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Abstract

A theoretical and computational study of the ion temperature gradient and η_i instabilities in tokamak plasmas has been carried out. In toroidal geometry the modes have a radially extended structure and their eigenfrequencies are constant over many rational surfaces that are coupled through toroidicity. These nonlocal properties of the ITG modes impose strong constraint on the drift mode fluctuations and the associated transport, showing a self-organized characteristic. As any significant deviation away from marginal stability causes rapid temperature relaxation and intermittent bursts, the modes hover near marginality and exhibit strong kinetic characteristics. As a result, the temperature relaxation is self-similar and nonlocal, leading to a radially increasing heat diffusivity. The nonlocal transport leads to the Bohm-like diffusion scaling. The heat input regulates the deviation of the temperature gradient away from marginality. The obtained transport scalings and properties are globally consistent with experimental observations of L-mode discharges.

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I. INTRODUCTION

The ion temperature gradient (ITG) instability and η_i -instability¹⁻³ are considered to play an important role for anomalous ion thermal transport across magnetic flux surfaces in many auxiliary heated tokamak plasmas and have been experimentally investigated to confirm this conjecture.^{4,5} Turbulent density fluctuations whose phase is rotating in the ion diamagnetic direction have been observed in several tokamaks,^{6,7} indicating the presence of the ion branch. The modern diagnostics to investigate the detail of tokamak plasma turbulence and its connection to anomalous transport has become available in the hot core region on large tokamak experiments by using, for example, the beam emission spectroscopy (BES) and the correlation reflectometer.⁷⁻⁹ The recent diagnostics results in TFTR L-mode discharges show that long wavelength fluctuations with $k_\theta \rho_i \leq 0.1 - 0.2$ (ρ_i : the ion Larmor radius) dominate and that the radial correlation length is in the range of 2-4 cm, which is much greater than the ion Larmor radius. Furthermore, recent efforts to determine local thermal transport of L-mode discharges show that the ion thermal diffusivity χ_i approximately scales as the Bohm-like diffusion rather than the gyroBohm.^{11,12} This implies that the long wavelength fluctuations whose scale length is related to the plasma equilibrium size are important to the ion thermal transport.

In addition to these results, it has been pointed out that some experimental plasma profiles show similarity to the theoretical marginal stability boundary of the ITG/ η_i -mode, indicating that the observed profile hardly goes significantly beyond marginal stability in the core region.¹³⁻¹⁵ The degree of excess of the ion temperature gradient over the critical gradient is small in the core and then increases at the edge, although the edge analysis is inconclusive due to the necessity of including other modes, especially the impurity gradient mode. Some discharges show that strong profile resilience occurs in the ion temperature

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profile in the L-mode and even during the L-H phase transition.^{13,16} As explained later, transport near marginal stability where the fluid approximation breaks down is complicated and has not been well understood.^{17,18} So far, many of theoretical and computational works have been carried out to explain tokamak thermal transport based on the fluid model and also on the annular torus model which assumes locally homogeneous turbulence devoid of global structures.^{14,19,20} However, such a fluid model and/or local treatment are insufficient to fully understand the experimental results described above and the global treatment that allows the radial turbulent energy convection in full toroidal geometry is desired.

Recently, we carried out toroidal particle simulation based on the TPC (Toroidal Particle Code) to study the characteristics of the toroidal ITG mode.²¹⁻²⁴ We found in those simulations that the instability and related turbulence in toroidal geometry is quite distinct from the one in 3D slab (or cylindrical) geometry, showing (i) that radially extended global modes which are localized to the outside of the torus encompassing many rational surfaces is developed, while cylindrical ones are localized on each rational surface, that is, the potential structure forms a well-defined radial profile; (ii) that the eigenfrequencies are nearly constant over the extended radial region; and in particular (iii) that the heat conductivity χ_i is an almost flat or rather increasing function of the minor radius as opposed to a decreasing trend in a cylindrical counterpart. Parker *et al.*²⁵ (also Lee *et al.*²⁶) studied the ITG driven turbulence by 3D gyrokinetic simulation and also found the global nature of turbulence which is consistent with the recent TFTR fluctuation measurements.⁹ These remarkable features are primarily due to the toroidicity-induced coupling of mode rational surfaces over a finite radial region and are showing the importance of the toroidal geometrical effect.

Motivated by these simulation results, in this work we investigate the characteristics of the plasma relaxation induced by the prominent ITG turbulence. From a series of TPC simulation runs, the typical radial correlation length L_r is found to be smaller than the minor radius a and/or the temperature scale length $L_T (\equiv |\partial \ln T_i / \partial r|^{-1})$, but greater than

the ion gyroradius ρ_i . Namely, the results imply that the short ion Larmor length and large plasma scale length couple with each other, providing the relation $L_r \propto \rho_s^{1-\alpha} a^\alpha$ and that the turbulent structure is nonlocally determined. When the radially extended modes are well sustained, the exponent α is estimated as $\alpha \sim 0.5$, which is supported by or at least consistent with the recent theoretical studies²⁶⁻²⁹ including the first-order envelope ballooning equation. Therefore, we can say the radially extended toroidal modes with exponent α is 0.5, the *ideal radial modes*.

Furthermore, the nonlinear stage of the global particle simulation shows that such a radially extended nonlocal fluctuation provides a strong constraint on the plasma so that the plasma relaxation and related profile are self-organized. The initial (or arbitrary) temperature profile $T_i(r)$, which gives the unstable ITG mode, tends to be an exponential functional form given by $T_0(r) \simeq \exp(-r/L_T)$. This happens in a fast time scale. Namely, a small temperature deviation from the organized profile $T_0(r)$, i.e. $\delta T(r) = |T_i(r) - T_0(r)|$, is quickly smoothed out over the finite radial region. Once such a (nonlocal) profile is achieved, the system evolves more slowly with the profile functional form being kept constant, showing self-similarity in the plasma relaxation. Such a constraint is considered to result from the fact that the temperature at two radial locations (i.e. $r = r_1$ and r_2) cannot be changed arbitrarily, because these two radial locations are correlated with each other through the nonlocal mode when the condition $\Delta r (\equiv |r_1 - r_2|) \leq L_r$ is satisfied. This is strong experimental evidence of such an effect.³⁰ The functional form $T_0(r)$ is found to be determined so that the characteristic parameter $\epsilon_T \equiv L_T/R$ that defines the instability source becomes spatially constant over this spatial extent. This will be discussed in Sec. III.

Another important characteristics obtained in the simulations is that the plasma profile is forced to relax toward the vicinity of marginal stability of the ITG mode as a manifestation of a strong constraint on the plasma. Namely, the instability is radially nonlocal and robust in toroidal plasmas so that the plasma parameters that characterize the instability source have

to relax until they sufficiently approach marginal stability. This is the case even when there is external heating. As long as there is such an energy input and the instability parameter stays near but slightly above marginal stability, sustained long wavelength fluctuations can cause heat flux with a large $\mathbf{E} \times \mathbf{B}$ step size even near the critical gradient. Although sometimes referred to as a result of transport caused by small scale fluctuations,¹¹ the hypothesis of *near marginal stability* should be considered to be a direct consequence of the nonlocal nature of the toroidal mode excitation and the related radial turbulent energy convection. Near marginal stability, the commonly used estimates based on the strong turbulence ordering break down. Thus transport near marginal stability has been even less understood, especially in toroidal geometry. This is in part due to the fact that near marginal stability, heat flux is principally sustained by the wave-particle kinetic resonance which has a complicated velocity space dependence due to the ion magnetic drift motion.¹⁸ Furthermore, the saturation mechanism of the nonlocal toroidal mode and related nonlinear processes might be quite different from the one in the fluid regime of strong robust instability. For example, a mechanism pointed out by Mattor¹⁷ is ion Compton scattering near marginal stability. Such a nonlinear kinetic effect may be important for long wavelength fluctuations which have an organizing character near the critical gradient. We call the relaxed state observed in the simulation, *the self-organized criticality*, a concept based on our observation that large scale vortices exert an organizing influence on the plasma, which forces the plasma profile to tend to the critical gradient and any deviation from that critical profile can be quickly transported away, and thus the collective drift modes sustain the plasma profile and the associated profile resilient transport. This concept is similar to Bak *et al.*,³¹ in which they argued the critical gradient of a sand box.

Based on the simulation results, we present a *critical gradient model* to describe transport near marginal stability which is subjected by self-organization. The model is incorporated with a quasilinear kinetic calculation of heat flux and consists of coupled dynamical equations

for the nonlocal turbulent energy W and the inverse temperature gradient $\mu(\equiv \epsilon_T^{-1})$. The model describes how far the profile can deviate from the critical gradient through the balance between the external heating power and the radial turbulent energy convection. As a steady state solution, we obtain thermal diffusivity χ_i whose expression includes the heating power P_{in} . The thermal diffusion is found to scale as Bohm-like near the critical gradient when $L_r \simeq \rho_i^{1-\alpha} a^\alpha$ and $\alpha = 0.5$, (that is, the geometrical mean of ρ_i and a) is satisfied. The exponent α signifies the strength of the dependence of the radial correlation length on the global scale length a . (When the exponent α tends to $\alpha = 0$ where the radial correlation length is an order of ρ_i , the thermal diffusion scales as gyroBohm, reflecting transport due to small size fluctuations.) Thus, the Bohm-like diffusion is basically obtained in toroidal geometry when the ideal radial modes are well established. The diffusion, however, may be changed into the gyroBohm diffusion by introducing a mechanism which disintegrates the ideal radial mode. This transport change may take place, for example, by the presence of a sheared plasma rotation.

The outline of this paper is as follows. In Sec. II, we first study the fundamental transport behavior of the ITG mode near marginal stability based on the local kinetic dispersion relation and quasilinear theory. Especially, we investigate the dispersion relation of the ITG mode in the complex frequency plane by performing analytical continuation from $\text{Im } \omega > 0$ plain to $\text{Im } \omega < 0$ one (ω : the ITG mode frequency) by using the coordinate transformation method. We show that the heat flux is sustained even if $\text{Im } \omega = 0$ due to the toroidal kinetic resonance, and this effect plays an important role in determining heat flux near the critical gradient. In Sec. III, we show typical simulation results of the ITG mode and describe the characteristics of the toroidal turbulence structure. In addition, we present a phenomenological relaxation theory in Sec. IIIC to describe the remarkable plasma relaxation phenomenon obtained by the simulation in terms of the reductive perturbation method. Based on the transport analysis, we explain the radially flat or rather increasing heat diffusivity observed

in Ref. 17. In Sec. IV, we present the critical gradient model and investigate the dynamical and static properties of transport near the critical gradient. In Sec. V, we provide the concluding remarks.

