To solve QCD at high-energy we must simultaneously find the hadronic states and the exchanged pomeron (IP) giving UNITARY scattering amplitudes.

Experimentally, the IP ~ a Regge pole at small $Q^2$ and a single gluon at larger $Q^2$. ($P^P$ - H1, dijets - ZEUS)

In the "solution" which I will describe, these "non-perturbative" properties of the IP are directly related to the non-perturbative confinement and chiral symmetry breaking properties of hadrons.

(See Phys. Rev. D58, 074008 for more details.)

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1. **THE CENTRAL IDEA** is to study complicated multi-Regge limits of multiparticle quark and gluon scattering amplitudes described by reggeon diagrams. We consider infra-red limits in which gluon mass(es) $M$ and quark mass(es) $m \to 0$ and look for hadrons and the IP to appear as (coupled) Regge pole bound states of reggeons, e.g.

![Diagram](image)

The $U(1)$ anomaly and Reggeon Field Theory will be essential for our understanding and analysis of the divergence structure we find.

2. **PRELIMINARIES**

We start with quarks scattering via reggeized gluon exchanges and end with hadrons scattering via pomeron exchanges. Since, high energy, are involved, we might expect the dynamics to be close to perturbation theory. Nevertheless, if we hope to obtain a "non-perturbative solution" there are many issues of principle that must be faced. We first briefly discuss some of them.

**Renormalons and Non-Perturbative Condensates**

It is well-known that starting with gluon and quark degrees of freedom immediately leads to problems. There is asymptotic freedom, but the S-Matrix is infra-red divergent and for off-shell Green's functions the infra-red region produces a wildly divergent perturbation expansion.

\[ \sum \propto \frac{\alpha_s^n n!}{n} \]

\( \Rightarrow \) an infinite set of vacuum parameters $\langle F_{\mu
u}^2 \rangle, \langle F_{\mu
u}^4 \rangle, \ldots$ (infra-red renormalons) which are undetermined perturbatively.

If a solution (via the lattice), we expect the vacuum parameters to be dominated by "non-perturbative" strong-coupling gauge fields. (Massive quarks should not significantly alter the physics. Glueball states, some associated with the pomeron, should be evident. If perturbation theory still applies, the BFKL pomeron should appear at large $Q^2$.)
Adding (massless) quarks reduces the significance of infra-red renormalons - instantons eventually give the leading Borel plane singularities.

**Borel Plane**

For $N_f = 16$ there are (probably) no renormalons - the first is known to disappear.

The $U(1)$ instanton / Dirac sea problem is the only "vacuum problem".

There is an infra-red fixed point for very small $\alpha_s$. Although gluon infra-red divergences remain, perturbation theory should have maximal validity.

- Our analysis most directly applies to the infra-red fixed point version of QCD - which shares many properties with the supersymmetric gauge theories that have been much studied recently.

- Current wisdom says that a "non-abelian Coulomb-phase" of QCD is realized.

- We will encounter the $U(1)$ anomaly "perturbatively" (within reggeized gluon interactions) and see that it produces a unitary hadron $S$-Matrix, with confinement and chiral symmetry breaking, even though there is no "confinement phase" for off-shell Green's functions.

- Because of the dynamical role of the anomaly massless quarks clearly play an essential role - glueballs may be (close to) absent!

**Avoiding the Vacuum**

"Light-Cone Quantization" suggests how a complicated non-perturbative vacuum might be avoidable at infinite momentum.

Consider the gauge choice $A_+ = 0$, with $P_-$ as the "Hamiltonian". The perturbative vacuum is an eigenstate of $P_-$.

The only subtlety is that the zero-mode Gauss' law condition

$$ [p_+^2 A_-(p_+, p_-)]_{p_+ = 0} = 0 $$

has to be imposed to obtain full gauge invariance. Perturbation theory can be obtained by initially discretizing $p_+$ and setting $A_-(p_- = 0) = 0$.

**IF** singular (at $p_+ = 0$) longitudinal gluons can produce "vacuum properties", a fundamental origin of the parton model may be obtained.

In the Regge limit, $A_-$ gluons are exchanged by (one or more) infinite momentum scattering states. A zero-mode component of such states will produce a related "wee-parton" component of the exchanged pomeron (and of other exchanged reggeon states).

In our analysis of partially-broken QCD, a $\delta(p_+)$ $\delta^2(p_-)$ wee-parton component is, indeed, present in all Regge exchanges and is responsible for confinement and chiral symmetry breaking in the S-Matrix.

In unbroken QCD there is a more elaborate wee-parton phenomenon involving the critical pomeron.
Massive Reggeized Gluons and Complimentarity

To initially construct S-Matrix Regge behavior via dispersion theory we must avoid infra-red divergences. Therefore, we start with massive gluons, i.e. spontaneously-broken QCD.

To reach unbroken QCD smoothly (no "phase-transition") fundamental representation "Higgs scalars" must be used and an ultraviolet cut-off kept.

