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**Simulation-Based Diagnostics and Control
for
Nuclear Power Plants**

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Simulation-Based Diagnostics and Control for Nuclear Power Plants

Abstract

The objective of the project was to develop and test a simulation-based diagnostics and control guidance system that can be used to diagnose and manage off-normal transient events in nuclear power plants. The research has focused on developing two diagnostic approaches suitable for detection and identification of faults involving multiple components, subject to uncertainties in system modeling and observations. The first approach is based on a fuzzy logic framework that can diagnose binary failures using a single-failure diagnostic knowledge base. Construction of the binary-failure knowledge base is accomplished through the use of macroscopic conservation relationships and a fuzzy inference structure is developed to determine the magnitude of faults and the associated certainty. In the second diagnostic approach, an adaptive Kalman filter algorithm is derived to yield information on the type and magnitude of feasible component transitions that can account for system observations. To obtain the likelihood of feasible component failures or degradations, a general probabilistic formulation is developed where statistical distributions associated with component reliability data are explicitly represented. Testing of the diagnostic algorithms has been performed through the analysis of simulated transient events for light water reactor systems. Preliminary studies have been conducted to develop Monte Carlo algorithms for flexible control of transient events.

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1. Introduction

Under U. S. Department of Energy Grant DE-FG02-92ER75712, effort was begun at the University of Michigan in 1992 to develop a reliable, efficient, and robust methodology which can be implemented on an online computer for diagnosis and control of complex sequences of transient events. Our program has focused on the use of power plant simulation models to detect and identify multiple failures or degradations subject to potential masking of conflicting symptoms. This feature of our diagnostic package is expected to be valuable for handling degradation of multiple components in advanced reactor designs, with enhanced passive safety characteristics, as well as for current generation of light water reactors (LWRs).

The first year (April 1992 – April 1993) of our project focused on the development of a fuzzy logic framework that can use a single-failure diagnostic knowledge base for diagnosis of binary failures. The algorithm combines single-failure diagnostic rules, obtained through an entropy minimax clustering algorithm,¹ into binary-failure rules through the use of macroscopic conservation relationships. The fuzzy inference structure² can account for uncertainties or fuzziness inherent in failures and measured symptoms for both single and binary failures. During the second year (April 1993 – April 1994) of our research program, we concentrated our effort on developing statistical methods³ that can diagnose multiple failures and degradations in arbitrary combinations of components and severity. The statistical approach makes use of available component reliability information to determine the most likely failures, conditioned on the current and past state of the system. We also performed preliminary studies on the feasibility of using stochastic optimization algorithms to obtain optimal solutions to general transient management problems. Our efforts during the first two years of our project are summarized in our progress reports.^{4,5}

Our initial effort in developing the statistical diagnostics method involved a Monte Carlo algorithm⁶ in the form of solution to a stochastic differential equation, where a dynamic event tree approach is used to yield the likelihood of multiple component degradations. In addition, we developed an adaptive Kalman filter structure³ that can fully account for statistical uncertainties in measured symptoms to generate the diagnostic information. During the third and final year, we extended our effort to develop a formal stochastic framework^{7,8} for transient diagnosis of dynamical systems, represented by uncertain measurements and subject to random component failures. This general framework allows us to integrate the failure diagnostic information from the adaptive Kalman filter with the component reliability data. Through this integration process, we generate the final

diagnostic information in the form of the probability of component degradations that could account for the observed plant data or system measurements .

Due to the unavailability of funding for the last year of our grant, only limited effort has been made in further development of the fuzzy logic diagnostic framework and flexible transient control and management tools that can be used in conjunction with our diagnostic tools. We present a summary of our research objectives in Section 2, followed by a detailed description in Sections 3 through 5 of work carried out during the past three years of our diagnostics and control project. Some concluding remarks are presented in Section 6 and a list of publications resulting from the project is included.

2. Objectives of the Research

The overall objective of our project was to develop a diagnostics and control guidance system for nuclear power plants. An emphasis has been made throughout our research to make full use of power plant simulation models that could accurately represent dynamics of complex systems and components characteristic of nuclear power plants. A fuzzy logic algorithm and a stochastic method have been developed in parallel to account for uncertainties in component state characterization and measured symptoms.

Reflecting the evolution and changing direction of our research, we reorganize our tasks somewhat from our original scope and highlight those areas where significant progress has been made. Thus, the description of research effort in the remainder of the report is presented according to the following tasks:

Task 1. Fuzzy Logic Framework for Transient Diagnosis

1.1. Diagnostic Rule Generation through Fuzzy Pattern Recognition

Develop fuzzy pattern recognition algorithms to generate knowledge bases for diagnosis of representative nuclear power plant transients, with proper representation of fuzziness and uncertainties in the transient data.

1.2. Fuzzy Logic Structure for Multiple Failure Diagnostics

Develop fuzzy logic algorithms for diagnosis of multiple failures, by combining the fuzzy single-failure diagnostic rules of Task 1.1 with the help of macroscopic conservation relationships.

1.3. Validation of the Fuzzy Diagnostics Algorithm

Test and validate the overall fuzzy diagnostics algorithm by simulating binary-failure events in pressurized water reactor (PWR) systems.

