Two Dimensional Beam Smoothing by Spectral Dispersion for Direct Drive Inertial Confinement Fusion

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Two dimensional beam smoothing by spectral dispersion for direct drive inertial confinement fusion

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ABSTRACT

Two dimensional smoothing by spectral dispersion is analyzed by using diffraction theory calculations. It is shown that by using standard frequency modulated light one can obtain bandwidth limited smoothing over integration times relevant to inertial confinement fusion (about 1 nsec) with modest induced beam divergence. At longer integration times one can obtain bandwidth limited smoothing by increasing the divergence and/or by using more advanced phase modulation methods.

Keywords: Beam smoothing, smoothing by spectral dispersion, inertial confinement fusion, direct drive.

1. INTRODUCTION

A number of approaches have been suggested to achieve the required level of target illumination uniformity for direct drive inertial confinement fusion (ICF) of ~1% RMS intensity variance.1-6 The use of random phase plates (RPP)2 in conjunction with angular dispersion of frequency modulated (FM) light (termed smoothing by spectral dispersion- SSD)4 holds much promise for direct drive ICF using mega-Joule class glass lasers because the near field beam quality can be maintained. This method has been analyzed and measured for dispersion in one dimension.4,5,6 However, smoothing by 1D SSD is insufficient to reach the uniformity required within the allowable divergence, and it is necessary to disperse the driver beam in both orthogonal dimensions (2D SSD). This can be accomplished by sequentially applying FM, dispersion by a grating, applying FM at a second frequency, and orthogonal dispersion by a second grating.7

In Fig. 1 the standard arrangement for 2D SSD is shown and includes an additional grating before each modulator to compensate for the temporal skew generated by the grating following each modulator (the latter grating disperses the modulator bandwidth).

\[ E_{out}(t) = E_{in}(t) \exp[i \beta_x \sin(2 \pi v_x (t + s_x x)) + i \beta_y \sin(2 \pi v_y (t + s_y y))] \]  (1)

Figure 1: The standard implementation of 2D SSD (after ref. 7).

In this case the field after modulation and dispersion is given by
where $\beta_x$ and $\beta_y$ are the modulation depths, and $s_x$ and $s_y$ are the temporal skews per unit length generated by the gratings in the two orthogonal directions.

In a simplified implementation of 2D SSD the first grating is omitted and the grating pair between the two modulators may be combined into one rotated grating. In this case the first modulator may be a single mode integrated modulator as is currently being implemented on the Beamlet laser (Fig. 2). This leaves a temporal skew in the beam equal to the period of the first modulator (which can be less than 100 psec). The second modulator must be a bulk modulator, since after dispersion of the bandwidth from the first modulator, propagation in a single mode device is not possible.

![Diagram](image)

Figure 2: A simplified version of 2D SSD to be implemented on the Beamlet laser. Without grating pre-compensation, the residual total x-direction skew on the output pulse is given by $1/v_x$.

2. ANALYSIS OF SMOOTHING BY SPECTRAL DISPERSION

After transmission through an RPP a temporally varying speckle pattern is produced at the focal plane (Fig. 3). In one perspective the speckle pattern moves over many decorrelation lengths, and the integrated intensity variance is thereby reduced according to $\sigma = 1/\sqrt{N}$, where $N$ is the number of uncorrelated speckle patterns. In this time domain perspective, it is clear that the maximum number of speckle patterns generated will always be limited by the bandwidth. Thus, independent of smoothing technique, $N \leq \Delta v \cdot t$, where $\Delta v$ is the full extent of the generated bandwidth.

![Diagram](image)

Figure 3: Geometry of the focusing arrangement. The driver beam, after passing through the phase plate, is focused to field $E(x,y)$ in the focal plane (dashed line).

In the frequency domain view point one can consider each FM sideband as generating a shifted speckle pattern. The shift is determined by the sideband frequency detuning and grating dispersion for each orthogonal direction. To obtain optimal smoothing each shifted speckle pattern must be uncorrelated with
all others; and then the fluence will asymptotically smooth to \( \sigma = 1/\sqrt{N} \), where \( N \) is the total number of sidebands. The minimum angular shift between adjacent sidebands required to achieve decorrelation can be obtained from the spatial autocorrelation as given by the theory of speckle: \(^8\)

\[
\int E(x,y)E^*(x + \Delta x,y + \Delta y)dxdy = \tilde{I} \text{sinc}(\Delta x / (F\lambda / D))\text{sinc}(\Delta y / (F\lambda / D))
\]