II. LOCAL DISPERSION RELATION AND QUASILINEAR HEAT FLUX

In this section, we investigate the local kinetic dispersion relation governing ∇p_i driven ITG mode. The meaning of “local” and its relation to the globalness of the phenomena are to become clearer in later sections. As we discussed in Sec. I, later we pay attention to transport near marginal stability of the ITG mode. In Sec. IIA, in order to investigate the dispersion behavior around the critical gradient, we perform analytical continuation on the complex frequency plane from $\text{Im } \omega > 0$ plane to $\text{Im } \omega < 0$ plane by introducing the coordinate transformation method developed in Ref. 32. We describe quasilinear heat flux which is available to analyze transport near marginal stability, including the stable plane ($\text{Im } \omega < 0$) in Sec. IIB and show the typical numerical results in Sec. IIC.

A. Local dispersion relation

The basic equation to describe the toroidal pressure gradient driven mode is the electrostatic linear gyrokinetic equation where the ion perturbation is given by $\delta f_i = -(e\hat{\phi}/T_i)f_M + \hat{g}\exp(i\mathcal{L})$ and $\mathcal{L} \equiv \mathbf{v} \times \mathbf{b} \cdot \mathbf{k}_\perp/\omega_{ci}$ ($\mathbf{b} \equiv B/B$ and $\omega_{ci} \equiv eB/M_i c$). Here, we employ a simple formula for the perturbed nonadiabatic amplitude $\hat{g}$ where a differential operator with respect to ballooning coordinate θ , i.e. $(rR)^{-1}\partial/\partial\theta$ is replaced by the local parallel wave number $k_\parallel$, as follow

$$\hat{g} = \frac{e\hat{\phi}}{T_i} J_0(\alpha) \frac{\Omega - \Omega_{*i}^T(\tilde{v}_\parallel^2 + \tilde{v}_\perp^2 + \eta_i^{-1} - 3/2)}{\Omega - (\tilde{v}_\parallel^2 + \tilde{v}_\perp^2/2 + \Omega_{ti}\tilde{v}_\parallel)} f_M, \quad (1)$$

where a circular MHD equilibrium is assumed and only circulating particles are taken into account. Note that Eq. (1) is defined in $\text{Im } \omega > 0$ plane. Normalizations are $\Omega \equiv \omega/\omega_d$,

$\alpha \equiv (2b_i)^{1/2}\tilde{v}_\perp$, $\Omega_{*i}^T \equiv \omega_{*i}^T/\omega_d$, $\Omega_t \equiv k_\parallel v_i/\omega_d (= k_\parallel R/k_\theta \rho_i)$, $\tilde{v}_\parallel = v_\parallel/v_i$ and $\tilde{v}_\perp = v_\perp/v_i$, where $v_i \equiv (2T_i/M_i)^{1/2}$, $b_i \equiv (k_\theta \rho_i)^2/2$, $\omega_{*i}^T \equiv -(k_\theta \rho_i)(v_i/2L_T)$, $\omega_d \equiv 2\epsilon_T \omega_{*i}^T$ [$\equiv -(k_\theta \rho_i)(v_i/R_0)$] is magnetic drift frequency and $f_M = n_0/(\pi^{3/2}v_i^3) \exp(-\tilde{v}_\parallel^2 - \tilde{v}_\perp^2)$ is an unperturbed ion distribution. The perpendicular wave number $k_\perp = k_\theta K(\theta, \hat{s})$ and the magnetic drift frequency $\omega_d = 2\omega_{*i}^T \epsilon_T G(\theta, \hat{s})$ are also estimated at $\theta = 0$ by choosing $K = G = 1$, where $\hat{s} = (rq^{-1}\partial q/\partial r)$ represents the shear parameter. The nonadiabatic ion charge density $\delta\rho_{i,na}$ is simply obtained for $\text{Im } \omega > 0$ by carrying out the $v_\parallel$ -integration as follows,

$$\delta\rho_{i,na} = -\frac{k_{di}^2}{4\pi} \hat{\phi} \int_0^\infty d\tilde{v}_\perp J_0^2(\alpha) \tilde{v}_\perp \exp(-\tilde{v}_\perp^2) H(\tilde{v}_\perp^2, \Omega), \quad (2)$$

$$H(\tilde{v}_\perp^2, \Omega) = \frac{1}{u(\tilde{v}_\perp^2, \Omega)} \left\{ \left[\Omega - \Omega_{*i}^T (\tilde{v}_\perp^2 + \eta_i^{-1} - 3/2) \right] [Z(u_1) + Z(u_2)] \right. \\ \left. - \Omega_{*i}^T (u_1 [1 + u_1 Z(u_1)] + u_2 [1 + u_2 Z(u_2)]) \right\}, \quad (3)$$

where $Z(\zeta)$ represents the plasma dispersion function, and $u_1 = u - \Omega_t/2$, $u_2 = u + \Omega_t/2$ and $u = (\Omega + \Omega_t^2 - \tilde{v}_\perp^2/2)^{1/2}$. The linear dispersion is then determined from the quasi-neutrality condition by assuming an adiabatic electron response $\delta\rho_{e,ad} = -k_{de}^2 \hat{\phi}/4\pi$ as

$$D(\Omega) = \frac{1}{\tau} + 1 + \int_0^\infty d\tilde{v}_\perp J_0^2(\alpha) \tilde{v}_\perp \exp(-\tilde{v}_\perp^2) H(\tilde{v}_\perp^2, \Omega) = 0 \quad (4)$$

where $\tau \equiv T_e/T_i$ represents the ratio of the electron temperature to ion.

Before performing analytical continuation to the $\text{Im } \omega < 0$ plane, we briefly discuss the physical picture of marginal stability. Around marginal stability, the denominator in Eq. (1) becomes zero along the curve of resonance ions and the resonant exchange of energy and momentum between the ions and the mode occurs. The resonance condition is expressed by a 1D elliptic curve in velocity space and given by

$$\left(\tilde{v}_\parallel + \frac{\Omega_t}{2} \right)^2 + \frac{\tilde{v}_\perp^2}{2} = \Omega + \frac{\Omega_t^2}{4}. \quad (5)$$

Here, the center of the ellipse locates at $\tilde{v}_\parallel = -\Omega_t/2$ ($\equiv -k_\parallel v_i/2\omega_d$), and the edges are at $\tilde{v}_\parallel = -\Omega_t/2 \pm \sqrt{\Omega + \Omega_t^2/4}$. Equation (5) is found to lead to the magnetic drift resonance

$\omega \sim \omega_d(v_{\parallel}^2 + v_{\perp}^2/2)$ for $\Omega_t = k_{\parallel}v_{\parallel}/\omega_d \ll 1$ case and the transit Landau resonance $\omega \sim k_{\parallel}v_i$ for $\Omega_t \gg 1$ case, which correspond to the so-called toroidal mode and slab mode, respectively. The transition property from the toroidal to the slab mode was studied by Kim and Horton.³² Note that the real eigenfrequency for the toroidal mode at around marginal stability is characterized by the lower frequency, i.e. the magnetic drift frequency ω_d , which is smaller than the ion diamagnetic drift frequency ω_{*i}^T .

In order to investigate the dispersion relation around marginal stability and perform the analytical continuation, we now introduce the coordinate transformation method.¹⁸ In this method, the coordinate is transformed from the present $(v_{\parallel}, v_{\perp})$ space to new one (μ, t) which is given by the relation

$$\begin{cases} \tilde{v}_{\parallel} = \sqrt{t}\mu - \Omega_t/2 \\ \tilde{v}_{\perp} = \sqrt{2t}\sqrt{1-\mu^2}, \end{cases} \quad (6)$$

with integral intervals $-1 \leq \mu \leq 1$ and $0 \leq t \leq \infty$. In this (μ, t) space, we see the resonance curve is mapped into a straight line where the usual one-dimensional Landau damping theory is applicable. Then the nonadiabatic ion charge density $\delta\rho_{i,na}$ is expressed in the following form

$$\delta\rho_{i,na} = -\frac{k_{di}^2}{2\pi^{3/2}} \hat{\phi} \int_{-1}^{+1} d\mu \int_0^{\infty} dt \frac{g(\mu, t)}{t - (\Omega + \Omega_t^2/4)}, \quad (7)$$

$$g(\mu, t) = t^{1/2} J_0^2(\alpha) \left[\Omega - \Omega_{*i}^T(\tilde{v}^2 + \eta_i^{-1} - 3/2) \right] \exp(-\tilde{v}^2), \quad (8)$$

and $\tilde{v}^2$ and α are now function of (μ, t) given by

$$\begin{cases} \tilde{v}^2(\mu, t) = 2t(1 - \mu^2) + (\sqrt{t}\mu - \Omega_t/2)^2 \\ \alpha(\mu, t) = k_{\theta}\rho_i\sqrt{2t(1 - \mu^2)}. \end{cases} \quad (9)$$

It is found in Eq. (7) that the resonance takes place only when the real frequency $\text{Re } \omega (= \omega_d \text{ Re } \Omega)$ satisfies the condition $\text{Re } \omega > \omega_{br}$, where ω_{br} is the *branch frequency* defined by $\omega_{br} \equiv -k_{\parallel}^2 v_i^2 / 4\omega_d$. Therefore, the branch cut is found to exist along $-\infty < \text{Im } \omega \leq 0$ at

from $\text{Re } \omega < \omega_{br}$ to $\text{Re } \omega > \omega_{br}$ on the $\text{Im } \omega < 0$ plane. Note that in the toroidal limit given by $k_{\parallel} \rightarrow 0$, the branch frequency tends to zero, i.e. $\omega_{br} \rightarrow 0$. On the other hand, in the slab limit given by $\omega_d \rightarrow 0$, the branch frequency tends to $\omega_{br} \rightarrow -\infty$, which means that the branch cut has effectively disappeared. The branch frequency ω_{br} is closely related to the damping rate by the phase mixing of the ballistic response as shown in Ref. 18.