Task 2. Statistical Framework for Transient Diagnosis

2.1. Stochastic Description of Dynamical Systems and System Diagnostics

Develop a general stochastic framework for representing dynamical systems subject to component failures or transitions. The resulting stochastic differential equation is solved with due accounting given for the statistical nature of component reliability data characterizing the component failures.

2.2. Adaptive System Modeling for Fault Diagnosis

Develop an adaptive Kalman filter algorithm that can identify and diagnose component failures, subject to uncertainties and fluctuations inherent in system observations. Combine the diagnostic information from the filter with solution to the stochastic differential equation of Task 2.1 to arrive at the probability of likely failures.

2.3. Validation of the Statistical Diagnostics Algorithm

Test and validate the overall statistical diagnosis algorithm through simulation of multiple component failures in the balance of plant (BOP) of a boiling water reactor (BWR) plant.

Task 3. Transient Management and Control Algorithm

Develop a Monte Carlo control algorithm as a general-purpose learning tool that can provide realistic estimates of feasible control maneuvers. Test the algorithm with a simple dynamical model with a known time-optimal solution.

3. Fuzzy Logic Framework for Transient Diagnosis (Task 1)

Due to the potential for masking of conflicting symptoms in highly nonlinear dynamics of nuclear power plants, the diagnosis of multiple failures and degradations presents one of the major challenges for development of realistic online diagnostic systems. Under Task 1, an emphasis was placed on the development of a fuzzy logic framework^{2,9} to combine single-failure diagnostic rules into a binary-failure diagnostic knowledge base (KB), through the use of macroscopic conservation equations. This requires a formulation of the single-failure KB in the form of fuzzy clusters or patterns, as described in Section 3.1. The actual algorithm for combining single-failure clusters into binary-failure clusters is discussed in Section 3.2. Application of the binary-failure cluster approach for diagnosis of component failures in PWR plants is described in Section 3.3

3.1. Diagnostic Rule Generation through Fuzzy Pattern Recognition

The performance of diagnostic expert systems including the Rx code¹⁰ is highly dependent on the ability of the diagnostic rules to correctly hypothesize failures or malfunctions. By minimizing the information entropy for clusters containing plant data or simulated plant data, we may group transients of similar characteristics in a systematic way

and derive single-failure diagnostic rules, connecting symptoms in cluster C_i , ($i = 1, \dots, J$) to failures A_j , ($j = 1, \dots, J$). The clusters will be characterized by the conditional probability $P(A_j|C_i)$ that failures A_j are located in cluster C_i . We represent the clusters as heuristic rules in the form of {if (condition) then (consequence) with certainty $\langle p \rangle$ }. Here the condition part is specified by the location of cluster C_i in the feature space while the consequence part is given in terms of event A_j , with the conditional probability $P(A_j|C_i)$ providing a measure $\langle p \rangle$ of the certainty of the diagnostic projection.

The diagnostic rules generated through the entropy minimax algorithm¹ are, however, limited to single-failure events, and uncertainties or fuzziness in the transient results are not explicitly represented in the pattern recognition process. To let a fuzzy membership function¹¹ represent the degree of certainty of a data point belonging to cluster C_i , we consider a distance measure¹² from the center of the cluster, where the cluster center is defined as an average of the distances of the entire data points belonging to C_i . Introducing the membership function allows us to account for the relative location and hence the importance of each data point to the cluster it belongs.

In addition to the introduction of fuzzy logic into diagnostic rules, we also need to reverse the inference direction of the knowledge base into cause-effect relationships in the causal or forward direction, i.e., {if (failure) then (symptoms)}. This recasting of diagnostic rules allows us to combine single-failure rules into multiple-failure rules, as discussed in Section 3.2. We use the Bayes theorem¹³ to reverse the inference direction and generate the forward conditional probability $P(C_i|A_j)$ of obtaining data points in cluster C_i given event A_j .

With fuzziness in symptoms and failures represented through a membership function, we use the Lukasiewicz logic^{14,15} to represent diagnostic rules in a two-way fuzzy logic structure. In the forward or causal direction, we represent the estimate of failures by fuzzy failure vector $\mathbf{a} = \{a_i\}$ and the location of a data point by fuzzy cluster vector $\mathbf{c} = \{c_j\}$, and consider a fuzzy matrix $\mathbf{T} = \{t_{ji}\}$, where t_{ji} is the truth value of the implication $\{A_i \Rightarrow C_j\}$ or the statement {if failure A_i occurs, symptoms are observed in cluster C_j }. We may represent this *modus tollens* implication in terms of the Lukasiewicz logic¹⁴

$$t_{ji} = \mu_T(A_i \Rightarrow C_j) = \min(1, 1 - a_i + c_j), \quad (1)$$

and obtain a solution for failure vector $\mathbf{a}$:

$$a_i = \min(1, c_j - t_{ji} + 1), \text{ or } a_i \leq c_j - t_{ji} + 1. \quad (2)$$

In the reverse or usage direction, we also apply the Lukasiewicz logic in terms of a fuzzy matrix $\mathbf{S} = \{s_{ij}\}$, where s_{ij} represents the truth value of the *modus ponens* implication {if symptoms are observed in cluster C_j , then failure A_i has occurred}, and obtain:

$$s_{ij} = \mu_S(C_j \Rightarrow A_i) = \min(1, 1 - c_j + a_i), \quad (3)$$

which yields another failure estimate:

$$a_i = \max(0, c_i - s_{ij} - 1), \text{ or } a_i \geq c_j + s_{ij} - 1. \quad (4)$$

Combining the two failure estimates and choosing m as the subscript j that maximizes $c_j + s_{ij}$ and n as the subscript j that minimizes $c_j - t_{ji}$ for a given i , we obtain the following interval estimate for the failure membership function:

$$a_i = \begin{cases} [\max(0, c_m + s_{im} - 1), \min(1, c_m - t_{mi} + 1)], & \text{if } s_{im} > 0, \\ [\max(0, c_m + s_{im} - 1), \min(1, c_n - t_{ni} + 1)], & \text{if } s_{im} = 0, \end{cases} \quad (5)$$

where the solution is expressed as a range of certainty.