(2)

where \( E \) is the speckle field in the focal plane, \( x \) and \( y \) are the focal coordinates, \( D \) is the near field (square) beam aperture, \( F \) is the final lens focal length, and \( \lambda \) is the wavelength on target. Thus, for \( \Delta x \) or \( \Delta y = F\lambda / D \), the correlation between a speckle pattern and its shifted copy is zero, or, in terms of the far field angular shift necessary, \( \Delta \theta_{\text{min}} = \pm \lambda / D \). Thus, the asymptotic smoothness, which is determined by the maximum number of uncorrelated speckle patterns, is limited by the laser divergence \( \theta_{\text{div}} \). In the case of 1D SSD the asymptotic smoothing limit is therefore given by \( \sigma = 1/\sqrt{\theta_{\text{div}} / (\lambda / D)} \), whereas for 2D SSD, for the same laser divergence, the maximum number of uncorrelated speckle patterns is increased quadratically, and thus the 2D SSD asymptotic smoothing limit is given by \( \sigma = 1/(\theta_{\text{div}} / (\lambda / D)) \). It should be emphasized that \( \lambda \) in this discussion refers to the wavelength on target. Because divergence is unchanged by harmonic conversion, the fundamental beam divergence in terms of its number times diffraction limited will be less than that of the harmonic beam by a factor equal to the harmonic number. Thus, for example, defining the diffraction limited divergence of the fundamental to be \( 2\lambda_{\text{fs}} / D \), a divergence of the fundamental equal to 5 times the diffraction limit implies a 2D asymptotic smoothing limit at \( 3\omega \) of \( 1/(2.5 \cdot 3) = 3.3\% \).

The necessary minimum spectral dispersion for optimal SSD is derived by requiring adjacent sidebands in each orthogonal direction to be separated by the decorrelation angle \( \Delta \theta_{\text{min}} \). Given that adjacent sidebands are separated by the modulation frequency \( \nu_{\text{mod}} \), one requires that \( \Delta \theta_{\text{min}} = \lambda / D = \nu_{\text{mod}} \cdot d\theta / dv \). The angular dispersion on target is related to the induced temporal skew per unit transverse length across the beam (s) by the simple relation \( d\theta / dv = s \cdot \lambda \), and therefore the required minimum dispersion can be stated simply in terms of the beam skew:

\[
s \cdot D = 1/\nu_{\text{mod}}
\]

(3)

or, in other words, the total skew across the beam must be equal to or greater than the FM period. For 2D SSD the temporal skew necessary in each orthogonal direction is determined by the respective modulator frequency.

In the case of 2D SSD one must use two modulators with incommensurate frequencies, otherwise, the two dimensionality of the SSD is not fully exploited. To see this effect, the interplay of the modulation frequencies in 2D SSD is calculated in the case where one frequency is 10 Gfh, and the second is 3.6, 5, 7.5, and 10 Gfh. The dispersion in each dimension is assumed such that adjacent FM sidebands are separated by \( \Delta \theta_{\text{min}} \). All smoothing simulations presented in this paper are calculated assuming a binary RPP with 128\(^2\) elements. The far field intensity is calculated at each time step from the Fourier transform of the product of the modulated and dispersed input field (Eq. 1) and the RPP transmission amplitude. The intensity is then integrated and the variance determined. Figure 4 shows the time variation of the effective number of decorrelated speckle patterns, \( N = 1/\sigma^2 \). The observed oscillatory behavior is a result of periodic cycling of speckle patterns owing to the incommensurate relationship between the two modulation frequencies \( \nu_x \) and \( \nu_y \). That is, if there exists a given time interval \( t \) such that \( (\nu_x - \nu_y)t = m \), where \( m \) is an integer, then the generated speckle patterns will repeat exactly, and the variance will increase periodically.
Figure 4: Variation of the effective number of decorrelated speckle patterns \(1/\sigma^2\) with time for 2D SSD. One modulation frequency is 10 GHz, and the other is 10 (dots), 5 (dash), 7.5 (dot-dash), and 3.6 GHz (solid). The bandwidth of each modulator is set at 500 GHz.

Alternatively, from a frequency domain perspective the overlap of harmonics of the two modulators leads to an equivalent consideration. I.e. if there exist integers \(m, n\) such that \(mv_x = nv_y\), then if one considers the interval \(t = n/v_x\) one finds \((v_x - v_y)t = n - m\), and the periodic condition is satisfied. This repetition effect can thus be minimized by selecting an incommensurate second frequency, such that integer multiples of the two frequencies are not coincident (as in Fig. 4, for the solid curve, corresponding to modulation frequencies of 3.6 and 10 GHz).

