with a diameter of 0.77 mm full width at half maximum (FWHM) intensity points containing approximately 60% of the energy and a low intensity component with a 4 mm diameter. The pulse duration was approximately 7 ns FWHM. For the 316 nm generation experiment, the second harmonic was split into two beams of approximately equal intensity. One beam was directed to pump the OPO and the second was sent through a telescope to collimate the beam to a divergence less than 120 μR adians FWHM, delayed to match the OPO temporal peak and directed to the mixing crystal.

For the 248-nm generation experiments the second harmonic beam was mixed with the residual 1.06 μm in a 1 cm long, critically phase matched, Type I BBO crystal to generate as much as 15 mJ of 355 nm. To accomplish this, the polarization of the residual 1.06 μm from the doubling was rotated in a

wave plate designed to be $\lambda/2$ at 1.06 μm and $\lambda/1$ at 532 nm. The third harmonic beam under this configuration was found to have a much smaller beam diameter than the 532 nm beam. For the mixing experiments we found it desirable to increase the Q-switch delay of the Nd:YAG laser to improve the beam quality at the expense of 532 and 355 nm energy. The second and third harmonics were separated using a dichroic beam splitter. The third harmonic is directed through a telescope that collimated and matched the beam diameter to the OPO beam at the mixing crystal. The remainder of the second harmonic pumps the OPO.

As shown in Fig. 1, the OPO is similar to that described by Hamilton². It consists of a ring cavity having three flat mirrors and a Type II, 0.9 cm long KTP crystal ($\theta=51^\circ$, $\phi=0^\circ$). The cavity resonates the signal wave in the near infrared (780 or 827 nm) and has an output coupling of 50%. The signal wave propagates as an extraordinary wave with polarization in the plane of the ring. One of the high reflectors is mounted on a piezoelectric transducer (PZT) to permit fine adjustment of the cavity length for injection seeding with either a continuous wave, single frequency wave titanium doped sapphire laser or diode laser. When injection seeded with 15 mW of power from the titanium-doped sapphire laser, the OPO linewidth is nearly transform limited. The intense portion of the pump beam is approximately 0.77 mm FWHM and the cavity length is approximately 7 cm in length yielding a Fresnel number of approximately 11 at 780 nm.

The OPO signal beam profile is nearly diffraction limited (.59 mRadian divergence FWHM) in the plane of the ring though the out of plane divergence is about twice diffraction limited. A 1 meter focal length lens positioned half a meter from the OPO produced a slightly focussing beam (-170 μ Radians FWHM) in the plane of the ring. The spatially integrated intensity has a smooth temporal profile approximately 5 ns FWHM although the intensity near the center of the beam shows strong back conversion when the OPO is driven much above threshold. A half wave plate rotates the polarization 90° for proper phase matching in the mixing crystal. Two dichroic mirrors are used to separate the signal beam from the pump and idler and to combine the signal beam with the Nd:YAG harmonics for the sum frequency stage.

The crystals used for the sum frequency stage are both BBO cut for Type I phase matching. The acceptance angle for this process is approximately 200 μ Radian because of the large birefringence of BBO. For the 316 nm generation the crystal length was 10 mm and the crystal was cut at 31° . The crystal for the 248 nm generation was 6.6 mm long cut at an angle of 46° . Note that two crystals were used in these experiments because of the small (5 mm) apertures of the crystals, but a single crystal should, in principle, be sufficient to cover the entire wavelength interval from 248 to 400 nm (47° to 26° internal angle, respectively). The OPO beam was approximately 0.71 mm FWHM in the critical direction and 0.94 mm FWHM in the orthogonal direction. The pump beam was larger having dimensions of 1.3 and 1.5 mm FWHM, respectively and about 60% of the total energy was contained in this spot.

Figure 2 shows the generated 316 nm energy versus the 532 nm energy with a 780 nm beam energy of 3.6 mJ. The uv output energy monotonically increases to a value of approximately 4.6 mJ representing a conversion of over 50% of the 780 nm photons. Note that the overall mixing efficiency (uv energy out/ total energy in) peaks at about 24% with a 532 nm energy of 10 mJ. The beam profile of the uv varies with 532 nm energy from a single lobe at a few mJ of drive energy to more structured profiles at energies in excess of 15 mJ. It is believed that this is due to back conversion of the uv into 532 nm (355) and 780 nm (827).

The results for the 248 nm generation were less impressive because of poor beam quality on the 355 nm beam and poor temporal and spatial overlap in the mixing crystal. For input energies of 1.4 mJ and 5 mJ at 827 and 355 nm, respectively, about 1.2 mJ of 248 nm light was generated representing a conversion of 25% of the 827 nm photons. Improvements in our 355 nm beam should lead to increased conversion efficiencies. Spectral analysis of the tunable uv showed that it had a linewidth less than 2 GHz.

In conclusion, we have demonstrated that OPO's can be efficiently converted to the uv by sum frequency mixing with the harmonics of a Nd:YAG laser in a BBO crystal. We have shown 50% conversion of the photons in the signal beam of a 532 nm pumped KTP OPO. We have also shown 25%

photon conversion efficiency for converting 827 nm OPO light to 248 nm light by mixing with 355 nm. This bandwidth of this light was measured to be less than 2 GHz.

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Figure 1. The experimental layout is shown above. Not shown are telescope optics for the 1.06 μm and 532 nm beams. A variable attenuator (VA) is used to vary the 532 nm energy at the mixing crystal. A 1 meter lens is used to reduce the OPO beam divergence and a half-wave plate (HWP) rotates its polarization.

Figure 2. The energy and OPO photon conversion efficiency are plotted versus the incident 532 nm energy. An input energy of 3.6 mJ at 780 was used.

Frequency shifts in injection-seeded optical parametric oscillators with phase mismatch

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We have observed a frequency shift in the output signal pulses relative to the seed frequency in an injection-seeded, singly resonant, critically phase-matched, pulsed optical parametric oscillator in which phase mismatch was intentionally introduced. The observed shifts can be large compared with the linewidth of the signal pulse, are approximately linear in phase mismatch, and increase with increasing pump fluence. We observe frequency shifts of as much as ± 400 MHz for our 532-nm-pumped, potassium titanyl phosphate ring optical parametric oscillator. For zero phase mismatch, we observe nearly transform-limited linewidths of less than 130 MHz. We compare the experimental data to a simple analytic model that overestimates the shifts because it ignores pump depletion. We also compare our measurements with a numerical model that calculates the two-dimensional, transient electric fields and the resultant spectral distributions while explicitly including walk-off, diffraction, and pump depletion. We find good agreement between the experimental data and the results of this model.

Singly resonant optical parametric oscillators (OPO's) with their broad tunability are promising sources for spectroscopic applications because they can be easily tuned to atomic and molecular absorptions. Unfortunately, the linewidths of pulsed OPO's are generally much broader than most Doppler- and pressure-broadened transitions. Narrower OPO linewidths can be obtained by dispersive elements in the cavity¹ or by injection seeding.²⁻⁹ If a seed source is available, injection seeding with a low-power, narrow-bandwidth source permits simple cavity designs and provides single-longitudinal-mode output in pulsed OPO's. Injection seeding also reduces the OPO threshold and improves the OPO efficiency, in part by reducing the OPO buildup time.

For applications in which precise frequency control is required, it is necessary to understand the mechanisms that affect the output frequency of injection-seeded OPO's. As with injection-seeded lasers,¹⁰ detuning between the seed frequency and the closest power oscillator cavity resonance can lead to frequency shifts in the OPO output. When the cavity is resonant with the injected signal seed wave, additional mechanisms can lead to a frequency shift between the injected signal and the signal output of a pulsed singly resonant OPO⁴: phase mismatch, nonresonant idler feedback, and nonlinear index of refraction. Each of these mechanisms impresses phase shifts upon the waves interacting with the crystal. Because the resonated wave experiences this phase shift, $\Delta\phi$, on each round trip, its frequency is shifted by approximately $\Delta\phi/\tau$, where τ is the cavity round-trip time.