In this case, gauge-invariant non-local operators (path-ordered line-integrals) that create states in the "confining phase" can be (approximately) factorized into products of gauge-invariant local operators (gluon fields) that create states in the "deconfined phase", i.e.

\[ \exp \int dx_\mu A_\mu \approx \prod \left( \frac{\delta}{\delta x_i} - gA(x_i) \right) \delta x_i \approx \prod gB(x_i) \delta x_i, \]

where \( B_\mu = \Omega^+ \left( \frac{1}{3} \partial_\mu - A_\mu \right) \Omega \) and \( \Omega(x) \) is a color triplet classical Higgs field satisfying \( \Omega^+ \Omega = 1 \). \( B_\mu(x) \) is a gauge-invariant (= unitary gauge) massive vector field.

(Note that the factorization is straightforward only if a finite product is involved, which is why a cut-off is required.)

The Regge limit produces gauge-invariant transverse momentum integrals in which a gauge-invariant cut-off can be imposed. Using two Higgs triplets and decoupling them successively gives \( SU(2) \) gauge symmetry first and then full \( SU(3) \) symmetry.

Abstract multi-Regge theory provides both the basis for pomeron RFT and the major extension of existing QCD calculations that is fundamental for our analysis. The formalism was developed before QCD was established, but all the assumptions made should be valid in the massive version of QCD from which we start.

MULTI-REGGE THEORY (Gribov, Pomeranchuk, Ter-Martirosyan, Stapp, ARW … )

Angular Variables

For an N-pt amplitude, introduce a tree diagram with 3-pt vertices. At every vertex, define Lorentz frames in which each of the entering \( Q_i \) takes a standard form. Define \( g_i \in \text{little group of } Q_i \) (\( \equiv SO(3) \)) relating frames at neighboring vertices. Using \( g_i = u_x(\mu_i)u_x(\theta_i)u_x(\nu_i) \) we can write

\[ M_N(P_1, \ldots, P_N) \equiv M_N(t_1, \ldots, t_{N-3}, g_1, \ldots, g_{N-3}) \equiv M_N(t_1, \ldots, z_1, \ldots, u_{12}, \ldots) \]

where \( t_i = Q_i^2, z_i = \cos \theta_i \), and \( u_{ij} = e^{i(\nu_j - \mu_i)} \).

Partial-wave Expansions

For a function \( f(g), g \in SO(3) \), we can write

\[ f(g) = \sum_{J=0}^{\infty} \sum_{|n|,|n'|<J} D^J_{nn'}(g) a_{Jnn'} \quad D^J_{nn'}(g) = e^{\nu} d^J_{nn'}(\theta) e^{\nu} \]

and so for an N-pt amplitude we can write

\[ M_N(t, g) = \sum_{J, n_1, n'_1} \prod_i D^J_{nn_i}(g_i) a_{Jn_1n'_1}(t_i) \]
Multi-Regge Limits

are defined as \( z_1, \ldots, z_{N-3} \to \infty \), \( \forall \ t_i, u_{ij} \) fixed. We also consider Helicity-Pole Limits in which some \( z_i \to \infty \) some \( u_{jk} \to \infty \), or 0 (\( \to \) fewer large invariants than the related multi-Regge limit).

Asymptotic Dispersion Relations

It is a deep result of Analytic S-Matrix Theory that the generalised Steinmann relations hold in multi-Regge limits \( \Rightarrow \) no overlapping channel double discontinuities and simple multiple discontinuity formulae. Dispersing in \( z_1, \ldots, z_{N-3} \) we obtain

\[
M_N(p_1, \ldots, p_N) = \sum_{C} M^G_N(p_1, \ldots, p_N) + M^0
\]

where \( M^0 \) is small asymptotically and

\[
M^G_N(p_1, \ldots, p_N) = \frac{1}{(2\pi i)^{N-3}} \int \frac{ds_1' \ldots ds_{N-3}' \Delta^C(t_1, u_1, s_1', s_2', \ldots, s_{N-3}')}{(s_1' - s_1)(s_2' - s_2) \ldots (s_{N-3}' - s_{N-3})}
\]

\( \Sigma_C \) is over sets of \((N-3)\) asymptotic cuts classified via "hexagrams" - crucial for multiparticle complex angular theory, e.g.

\[
\begin{align*}
2 \rightarrow 2 + 2
\end{align*}
\]

This is the simplest case. c.f. Fadin - Lipatov NLO RRP vertex

Froissart-Gribov Continuation of P-W Amplitudes

For each hexagraphe partial-wave amplitude \( \alpha J_{N,N}'(\xi) \), some \( J \) and \( n' \) can be continued to complex values using the Froissart-Gribov method.

Each \( J_i \leftrightarrow \) a horizontal line and each \( n_{ij} \leftrightarrow \) a vertical line of the hexagraphe. If a \( J_i \) line and a \( n_{ij} \) line are joined, these indices can not be independently continued to complex values.