Then the explicit form of $\delta\rho_{i,na}$, which is analytically continued to the $\text{Im } \omega < 0$ plane for $\text{Re } \omega > \omega_{br}$, is represented by the sum of the residual contribution $\delta\rho_{i,na}^{(res)}$ and nonresidual contribution $\delta\rho_{i,na}^{(nonres)}$ as $\delta\rho_{i,na} = \delta\rho_{i,na}^{(res)} + \delta\rho_{i,na}^{(nonres)}$. The residual part is obtained from Eq. (7) as follows

$$\delta\rho_{i,na}^{(res)} = -\frac{k_{di}^2}{\sqrt{\pi}} (i\sigma) \hat{\phi} \int_{-1}^{+1} d\mu g(\mu, t = \Omega + \Omega_i^2/4) = -\frac{k_{di}^2}{\sqrt{\pi}} (i\sigma) \hat{\phi} (\Omega + \Omega_i^2/4)^{1/2} \cdot \int_{-1}^{+1} d\mu J_0^2(\alpha) \exp(-\tilde{v}^2) [\Omega - \Omega_{*i}^T (\tilde{v}^2 + \eta_i^{-1} - 3/2)]. \quad (10)$$

Here, σ is given by $\sigma = 1/2$ for $\text{Im } \omega = 0$, and $\sigma = 1$ for $\text{Im } \omega < 0$, and

$$\begin{cases} \tilde{v}^2 = [(\Omega + \Omega_i^2/4)\mu - \Omega_i/2]^2 + 2(\Omega + \Omega_i^2/4)(1 - \mu^2) \\ \alpha = k_{\theta}\rho_i \sqrt{2(\Omega + \Omega_i^2/4)(1 - \mu^2)} \end{cases} \quad (11)$$

are employed. The nonresidual part $\delta\rho_{i,na}^{(nonres)}$ is given by

$$\delta\rho_{i,na}^{(nonres)} = -\frac{k_{di}^2}{\sqrt{\pi}} (i\sigma) \hat{\phi} \int_{-1}^{+1} d\mu \mathcal{P} \int_0^{\infty} dt \frac{g(\mu, t)}{t - (\Omega + \Omega_i^2/4)} \quad (12)$$

where $\mathcal{P}$ represents the principal value. At marginal stability, $\delta\rho_{i,na}^{(nonres)}$ becomes pure real and $\delta\rho_{i,na}^{(res)}$ becomes zero to satisfy Eq. (4). Assuming that thermal particles with $v_{\parallel} \simeq v_{\perp} \simeq v_i$ mainly contribute to resonance, we can estimate the real eigenfrequency by $\Omega \simeq 3/2$ for $\Omega_i < 1$ and $\Omega \simeq \Omega_i$ for $\Omega_i > 1$. Therefore, the transition from the toroidal mode to the slab mode where the parallel transit damping plays an important role takes place roughly at $\Omega_i = 1$.

B. Quasilinear heat flux

The fluctuating field induced by the instability causes relaxation in the plasma as observed in the simulation. The evolution of the background distribution function f_0 may be described by the quasilinear theory: $\partial f_0 / \partial t = \mathcal{V}[\mathbf{E}(\delta f_i), \delta f_i]$, where δf_i is the perturbed ion distribution, $\mathbf{E}$ the electric field generated by δf_i , and $\mathcal{V}$ is the usual quasilinear Vlasov operator. When we make a second moment of velocity of this quasilinear equation, we obtain an equation describing the background temperature evolution:

$$\frac{3}{2} \frac{\partial}{\partial t} (n_i T_i) + \nabla \cdot \mathbf{Q} = S. \quad (13)$$

Here, T_i refers to the background ion temperature and heat flux $\mathbf{Q}$ is the second moment of velocity of the term $\mathcal{V}[\mathbf{E}(\delta f_i), \delta f_i]$ and it takes the form

$$\hat{\mathbf{e}}_r \cdot \mathbf{Q} = \text{Re} \int dv \frac{mv^2}{2} \frac{c(\mathbf{E}^* \times \mathbf{B})}{B^2} \hat{g} \exp(i\mathcal{V}) = \frac{1}{2} (k_\theta \rho_i) n_i T_i v_i \left| \frac{e\phi}{T_i} \right|^2 \text{Re } G \quad (14)$$

where $\hat{\mathbf{e}}_r$ is the unit vector in the radial direction, and $G = G(\Omega, \epsilon_T, \eta_i, k_\theta \rho_i, k_\parallel R)$ represents normalized heat flux (of order unity). The thermal diffusivity χ_i is written in terms of the normalized fluctuation energy $W \equiv |e\phi / \langle T_i \rangle|^2$ as

$$\chi_i = \hat{\chi} W,$$

$$\hat{\chi} = (\rho_i v_i / 2 \tilde{T}_i^2) k_\theta L_T \text{Re } G, \quad (15)$$

and $\tilde{T}_i \equiv T_i / \langle T_i \rangle$. Here, $\langle T_i \rangle$ is the average temperature of the system. Normalized heat flux G is calculated in the same manner as the one for the ion charge density (see Eq. (7)) as

$$G = \frac{2i}{\sqrt{\pi}} \int_{-1}^{+1} d\mu \int_0^{+\infty} dt \frac{g(\mu, t) \tilde{v}^2}{t - (\Omega + \Omega_i^2/4)}, \quad (16)$$

which is defined for the $\text{Im } \Omega > 0$ plane.

The function G is also analytically continued to the $\text{Im } \Omega \leq 0$ plane when the condition $\text{Re } \omega > \omega_{br}$ is satisfied, and can be also represented by the sum of the resonant part $G^{(\text{res})}$

and the nonresonant part $G^{(\text{nonres})}$ as $G = G^{(\text{res})} + G^{(\text{nonres})}$, which are respectively given by

$$G^{(\text{res})} = -4\sqrt{\pi}\sigma \int_{-1}^{+1} d\mu g(\mu, t = \Omega + \Omega_i^2/4) \tilde{v}^2(\mu, t = \Omega + \Omega_i^2/4) = -4\sqrt{\pi}\sigma \left(\Omega + \frac{\Omega_i^2}{4} \right)^{1/2} \cdot \int_{-1}^{+1} d\mu J_0^2(\alpha) \tilde{v}^2 \exp(-\tilde{v}^2) \left\{ \Omega - \Omega_{*i}^T (\tilde{v}^2 + \eta_i^{-1} - 3/2) \right\}, \quad (17)$$

and

$$G^{(\text{nonres})} = \frac{2i}{\sqrt{\pi}} \int_{-1}^{+1} d\mu \mathcal{P} \int_0^{+\infty} dt \frac{g(\mu, t) \tilde{v}^2(\mu, t)}{t - (\Omega + \Omega_i^2/4)}. \quad (18)$$

At exact marginal stability ($\text{Im } \Omega = 0$), the nonresonant part of G does not contribute to heat flux because $G^{(\text{nonres})}$ is pure imaginary. However, the resonant part $G^{(\text{res})}$, which describes the wave-particle kinetic interaction, gives rise to heat flux with the fluctuation energy W determining the rate of flux through Eq. (14). This is explained as follows: As found in Eq. (9), at marginal stability, $\delta\rho_{i,na}^{(\text{res})}$ vanishes as a result of the detailed charge density balance in velocity space between particles with $\tilde{v} < \tilde{v}_c$ and those with $\tilde{v} > \tilde{v}_c$ along the elliptic curve given by Eq. (5), where $\tilde{v}_c$ is given by $\tilde{v}_c = (\omega/\omega_{*i}^T - \eta_i^{-1} + 3/2)^{1/2}$. This condition actually determines the real eigenfrequency. However, such a detailed balance as held for ion charge density fluctuations breaks down for heat flux, because the velocity moment for the heat is weighted by v^2 . Namely, near marginal stability, the wave emission from ions with $v/v_i > \tilde{v}_c$ is in balance with the wave absorption from $v/v_i > \tilde{v}_c$, but those particles with $v/v_i > \tilde{v}_c$ contribute more to the out-of-phase part of the ion pressure fluctuation, i.e. $\text{Im}(\phi_k^* \tilde{P}_i)$, than those ions absorbing the fluctuation energy. Thus, there is net thermal flux proportional to wave energy density $|e\phi/\langle T_i \rangle|^2$ and the excess contribution from higher energy ions. This implies that the finite external heat source δP_{in} can balance with such *kinetic heat flux*, i.e. heat flux which is kinetically sustained by wave-particle interaction, maintaining the plasma profile and linear toroidal eigenmode at exact marginal stability. Thus, the plasma is found to be still active even at marginal stability. Such *kinetic heat flux* is also important especially when we consider the dynamical/transient problem like

the boundary crossings of the stable/unstable plasma profile.

Near the critical gradient, the normalized heat flux G may be calculated from the kinetic resonance part of the pressure fluctuation as $G \simeq G^{(\text{res})} = g_1 + g_2/\epsilon_T$ where

$$g_1 = -2\sqrt{\pi}\Omega \left(\Omega + \Omega_i^2/4\right)^{1/2} \int_{-1}^{+1} d\mu J_0^2(\alpha) \exp(-\tilde{v}^2) \tilde{v}^m, \quad (19)$$

$$g_2 = \sqrt{\pi} \left(\Omega + \Omega_i^2/4\right)^{1/2} \int_{-1}^{+1} d\mu J_0^2(\alpha) \exp(-\tilde{v}^2) \tilde{v}^m \left(\tilde{v}^2 + \eta_i^{-1} - 3/2\right), \quad (20)$$

and $\tilde{v}^2$ and α are again given by Eq. (11). The eigenfrequency can be evaluated by $\Omega(=\omega/\omega_d) \simeq 3/2$ for the ideal toroidal mode when the condition $\Omega_i \ll 1$ is satisfied as discussed in Sec. IIB. In Eqs. (19) and (20) resonant pressure contributions, g_1 and g_2/ϵ_T partially cancel with each other according to the above discussion. However, for ϵ_T smaller than a critical value $\epsilon_T < |g_1|/g_2$, the resonant pressure fluctuation gives net positive thermal flux.

For the hydrodynamic regime where the mode frequency (growth rate) is given by $\omega^2 \simeq -\omega_{*i}^T \omega_d$, normalized heat flux G is directly evaluated from Eq. (16) by using the relation $|\Omega| = |\omega/\omega_d| \sim 1/\sqrt{2\epsilon_T} \gg 1$ as

$$G_h \simeq \frac{1}{\sqrt{2\pi\epsilon_T}} \int_0^{+\infty} d\tilde{v}_\perp^2 \int_{-\infty}^{+\infty} d\tilde{v}_\parallel J_0^2(\alpha) \cdot \left[\left(\tilde{v}^2 + \eta_i^{-1} - 3/2\right) - 2\epsilon_T \left(\tilde{v}_\parallel^2 + \Omega_i \tilde{v}_\parallel + \tilde{v}_\perp^2/2\right) \right] \tilde{v}^2 \exp(-\tilde{v}^2), \quad (21)$$

which leads to $G_h \simeq c_0/\sqrt{\epsilon_T}$ by neglecting the term proportional to ϵ_T . Here, c_0 is a function of $k_\theta \rho_i$ and given by

$$c_0 = \frac{1}{\sqrt{2}} \int_0^{+\infty} dv_\perp^2 J_0^2(\alpha) \left(\tilde{v}_\perp^4 - \tilde{v}_\perp^2/2\right) \exp(-v_\perp^2), \quad (22)$$

which is of order unity.