An explicit accounting is made in Eq. (5) for the contingency of $s_{im} = 0$ so that the upper bound is selected based on the observation of a cluster, not on the non-observance of a cluster corresponding to a failure event which may end up in any of two or more clusters. This modification to the inference algorithm improves the failure severity estimates compared with the algorithm reported earlier.⁴ We approximate the fuzzy estimates for the forward and reverse implications, t_{ji} and s_{ij} , by the conditional probabilities $P(C_j|A_i)$ and $P(A_i|C_j)$, respectively, although a fuzzy membership function or relationship, e.g., t_{ji} and s_{ij} , represents a *possibility*, not probability.¹⁴

The last step in our fuzzy inference structure involves representing failures of varying severity as separate elements of the failure vector $\mathbf{a}$ and combining fuzzy estimates of failure severity to obtain an average estimate for the failure severity together with an estimate for the certainty of the failure itself. This task is accomplished through a method often used in fuzzy control,¹⁶ where the membership function curve g_i for each failure severity A_i is reduced by the fuzzy failure estimate a_i and a center-of-mass average of the curve $a_i g_i$ yields the average failure severity. An estimate for the certainty of the failure event is obtained from the area under the curve $a_i g_i$.

3.2. Fuzzy Logic Structure for Multiple Failure Diagnostics

Combining the systematic fuzzy rule generation capability of Section 3.1 and the macroscopic conservation relationships representing the dynamics of power plant components or subsystems, we have generated a diagnostic KB applicable to binary-failure events in the primary loop and pressurizer of a PWR. In this approach, we use the macroscopic mass and energy balance equations to obtain estimates for the rate of change of mass and energy inventories, dM/dt and dH/dt , respectively, from monitored symptoms, e.g., pressure, temperature and water level. We then characterize transient events in terms of these macroscopic inventory derivatives rather than in terms of directly observable plant parameters. We may also synthesize binary-failure diagnostic rules from the fuzzy single-

failure diagnostic rules of Section 3.1 by combining single-failure clusters represented in terms of dM/dt and dH/dt .

This synthesis of binary-failure clusters is generally meaningful, because changes in macroscopic inventories due to two different events could be linearly superposed with due accounting for differences in inlet and outlet parameters, e.g., the enthalpy of the surge flow to or from the pressurizer. This type of linear superposition is not as justifiable if we were to use directly measured variables, e.g., pressure and temperature, to characterize data points in the feature space. Furthermore, through the use of macroscopic inventory derivatives, dM/dt and dH/dt , or alternatively the macroscopic inventory time constants, $d\log M/dt$ and $d\log H/dt$, we may represent the diagnostic rules with minimum sensitivity to errors in our initial estimates of the energy and mass inventories, $M(0)$ and $H(0)$, respectively. The actual synthesis is performed through vectorial addition of cluster centers, rather than individual data points of clusters, and a fuzzy membership can be readily determined for a measured data point mapped into the binary-failure feature space.

To synthesize the fuzzy matrix T of Eq. (2) for binary-failure diagnostics, we begin with single-failure probabilities $P(C_k|A_i)$ and calculate the forward conditional probability that a combination of events A_i and A_j corresponds to symptoms in a binary-failure cluster C_{mn} synthesized from single-failure clusters C_m and C_n :

$$P(C_{mn}|A_i A_j) = P(C_m|A_i)P(C_n|A_j) + P(C_m|A_j)P(C_n|A_i) . \quad (6)$$

Equation (6) properly represents the two possible ways that each binary-failure cluster C_{mn} may in general result from binary combinations of events A_i and A_j . With Eq. (6) providing estimates for the forward fuzzy matrix T , we may again apply the Bayes theorem to determine the reverse conditional probabilities $P(A_i A_j|C_{mn})$ as estimates for the fuzzy matrix S in Eq. (4). The fuzzy inference structure of Eqs. (2) and (4) is then used to determine the fuzzy estimate $\mu(A_i A_j)$ for the likelihood of simultaneous failures A_i and A_j . The Lukasiewicz logic is used once more to decompose this binary-failure estimate into individual single-failure estimates $a_i = \mu(A_i)$ and $a_j = \mu(A_j)$. The remainder of the fuzzy inference structure can then be followed, as in the single-failure case, to determine the overall estimates of the severity and certainty of each failure.

