CERAMIC COMPACTION MODELS: USEFUL DESIGN TOOLS OR SIMPLE TREND INDICATORS?

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INTRODUCTION

It is well-known that dry pressing of ceramic powders leads to density gradients in a ceramic compact resulting in non-uniform shrinkage during densification. This necessitates diamond grinding to final dimensions which, in addition to being an extra processing step, greatly increases the manufacturing cost of ceramic components. To develop methods to control and thus mitigate density variations in compacted powders, it has been an objective of researchers to better understand the mechanics of the compaction process and the underlying material and tooling effects on the formation of density gradients.

This paper presents a review of models existing in the literature related to the compaction behavior of ceramic powders. In particular, this paper focuses on several well-known compaction models that predict pressure and density variations in powder compacts.

GENERAL APPROACH TO COMPACTION MODEL DEVELOPMENT

In studying the compaction of ceramic powders, a common approach is to measure the increase in density of a powder mass as a function of applied pressure. A pressing die is filled with ceramic powder and compacted by a plunger attached to a load cell. In this manner, the density of the compact for any applied pressure can be calculated from the displacement of the plunger and the green density of the compact measured after ejection from the pressing die. Several researchers have
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used this method to establish pressure-density relationships for different pressing methods and ceramic powders.\textsuperscript{1-5} This type of experiment, however, only pertains to the \textit{average} properties of the compact. Moreover, the correlations observed between density and applied pressure are largely empirical.

To develop models which can predict density variations \textit{within} a powder compact, typically a pressure distribution is established as a function of the applied compacting pressure using a mechanics-based approach. A density distribution is then obtained by coupling this pressure distribution with a density-pressure relationship often obtained as outlined above. Trends in density distributions have been found to correlate directly with trends established in pressure distributions.

A common geometry used to develop a compaction model considers the uniaxial compaction (either single-action or double-action) of cylindrical powder compacts in a metal die. This geometry is easy to treat mathematically and generally consists of a compact of diameter $D$ and height $H$, as shown in Fig. 1a.

![Figure 1.](image)

(a) Geometry and compaction forces related to cylindrical powder compact in pressing die.

(b) Forces acting on an axial slice of height $dz$ at depth $z$.

An applied pressure, $P_0$, acting over the top surface of the cylinder, will cause the powder to compact in the axial direction under the force, $F_z$. The applied pressure
will also cause the compact to expand, but it is restricted by the die-walls, resulting in a radial force, $F_r$. This radial force is commonly related directly to the axial force by a parameter $\alpha$:

$$F_r = \alpha \cdot F_z$$  \hspace{1cm} (1)

In solids, the parameter $\alpha$ would be a function of the Poisson's ratio. For granular materials, $\alpha$ is an empirical factor representing bulk properties of the powder. The radial force against the die-wall subsequently generates a frictional force along the die-wall opposing the compacting force, $F_z$. This frictional force is related to the radial force by a powder/die-wall coefficient of friction, $\mu$:

$$F_\mu = \mu \cdot \alpha \cdot F_z$$  \hspace{1cm} (2)

In this geometry, a cylindrical coordinate system is used with $z$ in the axial direction and $r$ in the radial direction.

**JANSSEN ANALYSIS**

A related problem to compaction was analyzed 100 years ago by H. A. Janssen who was mainly concerned with pressure variations in grain silos. Although pressure variations in grain silos are not presently of direct concern, Janssen's work remains a classic analysis still used in many powder compaction studies today. Janssen's approach consisted of a force balance on an axial slice of a granular cylinder. The forces considered in this analysis are the axial force acting on the top surface of the slice, a reactive force acting on the bottom surface of the slice and a frictional force acting at the die-wall along the circumference of the slice (Fig. 1b). By solving the resulting differential equation, Janssen obtained the solution:

$$P_z = P_o \exp \left( -\frac{4\mu \alpha}{D} z \right)$$  \hspace{1cm} (3)

which states that the axial pressure at any depth in the compact, $P_z$, is equal to the applied compacting pressure, $P_o$, multiplied by a term which decreases exponentially with depth into the compact.

The exponential term in eq. (3) includes both material and geometrical parameters. Examination of this relationship reveals two important trends. Firstly, if there is no friction at the die-wall ($\mu = 0$), then, according to Janssen's equation, the exponential term equals unity and the axial pressure is equal to the applied pressure throughout the compact. As the coefficient of friction increases, however, the axial pressure decreases exponentially with depth into the compact. It is important to
note that this dissipation is entirely due to the opposing frictional force at the die-wall. Based on this result, it has long been advocated in the ceramics industry that die-wall friction be minimized by lubricants or high polishing of the die.