C. Numerical example of ITG dispersion relation

Here, we numerically investigate the linear dispersion relation and normalized heat flux in the complex ω plane based on the discussion in Sec. IIA and B. Figures 1(a) and (b)

illustrate the eigenfrequency normalized by the magnetic drift frequency $\text{Im } \omega/\omega_d$ (a) and $\text{Re } \omega/\omega_d$ (b) as a function of ϵ_T for different values of X ($\equiv k_{\parallel} \bar{v}_i/\omega_d = k_{\parallel} R_0/k_{\theta} \bar{\rho}_i$). Figure 1(c) shows the normalized heat flux G . Basic parameters are given by $k_{\theta} \bar{\rho}_i = 0.3$ ($\bar{\rho}_i \equiv \rho_i/\sqrt{2}$), τ ($\equiv T_e/T_i$) = 1 and ϵ_n ($\equiv L_n/R$) = 3.0. The ITG mode is continuously (with ϵ_T) connected with the damped mode for ϵ_T crossing the threshold value ϵ_{tc} ($= L_{TC}/R$) (for example $\epsilon_{tc} \simeq 0.31$ for $X = 0$ case). The damping of the mode originates from the magnetic drift-transit resonance along the ellipse given by Eq. (5). With an increase of X , the threshold value ϵ_{Tc} becomes small due to the effect of parallel transit resonance. Around the threshold $\epsilon_T \simeq \epsilon_{Tc}$, the real frequency $\text{Re } \omega/\omega_d$ is roughly given by 1.6 for the region $X \leq 0.6$, which is consistent with the discussion in Sec. IIB. Furthermore, for a small value of X ($X \leq 0.6$), $\text{Re } \omega/\omega_d$ is found to be insensitive to ϵ_T and ranges over $\text{Re } \omega/\omega_d \simeq 1.5 - 1.7$ around the threshold. This indicates that the ITG mode near the critical gradient is dominated by wave-particle resonance interaction covering the finite region around $\epsilon_T = \epsilon_{Tc}$. As seen in Fig. 1(c), the normalized heat flux $\text{Re } G$ is sustained as a finite positive value of $\epsilon = \epsilon_{Tc}$, as discussed in Sec. IIB. It is also found in Fig. 1 that the mode behavior qualitatively changes between $X \leq 0.8$ (region A) and $X \geq 1.0$ (region B). That is, in region A, the real frequency ω/ω_d begins to increase with ϵ_T whereas the mode is heavily damped. This change with increasing X results from the transition of the ITG mode from the toroidal branch (region A) to the slab one (region B) that suffers strong wave damping due to the particle transit resonance.

The dependence of the stability boundary ϵ_{Tc} and corresponding real frequency ω/ω_d on X is shown in Fig. 2(a) for a given value of $k_{\theta} \bar{\rho}_i = 0.3$. The mode frequency ω/ω_d is found to be almost constant ($\omega/\omega_d \simeq 1.6$) for the toroidal regime of $X \leq 0.6$, whereas in proportion to X (i.e. $\omega/\omega_d \simeq 2X$) for the slab regime of $X \geq 1.0$. Figure 2(b) also shows ϵ_{Tc} , ω/ω_d and normalized heat flux G as a function of $k_{\theta} \bar{\rho}_i$ for a given value of $k_{\parallel} R = 0.25$. The marked point (A) in Fig. 2(b) corresponds to the one in Fig. 2(a) where $X \simeq 0.42$. An eigenfrequency ($\omega/\omega_d \simeq 3/2$) is established constant for a wide range of $k_{\theta} \bar{\rho}_i$.

for the toroidal regime. Figures 1(c) and 2(b) show that for a small deviation of the ion temperature gradient from ϵ_{Tc} , i.e. $\delta\epsilon_T = \epsilon_T - \epsilon_{Tc}$, normalized heat flux G can be expanded as $G \simeq G_0(\epsilon_T = \epsilon_{Tc}) + G_1(\delta\epsilon_T)$ assuming $|\delta\epsilon_T|/\epsilon_{Tc} \ll 1$ where $G_1 = (\partial G_0/\partial \epsilon_T)_{\epsilon_T = \epsilon_{Tc}}$.

III. TOROIDAL PARTICLE SIMULATION

Here, we present simulation results from the TPC (Toroidal Particle Code²¹) both for the toroidal ITG mode which has a flat density profile ($L_n \rightarrow \infty$) and the η_i -mode, although we concentrate our discussion mainly on the ITG mode. At first, we briefly explain the TPC and typical simulation parameters we employed in Sec. IIIA. In Sec. IIIB, we show the simulation results and describe notable characteristics of the toroidal ITG mode and related plasma transport phenomena. In Sec. IIIC, we present the relaxation theory for the toroidal plasma to explain the observed transport phenomena which become the basis for our transport model.

A. Global toroidal code and simulation parameter

We carried out a series of particle simulations in full toroidal geometry using the TPC.²¹ In the TPC, the global toroidal coordinate (r, θ, φ) is employed where the surfaces of constant r are nested and a nonuniform radial discretization from $r = 0$ to $r = a$ (minor radius) is used. Magnetic fields are given by $B_\varphi = B_{\varphi 0} R_0/R$ and $B_\theta = r B_\varphi / R_0 q(r)$ where $q(r)$ is the safety factor and $R = R_0 + r \cos \theta$. An electrostatic potential is expanded for poloidal and toroidal coordinates as $\phi(r, \theta, \xi) = \sum_{mn} \phi_{mn}(r, t) \exp(-im\theta - in\varphi)$, which allows radial turbulent energy convection. In the present study, we employed the dynamics with full kinetic ions and adiabatic electrons. Therefore, the simulation excludes the nonadiabatic electron nature which originates from the trapped electron response. In future work, we consider the impurity ion effects which can be important at the edge $r/a \geq 0.8$ (actually also in the core).

Typical parameters employed here are as follows: the aspect ratio $R_0/a = 4$, the average ratio of ion plasma frequency to ion cyclotron frequency $\langle \omega_{pi}/\omega_{ci} \rangle = 1$, the average ratio of ion Larmor radius to minor radius $\langle \rho_i/a \rangle \simeq 0.02$ and magnetic shear parameter $\hat{s} = r q'/q \sim 1.0$. For a given profile of a safety factor $q(r)$ ($q(0) \simeq 0.4$ and $q(a) \simeq 2.5$), we take the toroidal mode $|n| = 1-12$ and corresponding poloidal harmonics $\pm m$, which ranges over the minor radius with the relation $q(r) = m/n$. In the present simulation, however, in order to model the heat reservoir near the plasma center, we excluded mode rational surfaces in the region $r/a \leq 0.2$. Therefore, little relaxation takes place near the plasma center and the temperature at $r \simeq 0$ remains nearly constant. The ion temperature is initially given by an arctan dependence $T_i(r) = -(T_i(0)/\pi)[\arctan(r - r_0)/l_T + \pi/2]$, while the ion density is uniform for ITG mode simulation. The electron temperature is given spatially constant with $T_e/\langle T_i \rangle \simeq 0.4$. The radial profile of the safety factor q and magnetic shear $\hat{s}$ are shown in Fig. 3. $T_i(0)$ (and also r_0 and l_T) is chosen so that the resultant temperature scale length $\epsilon_T \equiv L_T/R$ and the value of $k_\theta \langle \rho_i \rangle$ are favorable to excite the ITG mode for toroidal modes $n = 8 - 10$.

B. ITG mode simulation

In this section, we present simulation results of the toroidal ITG mode and describe the main characteristics to note. In Fig. 4, we show four snapshots of the electrostatic potential contour on (r, θ) poloidal cross-section: (a) linear stage before saturation, (b) after saturation (early nonlinear) and (c) steady state (fully developed nonlinear). In Fig. 5, we illustrate the temporal evolution of the potential energy of each toroidal mode $\sum_m \langle |\phi_{mn}(r)|^2 \rangle$, where $\langle \quad \rangle$ denotes the average over minor radius. As seen in the linear stage in Figs. 4(a), radially extended global modes which display a prominent ballooning structure are formed. The $n = 9$ toroidal mode is dominant in the linear stage and the $m = 6 - 12$ successive poloidal modes are coupling with each other by the toroidal effect to form a global ballooning

structure. Furthermore, as seen in Fig. 4, the potential structure is twisted, forming long thin eddy streamer in the radial direction and shows an up-down asymmetric feature. The twisting results from the mode structure following the shear magnetic field in such a way as to maintain maximum growth. After saturation, the twisted potential eddies partially break up, disconnecting the radial coupling. Such a break up starts usually from the region of the poloidal angle $0 < \theta \leq \pi/2$, where the potential structure is strongly twisted. From the end of the linear stage, some lower toroidal modes are excited, and through the transient stage after the saturation ($4000 < t\omega_{ci}^{-1} < 7000$), the dominant toroidal mode shifts down to $n = 6$, showing an inverse cascade. In the quasi-steady state ($t \geq 7000\omega_{ci}^{-1}$) where the total field energy stays almost constant, the electrostatic potential structure becomes rather turbulent, but still showing structures which have a long radial correlation length as seen in Fig. 4(d). In the saturated state, the structures are more isotropic than in the linear phase.

Figure 6 shows the frequency power spectrum as a function of minor radius at the steady state ($t = 7000 - 15000\omega_{ci}^{-1}$). The power spectrum is found to form a frequency band over a radial region which weakly depends on the radial coordinate. The dashed line in Fig. 6 shows the frequency that satisfies the condition $\omega = \omega_d(r)$, i.e. the local magnetic drift frequency. The turbulent frequency spectrum is rather narrow and the central frequency in the band is characterized by $\omega \simeq \omega_d(r)$. This shows that the steady state is near marginal stability, as we discussed in Sec. II. From the analysis, the typical radial mode width or radial correlation length L_r is estimated as $L_r \simeq 15 - 20 \langle \rho_i \rangle$, which is much longer than the typical ion gyroradius $\langle \rho_i \rangle$, but smaller than the minor radius a , satisfying the relation $\langle \rho_i \rangle \ll L_r \ll a$. The result indicates that the small ion Larmor length and the larger plasma scale length are coupling with each other. Thus, the toroidal geometry which leads to toroidal coupling of neighboring m -modes plays a crucial role in determining the radial mode structure of the electrostatic potential and related turbulence, which is not taken into account in the conventional dispersion analysis by using the zeroth order ballooning equation. In order to

describe the radial scale of the vortices from the coupling effect, we write the characteristic radial length as

$$L_r(\sim \pi/k_r) \simeq \rho_i^{1-\alpha} a^\alpha, \quad (23)$$

and now estimate α . We performed several parameter runs to determine the exponent α , including gyrokinetic ion simulations.³³ Based on these simulation studies, we concluded that the radial mode width L_r is approximately evaluated as the geometrical mean of ρ_i and a , that is, $\alpha = 0.5 \pm 0.2$, and therefore typically

$$k_r \simeq \pi/\sqrt{\rho_i a}. \quad (24)$$

For example, in Ref. 21, we also obtain $\pi/k_r \simeq \sqrt{a/\langle \rho_i \rangle} \rho_i \simeq 11\rho_i$ which is consistent with the obtained frequency power spectrum. This is consistent with the several theoretical studies²⁶⁻²⁸ including the first-order envelope ballooning equation approach and also with the BES measurement in TFTR.⁷ This empirical formula for the radial wavenumber is utilized in Sec. IV to derive the transport model caused by the toroidal ITG mode.