3. ASYMPTOTIC BEHAVIOR

At longer integration times, even for incommensurate frequencies, there is a significant departure in the smoothing performance from the ideal rate of \(1/\sigma^2 = t\Delta v\). This is a result of the nonuniform filling of the bandwidth and the induced divergence. Assuming that the dispersion is set such that all sidebands are separated by \(\Delta \theta_{\min}\), and that each of \(N\) sidebands has frequency \(v_j\) and spectral intensity \(I_j\), Gaussian averaging can then be used to obtain the total effective number of uncorrelated speckle patterns as a function of time:

\[
\sigma^{-2} = \frac{\left(\sum_{j=1}^{N} I_j\right)^2}{\sum_{j,j'=1}^{N} I_j I_{j'} \text{sinc}^2(v_j - v_{j'})t} \xrightarrow{t \to \infty} \frac{\left(\sum_{j=1}^{N} I_j\right)^2}{\sum_{j=1}^{N} I_j^2},
\]

where \(\text{sinc} x = \sin \pi x / \pi x\). For optimal bandwidth limited smoothing one desires a uniform and equally spaced spectrum, i.e. \(I_j = 1\) and \(v_j - v_{j'} = \Delta v(j - j')\). In this case, at an integration time equal to the modulation period, the effective number of speckle patterns \(1/\sigma^2\) reaches the maximum \(N\), and thus the variance reaches \(\sigma = 1/\sqrt{N}\). In contrast, for example, 1D SSD using simple FM results in sidebands of
nonuniform spectral intensity $J^2_0(\beta)$, where $\beta$ is the modulation depth. The asymptotic smoothing level can then be written:

$$\sigma^{-2} = \left[ \sum_{-\infty}^{\infty} J^2_0(\beta) \right] / \sum_{-\infty}^{\infty} J^4_0(\beta).$$  \hspace{1cm} (5)

For $\beta = 10 - 30$ one finds that $\sigma^{-2} = (1.3 - 1.5)\beta$, whereas the total number of sidebands is ~$2\beta$, and thus effectively only 65-75% of the sidebands are used. For 2D SSD this spectral filling factor is applicable to each dimension. Thus for 2D SSD with FM in each orthogonal dimension, the asymptotic smoothing level will reach only about 50% of the total number of sidebands.

4. ADVANCED PHASE MODULATION METHODS

Increasing the modulation depth, bandwidth, and/or beam divergence is one way to get improved smoothing, but this also generally leads to reduced laser performance. Significant smoothing improvement at longer integration times can be also be accomplished using better bandwidth filling. One simple method which improves spectral uniformity is to apply two modulation frequencies along each dispersion direction. Figure 5 shows the spectrum from a single FM (dots) and two combined FM's (solid curve), when averaged over the interval of the larger modulation frequency.

![Figure 5: Spectrum of a single FM (4 Ghz and depth of 20, dots) compared with the spectrum of two successive FM's when averaged over a 4 Ghz bandwidth (modulation frequencies of 4 and 0.25 Ghz, depths of 20, solid curve).](image)

The improvement in smoothing obtained with this method is shown in the calculation of Fig. 6, where it is compared with standard 2D SSD under similar bandwidth and divergence conditions. It should be noted that over the time interval relevant to smoothing for inertial confinement fusion targets (less than 1 ns), the bandwidth limit is nearly achieved even with the standard 2D SSD (i.e. a single FM in each direction).
Figure 6: Variation of $1 / \sigma^2$ with time in the case of standard 2D SSD using 8.2 and 3.4 G Hz modulation frequencies along the orthogonal directions, aggregate bandwidth of 500 G Hz, and induced divergence 25 times the diffraction limit (dash). Solid curve: a second modulation frequency is applied in each direction (0.3 G Hz and depth of 20), and the beam divergence and bandwidth are unaltered. The dotted curve is the bandwidth limit.

Figure 7: Spectral intensity of sidebands of periodic train of linearly chirped pulses (solid curve with dots), and FM of depth 20 (dashed curve). Both modulation methods have a repetition frequency of 10 G Hz.

Further improvement can be obtained with other advanced phase modulation forms that generate the desired uniform spectral distribution. For example, applying quadratic phase modulation in a periodic fashion to a train of nearly overlapping pulses, yields pulses with a linear frequency sweep. The sideband spectrum generated by this phase modulation is shown in the solid curve (and dots) of Fig. 7, where it is
compared to a FM spectrum of similar bandwidth and periodicity (dashed curve). The clear improvement in spectral uniformity is evidenced by the effective number of sidebands as determined by Eqs. (4) and (5): 46 and 28 for the periodic quadratic phase modulation and simple FM, respectively.