In this Letter we focus on the effect of phase mismatch on the frequency profiles and shifts of the output signal pulse for an injection-seeded pulsed OPO

under conditions in which cavity detuning, idler feedback, and nonlinear index of refraction contribute negligibly to the observed shifts. We compare experimental measurements with two models: (1) a simple analytic model that ignores pump depletion but allows simple estimates of the shifts and (2) a detailed numerical model including pump depletion, diffraction, and walk-off that calculates the time-dependent signal, idler, and pump fields and their spectral distributions.

Phase-velocity mismatch in the parametric process is defined as $\Delta kL = [k_p - (k_s + k_i)]L$, where k_p , k_s , and k_i are the pump, signal, and idler wave vectors, respectively. When phase mismatch is present in optical parametric amplification, the amplified wave experiences a phase shift because the wave vector of the polarization producing that amplification is not equal to that of the incident wave. The phase shift of the amplified wave (relative to the incident wave) increases with phase mismatch and amplification. It can be shown that in the undepleted-pump, high-gain limit¹¹ this shift approaches $\Delta kL/2$ with a corresponding frequency shift of $-\Delta kL/(2\tau)$.

The experimental setup is shown in Fig. 1. The pump beam is the spatially filtered 532-nm output of a frequency-doubled, Q-switched, injection-seeded Nd:YAG laser operating at 10 Hz. It is a collimated beam with a 0.6-mm FWHM and an 8-ns FWHM pulse width whose spatial and temporal profiles are well approximated by Gaussian distributions. The peak pump fluence is limited to 2.5 J/cm² (300 MW/cm²) to prevent damage to the optics and potassium titanyl phosphate (KTP) crystal. At this peak pump intensity we estimate that shifts that are due to nonlinear index of refraction, n_2 , are less than 15 MHz for $\Delta kL = 0$ from the value measured

Fig. 1. Experimental layout: the seed and pulsed signal outputs from the OPO are analyzed with a high-finesse étalon. Photodiode PD1 is sensitive primarily to the seed beam, and photodiode PD2 detects the pulsed signal beam from the OPO. PZT, piezoelectric transducer.

by DeSalvo *et al.*¹² for KTP. We ignore n_2 effects in the models.

The OPO is a singly resonant, three-mirror (flat-flat-flat) ring cavity similar to that used by Hamilton and Bosenberg.⁸ It consists of two 780-nm high reflectors and a 780-nm 51% reflective output coupler. This cavity resonates the signal wave at 780 nm but provides less than 0.1% feedback of the 1.67- μ m idler and 532-nm pump waves to minimize the effects of idler and pump feedback on the signal frequency. We estimate that frequency shifts that are due to idler feedback are only a few megahertz for our conditions. The cavity length is 6.7 cm, yielding a cavity mode spacing of 4.0 GHz. The KTP crystal is cut for critical phase matching ($\theta = 51^\circ$, $\phi = 0^\circ$) and is antireflection coated for 780 and 532 nm. The signal wave is an extraordinary wave polarized in the plane of the cavity experiencing a walk-off of 0.52 mm relative to the pump and idler waves in the 1-cm-long crystal. The pump and idler waves are ordinary waves polarized out of the plane of the ring. The seed and pump beams are aligned to within 0.5 mrad of the cavity optic axis for collinear phase matching. The phase mismatch is approximately linear with the external crystal angle for the small range of angles investigated here, i.e., $\Delta kL = (-4 \times 10^3/\text{rad})(\theta - \theta_0)$, where θ is the external crystal angle and θ_0 is the external phase-match angle. Precise determination of θ_0 is difficult when one is simply optimizing the energy output of the OPO; hence we have arbitrarily defined θ_0 to be that crystal angle for which there is zero frequency shift between the output signal and seed beams when the seeded OPO is operating near threshold (0.66 J/cm²). At higher pump fluence there is a small shift in the signal frequency at this angle.

We use a single-frequency (<30 MHz FWHM) cw titanium-doped sapphire laser to injection seed the OPO through the output coupler. Typically the seed beam contains 30–40 mW of power in a beam slightly larger than the pump beam. One of the high-reflectivity mirrors, mounted on a piezoelectric transducer, permits fine adjustment of the cavity length. The OPO cavity is locked to the seed frequency by a standard modulation technique. To prevent frequency jitter that results from the dither

in the cavity length imposed by this technique, we fire the Nd:YAG laser when the cavity is precisely resonant with the seed.⁴ The cavity is thus kept resonant with the seed frequency to within ± 15 MHz.

The seed beam and the pulsed signal beam from the OPO are expanded, collimated, and spectrally resolved with a high-finesse (>60) scanning Fabry-Perot étalon of 974-MHz free spectral range. An optical isolator located between the OPO and the étalon prevents feedback into the OPO and the seed laser. The transmitted seed power is detected 1 ms before the pump laser fires with an electrically filtered photodiode (PD1), sensitive primarily to cw signals. The much more intense output signal pulse is detected with a fast photodiode (PD2). To obtain the spectra, we slowly scan the étalon while the transmitted seed power and the OPO signal energy are simultaneously recorded with a computer.

Figure 2 shows the measured and calculated frequency shift, $\delta\nu = \nu_{\text{signal}} - \nu_{\text{seed}}$, where ν_{signal} is the peak of the output signal pulse spectral distribution and ν_{seed} is the seed frequency, for peak pump fluences of 1.6 and 2.5 J/cm² as a function of ΔkL . The range in the horizontal axis in these graphs corresponds to a total excursion of ± 1 mrad (0.06°) in the external crystal angle. The observed shifts are nearly linear in phase mismatch and have best-fit slopes of -90 and -30 MHz/rad for 1.6- and 2.5-J/cm² peak pump fluences, respectively. We have observed shifts of as much as ± 400 MHz from the seed frequency.

The dashed curves in Fig. 2 are the signal frequency shift (signal phase shift divided by the cavity round-trip time) calculated with the undepleted-pump, steady-state solutions to the three coupled-wave equations¹¹ averaged over the signal intensity. We assume that a signal seed, Gaussian in space, is amplified in a crystal pumped by a Gaussian beam in time and space. Note that this frequency shift is not simply the high-gain limit, $-\Delta kL/(2\tau)$. This simple model qualitatively predicts the pump dependence, magnitude, and sign of

Fig. 2. Experimentally measured frequency shifts (filled circles) as a function of phase mismatch for peak pump fluences of (a) 1.6 J/cm² and (b) 2.5 J/cm². The solid curves represent the shifts calculated with the numerical model described in the text, and the dashed curves represent the shifts calculated with the analytic model.

Fig. 3. Spectral and temporal (insets) profiles for the output signal pulse when $\Delta kL = 0$ and the peak pump fluence is (a) 1.6 J/cm² and (b) 2.5 J/cm². The dots are experimentally measured results, the solid curves are results from the numerical model, and the dashed curves are transforms of the temporal data assuming a constant phase.

the shift but overestimates the shift as a function of ΔkL because the undepleted-pump approximation is violated in actual OPO operation.

The solid curves in Fig. 2 represent the calculated shifts for the output signal pulse with a model¹³ that integrates the three coupled-wave equations,¹¹ including pump depletion, walk-off, and diffraction in the paraxial approximation. The incident pump field is taken to be Gaussian in space and time. The time-dependent two-dimensional electric fields of the signal, idler, and transmitted pump waves are computed by propagation of time slices out of each wave through the nonlinear crystal and around the ring. A time slice for each wave is equal to the cavity round-trip time (0.25 ns). The temporal Fourier transform of the fields yields the power spectrum of each wave.

Figures 3 shows the calculated (solid curves) and measured (dots) spectral and temporal profiles for the signal beam when $\Delta kL = 0$ for peak pump fluences of 1.6 J/cm² (2.4 times threshold) and 2.5 J/cm² (3.8 times threshold). At 1.6 J/cm² the measured spectral profile has a 127-MHz FWHM that is nearly equal to the 115-MHz FWHM of the transform of the 4.8-ns FWHM temporal profile assuming a constant phase and in good agreement with the 120-MHz FWHM predicted by the numerical model. This measurement suggests that there is little time dependence in the phase during the pulse.