3.3. Validation of the Fuzzy Diagnostics Algorithm

Testing and validation of our fuzzy diagnostics algorithm for the detection of single- and binary-failure events has been performed through analysis of PWR loop and pressurizer dynamics. Seven different classes of PWR failures were modeled with the MNSS code¹⁷ to generate 247 simulated data points for our diagnostics KB. For the pressurizer, we considered three events: leaks through power-operated relief valves

(PORVs) and safety valves, and inadvertent actuation of spray. For the primary loop, we represented small-break loss-of-coolant accidents (SBLOCAs), a leak in the makeup line and an accidental closing of the letdown line. The final class of transients included in the knowledge base is the no-failure case, the desired operational state. The 7 classes of transients were further divided by severity levels so to generate 13 failure events: (1) small PORV leak, (2) large PORV leak, (3) small safety valve leak, (4) medium safety valve leak, (5) large safety valve leak, (6) small spray actuation, (7) large spray actuation, (8) small SBLOCA, (9) medium SBLOCA, (10) large SBLOCA, (11) letdown line closure, (12) makeup line leak, and (13) no-failure transient.

The *Rg* code¹ was used to generate 10 single-failure clusters for the 13 failure events, in a four-dimensional feature space consisting of mass and energy inventory time constants for the pressurizer and primary loop. The four-dimensional rules are illustrated in Figures 1 and 2 as a set of two two-dimensional plots of cluster centers. The *Rg* code also provided $P(A_i|C_j)$ as an estimate for the single-failure matrix $S = \{s_{ij}\}$, which was then used to generate the single-failure matrix $T = \{t_{ji}\}$ through the Bayes theorem. Table I presents the S matrix values for the single-failure cluster centers along with the probability $P(C_i)$ of each cluster. Thirty-two binary-failure cluster centers, representing 52 different combinations of single-failure events, were obtained by linear superposition, together with the binary-failure matrix T calculated through Eq. (6). The Bayes theorem finally provided the binary-failure matrix S .

Figure 1. Single-failure cluster centers in the pressurizer feature space

Table I. Single-Failure Cluster Centers

Cluster	Probabilities	$P(C_i)$
C_A	$P(\text{Small Pressurizer PORV Leak} C_A) = 0.50$ $P(\text{Small Pressurizer Safety Valve Leak} C_A) = 0.50$	0.2429
C_B	$P(\text{Large Pressurizer PORV Leak} C_B) = 0.25$ $P(\text{Medium Pressurizer Safety Valve Leak} C_B) = 0.75$	0.0972
C_C	$P(\text{Large Pressurizer Safety Valve Leak} C_C) = 1.00$	0.0243
C_D	$P(\text{Small Pressurizer Spray Fails On} C_D) = 1.00$	0.1700
C_E	$P(\text{Large Pressurizer Spray Fails On} C_E) = 1.00$	0.0243
C_F	$P(\text{Let Down Line Closed} C_F) = 1.00$	0.0243
C_G	$P(\text{Medium Small Break Primary LOCA} C_G) = 1.00$	0.2186
C_H	$P(\text{Small Small Break Primary LOCA} C_H) = 0.857$ $P(\text{Make Up Line Leak} C_H) = 0.143$	0.1700
C_I	$P(\text{Large Small Break Primary LOCA} C_I) = 1.00$	0.0243
C_J	$P(\text{No Failure} C_J) = 1.00$	0.0040

Figure 2. Single-failure cluster centers in the primary loop feature space

Additional transient data, representing 15 single and 9 binary failures, were obtained with the MNSS code to test the diagnostics structure. The fuzzy inference mechanism is able to identify single failures with certainty $\langle p \rangle > 0.67$ and estimate the severity within

15%, while the binary failures are diagnosed with $\langle p \rangle > 0.63$ and severity within 20%. The inference mechanism is not able to distinguish between failure events that share the same cluster, e.g., a PORV leak and a small safety valve leak. In such cases, the inference algorithm generates hypothesis representing both of the possible failures.

Substantial effort was required during the past year to modify and debug the MNSS code to accurately represent SBLOCAs postulated for the primary loop. The MNSS model uses a lumped parameter representation of the core and steam generator, with a movable boundary between single- and two-phase regions of the steam generator. The primary loop model is coupled through a surge node to the pressurizer. In the process of implementing the leak flow model in the primary loop model, a number of coding errors or inconsistencies in the solution of fluid conservation equations were corrected. Since our fuzzy logic algorithm depends heavily on accurately estimating macroscopic mass and energy inventories of a system undergoing a postulated transient, it becomes critical that our simulation model solve the fluid conservation equations as accurately as possible.

4. Statistical Framework for Transient Diagnosis (Task 2)

The fuzzy logic diagnostics algorithm discussed in Section 3 yields the likelihood or *possibility* estimates for single and binary failures, through monitoring macroscopic mass and energy inventories for various power plant components and subsystems. The fuzzy logic structure accounts for fuzziness in monitored plant data, including, but not limited to, statistical uncertainties, and efficiently generates hypotheses that could explain the observed deviations from the expected or nominal performance of the system. The algorithm is in practice limited, however, to single and binary failures and cannot readily account the statistical reliability data for various components under consideration.

To remedy some of the limitations of the fuzzy logic diagnostics approach of Section 3, we have developed statistical algorithms that could utilize all available process information, including plant data or system measurements, system models, and individual component reliability data. Because such process data are in general subject to statistical fluctuations and uncertainties, we derive in Section 4.1 a general stochastic differential equation that describes the statistical evolution of uncertain system dynamics subject to random component failures or transitions. We include in Section 4.1 a Bayesian formulation that combines an approximate solution to the stochastic balance equation with the diagnostic information from the adaptive Kalman filter discussed in Section 4.2. The Kalman filter algorithm may be considered a practical implementation of the general stochastic balance equation following a component failure. We then discuss in Section 4.3 application of the

statistical diagnostics algorithm to the analysis of multiple component degradations in the BOP of a BWR plant.