Here, we further discuss the characteristics of the obtained toroidal mode structure. As in Fig. 4, we observe the twisted thin eddy structure with up-down asymmetry. Such poloidal asymmetry can be understood as a result of a rotation of the symmetry plane from a $\theta_0 = 0$ midplane by a finite angle $\theta_0 \neq 0$. The angle θ_0 is known as the Bloch shift parameter that represents the most unstable poloidal angle for given radial profiles. By solving the lowest order ballooning equation, we obtain the growth rate in terms of θ_0 as $\gamma_0(\theta_0) \sim \hat{\gamma}_0 \cos \theta_0$, where $\hat{\gamma}_0$ is a constant. Namely, $\theta_0 = 0$ provides the maximum growth rate from the local eigenvalue problem. However, the solution is obtained based on a local treatment of the ballooning equation which assumes spatial periodicity of the wave function. The global radial envelope structure, which is obtained by fully solving the (r, θ) 2D eigenvalue problem, is not taken into account. In order to see the realized Bloch angle more clearly, in Fig. 7 we show the contour of $\phi_n(r, \theta) \left(= \sum_m \phi_{mn}(r, t) \exp(-im\theta) \right)$, i.e. the toroidal Fourier components of

the electrostatic potential, for $n = 9$ (a) and $n = 10$ (b) which corresponds to Fig. 4(a). From these figures, the Bloch angle is evaluated as $\theta_0 \simeq -\pi/6$ with negative sign. The shift from $\theta_0 = 0$ arises from the equilibrium deviation of the ion magnetic drift frequency (i.e. $\partial\omega_d/\partial r$) or equivalently the second derivative of the temperature profile (i.e. $\partial^2 T_i/\partial r^2$), predicted by a recent study.²⁸ A detailed analysis of this profile effect for θ_0 and the global eigenvalue will be described in another paper.

We now investigate the characteristics of plasma transport subjected by the global ITG mode. Figure 8 shows the ion temperature profile plotted in a logarithmic scale at four different time steps. Although the amount of relaxation is small, a careful inspection shows that the initial *arctan* ion temperature profile which provides the unstable ITG mode tends to the one fit well by an exponential functional form $T_i(r) \simeq \exp(-r/L_T)$ over a finite radial portion ($\Delta r \sim 10 \langle \rho_i \rangle$). This relaxation happens up to $t = 2000 - 3000 \omega_{ci}^{-1}$. Once such a profile is achieved, the temperature profile evolves more slowly toward a relaxed state ($t \simeq t_4$ in Fig. 7) expanding the radial region which is fit to an exponential form. Note that the subscripts to t in this subsection are different from those used for scale expansion in the next subsection and subsequent sections. During relaxation, the exponential form has been intact, but only the temperature scale length L_T slowly changes as a function of time. Namely, the plasma is found to display *self-similarity* in the relaxation. In the quasi-steady state ($t \geq 10000 \omega_{ci}^{-1}$), further temperature evolution is weak, but the exponential form is kept. Note that steady state heat flux is sustained by the heat reservoir near the plasma center as we explained in Sec. IIIA.

Thus, the temperature profile and plasma transport are subjected to a strong constraint during the relaxation and also in the quasi-steady state. Such a constraint is explained by the potential structure of toroidal ITG turbulence whose radial correlation length is large. Such a nonlocal effect, which leads to radial turbulent energy convection by a large $\mathbf{E} \times \mathbf{B}$ step size, plays a crucial role to form a self-organizing character in toroidal mode driven plasma

transport. As already discussed in Fig. 6, the typical frequency of turbulent fluctuation in the quasi-steady state ($t \geq 7000\omega_\alpha$) is characterized by the magnetic drift frequency $\omega \sim \omega_d$, which is consistent with the profile near marginal stability. However, as seen in Fig. 4, the wave excitation and thermal transport have been maintained, maintaining the self-organizing character. Therefore, it is concluded that the relaxed turbulent state is sustained near but slightly above marginal stability in the present simulation. It is this small margin above marginal stability that drives the fluctuation and sustains transport.

The relaxed state observed above will depend on the heat reservoir (or equivalently the initial temperature around the plasma center). In order to study the relaxed state under a different condition, we performed the simulation by using a different initial temperature profile which has a Gaussian shape as shown in Fig. 8(a) (dashed line). Note that $T_i(0)$ is chosen to be higher than the case in Fig. 7 although the total kinetic energy in the torus is the same. The relaxed state after saturation ($t = 16000:\omega_\alpha^{-1}$) is illustrated. The initial Gaussian temperature profile is also found to relax to a new profile well fit by the exponential functional form. However, in this case, two radial portions which have different L_T lengths appear such that $T_i \propto \exp(-r/L_{T1})$ for an interior portion and $T_i \propto \exp(-r/L_{T2})$ for a more exterior region.

Figure 8(b) shows the corresponding frequency power spectrum $P(\omega, r)$ in the quasi-steady state. Although the relaxed temperature profile is different from the one in Fig. 7, the typical frequency of turbulent fluctuations is also given by $\omega \sim \omega_d$. However, the power spectrum is found to exhibit a different feature. That is, the power spectrum of the electric field is not clearly monochromatic but forms two or three segmented frequency bands which follow the overall $\omega = \omega_d(r)$ curve. The deviation of the local frequency from the eigenfrequency ω is within $O(\sqrt{\rho_i/a})\omega$, because the extent of the eigenmode is over $\sqrt{\rho_i a}$, while the global change of $\omega_d(r)$ is over a . Since wave emission has been maintained long after the saturation, this relaxed state is also considered to be slightly above marginal stability.

By carefully choosing the height of the potential contour in (r, θ) poloidal cross-section, we can see microscale transport which occurs within the macroscale self-organized temperature profile. Figure 9 shows the electrostatic potential in the quasi-steady state after the exponential temperature profile is established. It is found that the ITG mode excited in the interior region in the torus ($t \simeq 5600 \omega_{ci}^{-1}(a)$) is transported toward the outer region ($t \simeq 10200 \omega_{ci}^{-1}(c)$), showing a radial movement of potential vortices. Then, we see the potential vortices propagate back to the plasma interior from the torus exterior. Such behavior is explained as follows: a small temperature perturbation (i.e. a local bump or hole) which happens in one radial region on the self-organized profile excites a localized ITG mode. Heat flux induced by such mode excitation is convected out so that the temperature gets back to the smooth self-organized profile. The temperature perturbation which is expelled from one radial region causes another perturbation in a different radial region and there again induces the mode excitation. Resultantly, the mode excitation follows the temperature perturbation and moves back and forth in the torus so that the deviation from the self-organized profile is minimized. This relaxation oscillation accompanied by the radial movement of the active region and associated bursty mode excitation have a time scale related to $(\epsilon^2 \omega_d)^{-1}$, where ϵ^2 is a smallness parameter for the time scale we shall discuss in the next subsection. This is another evidence that the toroidal plasma has a strong constraint which binds the plasma resiliently to the relaxed state. This implies that one can construct relaxation theory of transport in this plasma.

At the end of this subsection we briefly show the results of the η_i -mode where the density profile is given by Gaussian form $n_i(r) \propto \exp(-r^2/2L_n^2)$ in Fig. 10. The temperature profile is initially given as either arctan form (a) or Gaussian form (b). The toroidal η_i -mode is also found to provide a constraint on the plasma profile and relaxation. This time the initial arctan temperature profile tends to relax to the one fit to the Gaussian profile in a self-similar manner. It is also found that the initial Gaussian temperature profile relaxes

without changing the functional form, as seen in Fig. 10(b). Note that the density profile has changed somewhat for the simulation by using the adiabatic electrons. Therefore, the constraint to form a Gaussian temperature profile means that the initial $\eta_i (= L_n/L_T)$ value changes (overall decreases) to become spatially independent over the radial region where the radially extended η_i -modes develop. It is also confirmed that even when the initial density profile is given as an exponential form, the arbitrary temperature profile is also self-organized and relaxes to the exponential form keeping the spatially constant η_i value.

C. Discussion and relaxation theory

The above observation implies that there exist at least two distinct time scales. Namely, let $T_0(r, t)$ be a global temperature profile which has, for example, an exponential form for ITG mode or Gaussian one for η_i -mode, as observed in Sec. IIIB. Also let $\delta T(r, t)$ be a small temperature deviation away from $T_0(r, t)$. Here, the global temperature profile $T_0(r, t)$ evolves slowly by keeping the functional form. However, the deviation $\delta T(r, t)$ from $T_0(r, t)$ is quickly smoothed out so that $\delta T(r, t) \rightarrow 0$ in a faster time scale.