At 2.5 J/cm² the measured spectral profile has a FWHM of 125 MHz, is shifted by +60 MHz, and is skewed toward zero frequency shift, indicating the presence of time-dependent structure on the signal phase. The numerical model accurately predicts a spectral profile with a FWHM of 119 MHz and a small shoulder on the low-frequency side. The model does predict the observed asymmetry and the shoulder on the low-frequency side, but it predicts a shift of only +22 MHz.

In summary, we have observed a shift in the frequency of the output signal pulse of an injection-seeded pulsed OPO relative to the seed beam when phase mismatch is present. The shift is approximately linear in phase mismatch and increases with increasing pump fluence. The observed line shapes are nearly transform limited at low pump fluences and show evidence of time-dependent phase variations at high pump fluences. We have developed a numerical model including pump depletion, diffraction, and walk-off that accurately predicts the observed frequency shifts as well as the temporal and spectral profiles.

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Comparison of a numerical model with measured performance of a seeded, nanosecond KTP optical parametric oscillator

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Abstract

We have constructed a numerical model of optical parametric oscillators that is appropriate for nanosecond or longer pulsed operation. We have also experimentally characterized the performance of a KTP ring optical parametric oscillator. We present here a description of the model and show that its predictions agree well with the observed oscillator performance. We compare spatial beam quality, spectra, efficiency, and full-beam and spatially-resolved temporal profiles. Backconversion of signal and idler light to pump is found to affect all aspects of performance.

1. INTRODUCTION

Nanosecond pulsed optical parametric oscillators (OPO's) hold great promise as sources of coherent yet widely tunable light. They were first demonstrated 30 years ago¹, but for many years their development was stymied by the lack of suitable nonlinear crystals, cavity optics, and pump lasers. The recent resurgence of activity in the field²⁻²⁰ has been stimulated by the development of new crystals such as potassium titanyl phosphate (KTP), lithium triborate (LBO) and beta barium borate (BBO), by the development of single-longitudinal-mode

pump lasers, and by progress in high-damage-threshold optics. Despite recent progress, these devices have yet to reach their potential as sources with narrow linewidth, broad tunability, and good beam quality. One of the reasons is that their behavior is more complex than is often appreciated. In contrast to lasers, OPO's are sensitive to the phase of the pump light because it can be impressed on the signal and idler waves. Additionally, they have no gain storage time so the single pass gain must be high if the pump pulse duration is limited to a few nanoseconds. Such high gain often results in strong conversion of signal and idler waves back into the pump wave. This backconversion can strongly influence the efficiency, beam quality, and spectra of OPO-generated light. Another distinction between lasers and OPO's is the use of critically phase matched crystals in OPO's. They introduce angular sensitivity of the gain with the consequence that there is an acceptance angle or maximum allowed angular spread of the three interacting waves imposed by the crystal in addition to that imposed by other cavity optics. Further, parametric oscillator performance is sensitive to cavity feedback of signal, idler, and pump waves and can be strongly influenced by small amounts of unwanted feedback.

Because of the daunting array of variables that must be considered in designing an OPO, we have developed a numerical model of OPO performance as a design tool that allows us to quickly alter any of the variables and study the resulting changes in beam quality, efficiency, time profiles, and spectra. To benchmark the model we have made a careful laboratory study of the performance of a particular OPO and compared it with the model's predictions. The model includes all of the relevant physics for a seeded, nanosecond OPO pumped by a single-frequency pump laser. We account for the nonlinear interaction in the crystal including pump depletion, birefringence, diffraction, realistic spatial and temporal beam profiles, signal or idler wave seeding of the oscillator cavity, arbitrary cavity-mirror reflectivities, and absorptive losses in the crystal. The laboratory measurements encompassed efficiencies, energy fluence transverse profiles, spatially-resolved and spatially-integrated power profiles, and output spectra, for carefully characterized operating conditions. We report here on the comparison between the model predictions and laboratory measurements and show that our

model is successful in describing actual OPO performance.

2. NUMERICAL MODEL

Despite the long history of OPO's and the recent flurry of research activity, there have been few published reports of modeling that is applicable to nanosecond OPO's. Because these devices operate in the transient regime, models developed for continuous wave OPO's cannot accurately predict their behavior. In one model that is appropriate to pulsed OPO's, Brosnan and Byer²¹ used analytic expressions for parametric gain to describe nondiffracting waves in the limit of low pump depletion. It was used to predict threshold pump fluences. Later variations on this model by Guha *et al.*²² and Terry *et al.*²³ add optical cavity modes and crystal birefringence to the analysis but retain the assumption of low pump depletion. Because of this approximation, these models predict only threshold energies or fluences. Once the pump exceeds threshold by a small amount, the assumption of low pump depletion is violated, so they cannot predict other properties of OPO's such as conversion efficiencies, power profiles, or beam quality. In a recent paper, Breteau *et al.*²⁴ numerically modeled a KTP, linear-cavity OPO pumped by 12 ns pulses from a Q-switched Nd:YAG laser. Their model is based on numerically integrating the frequency-mixing equations for plane waves. It neglects spatial beam profiles, walkoff, and diffraction. Nevertheless, by adjusting the value of the nonlinear coefficient d_{eff} , they achieve good agreement with the measured power profiles and efficiencies for a multi-longitudinal-mode OPO described in the same report. Our model takes the next step and includes transverse profiles, diffraction, and walkoff.

A conceptual description of our model follows. All radiation within the cavity is approximated by a series of time slices separated by the round-trip time for the OPO cavity. Evolution of these slices is calculated by solving the paraxial Maxwell equations in retarded time as the slices propagate around the OPO cavity. For each time slice, the transverse profile of the pump optical field is constructed on a rectangular mesh and propagated to the OPO's input mirror where it is combined with the pump light already in the cavity. The

resulting pump field is again propagated around the optical cavity back to the input mirror and this process is repeated for the duration of the pump pulse. The same procedure is applied to the seeded signal wave. The third wave, the idler, is generated entirely within the OPO cavity. All propagation includes diffraction handled by Fast Fourier Transform (FFT) methods, allowing us to track the phases and amplitudes of all three waves over their transverse profiles. The modeling of the nonlinear interaction of the three waves in the mixing crystal accounts for diffraction, linear absorption, phase velocity mismatch, and strong energy exchange among the three waves. We assume that the crystal is uniaxial. This is sufficient for biaxial crystals as well, if they are oriented for propagation in one of the principal planes, as is usually the case. The result is a record of the phase and amplitude of each optical field at the OPO input and output mirrors on a transverse spatial grid and a time grid (which is set by the round-trip time). From this time-log of the fields, we derive the powers as a function of time, the transverse intensity profiles at any propagation distance, beam quality, as measured by Siegman's^{25,26} M^2 , and wavefront tilt and curvature. We can also find the time development of the intensities at any point in the transverse profile, plus fluence profiles and spectra. The spectra are found by Fourier transforming the fields separately for each spatial location and summing them.

This time-slice approach assumes that all three waves have the same group velocity and is thus usually not appropriate for picosecond or shorter pulses. It also assumes that there are no frequency selective elements such as etalons or gratings in the cavity. Strictly speaking, it is appropriate only for nanosecond or longer, seeded OPO's. Even for this case, under some conditions, the model predicts sharp, picosecond-scale time variations. Clearly, caution must be exercised in interpreting such results. Fortunately, these sharp features are absent unless the OPO is driven far above threshold, at levels that are not of practical interest. Although the model has a bandwidth or pulse width limit imposed by ignoring group velocity dispersion, it still may be useful, if not exact, for OPO's that weakly violate our assumptions.