4.1. Stochastic Description of Dynamical Systems and System Diagnostics

We consider a general dynamical system represented by state vector $\mathbf{x}(t)$ at time t corresponding to component state $\mathbf{c}(t)$:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}, \mathbf{c}) + \mathbf{h}(\mathbf{x}, \mathbf{c})\mathbf{w}(t) \quad (7)$$

subject to statistical fluctuations comprising noise design matrix $\mathbf{h}(\mathbf{x}, \mathbf{c})$ and white Gaussian noise vector $\mathbf{w}(t)$. The system state $\mathbf{x}(t)$ are determined indirectly through noisy measurements

$$\mathbf{y}(t) = \mathbf{q}(\mathbf{x}) + \mathbf{v}(t), \quad (8)$$

where the noise vector $\mathbf{v}(t)$ is again assumed white Gaussian. For the purpose of system diagnostics, we further assume that the components are susceptible to random faults which are considered Markovian in nature. The objective in system diagnosis is to determine the component state $\mathbf{c}(t)$, i.e., the configuration for individual components, that could best account for the measurements $\mathbf{y}(t)$ of Eq. (8) subject to the system model of Eq. (7). Because the available tools and information represented by Eqs. (7) and (8) are in general uncertain, and to account for the possibility of nonunique fault signatures and fault symptom masking, it is appropriate to seek the probability $p(\mathbf{c}, t | \mathbf{y})$ that the system is operating with component state $\mathbf{c}$ conditioned on uncertain process observations $\mathbf{y}(t)$ at time t .

We begin with the joint probability density function (pdf) $p(\mathbf{x}, \mathbf{c}, \mathbf{y}, t)$ that the system state is in unit phase volume around $(\mathbf{x}, \mathbf{c}, \mathbf{y})$ at time t . Using the Bayes rule and recognizing that the observation $\mathbf{y}$ does not depend explicitly on $\mathbf{c}$, we obtain⁸

$$p(\mathbf{c}, t | \mathbf{y}) = \int \frac{p(\mathbf{y}, t | \mathbf{x})p(\mathbf{x}, \mathbf{c}, t)}{p(\mathbf{y})} d\mathbf{x} = \int \frac{p(\mathbf{x}, \mathbf{c}, t)p(\mathbf{x}, t | \mathbf{y})}{p(\mathbf{x}, t)} d\mathbf{x}. \quad (9)$$

With the recognition that $p(\mathbf{x}, t) = \int p(\mathbf{x}, \mathbf{c}, t) d\mathbf{c}$, we require two pdf's, $p(\mathbf{x}, \mathbf{c}, t)$ and $p(\mathbf{x}, t | \mathbf{y})$, to evaluate the desired probability $p(\mathbf{c}, t | \mathbf{y})$ through Eq. (9).

A governing equation for the joint pdf $p(\mathbf{x}, \mathbf{c}, t)$ may be obtained^{18,19} through the Chapman-Kolmogoroff equation with the assumption that $\mathbf{c}(t)$ evolves in time by making sudden jumps and remaining constant between jumps:

$$\begin{aligned} \frac{\partial}{\partial t} p(\mathbf{x}, \mathbf{c}, t) = & - \sum_j \frac{\partial}{\partial x_j} [\eta_j(\mathbf{x}, \mathbf{c}) p(\mathbf{x}, \mathbf{c}, t)] + \frac{1}{2} \sum_{j,k} \frac{\partial^2}{\partial x_j \partial x_k} [\sigma_{jk}(\mathbf{x}, \mathbf{c}) p(\mathbf{x}, \mathbf{c}, t)] \\ & + \int_{\mathbf{c}'} d\mathbf{c}' \mathbf{W}(\mathbf{c} | \mathbf{c}'; \mathbf{x}) p(\mathbf{x}, \mathbf{c}', t) - \Gamma(\mathbf{x}, \mathbf{c}) p(\mathbf{x}, \mathbf{c}, t) \end{aligned} \quad (10)$$

where η and σ are the first two moments of the probability of system transition, from state $\mathbf{x}$ at time t to state $\mathbf{x}'$ at time $t + \Delta t$, for a given component state $\mathbf{c}$:

$$\eta_j(\mathbf{x}, \mathbf{c}) = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \int_{-\infty}^{\infty} d\mathbf{x}' (x'_j - x_j) p(\mathbf{x}', \mathbf{c}, t + \Delta t | \mathbf{x}, \mathbf{c}, t) = \lim_{\Delta t \rightarrow 0} \left\langle \frac{\Delta x_j(t)}{\Delta t} \right\rangle, \quad (11)$$

$$\sigma_{jk}(\mathbf{x}, \mathbf{c}) = \lim_{\Delta t \rightarrow 0} \left\langle \frac{\Delta x_j(t) \Delta x_k(t)}{\Delta t} \right\rangle. \quad (12)$$