In order to explain the above feature by a more mathematical way, we present a relaxation theory by introducing a multiple time scale expansion in the quasilinear equation Eq. (13). A systematic expansion may be carried out by the well-known reductive perturbation method,³⁴ expanding in terms of a smallness parameter ϵ . We expand the time and spatial scales, temperature, and heat flux as follows:

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_0} + \epsilon^2 \frac{\partial}{\partial t_1} + \epsilon^4 \frac{\partial}{\partial t_2} + \dots, \quad (25)$$

$$\frac{\partial}{\partial r} = \frac{\partial}{\partial r_0} + \epsilon \frac{\partial}{\partial r_1} + \epsilon^2 \frac{\partial}{\partial r_2} + \dots, \quad (26)$$

$$T = T_0 + \epsilon T_1 + \dots, \quad (27)$$

$$Q = \epsilon Q_0 + \epsilon^2 Q_1 + \dots, \quad (28)$$

where $\mathbf{Q}_0 = -\chi_i^{(0)} \nabla_0 T_0$ and $\mathbf{Q}_1 = -\chi_i^{(0)} \nabla_0 T_1 - \chi_i^{(1)} \nabla_0 T_0 - \chi_i^{(0)} \nabla_1 T_0$ and T_1 is equivalent to the above discussed δT . In general each quantity is a function of both t_0 and t_1 and r_0 and r_1 (and higher order variables). T_0 is the above explained global temperature profile which exhibits self-similarity and is evolved only in a slower time scale t_1 . Therefore, we assume $\partial T_0 / \partial t_0 = 0$, where t_0 represents the fast time scale. From this expansion, we obtain a series of transport equations:

$$\mathcal{O}(\epsilon) : \frac{3}{2} n_i \frac{\partial T_1}{\partial t_0} + \nabla_0 \cdot \mathbf{Q}_0 = 0, \quad (29)$$

$$\mathcal{O}(\epsilon^2) : \frac{3}{2} n_i \frac{\partial T_0}{\partial t_1} + \nabla_0 \cdot \mathbf{Q}_1 + \nabla_1 \cdot \mathbf{Q}_0 = S, \quad (30)$$

by assuming that the density n_0 is maintained constant, where S is a heat source.

In Eq. (29), if there is finite divergence for $\mathbf{Q}_0$, i.e. $\nabla \cdot \mathbf{Q}_0(r) \neq 0$, it will force T_1 to rapidly relax such that $T_1 \rightarrow 0$. Thus, on average, we obtain $\partial T_1 / \partial t_0 \rightarrow 0$ and therefore $\nabla \cdot \mathbf{Q}_0 \rightarrow 0$, which leads to the condition

$$r Q_0(r) = r \chi_i^{(0)}(r) \frac{\partial T_0}{\partial r} \simeq \text{const}, \quad (31)$$

over a range of r such that r_0 belongs to a region with a given value of r_1 . Since we know from the toroidal simulation that the global T_0 profile is self-organized to be $T_0 \sim \exp(-r/L_T)$ for the ITG mode and $T_0 \sim \exp(-r^2/2L_T^2)$ for the η_i mode, the constraint in $\mathcal{O}(\epsilon)$ given by Eq. (31) requires that the thermal conductivity varies as

$$\chi_i^{(0)}(\text{ITG}) \sim \frac{\text{const.}}{r} \exp\left(\frac{r}{L_T}\right), \quad (32)$$

$$\chi_i^{(0)}(\eta_i) \sim \frac{\text{const.}}{r^2} \exp\left(\frac{r^2}{2L_T^2}\right). \quad (33)$$

The thermal conductivity $\chi_i^{(0)}$ given by Eqs. (32) and (33) is found to show the radially flat or rather increasing profiles which are in agreement with our simulation results.²¹ In the next order in ϵ , we obtain the slow variation of the background temperature $T_0(t_1)$ through

divergence of Q_1 in Eq. (30). Note that the slow variation is induced as a variation of the scale length which is given by $\partial T_0/\partial t_1$.

In order to evaluate heat flux Q , here we employ the quasilinear expression (Eq. (14)) given by

$$r_1 Q_0(r_1) = \frac{n_{ice}}{B} nq(r_1) |\phi(r_1)|^2 \text{Re } G, \quad (34)$$

where n is a given toroidal mode number and $k_\theta = nq(r_1)/r_1$ is used in Eq. (14). We emphasized here the subscript 1 to indicate the spatial dependence of parameters over a larger scale (r_1). In order to have the constraint given by Eq. (31) i.e. $rQ_0(r) = \psi(= \text{const.})$ in $\mathcal{O}(\epsilon)$, the radial profile of the electrostatic potential is self-consistently determined as follows

$$|\phi_0(r)|^2 = \frac{\psi B}{n_{icen} q(r) \text{Re } G}, \quad (35)$$

where $|\phi_0(r)|^2$ is also an order ϵ and compensates the radial dependence of q and G . The finite divergence over a small scale (r_0) of Q_1 as well as the divergence of Q_0 over a larger spatial scale (r_1) become the driving forces of the self-similar relaxation of the temperature T_0 which takes place in slower time scale t_1 . On the other hand, the transport of temperature perturbation (a local bump or hole represented by T_1) which appears on the self-organized profile for example, as seen in Fig. 9, results from heat flux Q_0 and is governed by Eq. (29). We shall investigate the properties of Eqs. (29) and (30) in more detail in the subsequent section.

IV. CRITICAL GRADIENT TRANSPORT MODEL

In this section, we further develop the relaxation theory touched upon in Sec. IIIC. In order to solve the transport equations (29) and (30), heat flux Q has to be determined self-consistently with the plasma properties, such as the fluctuation level, plasma temperature gradient etc., as seen in Eq. (14). In this quasilinear expression $|\phi|^2$ or its normalized quantity

W , appears. We have to therefore determine the equation that describes the evolution of W . Here, we restrict our attention to the toroidal ITG mode which is excited in a flat density profile. As seen in Sec. II, the ITG mode becomes linearly unstable for the temperature profile which satisfies the condition $\epsilon_T < \epsilon_{Tc}$. As seen in Fig. 1(b), the linear growth rate γ_L can be approximated around marginal stability by

$$\gamma_L = \gamma_0(\mu - \mu_c) \simeq - \left(\frac{\gamma_0}{\epsilon_{Tc}^2} \right) (\epsilon_T - \epsilon_{Tc}) (\equiv \bar{\gamma}_L), \quad (36)$$

where $\mu \equiv 1/\epsilon_T$ and $\mu_c \equiv 1/\epsilon_{Tc}$. Here, γ_0 is related to the gradient of γ_L with respect to ϵ_T at the threshold by

$$\gamma_0 = -\epsilon_{Tc}^2 \left(\frac{\partial \gamma_L}{\partial \epsilon_T} \right) = c_2 \epsilon_{Tc} \omega_d, \quad (37)$$

where the constant c_2 is estimated as $c_2 = \bar{\gamma}_L(\epsilon_T = 0)/\omega_d (= (\epsilon_{Tc}/\omega_d)(\partial \gamma_L/\partial \epsilon_T))$ from the linear dispersion analysis in Fig. 1(c). In the case of $X = 0$, we evaluate $c_2 \simeq 1.0$ and $\epsilon_{Tc} \simeq 0.31$, and then $\gamma_0 \simeq 0.31 \omega_d$. Note that Eq. (36) is introduced by modelling the numerical dispersion analysis in which the growing ITG mode continuously goes to a damped mode with $d^2 \gamma_L / d\epsilon_T^2 > 0$ at $\epsilon_T = \epsilon_{Tc}$. Here, we define the nonlinear total growth rate γ by

$$\gamma(W) = \gamma_L - \gamma_{NL}(W), \quad (38)$$

where γ_{NL} represents the rate of nonlinear damping which saturates the linear mode and we describe the damping rate as $\gamma_{NL} = \gamma_N W$. We determine γ_N by considering $\mathbf{E} \times \mathbf{B}$ trapping of a particle by the radially extended global mode. We adopt this as a candidate of the saturation mechanism. A derivation is described in Appendix A.

It should be emphasized that since the plasma relaxes and achieves the critical gradient state, the temperature gradient of the plasma is very close to that of marginal stability, i.e.

$$\mu - \mu_c \ll \mu_c. \quad (39)$$

As discussed in Eq. (29), if Eq. (39) is not fulfilled, the rapid temperature evolution enforces the plasma to quickly attain the condition (39) in a short time scale (t_0). Thus the temporal

smallness expansion parameter ϵ in Eq. (25) is related to Eq. (39) as

$$\epsilon^2 = \frac{\gamma_0}{\omega_d} (\mu - \mu_c). \quad (40)$$

The equation thus derived takes the following form:

$$\frac{\partial W}{\partial t_1} = 2[\gamma_0(\mu - \mu_c) - \gamma_N W]W. \quad (41)$$

This equation resembles the Landau-Ginzburg equation in the critical phenomena as in phase transition, where the critical slowing-down takes a very long growth time, just like in our present case of the critical gradient of temperature. The slowness of this evolution is explicitly written in Eq. (41) through the slow time scale (t_1). As is evident, the right-hand side ordering is consistent with the left-hand side, as Eqs. (40) and (25) indicate. The nonlinear saturation $\gamma_N W$ in this critical gradient problem is much below the level often discussed in the theoretical literature that discusses robust turbulent saturation, in which the time scale of t_1 can not come in. In the present theory the steady-state saturation of W is

$$W^{(\text{sat})} = \frac{\gamma_0}{\gamma_N} (\mu - \mu_c) = \frac{\omega_d}{\gamma_N} \epsilon^2, \quad (42)$$

at the order of ϵ^2 . Because the temporal evolution of W in Eq. (41) is in a slow scale (t_1), we can couple Eq. (41) with the slow scale transport equation (30). In many works where the level of $\mu - \mu_c$ is high, there exists no time scale separation between t_0 and t_1 and thus no transport separation between Eqs. (29) and (30). Thus the transport equation in those works do not have separate terms as $\nabla_0 \cdot \mathbf{Q}_1$ and $\nabla_1 \cdot \mathbf{Q}_0$, as we shall see in the below.

The transport phenomenon we described in Sec. III has the relaxed state over the spatial scale $(\rho_i a)^{1/2}$ (or more precisely $\rho_i^{1-\alpha} a^\alpha$). Within this scale the spatial variations of T_0 , $|\phi|^2$ etc. over r_0 are zero as they are smoothed out, i.e. $T_0(r_1)$, $|\phi(r_1)|^2$ [not $T_0(r_0, r_1)$, $|\phi(r_0, r_1)|^2$]. In particular from Eq. (29) the steady-state condition, or precisely over the time scale t_1

(beyond t_0), $\nabla_0 \cdot \mathbf{Q}_0 = 0$ means

$$-r\chi_i^{(0)}T_0\mu = \text{const} = \psi(r_1), \quad (43)$$

where $\mu \equiv -(\partial T_0/\partial r_0)/T_0$ is used and the constancy of the left-hand side means constant over the period of $\sqrt{\rho_i a}$, but beyond that period it may be a slowly varying function of r_1 , thus written as $\psi(r_1)$. The amount of ψ determines the magnitude of heat flux, which needs an additional equation, as is considered by Eq. (30). These are the consequences of the self-organized critical state due to the presence of the extended modes over $\sqrt{\rho_i a}$. Thus the spatial smallness expansion parameter ϵ should be $(\rho_i/a)^{1/2}$ [or more precisely $(\rho_i/a)^{1-\alpha}$ and a more detail on this point will be discussed in the final section].