Sommerfeld-Watson Representations

The continued F-G amplitudes allow (partial) integral representations to be written e.g. for the 6pt hexagraphe

\[
M^H_6(z_1, z_2, z_3, u_1, u_2, t_1, t_2, t_3)
\]

\[
= \frac{1}{8} \int_{C_{n_2}} \frac{dn_2 u_{2n_2}^n}{\sin \pi n_2} \int_{C_{n_1}} \frac{dn_1 u_{1n_1}^n}{\sin \pi (n_1 - n_2)} \int_{C_{J_1}} \frac{dJ_1 d_{J_0, n_1}^1(z_1)}{\sin \pi (J_1 - n_1)} \times \sum_{J_{2-n_1}=N_{1}=0}^{\infty} \sum_{J_{3-n_2}=N_{3}=0}^{\infty} d_{n_1^2 n_2^2 n_3^2}^5 d_{n_2^3 n_3^3}^5 a^H_{N_2 N_3}(J_1, n_1, n_2, \xi)
\]

These representations give the asymptotic behaviour in all multi-Regge and helicity-pole limits. The integrals reflect all asymptotic cuts \( \Rightarrow \) convergence of the remaining sums. In a "maximal" helicity-pole limit (all \( u_{jk} \to \infty \) or 0) only a single F-G amplitude contributes, e.g. Regge poles at \( J_1 = \alpha_1, J_2 = n_2 = \alpha_2, J_3 = n_3 = \alpha_3 \) give

\[
M^H_6 \sim z_1^{\alpha_1} (u_1 z_2)^{\alpha_2} (u_2 z_3)^{\alpha_3} a^H_{0,0}(\alpha_1, \alpha_2, \alpha_3, \xi)
\]

\( z_1, u_1, u_2 \to \infty \)
Multiparticle $t$-channel Unitarity in the $J$-plane

We consider hexagraph amplitudes $H_L$ and $H_R$ in the $2M$-particle contribution to the unitarity equation in some $t$-channel, e.g.

\[ H_L \quad 2M \text{ particle state} \quad H_R \]

The $2M$-particle phase-space can be written in terms of angular variables as $i \int dp(t, t_1, \ldots) \int d\theta g_j \, d\theta g_j$ so that the unitarity equation becomes

\[ M^+(g) - M^-(g) = i \int dp \int d\theta g_j M_{H_L}^{+}(g_L, g_1, \ldots) M_{H_R}^{-}(g_L, g_1, \ldots) + \cdots \]

Partial-wave projection $\rightarrow a^+_J - a^-_J = i \int dp \, \Sigma_{n_1, n_2} a^+_J a^-_{J, n_2} + \cdots$

The hexagraph product can be continued to complex $J \rightarrow$

\[ a^+_J - a^-_J = i \int dp \left( \frac{dn_1 dn_2}{\sin \pi (J - n_1 - n_2)} \right) \left( \frac{dn_3 dn_4}{\sin \pi (n_3 - n_4)} \right) \cdots \sum a^+_J a^-_{J, n_2} + \cdots \]

"Nonsense poles" at $J = n_1 + n_2 - 1$, $n_1 = n_3 + n_4 - 1$, combined with Regge poles in $a^+_J a^-_{J, n_2}$ and $\int dp \rightarrow$ Regge Cuts

- at $J = \alpha_M(t) = M \alpha(t/M^2) - M + 1$ for $M$ identical reggeons.

Reggeon Unitarity

In ANY multiparticle $F-G$ amplitude, in ANY $J$-plane, the regge cut discontinuity due to $M$ Regge poles $\alpha = (\alpha_1, \alpha_2, \ldots \alpha_M)$ is given by

\[ \text{disc} \quad a_N(\tau, J) = \alpha_M \int \frac{d\tau}{\sin \frac{\pi}{2} (\alpha_1 - \tau_1) \cdots \sin \frac{\pi}{2} (\alpha_M - \tau_M)} \]

where (after extracting threshold factors)

\[ \int d\tau (t_1, \ldots, t_J) = \int \prod \frac{dt_j \lambda^{-1/2}(t_1, t_2) \lambda^{-1/2}(t_1, t_3, t_4) \cdots \lambda^{-1/2}(t_1, t_J, t_{J-1}, t_{J-2}) \cdots}{dt_j \lambda^{-1/2}(t_1, t_2)} \]

Using $\int d\tau (t_1, \ldots, t_J) = 2 \int d^2 k_1 d^2 k_2 \delta(k - k_1 - k_2)$, with $t_i = k_i^2$, the discontinuity formula can be rewritten in terms of two dimensional $k_1$ integrations $\Rightarrow$ high-energy scattering is described by "reggeon diagrams", anticipating the results of direct high-energy calculations.

The power of the "reggeon unitarity equations" is that they hold in every complex angular momentum and helicity plane and control multi-regge exchanges in all multiparticle amplitudes.

When first proposed in 1965 they were a remarkable "non-perturbative" generalization of results found in lowest-order field theory models of Regge cut behaviour in elastic scattering.