The key to implementing the model is integrating the signal, idler, and pump waves

through the mixing crystal allowing for birefringence, pump depletion, and diffraction. This problem can not be solved analytically so we use numerical methods similar to those described in the literature²⁷⁻³⁰. Our methods are documented in an earlier paper³¹. Briefly, in a birefringent crystal, the Poynting vector for waves with extraordinary polarization is tilted by the walkoff angle, ρ , relative to its k vector. In our laboratory coordinate system, the optic axis of the uniaxial crystal lies in the x-z plane. The k vectors nominally point in the z direction, so x-polarized, or extraordinary, light walks off in the x direction, but y-polarized, or ordinary, light does not walk off. In the paraxial approximation, ignoring the slight asymmetry of diffraction in the x and y direction, the mixing equations take the form

$$\frac{\partial \varepsilon_j(x, y, z, t)}{\partial z} = \frac{i}{2k_j} \left[\frac{\partial^2 \varepsilon_j(x, y, z, t)}{\partial y^2} + \frac{\partial^2 \varepsilon_j(x, y, z, t)}{\partial x^2} \right] - \tan(\rho_j) \frac{\partial \varepsilon_j(x, y, z, t)}{\partial x} + \wp_j(x, y, z, t) - \alpha_j \varepsilon_j(x, y, z, t) \quad (1)$$

where j indexes the frequency (signal, idler, or pump). The complex variable ε is a Fourier component of the optical electric field, E_j , defined by

$$E_j = \frac{1}{2}(\varepsilon_j e^{-i(\omega_j t - k_j z)} + \varepsilon_j^* e^{i(\omega_j t - k_j z)}). \quad (2)$$

The linear loss in the crystal is α and the nonlinear polarization drive term $\wp_j(x, y, z, t)$ is defined by

$$\wp_s(x, y, z, t) = i \frac{d_{\text{eff}} \omega_s}{c n_s} \varepsilon_p(x, y, z, t) \varepsilon_i^*(x, y, z, t) e^{i \Delta k z} \quad (3a)$$

$$\wp_i(x, y, z, t) = i \frac{d_{\text{eff}} \omega_i}{c n_i} \varepsilon_p(x, y, z, t) \varepsilon_s^*(x, y, z, t) e^{i \Delta k z} \quad (3b)$$

$$\wp_p(x, y, z, t) = i \frac{d_{\text{eff}} \omega_p}{c n_p} \varepsilon_i(x, y, z, t) \varepsilon_s(x, y, z, t) e^{-i \Delta k z} \quad (3c)$$

where

$$\Delta k = k_p - k_s - k_i. \quad (4)$$

These equations are integrated through the crystal by transforming to retarded time coordinates where $z = ct/n$. Thus z and t are not independent variables. Instead, t can

be considered an index on the time slices. Fourier transforming the electric fields and polarization terms in the transverse dimension, using

$$\epsilon_j(x, y, z, t) = \int_{-\infty}^{\infty} \tilde{\epsilon}_j(s_x, s_y, z, t) \exp[i2\pi(s_x x + s_y y)] ds_x ds_y \quad (5)$$

$$\wp_j(x, y, z, t) = \int_{-\infty}^{\infty} \tilde{\wp}_j(s_x, s_y, z, t) \exp[i2\pi(s_x x + s_y y)] ds_x ds_y \quad (6)$$

and substituting these definitions of $\epsilon_j(x, y, z, t)$ and $\wp_j(x, y, z, t)$ into Eq. 1, we arrive at the following equation for the propagation of the individual spatial-frequency component waves:

$$\frac{\partial \tilde{\epsilon}_j(s_x, s_y, z, t)}{\partial z} = -i \left[\frac{2\pi^2}{k_j} (s_x^2 + s_y^2) + 2\pi s_y \tan(\rho_j) \right] \tilde{\epsilon}_j(s_x, s_y, z, t) + \tilde{\wp}_j(s_x, s_y, z, t). \quad (7)$$

We now have three coupled first-order ordinary differential equations describing the change in each spatial-frequency component of the fields as they propagate through the crystal. The coupling is via the nonlinear interaction term $\tilde{\wp}_j(s_x, s_y, z, t)$.

We integrate Eq. 7 using the Cash-Karp Runge-Kutta algorithm³². At the beginning of each z step, the $\tilde{\epsilon}_j(s_x, s_y, z, t)$ are Fourier transformed to give $\epsilon_j(x, y, z, t)$. These are used in Eq. 3 to calculate the polarization drive terms $\wp_j(x, y, z, t)$ which are Fourier transformed to obtain the $\tilde{\wp}_j(s_x, s_y, z, t)$'s of Eq. 7. The x-y spatial grid is typically 32×32 or 64×64 , and the integration of a single time slice through the crystal is performed in approximately 32 steps. The number of time slices is typically 75. Run time on a Pentium-based computer is of the order of 1000 s.

In a previous paper³¹, we showed formulas for computing the beam quality factor M^2 , the spot size, and other beam parameters as a function of time from this type of modeling. While useful in illuminating the dynamics of the OPO, these quantities are difficult to measure for nanosecond pulses. However, their time-integrated counterparts are measurable and are usually the quantities of interest. Thus it would be useful to define similar time-integrated quantities to characterize the fluences, or time-integrated intensities. We present here formulas for calculating these. The derivation is similar to our earlier one except the transverse moments of the optical pulse in both the spatial and spatial-frequency domains

are now based on the intensity integrated over time (the energy fluence distribution). We start by defining the quantity U as

$$U = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\varepsilon(x, y, t)|^2 dx dy dt = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\tilde{\varepsilon}(s_x, s_y, t)|^2 ds_x ds_y dt. \quad (8)$$

where s_x is $\frac{k_x}{2\pi}$ or the x transverse component of the spatial-frequency vector. The first moments in spatial and spatial-frequency domains are now defined in terms of fluence rather than intensity:

$$\bar{x}(z) = \frac{1}{U} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x \left[\int_{-\infty}^{\infty} |\varepsilon(x, y, z, t)|^2 dt \right] dx dy, \quad (9)$$

$$\bar{s}_x = \frac{1}{U} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s_x \left[\int_{-\infty}^{\infty} |\tilde{\varepsilon}(s_x, s_y, t)|^2 dt \right] ds_x ds_y. \quad (10)$$

One can show that the first spatial moment propagates according to

$$\bar{x}(z) = \bar{x}(0) + \lambda z \bar{s}_x. \quad (11)$$

Similarly, the fluence-based variances are given by

$$\sigma_x^2(z) = \frac{1}{U} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [x - \bar{x}(z)]^2 \left[\int_{-\infty}^{\infty} |\varepsilon(x, y, z, t)|^2 dt \right] dx dy, \quad (12)$$

$$\sigma_{s_x}^2 = \frac{1}{U} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [s_x - \bar{s}_x]^2 \left[\int_{-\infty}^{\infty} |\tilde{\varepsilon}(s_x, s_y, t)|^2 dt \right] ds_x ds_y. \quad (13)$$

The x variance can be shown to propagate according to

$$\sigma_x^2(z) = \sigma_x^2(0) - [A_x(0) + 2\lambda \bar{x}(0) \bar{s}_x] z + \lambda^2 \sigma_{s_x}^2 z^2 \quad (14)$$

where

$$A_x(z) = \frac{\lambda}{\pi U} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x \left(\int_{-\infty}^{\infty} dt \operatorname{Im} \left[\varepsilon(x, y, z, t) \frac{\partial \varepsilon^*(x, y, z, t)}{\partial x} \right] \right) dx dy. \quad (15)$$

The formulas for the fluence-based minimum variance or beam waist, σ_{ox}^2 , beam quality factor, M^2 , and radius of curvature in the x dimension become

$$\sigma_{ox}^2 = \sigma_x^2(z) - \frac{[A_x(z) + 2\lambda \bar{x}(z) \bar{s}_x]^2}{4\lambda^2 \sigma_{s_x}^2} \quad (16)$$

$$M_x^2 = 4\pi \sigma_{ox} \sigma_{s_x} \quad (17)$$

$$R_x(z) = \frac{-2\sigma_x^2(z)}{A_x(z) + 2\lambda \bar{x}(z) \bar{s}_x}. \quad (18)$$

Similar equations describe the beam properties in the transverse y dimension.