With these considerations, the transport equation coupled to Eq. (41) is derived from Eq. (30) as

$$\frac{3}{2} \frac{\partial T_0}{\partial t_1} = \frac{1}{r} \frac{\partial}{\partial r_0} \left[r\chi_i^{(0)} \left(\frac{\partial T_1}{\partial r_0} + \frac{\partial T_0}{\partial r_1} \right) + r\chi_i^{(1)} \frac{\partial T_0}{\partial r_0} \right] + \frac{1}{r} \frac{\partial}{\partial r_1} \left(r\chi_i^{(0)} \frac{\partial T_0}{\partial r_0} \right) + \frac{P_{in}(r_1)}{n_i}, \quad (44)$$

where the first bracketed terms arose from $\nabla_0 \cdot \mathbf{Q}_1$ and the second bracketed term from $\nabla_1 \cdot \mathbf{Q}_0$ and the source term S is given by the external input power per unit volume P_{in} . In Eq. (44) the term $\partial T_1/\partial r_0$ may be put to be zero, as the temperature T_1 quickly relaxes to zero in time t_0 according to Eq. (29).

The stationary solution of Eq. (44) is sought by

$$R \frac{\partial}{\partial r_0} \left(r\chi_i^{(0)} \frac{\partial T_0}{\partial r_1} \right) - \frac{\partial}{\partial r_1} \left(r\chi_i^{(0)} T_0 \mu \right) = \frac{r P_{in}}{n_i}, \quad (45)$$

where we dropped the term proportional to $\chi^{(1)}$ (though possible to keep) for simplicity. By commuting ∂_{r_0} and ∂_{r_1} and using Eq. (43), Eq. (45) may be written as

$$\frac{\partial}{\partial r_1} (\ln \mu \psi) = -\frac{r R P_{in}(r_1)}{n_i} \psi(r_1), \quad (46)$$

or

$$-r\chi_i^{(0)}\mu^2(r_1)T_0(r_1) = C \exp \left[\int_r^a \frac{r_1 P_{in}(r_1)}{n\psi(r_1)} dr_1 \right], \quad (47)$$

where $C = \mu(a)\psi(a)$. Equation (47) is an implicit integral equation in which ψ is related to μ through Eq. (43) and χ is related to W through Eq. (15). The quantity W is now governed by Eq. (41). Thus Eqs. (41) (with $\partial/\partial t_1 = 0$) and (47) constitute a complete set of the coupled equations that self-consistently describe a steady state realized by the interplay between plasma fluctuations and heat transport in the self-organized critical plasma with the auxiliary defining equations (15) and (43). A few simplifications and assumptions employed in their derivations are mentioned above. One of the main differences in the present transport theory from the conventional theory is the presence of μ^2 instead of μ in Eq. (47). In the future we will investigate the influence of the factor μ^2 on the resiliency of the temperature profile in a self-organized critical tokamak plasma. Another important distinction of the present theory is that the slow time scale of transport is now legitimately tied to the slow time scale of the Landau equation describing the wave growth and damping. As commented earlier, unless the instability is near marginal, such assignment of time scale is not possible. A third distinct property of the present theory is that the slow time scale is now tied to the nonlocal spatial scale ($\sqrt{\rho_i a}$) (beyond the local scale ρ_i) so that the equations describe the nonlocal (but yet not global) spatial behavior: We have to now recognize three distinct spatial scales for the problem: ρ_i , $\rho_i^{1-\alpha} a^\alpha$, and a (a may be substituted by L_T).

As a simple case, we consider the steady state where the first term of the right-hand side in Eq. (45) is dropped. This analysis is useful to investigate the parametric dependence of (μ, W) and the resultant heat diffusivity scaling. In the steady state ($\partial/\partial t_1 \rightarrow 0$), we obtain the quadratic equation with respect to μ :

$$\mu^2 - \mu_c \mu - \frac{\bar{p}}{g} \left(\frac{\gamma_N}{\gamma_0} \right) = 0, \quad (48)$$

where $\hat{g} \equiv \hat{\chi}/r^2$ and $\bar{p}$ represents normalized heat flux given by

$$\bar{p} = \frac{F(r)}{n_i T_0} \left(\frac{a^2 R}{r^3} \right), \quad (49)$$

where $F(r) = a^{-2} \int_0^r r P_{in}(r) dr$. When $P_{in}(r)$ is spatially uniform, Eq. (49) becomes $\bar{p} = (P_{in}/n_i T_0)(R/2r)$. γ_N/γ_0 in Eq. (48) is estimated from Eqs. (37) and (A2) as

$$\frac{\gamma_N}{\gamma_0} \sim \left(\frac{c_1}{c_2}\right) \frac{\mu_c (k_r R)^2}{\tilde{T}_i^2} \simeq \left(\frac{c_1}{c_2}\right) \left(\frac{\mu_c}{\pi^2 \tilde{T}_i^2}\right) \frac{R^2}{\rho_i^{2(1-\alpha)} a^{2\alpha}}. \quad (50)$$

Equation (48) leads to a solution for μ and W as

$$\mu = \frac{\mu_c}{2} \left[1 + \sqrt{1 + \frac{4\bar{p}}{g\mu_c^2} \left(\frac{\gamma_N}{\gamma_0}\right)} \right], \quad (51)$$

$$W = \left(\frac{\gamma_0}{\gamma_N}\right) (\mu - \mu_c). \quad (52)$$

Equation (51) changes its character depending on different values of $(4\bar{p}/g\mu_c^2)(\gamma_N/\gamma_0)$ ($\equiv \delta$: inflection parameter). Since $\bar{p}/g\mu \propto P_{in}r/n_i T_0^{3/2}$ and $\gamma_N/\gamma_0 \propto T_0^{-1/2}$ assuming $\hat{\chi} \propto T_0^{1/2}$ and uniform input power, $\delta \propto P_{in}r/n_i T_0^2$ is found. Namely, for weak heating and/or high temperature regime (i.e. $\delta < 1$),

$$\mu = \mu_c + \frac{\bar{p}}{g\mu_c} \left(\frac{\gamma_N}{\gamma_0}\right), \quad (53)$$

$$W = \frac{\bar{p}}{g\mu_c}, \quad (54)$$

where (μ, W) is a linear function of $\bar{p}$. On the other hand, for the strong heating case and/or high temperature region, (μ, W) becomes

$$\mu = \left(\frac{\bar{p}}{g}\right)^{1/2} \left(\frac{\gamma_N}{\gamma_0}\right)^{1/2}, \quad (55)$$

$$W = \left(\frac{\bar{p}}{g}\right)^{1/2} \left(\frac{\gamma_0}{\gamma_N}\right)^{1/2}, \quad (56)$$

where (μ, W) increases in proportion to $\bar{p}^{1/2}$. The deviation of the inverse gradient μ from μ_c at the inflection power is found from Eq. (50) to be

$$\delta\tilde{\mu} \left(\equiv \frac{\mu - \mu_c}{\mu_c} \right) = \left(\frac{\sqrt{2} - 1}{2} \right). \quad (57)$$

That is, 21% deviation from the critical gradient is found. To know the radial dependence of (μ, W, χ_i) , it is worthwhile to investigate how (P_{in}, T_0, n_i) depends on (μ, W, χ_i) in Eqs. (53)–(56). In each regime, we obtain

$$(\delta\mu \propto rP_{in}/n_iT_0^2, \quad W \propto rP_{in}/n_iT_0^{3/2}, \quad \chi_i \propto rP_{in}/n_iT_0) \quad (58)$$

for $\delta < 1$ case and

$$(\delta\mu \propto r^{1/2}P_{in}^{1/2}/n_i^{1/2}T_0, \quad W \propto r^{1/2}P_{in}^{1/2}/n_i^{1/2}T_0^{1/2}, \quad \chi_i \propto r^{1/2}P_{in}^{1/2}/n_i^{1/2}) \quad (59)$$

for $\delta > 1$ case. In both regimes, the deviation from the critical gradient $\delta\mu$ and the fluctuation energy W increase as T_0 decreases. Resultantly, the thermal diffusivity χ_i increases with decrease of T_0 . In many tokamak experiments and our toroidal simulation, the radially increasing χ_i is observed and this tendency is consistent with the present theoretical results.

For the $\bar{p} \gg \bar{p}_i$ case, we obtain the heat diffusivity scaling resulting from the mixed time scale determined by the intrinsic χ_i and the heating rate as

$$\chi_i = c_5 \left(\frac{c}{e}\right)^{1/2} \left(\frac{P_{in}}{n_i B}\right)^{1/2} \rho_i^{1/2-\alpha} a^\alpha \frac{(L_i L_{TC})^{1/2}}{R} (G^{(res)})^{1/2}, \quad (60)$$

where $c_5 \simeq (c_2/2c_1)^{1/2}(k_\theta \rho_i)^{1/2}/\pi \sim 0.26$ is a constant. Transport is Bohm-like (for $\alpha = 0.5$) and χ_i is in proportion to $P_{in}^{1/2}$, showing the typical L-mode scaling.

V. CONCLUDING REMARKS

Based on the particle simulation in full toroidal geometry, we investigate the remarkable characteristics of the toroidal ITG mode and related plasma relaxation. The primary simulation results are summarized as follow:

1. Due to the toroidicity-induced coupling, the radially extended nonlocal mode and turbulence, which have the long radial correlation length scaling scale as $L_r \propto \rho_i^{1-\alpha} L_T^\alpha$, are induced, showing the coupling between the short ion Larmor length and plasma

scale size. Subjected by such nonlocal turbulence, the global temperature relaxation was observed.

2. Radially extended nonlocal turbulence provides the strong constraint on the plasma profile so that the plasma is self-organized. As a result of that, the plasma shows profile resilience and self-similarity in the relaxation. For the ITG mode case with the flat density profile, the self-similar functional form is approximately given by the exponential $T_i(r, t_1) \propto \exp(-r/L_T(t_1))$, and only scale length L_T changes in a slow time scale during the relaxation.
3. Due to such a constraint on the plasma, the plasma profile is difficult to sustain much above marginal stability, and forced to relax until it sufficiently approaches the critical gradient. The relaxed state forms self-organized criticality and how far the plasma profile can deviate from the critical gradient is determined by the balance between radial turbulent energy transport and external heating. Any temperature bump away from the extended mode region built up from this process contributes to reshuffling of eigenmodes and subsequent intermittent bursts.