A calculation using quadratic phase modulation demonstrates that bandwidth limit smoothing performance can be obtained over integration times of many nanoseconds, and asymptotic levels of ~2% can be achieved (Fig. 8).

Figure 8: Variation of $1/\sigma^2$ with time for 2D SSD, where instead of standard FM, quadratic phase modulation (a linear frequency sweep) is used in each direction. The driver field is taken to be a periodic train of nearly overlapping pulses, each with a linear frequency sweep. The repetition frequencies are 10 and 2.45 GHz in the two directions, the total bandwidth is 550 GHz, and the induced divergence is 30 times diffraction limited. The dashed curve shows the ideal smoothing limit for a bandwidth of 550 GHz.

5. SMOOTHED FLUENCE DISTRIBUTION

A distinct feature of 1D SSD is the appearance of streaks in the integrated intensity distribution. This is easily understood since the generated speckle patterns are constrained to move along the dispersion direction. It is of interest to examine the smoothed intensity distribution for 2D SSD as well. In Fig. 9, images of the integrated intensity distribution are compared for 1D (top left) and 2D SSD (top right), where the phase modulation method is simple FM. Horizontal streaks are clearly seen in the 1D SSD image, whereas for 2D SSD streaks now appear in both the horizontal and vertical directions, forming a weave pattern. These patterns can be understood in terms of the far-field distribution of the sidebands for 1D and 2D SSD. In the case of 1D SSD, the sidebands are dispersed into a row of far field spots (each of which generates a shifted speckle pattern), whereas for 2D SSD, the far field pattern is a rectangular array of regularly spaced spots, and hence the observed fluence distribution. If it is desired to reduce the regularity of the fluence distribution (i.e. to eliminate the horizontal and vertical streaks), then one must use a more complicated phase modulation technique to eliminate the regularity of the sidebands. One such approach is to use two successive FM's in each of the orthogonal directions (as in the calculation of Fig. 6). In the lower panels of Fig. 9, standard 2D SSD (left) is compared with this double FM method (right). As can be seen, the horizontal and vertical streaks are mostly eliminated with the double FM technique.
Figure 9: Integrated far field intensity patterns of 1D (top left) and 2D SSD (top right), using simple FM. The width of each top image is 58λ/D. Bottom panels compare, at twice the magnification (a width of 28λ/D), the image of standard 2D SSD (left) with that of 2D SSD using two FM’s in each orthogonal direction (as in Fig. 6). The images are contrast enhanced. The total variances are 19%, 3.3%, and 2.4%, and the integration times are one modulation period, 6 ns, and 10 ns for 1D SSD, standard FM 2D SSD, and 2D SSD with double FM in each direction, respectively.

6. SPATIAL SPECTRUM OF THE SMOOTHED FLUENCE DISTRIBUTION

For direct drive ICF it is not only important what the total variance of the fluence is, but the spatial spectrum imprinted on the target is crucial as well. Therefore it is of interest to examine the spatial spectrum of the smoothed fluence distribution. For a single speckle pattern generated from a uniformly illuminated square aperture the spatial power spectrum of the fluence \( \tilde{U}(x,y) \) is given by

\[
|\tilde{U}(f_x, f_y)|^2 = \Lambda(f_x/f_{\text{max}})\Lambda(f_y/f_{\text{max}}) + f_{\text{max}}^2 \delta(f_x, f_y),
\]  

(6)
where $A(x) \equiv 1-|x|$ for $|x| \leq 1$ and 0 for $x > 1$, and $f_{\text{max}} \equiv D / F \lambda$. The non-zero average speckle intensity contributes the $\delta$-function at $f_x = f_y = 0$ to the spatial spectra, and is omitted for simplicity in the following analysis. The spatial spectrum of the asymptotic fluence distribution of 2D SSD is easily found in a frequency domain approach when the modulation frequencies are incommensurate and adjacent sidebands are uncorrelated (i.e., they are separated by $\Delta \theta_{\text{min}}$). For this case, in each dimension of dispersion one has that the asymptotic fluence is the sum of identical shifted speckle patterns of relative intensity given by the corresponding spectral sideband intensities ($I_{xj}$ and $I_{yk}$ for the two dimensions, respectively). Thus for 2D SSD one has that the asymptotic fluence is given by