Summarizing the numerical calculations of the OPO: We numerically integrate the wave equations, Eq. 7, for pancakes of light propagating through the crystal and around the cavity. The wave equations include all the relevant physics of the problem including crystal birefringence, diffraction, and pump depletion. The result of this calculation is a record of the electric field amplitude and phase for each of the three waves exiting the OPO cavity, from which properties such as spectra, time profiles, spatial profiles, and beam quality are calculated.

3. LABORATORY MEASUREMENTS

In this section we describe our laboratory OPO and explain how the measurements were made. We chose as an OPO cavity the simple three-mirror ring configuration shown in Fig. 1, rather than a two-mirror linear configuration, primarily to reduce unwanted feedback of the pump and idler waves and thus ensure that the cavity is truly singly resonant. This geometry also reduces unwanted optical feedback to the seed laser, and tends to average signal beam inhomogeneities in the plane of the ring due to image reversals on every round trip. The three cavity mirrors are flat and arranged so that the total cavity length is 6.7 cm. The signal, idler, and pump wavelengths are 780 nm, 1673 nm, and 532 nm, respectively. At 780 nm, two of the mirrors are high reflectors while the third has 51% reflectivity. One of the high reflectors is mounted on a piezoelectric transducer to allow fine adjustments of the cavity length in order to resonate the 780 nm seed light. The round trip loss at the pump and idler wavelengths is greater than 99.9%. The nonlinear medium is a 1 cm long KTP (potassium titanyl phosphate) crystal cut at $\theta = 51^\circ$, $\phi = 0^\circ$ for Type II phase matching. It is mounted on a rotation stage so it can be rotated about an axis perpendicular to the plane of the ring. The crystal is antireflection coated for 780 and 532 nm. The resonated signal wave at 780 nm is polarized in the plane of the ring cavity and propagates through the crystal as an extraordinary wave. The pump and idler waves are polarized perpendicular to

the plane of the ring and are ordinary waves. Thus the critical direction for phase matching coincides with the plane of the ring. The signal wave walkoff is 0.51 mm in the plane of the resonator.

Fig. 2 shows a schematic of the entire experiment. The OPO is pumped by spatially-filtered 532 nm pulses. The 1064 nm light from the oscillator of a Q-switched, injection-seeded Nd:YAG laser (Continuum NY61) is frequency doubled in either a 2 cm long LBO crystal or an 8 mm long KTP crystal, and the second harmonic light is focused with a 1 m focal length lens onto a 400 μ m diameter diamond pinhole for spatial filtering. A telescope collimates the light to a 0.6 mm (FWHM) beam with a pulse energy of up to 12 mJ. This corresponds to pump fluences of up to 2.5 J/cm² (300 MW/cm²). The duration of the 532 nm pulse is set to 6-8 ns FWHM, and is controlled by varying the Q-switch delay of the Nd:YAG laser. The resulting pump pulse has a slightly shorter rise than fall time, and a fluence profile closely approximated by a Gaussian distribution. Figure 3 shows measured time and fluence profiles fit by Gaussians.

The OPO is injection-seeded with a cw, single-mode Ti:sapphire laser (Schwartz Electro-Optics model Titan CWBB) pumped by an argon ion laser (Spectra Physics model 2020). The Ti:sapphire laser produces light with a near-TEM₀₀ mode that is magnified, collimated, and passed through an iris to produce a seed beam with an Airy central disk with a FWHM of 1 mm at the OPO, and a power of typically 10-40 mW. This is much larger than the approximately 10 nW minimum that we find is required to seed the OPO. The measured linewidth of the seed laser is less than 30 MHz. The OPO cavity is adjusted to resonate the seed light using a standard cavity-length dithering technique employing a lock-in stabilizer (Lansing 80.215). When measuring the OPO output spectra, we prevented frequency jitter caused by cavity length dither by firing the Nd:YAG laser when the OPO cavity was exactly resonant with the seed light.

The light generated by the OPO is diagnosed using a variety of instruments. CCD cameras connected to a beam profiling system (Big Sky Analyzer Plus) record fluence profiles of the signal and depleted pump beams 30 cm away from the output coupler. Fast photodetec-

tors (Hamamatsu R1193 and 1328U, and New Focus 1611) record both spatially-integrated and spatially-resolved power profiles for the incident pump, depleted pump, signal, and idler beams. The OPO signal energy is measured with a pyroelectric detector (Laser Precision Rj-7200) and the incident pump energy is measured with a calorimeter (Scientec 300100). Spectra of the signal and pump (incident and depleted) waves are obtained using high finesse (>50) scanning Fabry-Perot etalons with free spectral ranges of approximately 1 GHz.

Beam quality measurements are performed using the beam profiling system mentioned above to record profiles at various positions through a mild focus formed by a long focal length lens. Each profile is analyzed to obtain a waist sizes, w_x and w_y , in the two transverse dimensions, x and y . The variation of the waist size with propagation distance, z , is then fit to the following expression²⁶ that describes propagation of a beam with a beam quality factor of M^2 :

$$w^2(z) = w_o^2 + \left[\frac{M^2 \lambda (z - z_o)}{\pi w_o} \right]^2 \quad (19)$$

where z_o is the position of the minimum waist size, w_o , and λ is the wavelength. Note that a beam with a Gaussian spatial distribution and uniform phase front has $M^2 = 1$ while real beams have $M^2 \geq 1$.

4. RESULTS AND DISCUSSION

A. OPO Operating Parameters

Apart from the efficiency, threshold, and beam quality measurements, we compare model and laboratory results at two values of the pump fluence; the lower is 1.6 J/cm² or 2.3 times threshold, and the higher is 2.5 J/cm² or 3.5 times threshold. The other parameters of importance are described in Appendix A along with estimates of their experimental uncertainties. These comprise all the parameters used in the model. We used only measured input parameters, making no adjustments to better match the actual performance of the OPO. In addition, all comparisons are on absolute scales unless indicated otherwise.

B. Threshold and Efficiency

The measured and predicted signal energy and efficiency are plotted in Fig. 4, for both seeded and unseeded operation of the OPO. Each data point represents a single laser pulse. One benefit of seeding that is apparent in these plots is a reduction of the threshold pump energy. This is expected since the high seed power (30 mW) produces a signal intensity in the OPO cavity that is about six orders of magnitude above the quantum noise value of one photon per mode. The oscillator thus requires less gain to reach threshold when it is seeded. Fig. 4 also illustrates a reduced shot-to-shot variation of the OPO output energy with seeding.

The seeded (unseeded) signal efficiency, defined as the signal energy out divided by the pump energy in, peaks at about 29% (21%) corresponding to a quantum efficiency (signal photons out/pump photons in) of about 43% (31%). The model reproduces the seeded operation of the OPO, but is less successful in predicting the unseeded performance. This is presumably due to the multiple longitudinal mode operation of the unseeded OPO which is not accounted for in the numeric model. We assume a monochromatic seed wave whereas the unseeded device is seeded by quantum noise at the signal and idler wavelengths. At the observed unseeded linewidth of about 3 cm^{-1} , our approximation of zero group velocity dispersion begins to fail. In the remainder of this paper we will consider only seeded operation of the OPO.

C. Fluence Profiles

Contour plots comparing measured and calculated energy fluence profiles for the signal and depleted pump beams are presented in Figs. 5 and 6 for the low (1.6 J/cm^2) and high (2.5 J/cm^2) pump fluences respectively. Recalling that the incident pump beam is nearly Gaussian, it is clear from the depleted-pump contours (Figs. 5 (a) and 6 (a)) that the parametric process has distorted the pump beam asymmetrically with respect to the critical

and noncritical planes. In addition, the distortion is greater at the higher pump level. The signal beam is relatively unstructured at the lower pump fluence but develops side lobes in the noncritical direction at the higher fluence in both the model and experimental profiles (Figs. 6 (c) & (d)).