Given the reggeization of gluons and quarks, the (essentially) factorizing nature of the equations implies the very powerful consequence that the multi-regge behaviour of all QCD multiparticle amplitudes is built up from elementary components, many of which are already known from existing calculations.
4. POMERON REGGEON FIELD THEORY (Gribov)

The formulation of RFT can be based entirely on reggeon unitarity. If the IMP is a Regge pole, as suggested by experiment, all multi-IMP exchanges and interactions must satisfy the reggeon unitarity equations.

For a single t-channel, we can write an effective lagrangian in terms of fields $\tilde{\phi}(x,y)$ and $\phi(x,y)$ that, respectively, create and destroy pomeron ($x =$ impact parameter, $y =$ rapidity)

$$\mathcal{L}(\tilde{\phi}, \phi) = \frac{1}{2}i\tilde{\phi}^T \partial_y \phi - \Delta_0 \tilde{\phi} \phi + \alpha_0' \nabla \tilde{\phi} \cdot \nabla \phi + \alpha_0'' \nabla^2 \tilde{\phi} \cdot \nabla^2 \phi$$

$$\cdots + \frac{-i}{2} \left[ r_0 \tilde{\phi}^2 \phi + r_0 \tilde{\phi} \phi^2 + r_0 \tilde{\phi} \phi \nabla^2 \phi + \cdots \right]$$

$$+ \frac{1}{6} \left[ \lambda_0 \tilde{\phi}^3 \phi + \lambda_0 \tilde{\phi} \phi^3 + \cdots \right] + \cdots.$$  

The free part of the lagrangian (i.e. $\frac{1}{2}i\tilde{\phi}^T \partial_y \phi - \Delta_0 \tilde{\phi} \phi$) corresponds to the Regge pole propagator

$$[J - 1 + \Delta_0 + \alpha' k_\perp^2 + \alpha'' k_\perp^4]^{-1}$$

The set of "reggeon diagrams" (in $k_\perp$ space) with reggeon propagators and a complete set of interactions (the higher-order interactions require a $k_\perp$ cut-off) provide

A GENERAL SOLUTION OF REGGEON UNITARITY.

The Critical Pomeron (Migdal, Polyakov, Aharanov, Bronzan)

Expanding in powers of $\epsilon = 4 - D$ ($D = k_\perp$ dimension) a renormalization group fixed-point solution is found in which $\alpha_\lambda(0) = 1$ and total cross-sections rise asymptotically

$$\sigma_T \sim \ln \eta, \quad \eta = \frac{\epsilon}{12} + O(\epsilon^2)$$

The resulting "Critical IMP" description of amplitudes and cross-sections satisfies all known unitarity constraints in both the t-channel and the s-channel.

What is the "new phase" appearing at the critical point?

The Super-Critical Pomeron

Close to the critical point all relevant interactions are contained in the triple IMP lagrangian

$$\mathcal{L}_R = \frac{1}{2}i\tilde{\phi}^T \partial_y \phi - \alpha' \nabla \tilde{\phi} \cdot \nabla \phi - \Delta_0 \tilde{\phi} \phi - \frac{1}{2}i\gamma_0 \left[ \tilde{\phi}^2 \phi + \tilde{\phi} \phi^2 \right]$$

To define a theory with $\Delta_0 < 0$ ($\alpha_\lambda(0) > 1$) we look for a classical field configuration minimizing the "potential"

$$V(\tilde{\phi}, \phi) = \Delta_0 \tilde{\phi} \phi + \frac{i\gamma_0}{2} (\tilde{\phi}^2 \phi + \tilde{\phi} \phi^2)$$

We can define a "Super-Critical" solution by using the stationary point $\phi = \tilde{\phi} = \frac{2i\Delta_0}{3\gamma_0}$ to introduce a "IMP condensate". ($\Delta_0 \to -\Delta_0/3$)

(The solution we describe was very controversial 20 years ago - although it was supported by Gribov!)

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The condensate generates new classes of RFT diagrams whose physical interpretation is apriori not apparent. E.g. diagrams such as

\[ \begin{array}{c}
\text{2 IP propagator} \\
\longrightarrow \\
\text{condensate} \\
\end{array} \quad \leftrightarrow \\
\begin{array}{c}
\text{odd signature reggeon}
\end{array} \]

The zero energy 2 IP propagators \((\Delta_0 - 2\alpha'_I k_1^2 + \cdots)^{-1}\) produce \(k_\perp\) poles that can be interpreted as particle poles. Consequently, as illustrated, the diagram describes a IP transition to 2 odd-signature reggeons with trajectory degenerate with that of the IP.

In general, divergences in rapidity produced by \(\Delta_0 < 0\) are converted to vector particle divergences in \(k_\perp\). All diagrams are interpretable in terms of IP’s and reggeons with singular interactions due to vector particle poles.

\(\Rightarrow\) The supercritical phase characterized by the vacuum production of IP’S \(\leftrightarrow\) “DECONFINEMENT OF A VECTOR PARTICLE” ON THE IP TRAJECTORY !!