$$U(x,y) = I_s(x,y) \otimes \sum_j I_{xj} \delta(x - \Delta x \cdot j) \otimes \sum_k I_{yk} \delta(y - \Delta y \cdot k) ,$$

where $I_s(x,y)$ is the single frequency speckle pattern, $\otimes$ denotes convolution, and $\Delta x = \Delta y = F \lambda / D = 1 / f_{\text{max}}$ is the minimum focal plane shift necessary to decorrelate adjacent sidebands. The spatial power spectrum of Eq. (7) is found to be

$$|\hat{U}(f_x,f_y)|^2 = \Lambda(f_x / f_{\text{max}}) \Lambda(f_y / f_{\text{max}}) |\mathcal{S}(\sum_j I_{xj} \delta(x - j \Delta x))|^2 |\mathcal{S}(\sum_k I_{yk} \delta(y - k \Delta y))|^2 ,$$

where $\mathcal{S}$ denotes Fourier transform. Thus, one sees that the spatial spectral behavior is separable in the two dispersion directions and that the spectrum in each direction is the product of the single speckle spectrum (Eq. 6) and the Fourier transform of the shift pattern in the focal plane. As a simple example, in the case of optimal smoothing, where $I_{xj} = 1$ for $1 \leq j \leq N$, one obtains

$$|\mathcal{S}(\sum_j I_{xj} \delta(x - \Delta x \cdot j))|^2 = \sin^2(\pi Nf_x / f_{\text{max}}) / \sin^2(\pi f_x / f_{\text{max}}) .$$

In the case of SSD using FM one can simply evaluate Eq. (8) with $I_j = J_2^2(\beta_x)$, or equivalently it can be shown that the Fourier transform in Eq. (8) yields exactly the spatial autocorrelation of the modulated field $E(x,t) = \exp[i \beta_x \sin(2 \pi v_x (u + s_x x))]$ (independent of $t$) with $s_x v_x = 1 / D$ and the Fourier focal plane substitution $x \rightarrow f_x \lambda F$. That is,

$$|\mathcal{S}(\sum_j I_{xj} \delta(x - \Delta x \cdot j))|^2 = |\exp[i \beta_x \sin(2 \pi x / D)] \otimes \exp[-i \beta_x \sin(2 \pi x / D)]|^2_{x = f_x \lambda F} .$$

The above analysis is demonstrated in the binary RPP diffraction calculations of Fig. 10, which shows the spatial spectrum of the asymptotic fluence distribution for 1D and 2D SSD. In the 1D case (top curves), the spatial spectra are separable in the dispersion and orthogonal directions, and are markedly different. This corresponds to the appearance of streaks in 1D SSD, in that in the dispersion direction the spectral energy is concentrated at low frequency, whereas orthogonally, the spectral shape is identical to that of an unsmoothed speckle pattern (Eq. 6). It should be emphasized that the scales for these two curves are not equal, and that the integrated spectrum in each direction is equal. For 2D SSD (bottom curves) the spatial spectrum is again separable into $x$ and $y$ dependencies. As can be seen from these curves, in the case that the modulation depths are equal, the $x$ and $y$ spectra are identical within the statistical error and are both given by the result of Eq. (10). For 2D SSD the spectral energy is concentrated at low frequency in both directions, which corresponds to appearance of streaks in both dispersion directions (as in Fig. 9).
7. CONCLUSIONS

It has been shown that 2D SSD using FM in both orthogonal directions can accomplish bandwidth limited smoothing for times ~ 1 nsec. These results were obtained assuming a bandwidth of 500 GHz and divergence of 25 times diffraction limit on target. Streaks appear along the dispersion directions and correspond to the concentration of the smoothed speckle energy at low spatial frequency. To achieve the bandwidth limit for integration times longer than 1 nsec one can either increase the divergence of the beam, or implement a phase modulation technique which results in a more uniform spectral distribution. Applying a second FM in each direction gives significant improvement in this regard, and also eliminates streaks from the integrated far field intensity distribution. The use of linear chirped pulses results in an almost completely uniform spectrum, achieves nearly bandwidth limited smoothing performance for ~5 nsec, and asymptotic smoothing levels of ~2%.

Finally, it should be noted that all results discussed here are for a single driver beam, and that the averaging effects of the overlap of 192 beams (effectively ~50 beams illuminate any single point) and two polarizations is expected to further reduce the intensity variance on target by a factor of \( \sqrt{100} = 10 \). Therefore, the ~1% variance required for direct drive ICF can be accomplished by standard 2D SSD with bandwidth 500 GHz in an integration time of ~ 200 psec.
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9. REFERENCES

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