Secondly, if the pressure decreases exponentially through the compact, then the aspect ratio of the compact must also have a significant influence. To show this effect, we can use eq. (3) to calculate the axial pressure at the bottom of a compact for different aspect ratios (H/D, where z = H), assuming a constant die-wall coefficient of friction of 0.25 and applied compaction pressure of 68.9 MPa (10,000 psi). For example, for a compact of aspect ratio 0.1, equation (3) gives an axial pressure at the bottom of the compact, $P_H$, of 65.6 MPa (9,510 psi). In terms of a pressure transmission ratio (axial pressure/applied pressure), 95% of the applied pressure is transmitted to the bottom of the compact in this case. For a compact aspect ratio of 1, $P_H = 41.8$ MPa (6,065 psi); 61% of the applied pressure is transmitted to the bottom of the compact. For an aspect ratio of 4, $P_H = 9.3$ MPa (1,350 psi); only 13.5% of the applied pressure is transmitted to the bottom of the compact! Obviously, if 87% of the compacting pressure has been dissipated at the bottom of the highest aspect ratio compact, then this bottom region will compact to a much lower green density leading to considerable shrinkage differences in the component. This is a major reason why only low aspect ratio components are die-pressed in the ceramics industry.

Explicit in Janssen’s analysis is the fact that the axial pressure at any depth into the compact is constant along that plane. That is, there is no variation in pressure with respect to radius from the center of the plane out to the die-wall.

**UNCKEL ANALYSIS**

Fifty years later, Unckel conducted numerous experiments to investigate the formation of density gradients in metal powder compacts. In one set of experiments, copper powder was compacted in a metal die to two different final aspect ratios using the same applied pressure. After compaction, longitudinal slices were machined out of the compacts, smaller 1 cm$^3$ cubes were then machined from these slices and their densities measured. Figure 2 shows the measured densities of each of the cubes as a function of position in the compact. Measured densities are shown from the central axis of the compact out to the die-wall from the top to the bottom of the compact.

Considering first the results of the higher aspect ratio compact (H/D = 1), two distinct trends are observed. At each of the three radial positions, the density decreases in the axial (z) direction from the top to the bottom of the compact. This trend qualitatively follows that predicted by Janssen’s model. In the radial direction, the density increases from the central axis to the die-wall at the top of the compact; whereas at the bottom of the compact, the density decreases with radius from the central axis out to the die-wall.
In the lower aspect ratio compact (H/D = 0.3), similar trends in density are observed with respect to radius: at the top of compact, density increases from the central axis to the die-wall; at the bottom of the compact, density decreases from the central axis to the die-wall. In the axial direction, the density decreases from the top to the bottom of the compact in the near die-wall region as predicted by the Janssen model, but in the bulk of the compact, the density increases, then decreases in the axial direction. Thus, the maximum density occurs in the bulk of the compact.

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<thead>
<tr>
<th>H/D = 1</th>
<th>H/D = 0.3</th>
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<tbody>
<tr>
<td>Central Axis</td>
<td>Central Axis</td>
</tr>
<tr>
<td>Top of compact</td>
<td>Die-wall</td>
</tr>
<tr>
<td>7.36</td>
<td>7.50</td>
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<tr>
<td>7.35</td>
<td>7.54</td>
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<td>7.31</td>
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Figure 2. Measured densities (g/cm³) of copper cubes machined from powder compacts pressed under 589 MPa (85,000 psi) to two different aspect ratios [from Ref. 7].

Similar trends to those found by Unckel were also found by Kuczynski and Zaplatynskyj in compaction tests conducted with nickel powder and by Train using MgCO₃ powder. It should be noted that these similarities were found for both metal powder compacts in which work hardening and plasticity effects are important, as well as for ceramic compacts such as magnesium carbonate, in which such effects are considered negligible.

THOMPSON ANALYSIS

A mathematical model to account for this observed radial distribution in density was developed 36 years later by Thompson. Thompson began his analysis with the equilibrium equations for a cylindrical compact which include axial stress and shear.
stress terms. Rearranging one of the equations, Thompson defined a relationship for the axial stress with respect to the axial direction:

\[
\frac{\partial \sigma_z}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r}(r \tau)
\]  

(4)

where \( \sigma_z \) is the axial stress and \( \tau \) is the shear stress. Based on Unckel's experimental data, he then empirically fit an explicit radial dependence to the axial stress,

\[
\sigma_z(r, z) = r^2 f(z) + C(z)
\]

(5)

where \( C(z) \) represents the stress along the central axis. Assuming that the shear stress is also some function of both \( r \) and \( z \), Thompson solves for the equilibrium equation (eq.(4)) by setting boundary conditions for the shear stress: at the die-wall (\( r = R \)), the shear stress is equal to the familiar relationship of the radial force times the coefficient of friction (see eq. (2)). Because of radial symmetry, Thompson states that the shear stress is zero along the central axis. He then concludes by inspection of eq. (4) that the axial stress along the central axis must be constant, independent of \( z^* \). Thus, \( C(z) \) is equal to \( C_0 \). The resulting equation,

\[
\sigma_z(r, z) = B \left( \frac{r^2}{R^2} \right) \exp \left( -\frac{4 \mu \alpha}{R} z \right) + C_0 \left( 1 - \frac{r^2}{R^2} \right)
\]

(6)

contains two terms. The first term includes the same exponential term found in Janssens's analysis (eq. (3)). Additionally, there is a radial term which includes the radial distance from the central axis, \( r \), and the radius of the compact, \( R \). This radial term also influences the exponential term.