Is this phase realized in QCD ??? The appearance of a “reggeized gluon” i.e. a reggeized vector particle) strongly suggests the spontaneous breaking of a gauge symmetry ! \(\text{SU}(3) \rightarrow \text{SU}(2) \Rightarrow\)

5. REGGEON DIAGRAMS FOR (spontaneously-broken) QCD

(Padin, Kuraev, Lipatov, Cheng, Lo, Broozen, Sugar, Bartels ...)

Leading-log calculations \(\rightarrow\) “gluon reggeizes” \(\leftrightarrow\) pole at \(J = 1 - \Delta t\) in the \(J\)-plane. Non-leading logs \(\rightarrow\) reggeon diagrams.

Gluon reggeon diagrams involve \(k_\perp\) integrals, just like IP RFT diagrams, but gluon particle poles are present in addition to reggeon propagators, e.g. 2 reggeon state -

\[ \int \frac{d^3k_1}{(k_1^2 + M^2)} \frac{d^3k_2}{(k_2^2 + M^2)} \frac{d^3(k_1 + k_2 - k_1 - k_2)}{J - 1 + \Delta(k_1^2) + \Delta(k_2^2)} \]

Reggeon Unitarity \(\Rightarrow\) a complete set of reggeon diagrams appear in higher-orders.

BFKL \(\leftrightarrow\) 2-reggeon unitarity, i.e. iteration of the 2-reggeon state

\[ \begin{array}{c}
\text{R}_{22} \text{R}_{33} \\
\end{array} + \begin{array}{c}
\text{R}_{22} \\
\end{array} + \begin{array}{c}
\text{R}_{22} \text{R}_{33} \\
\end{array} + \cdots \]

with \(R_n = ([k_1^2 + M^2](k_1^2 + M^2) + (k_1^2 + M^2)(k_1^2 + M^2)) / (k_1^2 - k_1^2 + M^2) + \cdots \)

When \(M \rightarrow 0\), infra-red divergences exponentiate to zero all diagrams with non-zero \(t\)-channel color. If \(\alpha_s \rightarrow \infty\), color zero reggeon amplitudes scale canonically \((\sim k_\perp^2)\) when \(\forall k_\perp \rightarrow 0\), \(\Rightarrow\) no confinement.

To find the IP in QCD, we must see the odd-signature gluon particle poles disappear completely, leaving only an even signature IP. In part, we need to find the reverse of the RFT phase-transition to the Supercritical IP. \(\text{The correct scattering states will be crucial.}\)
It will be essential to study hadrons and the IP simultaneously (as reggeon bound states). The simplest kinematics for this purpose is that of "maximal helicity-pole limits".

Reggeon Diagrams for Maximal Helicity-Pole Limits

We noted that S-W representations show maximal helicity-pole limits are described by a single F-G amplitude. Consequently, reggeon unitarity implies all such limits can be described by reggeon diagrams involving two-dimensional "transverse momenta".

(Because the large momenta involved can not be confined to a single light-like plane, the \( k_\perp \)-planes involved contain lightlike momenta. This is why the \( k_\perp \) infra-red divergences we will find are related to the \( k_\perp = 0 \) problems of light-cone quantization.)

E.g. for an 8-pt amplitude define "angular variables" \( z_i \) and \( u_i \) as illustrated and study the maximal "helicity-flip" limit

\[
\begin{align*}
z_0, u_1, u_2^{-1}, u_3, \text{ } u_4^{-1} \rightarrow \infty
\end{align*}
\]

The behavior of invariants is

\[
\begin{align*}
P_1\cdot P_2 &\sim u_1 u_2^{-1}, \quad P_1\cdot P_3 \sim u_1 z_0 u_3, \quad P_2\cdot P_4 \sim u_2^{-1} u_4^{-1}, \quad P_1\cdot Q_3 \sim u_1 z_0, \quad P_1\cdot Q_4 \sim z_0 u_4^{-1},
Q_1\cdot Q_3 &\sim z, \quad P_4\cdot Q_1 \sim z_0 u_4^{-1}, \quad P_1\cdot Q_6, P_2\cdot Q_9, P_3\cdot Q_9, P_4\cdot Q_0 \text{ finite}
\end{align*}
\]

(In the "non-flip" limit \( u_2, u_4 \rightarrow \infty \), the invariants \( P_1\cdot P_2, P_3\cdot P_4 \rightarrow \infty \).)

The limit \( z_0, u_1, u_2^{-1}, u_3, u_4^{-1} \rightarrow \infty \) is determined by 5 conjugate \( J \)-planes \( (J_i = n_i, \ i = 1, \ldots, 4) \).

Reggeon unitarity \( \Rightarrow \) elastic scattering reggeon interactions appear in each \( J_i \)-plane. Therefore, the limit is described by a set of reggeon diagrams of the form

where \( \bigotimes \) contains all elastic scattering reggeon diagrams. There are also crucial new elements, the \( T^F \) couplings, which contain new "reggeon helicity-flip" vertices.