Insight gained from the data and model lead to the following general explanation of the structure in these profiles: The asymmetry between the critical and noncritical planes is due to walkoff of the resonated signal beam. This leads to a small acceptance angle (0.88 mrad) in the critical plane that restricts the range of off-axis k vectors that have gain. At the lower pump fluence the signal beam is single-peaked (like the pump) and elliptical in shape because of different divergences in the critical and noncritical planes. At the higher fluence the signal and idler generated in the center of the pump beam can completely deplete the pump beam and backconvert to generate new pump. Thus the signal beam is somewhat depleted on axis due to backconversion. The amount of backconversion varies with position in the beam, and this spatial modulation creates off-axis k vectors. The small acceptance angle in the critical plane means the off-axis k vectors in that direction see much lower gain than those in the noncritical direction. Hence the lobes grow more readily in the noncritical plane.

D. Beam Quality

The signal beam contours qualitatively indicate that the beam quality degrades as the pump fluence increases, and that the beam quality is better in the critical plane than in the noncritical plane. We quantified these effects by measuring the beam quality factor M^2 in both the critical and noncritical planes. These results are shown in Fig. 7 along with calculated values of M^2 (solid lines). The beam quality in the critical plane is much better than in the noncritical plane at the higher fluence of 2.5 J/cm². This asymmetry in beam quality can be important for applications requiring subsequent nonlinear conversion of the OPO output.

E. Power

Figures 8 and 9 display measured and computed full-beam power profiles at pumping levels of 1.6 J/cm^2 and 2.5 J/cm^2 . As expected, the turn-on time at the higher pump fluence is earlier than the lower pump fluence. The signal and idler profiles are smooth single peaks for both pump fluences. The depleted pump profiles, however, show a secondary peak centered near 0 ns that indicates backconversion. This peak is larger at the higher pump level, as expected.

Agreement between experiment and model is good at this level. A more stringent test of the model's accuracy, and stronger evidence of backconversion is provided by spatially-resolved power profiles. These are shown in Figs. 10, 11, and 12 for a pump fluence of 1.6 J/cm^2 at various locations within the pump, signal, and idler beams. They are obtained by imaging the OPO output onto a small aperture and recording the transmitted power using fast detectors. The depleted pump profiles (Fig. 10) show clear evidence of backconversion. The time profile of the center of the pump beam shows a strong backconversion peak centered at 1 ns. Profiles in the wings of the pump beam show much less backconversion due to the smaller pump intensities and resulting lower gain. Thus the pump is converting to signal and idler more efficiently in the wings than in the center of the pump beam. Data obtained at a pump level of 2.5 J/cm^2 (not shown) show an even greater backconversion peak at the center of the pump beam. The spatially-resolved power profiles of the signal and idler are rather unstructured compared with the depleted pump profiles but also agree with the model reasonably well.

F. Spectra

Spectra of the entire (spatially integrated) signal beam are shown in Fig. 13 for pump levels of 1.6 J/cm^2 and 2.5 J/cm^2 . The signal spectrum at the lower pump level is quite close to the Fourier transform of the spatially-integrated time profile of the signal, i.e. the signal is

very nearly "transform limited". At the higher pump level, however, the spectrum deviates significantly from the transform limit. The laboratory spectrum is shifted and broadened. These changes result from time-dependent phase shifts, created by backconversion, that vary across the spatial profile of the signal beam. Because the whole-beam spectrum is the sum of spectra for all the spatial locations of the beam, and each of these locations can have different amplitude and phase time profiles, deviations from the transform of the whole-beam time profile are not surprising.

Figure 14 shows spectra for the entire pump beam corresponding to the signal spectra of Fig. 13. In contrast to the signal spectra, both pump spectra show significant deviation from the transform of the whole-beam time profile. One would expect the pump spectra to show stronger backconversion effects because the pump beam's time profiles vary significantly with transverse position in the beam as we showed in Fig. 10. In addition, since the backconversion peak is produced with a π phase shift relative to the incident pump, one expects a doublet-like spectrum in the heavily backconverted parts of the beam. The shoulders on the sides of the spectra in Fig. 14 are clear evidence of this doublet.

G. Nonzero Phase Mismatch

All the data presented above were taken with zero phase mismatch. It is instructive to look at the OPO performance when the phase mismatch is nonzero. Figure 15 shows spatial contours of the pump (a&b) and signal (c&d) at a pump level of 2.5 J/cm^2 with $\Delta kL = -0.64$. Figure 16 is the same but at $\Delta kL = +2.88$. We find that the depleted pump beam is defocussing (focusing) for negative (positive) phase mismatch. This is quite noticeable in Fig. 16a where the measured depleted pump beam has a higher peak fluence (3.2 J/cm^2) than the incident pump beam (2.5 J/cm^2). Focusing and defocussing are the result of curvature impressed on the wavefront by intensity dependent phase shifts associated with nonzero Δk^{31} .

The spectrum of an injection-seeded OPO with phase mismatch is an interesting topic that has been briefly discussed previously^{11,33}. The existence of phase mismatch causes a

phase shift of the resonated wave of order ΔkL on each pass through the nonlinear crystal. Because the phase shift occurs on each round trip, the amplified resonated wave is frequency shifted relative to the seed wave. Figure 17 shows the signal spectrum for negative, zero, and positive values of Δk at a pump level 1.6 J/cm^2 . The measured sign of the frequency shift is opposite that of Δk . These shifts are approximately linear in Δk , and increase with increasing pump fluence³³. We also find that the shifted peaks are broadened relative to the unshifted, $\Delta k=0$, peak. It should be noted that even modest values of ΔkL , which diminish the output energy by only 10%, are sufficient to cause frequency shifts comparable to the linewidth of the pulsed light. Such shifts could be important in high-resolution spectroscopic applications. At higher pump fluences, the peaks are not only broadened but can develop significant structure, as illustrated in Fig. 18, which shows the signal spectrum at a pump level 3.6 times threshold with small negative phase mismatch.

The agreement between model calculations and experiments with nonzero phase mismatch are generally worse at the higher fluence, as seen in comparing Figs. 17 and 18. We have also observed, in both the laboratory and the model, features in the power profile of the depleted pump at the center of the pump beam that occur on the time scale of a cavity round-trip time. Because the model uses the round-trip time as the time increment, we may not be resolving all the time features, and thus not be accurately predicting spectra. The laboratory spectra are recorded using flat mirror etalons, and are probably more accurate than the model.

5. CONCLUSIONS

We have developed a model of nanosecond, injection-seeded OPO's that includes all the relevant physics of these devices including walkoff, diffraction, and pump depletion. We have also built a laboratory ring-cavity, KTP OPO and carefully characterized all of the physical parameters relevant to its performance. We presented here a comparison of laboratory measurements and model predictions of OPO efficiency, thresholds, spatially-resolved and

full-beam power profiles, signal and pump spectra, fluence profiles, and signal beam quality. In our comparison, we used only measured values for the input parameters to the model. We did not vary them to improve agreement between model and experiment yet we find good qualitative agreement of model and experiment in all cases, and we usually have good quantitative agreement as well.

Our major conclusion concerning the operation of nanosecond OPO's is that backconversion (conversion of signal and idler back to pump) affects all aspects of performance. It limits OPO efficiency, and degrades the spectrum and beam quality. Further, backconversion is almost always present in nanosecond OPO's because the single-pass gain must be very high in order to reach threshold during a single pulse. Once threshold is reached, however, the high gain usually completely depletes the center of the pump beam and allows the signal and idler to backconvert. The effects of backconversion are minimized at pump levels just above threshold.

Based on our results, we conclude that conversion efficiency can be rather high (quantum efficiency of about 50%), but levels off at high pump fluences due to backconversion. Further, the quality of the output beam degrades at higher pump fluences, and is generally better in the critical plane than in the noncritical plane, due to the crystal's small acceptance angle in the critical plane. In addition, the output spectra can be broadened and shifted at high pump fluences, and phase mismatch induced shifts can be greater than the linewidth of the OPO output.

We are convinced by the agreement between model and experiment that the model should prove a useful tool in designing and developing improved OPO's. We intend to use it to explore methods of improving OPO beam quality, such as new resonator designs, pump geometries, etc.