In Eq. (10), $W(\mathbf{c}|\mathbf{c}'; \mathbf{x})$ is the probability per unit time of transition from component state $\mathbf{c}'$ to $\mathbf{c}$ for a given system state $\mathbf{x}$ and $\Gamma(\mathbf{c}; \mathbf{x})$ is the total probability per unit time of leaving component state $\mathbf{c}$:

$$\Gamma(\mathbf{x}, \mathbf{c}) = \int d\mathbf{c}' W(\mathbf{c}'|\mathbf{c}; \mathbf{x}). \quad (13)$$

To be able to use the stochastic differential equation (10) in the overall diagnostic expression of Eq. (9), we need to introduce an approximation that the system evolves deterministically for a constant component state. This assumption allows us to drop the diffusive term in Eq. (10):

$$\frac{\partial}{\partial t} p(\mathbf{x}, \mathbf{c}, t) = - \sum_j \frac{\partial}{\partial x_j} [\eta_j(\mathbf{x}, \mathbf{c}) p(\mathbf{x}, \mathbf{c}, t)] + \int_{\mathbf{c}'} d\mathbf{c}' W(\mathbf{c}|\mathbf{c}'; \mathbf{x}) p(\mathbf{x}, \mathbf{c}', t) - \Gamma(\mathbf{x}, \mathbf{c}) p(\mathbf{x}, \mathbf{c}, t). \quad (14)$$

The second pdf $p(\mathbf{x}, t|y)$ required for evaluation of Eq. (9) may be obtained in principle, provided we are given the pdf associated with the observation $y(t)$ in Eq. (8). The actual evaluation of this conditional pdf may be accomplished most conveniently through an adaptive Kalman filter algorithm discussed in Section 4.2. The filter algorithm may be considered a solution to the general stochastic balance equation (10), with constant component state $\mathbf{c}$.

For the system equation (7), we formally write the solution $\mathbf{x}(t) = \mathbf{g}(\mathbf{u}, \mathbf{c}, t)$, corresponding to the initial condition $\mathbf{x}(0) = \mathbf{u}$, and note that

$$\eta_j(\mathbf{x}, \mathbf{c}) = f_j(\mathbf{x}, \mathbf{c}) \quad (15)$$

in Eq. (14). The integro-differential equation (14) may be converted^{8,20} to an integral form for the joint pdf $p(\mathbf{x}, \mathbf{c}, t)$:

$$p(\mathbf{x}, \mathbf{c}, t) = \int d\mathbf{u} \delta[\mathbf{x} - \mathbf{g}(\mathbf{u}, \mathbf{c}, t)] \exp \left\{ - \int_0^t ds \Gamma[\mathbf{g}(\mathbf{u}, \mathbf{c}, s), \mathbf{c}] \right\} p(\mathbf{u}, \mathbf{c}, 0) \\ + \int_0^t dt' \int d\mathbf{u} \int d\mathbf{c}' \delta[\mathbf{x} - \mathbf{g}(\mathbf{u}, \mathbf{c}, t - t')] \exp \left\{ - \int_{t'}^t ds \Gamma[\mathbf{g}(\mathbf{u}, \mathbf{c}, s - t'), \mathbf{c}] \right\} W(\mathbf{c}|\mathbf{c}'; \mathbf{u}) p(\mathbf{u}, \mathbf{c}', t'). \quad (16)$$

Once the system state evolution $\mathbf{x}(t)$ is known, we may use a Monte Carlo algorithm representing Eq. (16), and solve for the pdf $p(\mathbf{x}, \mathbf{c}, t)$, corresponding to each feasible component state trajectory $\mathbf{c}(t)$. Testing of the Monte Carlo algorithm⁶ for a simple dynamical system indicates large computational requirements to account for multiple discrete component states and the difficulty to represent uncertainties in system states

directly measured or indirectly estimated through Eq. (8). To circumvent these difficulties associated with an essentially dynamic event tree approach,²⁰ we introduce further approximations that the component transitions or failures take place over a short diagnostic time interval t so that $t\Gamma(\mathbf{x}, \mathbf{c}) \ll 1$. This certainly is a reasonable approximation for highly reliable components required for nuclear power plants and allows us to simplify Eq. (16):

$$p(\mathbf{x}, \mathbf{c}, t) = \int d\mathbf{u} \delta[\mathbf{x} - \mathbf{g}(\mathbf{u}, \mathbf{c}, t)] \exp[-t\Gamma(\mathbf{u}, \mathbf{c})] p(\mathbf{u}, \mathbf{c}, 0) \\ + t \int d\mathbf{u} \int d\mathbf{c}' \delta[\mathbf{x} - \mathbf{g}(\mathbf{u}, \mathbf{c}, t)] W(\mathbf{c}|\mathbf{c}'; \mathbf{u}) p(\mathbf{u}, \mathbf{c}', 0). \quad (17)$$

This approximate solution for the joint pdf $p(\mathbf{x}, \mathbf{c}, t)$ may then be used in Eq. (9), together with the Kalman filter solution for the conditional pdf $p(\mathbf{x}, t|y)$ from Section 4.2, to yield the final diagnostic information $p(\mathbf{c}|y)$, i.e., the probability of component transition or degradation $\mathbf{c}(t)$ given system observation $y(t)$. In this statistical diagnostics algorithm, we are able to explicitly account for the system dependence of component reliability data for $W(\mathbf{c}|\mathbf{c}'; \mathbf{x})$ and $\Gamma(\mathbf{x}, \mathbf{c})$, e.g., dependence of the failure rate of a valve on temperature.