Reggeon Helicity-flip Vertices

The structure of these vertices, which will play a central role in our discussion, can be studied most simply in a triple-Regge "helicity-flip limit". We consider the

"maximal helicity-pole limit"

\[
\begin{align*}
z_0, u_1, u_2^{-1} \rightarrow \infty.
\end{align*}
\]

Large invariants are

\[
\begin{align*}
s_{13} &\sim z_3 u_1, \quad s_{23} \sim z_3 u_2^{-1},
\end{align*}
\]

\[
\begin{align*}
s_{11'3} &\sim z_3, \quad s_{12} \sim u_1 u_2^{-1}
\end{align*}
\]

This limit can be realized in terms of light-cone momenta as follows.
First consider three quarks scattering via gluon exchange with a quark loop triple-gluon coupling. If the loop remains at rest, the three large light-cone momenta pick out the corresponding $\gamma$ matrices in the loop,

$$
\gamma \rightarrow g^6 \frac{P_1 P_2 P_3}{t_1 t_2 t_3} T_F
$$

where $t_i = Q_i^2$, and the helicity-flip vertex $T_F$ is a component of the full triangle diagram, i.e. $T_F = \Gamma_{1+1-3+} (q_1, q_2, q_3)$, where $\gamma_{\pm} = \gamma_0 \pm \gamma_i$ and

$$
\Gamma_{\mu \nu \sigma} = i \int \frac{d^4 k}{(q_1 + k)^2 - m^2} \frac{\epsilon_{\mu \nu \sigma}(q_1 + k + m)\gamma_{\nu}(q_2 + k + m)\gamma_{\sigma}(q_3 + k + m)}{((q_1 + k)^2 - m^2)((q_2 + k)^2 - m^2)((q_3 + k)^2 - m^2)}
$$

Consider the $O(m^2)$ chirality-violating part of $\Gamma_{1+1-3+}$, which we denote by $T_F m^2$. It will be very important that for $T_F m^2$, the limits $q_1, q_2, q_3 \rightarrow Q$ and $m \rightarrow 0$ do not commute, i.e.

$$
T_F m^2 \sim T_0^F = Q i m^2 \int \frac{d^4 k}{(k^2 - m^2)^3} = R Q
$$

where $R$ is independent of $m$. This is an "infra-red anomaly" due to the triangle Landau singularity!

6. THE INFRA-RED ANOMALY IN HELICITY-FLIP VERTECES

After color factors are included, all diagrams summed, and signatured amplitudes formed, the odd-signature of the reggeized gluon implies the "infra-red anomaly" $T_0^F$ cancels in the helicity-flip triple reggeon vertex.

To obtain higher helicity-flip vertices we evaluate appropriate multiple discontinuities of multi-gluon exchange amplitudes.

(Ξ performing $k^\pm$ integrations by putting quark lines on-shell.)

The triangle diagram, with $\gamma_{\pm}$ couplings, again appears. E.g.

But is the infra-red anomaly $T_0^F$ present?

Usually helicity conservation $\Rightarrow$ color parity $C_c \ (A^i_{ab} \rightarrow -A_{ba})$ for color matrices = signature $\tau$ (odd/even reggeon no.) for reggeon states.

But, because $T_0^F$ is linear in the $Q_i$, an odd number of reggeon states with "ANOMALOUS COLOR PARITY", i.e. $C_c \neq \tau$ must be involved.

E.g. the "anomalous odderon" (ANO) reggeon state, i.e. the 3-reggeon state with color factor $f_{ijk} A^i_\mu A^j_\nu A^k_\rho$, this has $C_c = -\tau = +1$ (c.f. the winding-number current $K_\mu = \epsilon_{\mu \nu \gamma \delta} f_{ijk} A^i_\nu A^j_\gamma A^k_\delta$).
are many discontinuities of many quark-loop Feynman diagrams contributing to multi-reggeon vertices. ANO vertices that contain $T^F_3$ also contain the ultra-violet anomaly e.g. the triple ANO vertex.

The anomaly is present since $V_{123} \sim \gamma_1 \gamma_1 \gamma_3 \sim i\gamma_5 \gamma_2 + \ldots$ and the other two couplings contain just two distinct $\gamma_\pm$-matrices.

Because the vertices appearing in helicity-flip (and more general multi-Regge) limits contain reduced four-dimensional (quark) loop integrals, it is clear that ultra-violet, power-like, divergences are likely to be present, in particular in the triangle diagram we are discussing.

$\Rightarrow$ underlying diagrams increase with energy faster than reggeon diagrams $\Rightarrow$

non-unitary high-energy behavior ?? (needs study !)

To remove all potential ultra-violet divergences (and the anomaly) we use Pauli-Villars fermions with mass $m_A \to \infty$ after $m \to 0$.

$\Rightarrow$ GAUGE INVARIANCE (i.e. reggeon Ward identities) IS SATISFIED FOR $m \neq 0$. But, the theory is non-unitary in the quark sector for $|p| \gtrsim m_A$.

( The $m = 0$ theory will be unitary !!