Finally, we note that the primary limitations of the model are that it does not allow for frequency selective intracavity elements, and it does not handle unseeded operation accurately. Presently it does not resolve time structure shorter than the round-trip time of the cavity but this can be rectified by interleaving time slices.

6. APPENDIX A. TYPICAL OPO PARAMETERS IN MODEL AND MEASUREMENTS

Table I lists typical operating parameters used in modeling the OPO. All of these were measured or estimated for the laboratory device. Here we describe each parameter and discuss the measurement methods and uncertainties.

L_{crystal} is the physical length of the KTP crystal. d_{eff} is the effective nonlinear coefficient appropriate for the propagation angle and polarizations used in our device. This was found by measuring single pass gain in the actual crystal used in the OPO. We estimate the uncertainty of the measurement at 5%. Δk is the wavevector mismatch in the crystal. It was calculated as a function of angle using Sellmeier equations³⁴ that accurately describe the angular tuning of KTP OPO's. The zero point of Δk was located by finding the angle of minimum threshold. n_2 is the nonlinear coefficient of the refractive index, for which a value 2.4×10^{-15} has been given by DeSalvo et al³⁵. We find that including this has no effect on OPO model results so it is normally set to zero. The pump energy U_{pump} is accurate to 5%. The pump beam is slightly elliptical so we specify two orthogonal beam radii. The radius, R_{pump}^i , is the $1/e^2$ intensity radius in the plane of the ring, and R_{pump}^o is the radius out of the plane. The uncertainty of the Gaussian fits to the actual beam profile is about 10%. The refractive indexes, n , and the walkoff angles, θ , are all derived from the Sellmeier equations cited above. The δ^i 's are displacements from the ring cavity axis in the plane of the ring. $\delta_{\text{sig.seed}}^o$ is the displacement of the seed beam from the pump beam perpendicular to the plane of the ring. The uncertainty is about 0.1 mm. R_{signal} is the radius ($1/e^2$) of the seed beam, accurate to 10%. The seed power $P_{\text{sig.seed}}$ is accurate to 10%. The number of grid points N_x and N_y are typically 64. We vary these from 32 to 128 to check convergence of the model results. X_{max} and Y_{max} are half the full spatial extent of the model grid. L_{ring} is the full physical length of the cavity. $L_{\text{crystal leg}}$ is the length of the leg of the ring containing the crystal. This parameter is used only to calculate the curvature of the pump beam at the input mirror necessary to produce the specified pump spot size at the center of

the crystal. For the beam sizes used here, this parameter is not important. The parameter O/E specifies whether the optical cavity has an odd or an even number of mirrors. For an odd number, the beams reflect in the plane of the ring on each round trip, while they do not for an even number of mirrors. Here we use only an odd number of mirrors. The R's are the reflectivities of the three cavity mirrors. The pump values for R1 (the pump input mirror) and R2 (signal output mirror) have been measured, as has the signal value for R2. The remainder are estimates. The α 's are linear absorption coefficients in the crystal. The pump value is deduced from a measurement of the crystal transmission. The others are known to be small. The crystal face reflectivities are RC. The crystal is antireflection coated for the signal and pump but not for the idler. The signal reflectivity is 1% or less but the pump reflectivity is about 2%. The idler reflectivity is unknown. The phase shifts, ϕ , are in two parts: The first, ϕ_1 , is the shift over the path from the input mirror to the crystal; the second is from the crystal output face to the input mirror. Only the signal phase is important here because it is the only wave resonated. This parameter is nonzero only when the cavity is not resonant with the seed light. It is always zero for this work. The start time and stop time are specified by T_{start} and T_{stop} measured in cavity round trip time. The tilt of the pump relative to the cavity axis, $\text{Tilt}_{\text{pump}}$, is always zero. The two-photon absorption coefficient, β , is 0.1 cm/GW^{35} and we set it to zero. The duration of the pump pulse (FWHM intensity), τ_{pump} , is about 7.0 ns. This can vary from day to day. It is routinely measured to 8% and the actual value is used in comparisons.

7. ACKNOWLEDGMENTS

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FIGURES

Fig. 1. Schematic diagram of laboratory KTP ring OPO.

Fig. 2. Schematic diagram of OPO experiment.

Fig. 3. Power (a) and fluence (b) profiles of the incident pump beam. The dots are the experimental data and the solid curves are least squares fits of Gaussians to the experiment.

Fig. 4. The measured and predicted output (a) signal energy and (b) efficiency are shown for both seeded and unseeded operation. Note that the threshold is reduced and efficiency increased when the OPO is seeded.

Fig. 5. Measured (a & c) and calculated (b & d) contour plots of the fluence spatial profiles of the depleted pump (a & b) and signal (c & d) beams measured 30 cm from the OPO output mirror when the OPO is pumped at 1.6 J/cm^2 (2.3 times threshold), and the phase mismatch is zero. The peak measured (calculated) depleted pump fluence is 1.17 ($.84$) J/cm^2 , and each contour is separated by 0.1 J/cm^2 . The peak measured (calculated) signal fluence is 0.67 (0.53) J/cm^2 and each contour is separated by 0.05 J/cm^2 .

Fig. 6. Measured (a & c) and calculated (b & d) contour plots of the fluence spatial profiles of the depleted pump (a & b), and signal (c & d) beams measured 30 cm from the OPO output mirror, when the OPO is pumped at 2.5 J/cm^2 (3.5 times threshold), and the phase mismatch is zero. The peak measured (calculated) depleted pump fluence is 2.27 (1.55) J/cm^2 and the contours are separated by 0.25 J/cm^2 . The peak measured (calculated) signal fluence is 0.95 (0.97) J/cm^2 and each contour is separated by 0.1 J/cm^2 .

Fig. 7. Beam quality factor M^2 for various pump fluences. The symbols are experimental data and the solid lines are model predictions. The better beam quality in the critical direction is caused by the strong angular dependence of the gain in the critically phase matched KTP crystal.

Fig. 8. Spatially-integrated power profiles of the (a) incident and depleted pump, (b) signal and (c) idler beams when the OPO is pumped at 1.6 J/cm^2 (2.3 times threshold).

Fig. 9. Spatially-integrated power profiles of the (a) incident and depleted pump, (b) signal and (c) idler beams when the OPO is pumped at 2.5 J/cm^2 (3.6 times threshold). Note: An experimental profile for the idler is not available.

Fig. 10. Measured fluence profile of the depleted pump and spatially-resolved power profiles at locations in the beam indicated by the arrows. Shown are waveforms for the incident pump (crosses), depleted pump (filled circles) and calculated depleted pump (dashed curves). Incident pump fluence is 1.6 J/cm^2 (2.3 times threshold).

Fig. 11. Measured fluence profile of the signal and spatially-resolved power profiles at locations in the beam indicated by the arrows. Shown are waveforms for the measured (filled circles) and calculated (dashed curves) signal waveforms. Incident pump fluence is 1.6 J/cm^2 (2.3 times threshold). The signal peak fluence is 0.54 J/cm^2 .

Fig. 12. *Calculated* energy fluence profile of the idler and spatially-resolved power profiles at locations in the beam indicated by the arrows. An experimental spatial profile was not available. Shown are the waveforms for the measured (filled circles) and calculated (dashed curves) idler waveforms. Incident pump fluence is 1.6 J/cm^2 (2.3 times threshold). The idler peak fluence is calculated to be 0.2 J/cm^2 .

Fig. 13. Spectra of the signal for pump levels of 1.6 J/cm^2 (a) and 2.5 J/cm^2 (b). The filled circles are the measured spectra and the solid lines are the model predictions. The dashed lines show the Fourier transform of the spatially-integrated time profiles.

Fig. 14. Spectra of the depleted pump at pump levels of 1.6 J/cm^2 (a) and 2.5 J/cm^2 (b). The filled circles are the measured spectra and the solid lines are the model predictions. The dashed lines show the Fourier transform of the spatially-integrated time profiles.