4.2. Adaptive System Modeling for Fault Diagnosis

During the monitoring of system dynamics through Eq. (7), if no component state transition has occurred, then one would expect to track the noisy system observations to some acceptable level of accuracy. Upon onset of some component fault(s), however, agreement between observed and computed system trajectory will begin to deviate. Such a deviation is the manifestation of the modeling deficit of unknown origin, i.e., the present system model is no longer sufficiently representative of actual system behavior. The fault diagnosis then consists of relating the modeling deficit to component state transitions.

As suggested in Section 4.1, we assume that the component state undergoes random jumps and stays constant between jumps, and augment the system model of Eq. (7) with a stochastic differential equation for the constant but uncertain component state between transitions:

$$\dot{\mathbf{c}}(t) = \mathbf{k}(\mathbf{x}, \mathbf{c})\mathbf{u}(t), \quad (18)$$

where $\mathbf{k}(\mathbf{x}, \mathbf{c})$ is a noise design matrix and $\mathbf{u}(t)$ is a white Gaussian noise source, uncorrelated with $\mathbf{w}(t)$ of Eq. (7). Linearizing the nonlinear system model $\mathbf{f}(\mathbf{x}, \mathbf{c})$, we may write the overall system equation:

$$\dot{\mathbf{z}} = \begin{pmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{c}} \end{pmatrix} = \begin{pmatrix} \mathbf{f}(\mathbf{x}, \mathbf{c}) \\ \mathbf{0} \end{pmatrix} + \begin{pmatrix} \mathbf{h}(\mathbf{x}, \mathbf{c})\mathbf{w}(t) \\ \mathbf{k}(\mathbf{x}, \mathbf{c})\mathbf{u}(t) \end{pmatrix} \approx \mathbf{A}(\mathbf{z})\mathbf{z} + \mathbf{S}(\mathbf{z})\mathbf{n}(t), \quad (19)$$

where

$$S(z) = \begin{pmatrix} h(z) & \emptyset \\ \emptyset & k(z) \end{pmatrix}, n(t) = \begin{pmatrix} w(t) \\ u(t) \end{pmatrix}. \quad (20)$$

With constant c , the stochastic differential equation (10) may be reduced to the Fokker-Planck equation¹⁸ for the probability $p(z,t)$ associated with the combined system state $z(t)$:

$$\frac{\partial}{\partial t} p(z,t) = - \sum_j \frac{\partial}{\partial z_j} [f_j(z) p(z,t)] + \frac{1}{2} \sum_{j,k} \frac{\partial^2}{\partial z_j \partial z_k} [\sigma_{jk}(z) p(z,t)] \quad (21)$$

where

$$\sigma_{jk}(z) = \lim_{\Delta t \rightarrow 0} \left\langle \frac{\Delta z_j(t) \Delta z_k(t)}{\Delta t} \right\rangle = [S(z) Q S^T(z)]_{jk} \text{ with } Q = \langle n n^T \rangle. \quad (22)$$

Although Eq. (21) may be directly solved together with Eqs. (19) and (8), the solution may, in practice, be obtained conveniently by using the discrete Kalman filter algorithm.²¹ Thus, in terms of the state transition matrix $\Phi(k|k-1)$ at time step k , we obtain for the mean value $\hat{z}$ of the state estimate and the associated covariance $P = \langle (z - \hat{z})(z - \hat{z})^T \rangle$:

$$\hat{z}(k|k-1) = \Phi(k|k-1) \hat{z}(k-1), \quad (23a)$$

$$P(k|k-1) = \Phi(k|k-1) P(k-1) \Phi^T(k|k-1) + S(k) Q(k) S^T(k). \quad (23b)$$

Once measurements $y(k)$ are taken, using the linearized form of the measurement equation (8):

$$y(k) = H(k) \hat{z}(k|k-1) + v(k), \quad (24)$$

we may obtain an updated estimate $\hat{z}(k)$ of the state and the covariance $P(k)$ in terms of the Kalman gain matrix in standard Kalman filter structure.²¹

In terms of the optimal system estimate $\hat{z}(k)$ and covariance $P(k)$, we may proceed to estimate²¹ the covariance T associated with the residual $\xi(k) = y(k) - H(k) \hat{z}(k)$:

$$T = H \Phi P \Phi^T H^T + H S Q S^T H + R, \quad (25)$$

where R is the covariance matrix of the measurement noise, i.e., $R = \langle v v^T \rangle$. Assuming the residual ξ is Gaussian, we may evaluate the consistency between the expected process dynamics and observed process dynamics by monitoring the chi-squared statistic¹⁰

$$d^* = \xi T^{-1} \xi^T. \quad (26)$$

If $d^* > \chi_{\Omega}^2$, where Ω is the desired test significance, then we declare that a modeling deficit exists, i.e., a component state transition has occurred, resulting in a statistically significant deviation between observed and predicted process variables.