The "infra-red anomaly", in regularized vertices $T^{F,m^2}$ gives

$$T^{F,m^2} = T^{F,m^2} - T^{F,m^2}_A \sim Q^2 , \quad T^{F,0} = T^{F,m^2}_A \sim -RQ$$

$Q \to 0$

The "anomalous" behavior of $T^{F,0}$, as $Q \to 0$, produces a new infra-red divergence, as $m \to 0$, in color zero massless reggeon diagrams where $\forall Q_i \sim 0$ is part of the integration region.

The Infra-red Divergence due to the Anomaly

For such a divergence to occur $T^F$ must be a disconnected component of a vertex coupling distinct reggeon channels e.g.

\[ \int \frac{d^2Q}{Q^3} \left( \equiv \int \frac{d^2Q}{Q^2 Q_1 Q_2 (Q-Q_1)^2 (Q-Q_2)^2} \right) \times V_1(Q) V_4(Q) V_5(Q) V_6(Q) T^F(Q) T^F(Q) \]

$\times$ [REGULAR VERTICES AND REGGEON PROPAGATORS]

Ignoring the $V_i$ and taking $T^{F,0}(Q) \sim Q$ gives $\int d^2Q/Q^6 \times Q^2 \equiv \int dQ/Q^4$. Clearly, this divergence could be canceled by the behavior of $V_i$ and/or other diagrams.
SU(2): SYMMETRY AND CONFINEMENT

We can avoid the possibility of a diagrammatic cancelation if we only partially restore the gauge symmetry to SU(2) ⊂ SU(3). The gluon spectrum is then

- SU(2) triplet, mass = 0
- 2 SU(2) doublets, mass = 2M/3
- SU(2) singlet, mass = M

The trajectory of the SU(2) singlet is clearly infra-red finite. We again consider reggeon diagrams of the form

![Reggeon Diagram]

We also impose the “initial condition” that $V_1(Q), V_2(Q) \neq 0$ when $Q \to 0$. For the remaining vertices, gauge invariance ($\leftrightarrow$ reggeon Ward identities) $\to V_i(Q) \sim Q \ i \neq 1, 2$

If we choose
- $\Rightarrow$ SU(2) singlet combination of massless reggeons with $C_\tau = -\tau = +1$ ($\equiv$ ANO)
- $\Rightarrow$ SU(2) singlet massive reggeized gluons.

there is an overall logarithmic divergence !!

Similarly, for a general diagram with $n$ $T^F$ vertices, if $V_1(Q), V_2(Q) \to 0$, when $Q \to 0$, and $V_i(Q) \sim Q$ for the remaining vertices,

$\int \frac{d^2Q}{Q^2} \left[ \int \frac{d^2Q}{Q^4} \right]^n [V(Q) T^F(Q)]^n$ $\to$ an overall logarithmic divergence (as $m \to 0$). The initial condition $V_1, V_2 \neq 0 \leftrightarrow$ imposing a $k_L = 0$ (“wee parton”) ANO component on two of the scattering reggeon states

Extracting “physical amplitudes” from the coefficient of the divergence gives “a confining reggeon S-Matrix”

$\leftrightarrow$ an ANO component, with $k_L = 0$, for all reggeon states

$\leftrightarrow$ a wee-parton ANO (winding-number ?) “condensate”.

$\leftrightarrow$ no massless multigluon states + “completeness”

$\leftrightarrow$ CONFINEMENT !!!
SU(3) gauge symmetry → SU(2) introduces a mass scale separating “partons” in a physical state into a universal wee-parton “condensate” and a dynamical “normal” parton component containing massive reggeons (and quark bound states - see next page).

=> a very simple “parton model” for physical scattering amplitudes.

In the simplest processes, the dynamical partons scatter via perturbative interactions while the infra-red quark triangle anomaly provides the “scattering” of the condensate.

\[ \text{A physical amplitude} \]

The condensate has the crucial property that it switches the signature of the normal parton component of reggeon states.

Note that this argument works only if the \( q\bar{q} \) pair carries a quantum number that ensures there is no mixing with daughter multigluon states

=> there is no massless \( \eta' \)?
8. SUPERCRITICAL → CRITICAL IP AND SU(2) → SU(3)

Reggeization of the SU(2) singlet gluon depends on the cancelation

\[ \alpha_+ + \alpha_- + \alpha_0 = 0 \]

where now \( \alpha_\pm \) = massive SU(2) doublet.) This cancelation also \( \leftrightarrow \) “reggeization” in the SU(2) singlet \( \tau = +1 \) channel. In the condensate this gives a \( \tau = -1 \) Regge pole partner for the IP - the massive SU(2) singlet vector particle lies on the trajectory.

\( \leftrightarrow \) a fundamental feature of the super-critical phase.

In addition to coupling multi-reggeon states in which all reggeons have \( k_\perp = 0 \), the anomaly will also couple states with subsets having total \( k_\perp = 0 \). (\( \leftrightarrow \) interactions of the condensate with normal partons.) This generates the vacuum production of \( \alpha_\tau \)'s (\( \leftrightarrow \) singular interactions) that is another characteristic of the supercritical phase, e.g.