Fig. 15. Measured (a & c) and calculated (b & d) contour plots of the energy fluence profiles of the depleted pump (a & b) and signal (c & d) beams when a phase mismatch is intentionally introduced by rotating the KTP crystal. The OPO is pumped at 2.5 J/cm^2 and the phase mismatch, ΔkL , is -0.64 . The peak measured (calculated) depleted pump fluence is $1 \text{ (} 1.0 \text{) J/cm}^2$ and the contours are separated by 0.1 J/cm^2 . The peak measured (calculated) signal fluence is $0.25 \text{ (} 0.53 \text{) J/cm}^2$ and the contours are separated by 0.05 J/cm^2 .

Fig. 16. Measured (a & c) and calculated (b & d) contour plots for the energy fluence profiles of the depleted pump (a & c) and signal (c & d) beams when a phase mismatch is intentionally introduced by rotating the KTP crystal. The OPO is pumped at 2.5 J/cm^2 , and the phase mismatch, ΔkL , is $+2.88$. The peak measured (calculated) depleted pump fluence is $3.1 \text{ (} 2.65 \text{) J/cm}^2$ and the contours are separated by 0.25 J/cm^2 . The peak measured (calculated) signal fluence is $0.42 \text{ (} 0.76 \text{) J/cm}^2$ and the contours are separated by 0.1 J/cm^2 .

Fig. 17. Spectra of the signal beam for different values of phase mismatch. The measured (filled circles) and calculated (solid curves) profiles are shown for $\Delta kL =$ (a) $+2.1$, (b) 0.0 , and (c) -1.65 . The OPO is pumped at 1.6 J/cm^2 .

Fig. 18. Spectrum of the signal beam with $\Delta kL = -0.64$ and the OPO pumped at 2.5 J/cm^2 .

TABLES

Table 1. Typical OPO Parameters

Parameter	Value	Parameter	Value
L_{crystal} [mm]	10.0	$d_{\text{eff.}}$ [pm/V]	2.9
Δk [cm^{-1}]	-2.14 to +2.88	n_2 [cm^2/W]	0.0
U_{pump} [mJ]	7.4 and 11.6	λ_{pump} [nm]	532
λ_{signal} [nm]	780	λ_{idler} [nm]	1673.23
R_{pump}^i [mm]	0.51	R_{pump}^o [mm]	0.58
n_{signal}	1.81625	n_{idler}	1.73462
n_{pump}	1.79030	θ_{signal} [mrad]	51.0
θ_{idler} [mrad]	0.0	θ_{pump} [mrad]	0.0
δ_{pump}^i [mm]	0.0	$\delta_{\text{sig.seed}}^i$ [mm]	0.0
$\delta_{\text{sig.seed}}^o$ [mm]	0.0	R_{signal} [mm]	0.73
$P_{\text{sig.seed}}$ [W]	0.025	N_x	32
N_y	32	$X_{\text{max.}}$ [mm]	1.4
$Y_{\text{max.}}$ [mm]	1.5	L_{ring} [mm]	67
$L_{\text{crystal leg}}$ [mm]	25	O/E	Odd
$R1_{\text{signal}}$	0.99	$R1_{\text{idler}}$	0.01
$R1_{\text{pump}}$	0.04	$R2_{\text{signal}}$	0.51
$R2_{\text{idler}}$	0.01	$R2_{\text{pump}}$	0.18
$R3_{\text{signal}}$	0.99	$R3_{\text{idler}}$	0.01
$R3_{\text{pump}}$	0.04	α_{signal} [mm^{-1}]	0.0
α_{idler} [mm^{-1}]	0.0	α_{pump} [mm^{-1}]	0.00513
RC_{signal}	0.01	RC_{idler}	0.01
RC_{pump}	0.02	$\phi1_{\text{signal}}$ [rad]	0.0
$\phi1_{\text{idler}}$ [rad]	0.0	$\phi1_{\text{pump}}$ [rad]	0.0

$\phi_{2\text{signal}}[\text{rad}]$	0.0	$\phi_{2\text{idler}}[\text{rad}]$	0.0
$\phi_{2\text{pump}}[\text{rad}]$	0.0	$T_{\text{start}}[\text{round trips}]$	-40
$T_{\text{stop}}[\text{round trips}]$	40	$\text{Tilt}_{\text{pump}}[\text{mrad}]$	0.0
$\beta_{\text{pump}}[\text{mm/GW}]$	0.0	$\tau_{\text{pump}}[\text{ns}]$	7.0

Fig. 1 Smith, Alford, Raymond and Bowers "Comparison of a numerical model with measured performance of seeded, pulsed KTP optical parametric oscillator"

Fig. 2 Smith, Alford, Raymond and Bowers "Comparison of a numerical model with measured performance of seeded, pulsed KTP optical parametric oscillator"

Fig 3 Smith, Alford, Raymond and Bowers "Comparison of a numerical model with ..."

Fig 4 Smith, Alford, Raymond and Bowers "Comparison of a numerical model with ..."

Fig 5 (a) and (b) Smith, Alford, Raymond and Bowers "Comparision of a numerical model with measured performance of seeded, pulsed KTP optical parametric oscillator"

Fig. 5 (c) and (d) Smith, Alford, Raymond and Bowers "Comparision of a numerical model with measured performance of seeded, pulsed KTP optical parametric oscillator"

Fig. 6 (a) and (b) Smith, Alford, Raymond and Bowers "Comparison of a numerical model with measured performance of seeded, pulsed KTP optical parametric oscillator"

Fig. 6 (c) and (d) Smith, Alford, Raymond and Bowers "Comparison of a numerical model with measured performance of seeded, pulsed KTP optical parametric oscillator"

Figure 7, Smith, Alford, Raymond, & Bowers

'Comparison of a numerical model with measured performance of a seeded, nanosecond KTP optical parametric oscillator'

Fig 8 (a) Smith, Alford, Raymond and Bowers "Comparison of a numerical model with ..."

Fig 8 (b) Smith, Alford, Raymond and Bowers "Comparison of a numerical model with ..."

Fig 8 (c) Smith, Alford, Raymond and Bowers "Comparison of a numerical model with ..."

Fig 9 (a) Smith, Alford, Raymond and Bowers "Comparison of a numerical model with ..."

Fig 9 (b) Smith, Alford, Raymond and Bowers "Comparison of a numerical model with ..."

Fig 9 (c) Smith, Alford, Raymond and Bowers "Comparison of a numerical model with ..."

Fig. 10 Smith, Alford, Raymond and Bowers "Comparison of a numerical model with measured performance of seeded, pulsed KTP optical parametric oscillator"

Fig. 11 Smith, Alford, Raymond and Bowers "Comparison of a numerical model with measured performance of seeded, pulsed KTP optical parametric oscillator"

Fig. 12 Smith, Alford, Raymond and Bowers "Comparison of a numerical model with measured performance of seeded, pulsed KTP optical parametric oscillator"

Fig 13 Smith, Alford, Raymond and Bowers "Comparison of a numerical model with ..."

Fig14 (a) and (b) Smith, Alford, Raymond and Bowers "Comparison of a numerical model with ..."

Fig 15 (a) and (b) Smith, Alford, Raymond and Bowers "Comparison of a numerical model with measured performance of seeded, pulsed KTP optical parametric oscillator"

Fig 15 (c) and (d) Smith, Alford, Raymond and Bowers "Comparison of a numerical model with measured performance of seeded, pulsed KTP optical parametric oscillator"

Fig. 16 (a) and (b) Smith, Alford, Raymond and Bowers "Comparison of a numerical model with measured performance of seeded, pulsed KTP optical parametric oscillator"

Fig. 16 (c) and (d) Smith, Alford, Raymond and Bowers "Comparison of a numerical model with measured performance of seeded, pulsed KTP optical parametric oscillator"

Fig17 Smith, Alford, Raymond and Bowers "Comparison of a numerical model with ..."

Fig 18 Smith, Alford, Raymond and Bowers "Comparison of a numerical model with ..."

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