Thompson obtains a density distribution within a powder compact by coupling the mathematical stress distribution with a general empirical stress(pressure)-density relationship.\(^{11}\) The resulting density distribution, shown in Fig. 3, predicts a decreasing density in the axial direction in the near die-wall region similar to Janssen's model, as well as increasing density with respect to radius at the top of the compact and decreasing density with respect to radius at the bottom of the compact, similar to trends found experimentally (which must be true since experiments formed the basis of the radial stress distribution, i.e. eq. (5)). However, the model predicts a constant density along the central axis which does not match the experimental results.

\*While it is true that the shear stress is zero at \( r=0 \), it does not follow that the derivative of \( \sigma_z \) with respect to \( z \) must also be zero along the central axis. Thompson's approach is valid only for a limited case; a more general mechanics analysis reveals a non-zero derivative of \( \sigma_z \) at \( r=0 \), i.e. \( \partial(r \tau)/\partial r \neq 0 \), indicating that \( C(z) \) is indeed a function of \( z \) and not constant.
SOIL MECHANICS APPROACH

A different approach to Thompson's analysis of compaction comes from soil mechanics research where a powder compact is treated as an assemblage of granular material. This approach is often used when material yields or fails in shear as particles slide past each other. A common description of this shear behavior is the Coulomb yield criterion:

$$\tau_f = C + \sigma_f \tan \phi$$ \hspace{1cm} (7)

where the shear strength of granular material, $\tau_f$, is equal to a term $C$, representing the cohesive strength of the particles plus a term combining the axial stress at yield, $\sigma_f$, and an interparticle friction term, $\tan \phi$, where $\phi$ also represents the angle of the plane along which shear occurs. Schwartz and Weinstein used this approach along with a finite element analysis to solve for the same equilibrium equation used by Thompson. However, similar to Thompson, the radial distribution of the axial stress was obtained from an empirical fit to Unckel's data, resulting in a qualitatively similar stress distribution to Thompson's solution with a constant stress along the central axis.
A common denominator in the models we have reviewed is that the stress variation in a compact is generated solely from a die-wall frictional force. Regardless of the approach used to analyze the stress or density distribution, no present model can account for the complex nature of density variations found experimentally in the bulk of powder compacts. The assumption that die-wall friction is the only contributor to the variations in stress and density within a compact appears insufficient. This observation leads to the question: what would happen during powder compaction if die-wall friction were eliminated?

To address this question, preliminary tests were conducted at Sandia National Laboratories in which Al₂O₃ powder was compacted isostatically in a thin latex bag at 68.9 MPa (10,000 psi). After compaction, a section from the middle of the compact was analyzed under a scanning electron microscope using image analysis software to measure the areal density on a plane of the compact. Image analysis was conducted at four points in the center of the compact and along four radial lines out to the edge of the sample. The measured densities as a function of radial position showed a statistically significant difference in percentage theoretical density from the center of the compact (51.7 ± 1.5 %) to the edge of the compact (60.2 ± 1.2 %). These results show that density gradients were generated even with no die-wall frictional force. Certainly, isopressing introduces a different stress-strain path in powder compaction than uniaxial pressing. However, it has been common belief in the ceramics industry that isopressing results in uniformly dense green compacts. Analogous to bridging hoop stresses which form during isopressing to shield the central region of the compact, leading to lower green densities, the formation of bridging columns or networks has been reported for uniaxial pressing operations, also leading to density variations.¹³

CONCLUSIONS

Numerous models exist in the literature for predicting the compaction behavior of ceramic powders. The models illustrated in this review represent the basic approaches used in many of the models. Although many models have been incorporated into finite element, computer-aided design programs to predict the density variations in powder compacts, it is obvious that the underlying mechanics models describing compaction are inadequate to accurately predict the stress distribution in a compact. Regardless then of the form of the stress-density relationship, it will not be possible to accurately predict the density distribution in a powder compact with present models. Isostatic pressing experiments conducted at Sandia National Laboratories support the idea that additional dissipative forces or shielding mechanisms to the die-wall frictional force must be taken into account in any model which aims to accurately predict density gradients in ceramic powder compacts.
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