![Diagram](image)

where, \( \cdots \) is a normal reggeon state and \( \cdots \) \( \leftrightarrow \) IP.

Consequently, all features of the supercritical phase of RFT are present in our solution of partially-broken QCD.

**Restoring SU(3) Gauge Symmetry** (decoupling a scalar triplet) is straightforward with a \( k_\perp \)-cut-off (\( \Lambda_\perp \)). Since the SU(2) singlet mass \( M \to 0 \Rightarrow \alpha_\tau(0) \to 1 \), does this give the critical IP ??

Within supercritical RFT, as \( \alpha_\tau(0) \to 1 \), the condensate simultaneously disappears and the odd-signature partner for the IP also decouples.

If the mapping partially-broken QCD \( \leftrightarrow \) supercritical RFT is complete, both \( M \) and \( \Lambda_\perp \) are “relevant parameters” and \( \exists \Lambda_{\perp c} \) for which

\[ M \to 0 \Rightarrow \text{both critical IP scaling and SU}(3) \text{ gauge symmetry}. \]

\( \leftrightarrow \) wee-parton condensate disappears, leaving a universal wee parton distribution within hadrons producing all vacuum properties and merging smoothly with “constituent” partons. (\( \equiv \) Feynman’s parton model.)

Also RFT \( \Rightarrow \) varying \( \Lambda_\perp \) can produce the phase transition,

\[ \Lambda_\perp > \Lambda_{\perp c} \leftrightarrow \text{subcritical IP}, \ \Lambda_\perp < \Lambda_{\perp c} \leftrightarrow \text{supercritical IP}. \]

To obtain a non-trivial result when \( \Lambda_\perp \to \infty \) requires \( \Lambda_{\perp c} \to \infty \). (This corresponds to a very special quark content - see below !)

\[ \text{That SU}(3) \text{ symmetry is not broken in the supercritical phase} \Rightarrow \text{the direction of the wee-parton condensate in SU}(3) \text{ must be averaged over. In effect we are trying to construct a solution with SU}(3) \text{ symmetry by first obtaining a non-trivial solution for the anomaly (\( \equiv \) the topological instanton problem) in an SU}(2) \text{ subgroup and then integrating over the subgroup direction (\( \equiv \) an instanton parameter).} \]
In the SU(3) limit both the SU(2) singlet condensate and the quark pair project on SU(3) singlet and octet states.

$$\Pi = 0 + 8$$

$$C_e = +1 \quad C_o = -1$$

implies that (locally) we have “finite volume RFT”. If there is also a $$\Lambda_\perp$$ then, the finite volume will keep the theory supercritical as SU(3) symmetry is restored.

$$C_e = -1 \quad C_o = -1$$

The IP carries only the -1 state and scatters the +1 state into the -1 state.

$$C_e = -1 \Rightarrow \text{glueballs hard to find and also the } C_e = +1 \text{ two-gluon BFKL Pomeron does not contribute!}$$

In DIS diffraction, large $$Q^2$$ acts as a (local) lower $$k_\perp$$ cut-off. This implies that (locally) we have “finite volume RFT”. If there is also a $$\Lambda_\perp$$ then, the finite volume will keep the theory supercritical as SU(3) symmetry is restored.

$$\Rightarrow \text{IP } \sim \text{ a single (reggeized) gluon.}$$

Note that $$\Lambda_\perp << \Lambda_{\perp e} \Rightarrow \text{constituent quark hadrons interacting via a massive “composite gluon” (exchange degenerate IP).}$$

- c.f. the constituent quark model - that many people would like to derive via light-cone quantization.

When is this Solution Realized in QCD??

Our high-energy hadron S-Matrix is produced by massless gluons and quarks. Conventionally, non-perturbative strong-coupling effects are expected to eliminate massless gluons for $$k_\perp < \lambda_{QCD}$$.

Recall $$N_f = 16 \rightarrow$$ a weak-coupling infra-red fixed point. Also, with $$N_f = 16$$, SU(3) symmetry can be broken to SU(2) with an asymptotically-free scalar field $$\rightarrow \Lambda_{\perp e} = \infty$$,

$$\Rightarrow \text{critical IP scaling occurs } \forall k_\perp$$

(in principle) can match smoothly with perturbative QCD.

We conclude that, for our high-energy solution to apply, there should be an additional quark sector in QCD - WHERE IS IT??

The “Higgs sector” of the Standard Model could be produced by a flavor doublet of color sextet quarks ($\leftrightarrow$ the electroweak scale is a QCD scale).

In the $$\beta$$-function, $$N_f^2 = 6 + N_f^2 = 2 \equiv N_f^2 = 16$$!!

We expect that, to obtain our solution, the hadron S-Matrix should be defined initially with massless quarks (including sextets). Mass effects should then be added within the S-Matrix.

Observing our solution, (i.e. “single gluon” supercritical IP behavior) at HERA $$\Rightarrow \exists$$ a further quark sector, i.e. new QCD physics, above the (diffractive) $$Q^2$$ range seen at HERA.