The characteristic of near marginal stability (or critical gradient) is a direct consequence of the nonlocal nature of the mode excitation (item (1)) and the related strong constraint on the plasma (item (2)). That is, due to the nonlocal nature of the mode which has the radial long correlation length, the $\mathbf{E} \times \mathbf{B}$ step size becomes large. This is the reason why we obtain thermal transport scaling as the Bohm-like diffusion when the radial mode is well established as $L_r = \rho_i^{1-\alpha} a^\alpha$ with $\alpha \sim 0.5$ near the critical gradient. Namely, the concepts of the *Bohm-like* (or large scale nonlocal) *diffusion* and the *near critical gradient* coincide with each other and belong to one physical category.

In the present theory it does not necessarily mean that the Bohm-like diffusion causes more transport than the gyroBohm diffusion, although the diffusive step size is far greater in

the former than the latter. This is because precisely due to the great radial correlation length of the Bohm-like case the plasma relaxes close to marginality and plasma fluctuations remain low and the coefficient of $\rho_i v_i$ (Bohm-like diffusion) becomes small (see Eq. (15)). On the other hand, let us consider a case, for a certain reason, in which a strong instability develops before the temperature gradient is washed away. In such a case, the fluctuation level of the turbulence would become high enough to cause a secondary instability³⁵ that could lead to disintegration of the radially extended nonlocal modes. In this case, the radial scale length of the turbulence may be reduced from $(\rho_i a)^{1/2}$ (i.e. $\alpha = 0.5$ in Eq. (23)) to ρ_i (i.e. $\alpha = 0$) and thus we would get the gyroBohm scaling as found in Eq. (60). When this happens, even if the inverse temperature gradient μ increases substantially above the critical value μ_c in Eq. (52), the increase of γ_N due to the change (decrease) of α leaves the level of the fluctuation energy W , and thus χ_i is not significantly affected. The parametric dependence of (μ, W) and χ_i on the exponent α would be discussed in a followup paper. However, such an effect might explain the "puzzle" that even in so-called "supershot" discharges⁷ in which the temperature is sharply peaked in the plasma center the heat diffusivity was observed to be not greatly changed over the value for shots with less peaked temperatures. If there is an agent to reduce the toroidicity-induced radial coupling such as shear flow in the plasma, it can lead to a radial step size smaller than the nonlocal case. Such a possibility has been explored recently.

Based on the two-field critical gradient model which takes into account self-organization of the plasma profile, we investigate the deviation of the temperature gradient from the critical one as a function of the external heating power. Our theory recognizes that the slow time scale of the transport in the energy equation can be tied to the fluctuation growth/damping time scale in the Landau equation only when the plasma relaxes toward the near marginality (the critical gradient). Our theory also recognizes that because the radial extent of the toroidicity-coupled mode structure is much greater than the individual mode wavelength ($\sim$

ρ_i), the spatial self-organization (such as a smooth self-similar temperature profile) results: this large spatial diffusive length allows one to realize the slow wave and transport time scale under the critical temperature gradient condition. Thus the self-organized critical gradient phenomenon we discovered ties the spatial extent of the nonlocal mode/diffusion structure, the slow temporal scale of the wave growth and transport, and the closeness of the relaxed temperature gradient to marginality all together. There may be a possibility that the plasma's self-organizing tendency tries to force the ordering of the spatial size of the vortices to better match that of the temporal scale, which demands the exponent α to tend to

$$\alpha = 1 - \frac{1}{2} \frac{\ln(\hat{\gamma}_0 \delta \mu)}{\ln \sqrt{\rho_i/a}}, \quad (61)$$

where $\hat{\gamma}_0 \equiv \gamma_0/\omega_d$. That is, if the self-organization is such, then α will be adjusted from the value 1/2 to the one that balances the difference between the two (logarithms of the) temporal and spatial smallness parameters $\ln(\hat{\gamma}_0 \delta \mu)$ and $\ln \sqrt{\rho_i/a}$. However, an alternative possibility, the detachment of spatial and temporal scales, may occur if the imbalance becomes lopsided; this needs further investigation. Finally, it is also instructive to know that it appears that all (or nearly all) stars on the main sequence maintain the critical temperature gradient for the global (and thus nonlocal) superadiabatic convection. This is, we can surmise, similar to our present theory of tokamak ITG transport, because if the temperature gradient is much above marginality, it would cause too strong convection of a global scale in the stellar convection zone and would quickly cool and relax back to marginality. On the other hand, it appears that more local short wavelength convective modes are robustly unstable.

In conclusion, the recent experimental results of transport studies of L-mode discharges in four major tokamaks JET,¹¹ TFTR,¹⁰ D-IIID,¹² and JT-60U³⁶ show the Bohm-like diffusion in the ion thermal diffusivity. Our theory and simulation indicate the Bohm-like diffusion, based on the radially extended mode structure and consequent critical gradient phenomenon

as described in the text.

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VI. APPENDIX: NONLINEAR DAMPING RATE

The nonlinear damping rate γ_{NL} which appears in Eq. (38) may be written as

$$\gamma_{NL} = \gamma_N W \simeq k_r^2 D(W), \quad (A1)$$

by assuming the condition $|\gamma| \ll |\gamma_L| \sim |\gamma_{NL}|$ where $\gamma(W)$ is the total growth rate given by Eq. (38). Here, $D(W)$ is estimated by $D(W) = \delta\xi_r^2/\delta t$, where $\delta\xi_r = ck_\theta|\phi|/B_0\omega$ represent the $\mathbf{E} \times \mathbf{B}$ step size due to the fluctuating field. If we estimate the phase decorrelation time simply by $\delta t_{\text{decorr.}} \sim \omega_d^{-1}$, γ_N is given by

$$\gamma_N \simeq \frac{c_1}{\tilde{T}_i^2} R^2 k_r^2 \omega_d = \frac{c_1}{\pi^2 \tilde{T}_i^2} \frac{R^2 \omega_d}{\rho_i^{2(1-\alpha)} a^{2\alpha}}, \quad (A2)$$

where $c_1 \sim 1/9$ and $W = |e\phi/\langle T_i \rangle|^2$. In deriving Eq. (A2), we used the relation $\omega \sim (3/2)\omega_d$ by assuming the toroidal branch near the critical gradient. As seen in Eq. (A2), γ_N is proportional to k_r^2 and therefore sensitively depends on the exponent α . We estimate the saturation level of the electrostatic potential based on the $\mathbf{E} \times \mathbf{B}$ trapping by simply using the wave-breaking condition. The later steady state is determined from the power input, as we shall see in Eqs. (40)–(41). From the condition, $\delta\xi_r \sim \pi/k_r$ ($\xi_r \equiv ck_\theta|\phi|/B\omega$ is the $\mathbf{E} \times \mathbf{B}$ radial step size), we estimate the fluctuation energy by

$$W \simeq \frac{9\pi^2 \tilde{T}_i^2}{(k_r R)^2} \quad (A3)$$

where $\omega \sim (3/2)\omega_d$ is used. This level of saturation should be considered to be the maximum level that can be achieved without the feedback of the evolution of W to the background temperature profile, perhaps corresponding to the initial overshooting regime.

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FIGURE CAPTIONS

FIG. 1. Growth rate $\text{Im } \omega/\omega_d$ (a), real part of the frequency $\text{Re } \omega/\omega_d$ of the ITG mode versus $\epsilon_T \equiv L_T/R$ for different value of $X \equiv k_{\parallel} \bar{v}_i/\omega_d$. Parameters are given by $k_{\theta} \bar{\rho}_i = 0.3$ ($\bar{\rho}_i \equiv \rho_i/\sqrt{2}$), $\tau (\equiv T_e/T_i) = 1$ and $\epsilon_n (\equiv L_n/R) = 3.0$. Figure (c) show the corresponding normalized heat flux G . Arrows in Figs. 1 (c) and (d) show the position of $\text{Im } \omega = 0$. Arrows in Figs. 1(b) and (c) indicate the point at the critical gradient corresponding to the ones in Fig. 1(a).

FIG. 2. (a) Dependence of stability boundary ϵ_{Tc} (solid line) and corresponding real frequency ω/ω_d (dashed line) on X for given $k_{\theta} \bar{\rho}_i = 0.3$, and (b) dependence of ϵ_{Tc} (solid line), ω/ω_d (dashed line) and corresponding G (dash-dotted line) on $k_{\theta} \bar{\rho}_i$ for given $k_{\parallel} R = 0.25$. Critical gradient locates at $k_{\theta} \bar{\rho}_i \simeq 0.35$ where $X \simeq 0.36$ for $k_{\parallel} R = 0.25$.

FIG. 3. Radial variation of (a) safety factor q and (b) shear parameter $\hat{s}$.

FIG. 4. Potential structure in the poloidal cross-section $\phi(r, \theta, \varphi = \varphi_0)$ at (a) $t = 2600\omega_{ci}$ (early linear stage), (b) $3400\omega_{ci}$ (before saturation), (c) $5600\omega_{ci}$ (after saturation) and (d) $11200\omega_{ci}$ (steady state). Solid line and dashed line represent positive and negative potential value, respectively. The arrow represents the direction of the poloidal potential rotation by ion diamagnetic drift.

FIG. 5. Contour plot of real frequency power spectrum ω/ω_{ci} as a function of minor radius in the steady state during $6000\omega_{ci}^{-1} \leq t \leq 15000\omega_{ci}^{-1}$. Plus and minus signs of ω correspond to electron and ion drift sides, respectively. Dashed line represents the one which satisfies the condition $\omega/\omega_{ci} = 2k_{\theta} \bar{\rho}_i^2/R$.

FIG. 6. Potential structure (toroidal Fourier component) in the poloidal cross-section $\phi_n(r, \theta, \varphi)$ for $n = 9$ (a) and $n = 10$ at $t = 11400$.

FIG. 7. Time evolution of the ion temperature profile plotted by logarithmic scale at four time steps. Dashed line represents the initial profile.

FIG. 8. (a) Temperature profile plotted by logarithmic scale at $t = 16000\omega_{ci}^{-1}$. Dashed line represents the initial profile which has a Gaussian shape.

(b) Contour plot of real frequency power spectrum ω/ω_{ci} as a function of minor radius in the steady state $8000\omega_{ci}^{-1} \leq t \leq 16000\omega_{ci}^{-1}$. Dashed line represents the one which satisfies the condition $\omega/\omega_{ci} = 2k_\theta \bar{p}_i / R$.

FIG. 9. Potential structure in the poloidal cross-section $\phi(r, \theta, \varphi = \varphi_0)$ at six different time steps in the steady state. Parameters are the same as in Fig. 8.

FIG. 10. Ion temperature profile of η_i -mode plotted by logarithmic scale as a function of r^2 , for two different initial profiles; (a) Gaussian profile and (b) arctan profile. Density profile is given by Gaussian profile.

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Fig. 10(b) Y. Kishimoto et. al.