Upon detection of a system anomaly of unknown origin, one would then like to model the uncertain system behavior through the combined system equation (19). Since one can show²¹ that, of all possible residuals associated with a given process model and observation, the most likely residual ξ_m is given by $\xi_m \xi_m^T = T$, we may then choose⁸ the noise source Q of Eq. (22) so that the uncertainty represented by Q will be on the order of the modeling deficit $\xi \xi^T - T$. Such a choice of Q will increase the covariance matrix P

associated with the combined system estimate $\mathbf{z}$ to reflect the observed modeling deficit. Each uncertain system model represented in this adaptive fashion will then correspond to one unique fault hypothesis regarding the component transition $\mathbf{c}' \rightarrow \mathbf{c}$. Through this adaptive Kalman filter process, we obtain the magnitude of the fault that could achieve consistency between observed system behavior and the predicted system dynamics adapted for the hypothesized fault. Our stochastic approach thereby avoids the need to explicitly model every possible component fault, in both type and magnitude, in a large number of deterministic simulations.

For each component fault or transition $\mathbf{c}$, estimated through the adaptive Kalman filter, we may use the normal distribution $N(\hat{\mathbf{z}}, \mathbf{P})$ to determine the conditional pdf $p(\mathbf{c}, t | \mathbf{y}) = N(\hat{\mathbf{c}}, \mathbf{V})$ required to complete our diagnostics through Eq. (9). Here $\mathbf{V}$ is the submatrix of the covariance matrix $\mathbf{P}$ corresponding to $\mathbf{c}$. The actual evaluation of Eq. (9), accounting fully for the statistical distribution of various parameters, including the transition rate $\mathbf{W}(\mathbf{c} | \mathbf{c}'; \mathbf{x})$, is handled through a stratified Monte Carlo algorithm.⁸

4.3. Validation of the Statistical Diagnosis Algorithm

To illustrate and demonstrate the usefulness of our statistical diagnosis algorithm, we consider diagnosis of a simulated transient in the BOP of a BWR plant. The system of interest consists of a high pressure (HP) turbine, fed through a main steam admission valve, and exhausting wet steam to a steam dryer. The saturated steam from the dryer then passes through a reheater, which is fed bleed steam via a tap in the main steam line, and into a low pressure (LP) turbine. Steam is also bled from the LP and HP turbines, combined with condensed steam from the reheater, and passed through a series of HP and LP feedwater heaters (FWHs). A simple time-lag model²² consisting of 11 nonlinear differential equations, for 11 system variables $\mathbf{x}$, 9 component characteristics ω and 5 system observations $\mathbf{y}$, is used to represent the BOP system. A combination of component characteristics ω_i , ($i = 1, \dots, 9$), listed in Table II, defines a particular component state $\mathbf{c}$.

The manifestation of a simulated transient resulting from a simultaneous decrease of 5% in LP turbine bleed ω_2 and a 10% increase in reheater steam valve flow area ω_4 on two system observations, LP turbine torque y_1 and HP FWH exit temperature y_5 , (with 1% white noise superimposed) is represented in Figures 3 and 4. We limit our diagnosis to not more than three simultaneous failures, resulting in a total of 130 adaptive filter runs or hypotheses to be tested, out of a total of 512 possible hypotheses. Through application of hypothesis testing techniques¹³ to the filter results, we arrive at 10 feasible, unique component states, two of which, $\mathbf{c}_3$ and $\mathbf{c}_6$, correctly describe the actual simulated fault sequence.

Table II. Component Characteristics for the BOP Fault Diagnosis

Component Characteristic	True Value (Faulted)	Estimated Value	
		Feasible Component State c_3	Feasible Component State c_6
ω_1 = HP steam bleed (%)	8.800×10^0	$8.79 \times 10^0 \pm 4.4 \times 10^{-2}$	$8.80 \times 10^0 \pm 4.4 \times 10^{-2}$
ω_2 = LP steam bleed (%)	2.215×10^1	$2.30 \times 10^1 \pm 9.9 \times 10^{-2}$	$2.20 \times 10^1 \pm 3.8 \times 10^{-1}$
ω_3 = main steam valve area (m^2)	5.247×10^{-2}	$5.25 \times 10^{-2} \pm 5.8 \times 10^{-5}$	$5.24 \times 10^{-2} \pm 6.2 \times 10^{-5}$
ω_4 = reheater steam valve area (m^2)	7.338×10^{-4}	$6.92 \times 10^{-4} \pm 1.1 \times 10^{-6}$	$7.43 \times 10^{-4} \pm 2.3 \times 10^{-5}$
ω_5 = reheater heat transfer coefficient (J/kg·K)	7.956×10^4	$7.97 \times 10^4 \pm 3.1 \times 10^2$	$7.96 \times 10^4 \pm 3.9 \times 10^2$
ω_6 = HP FWH heat transfer coefficient (J/kg)	7.590×10^5	$7.60 \times 10^5 \pm 3.7 \times 10^3$	$7.59 \times 10^5 \pm 3.7 \times 10^3$
ω_7 = LP FWH heat transfer coefficient (J/kg)	8.030×10^5	$8.03 \times 10^5 \pm 3.7 \times 10^3$	$8.03 \times 10^5 \pm 3.7 \times 10^3$
ω_8 = HP turbine efficiency (%)	8.600×10^1	$8.61 \times 10^1 \pm 3.7 \times 10^{-1}$	$8.61 \times 10^1 \pm 3.7 \times 10^{-1}$
ω_9 = LP turbine efficiency (%)	8.300×10^1	$8.35 \times 10^1 \pm 1.1 \times 10^{-1}$	$8.30 \times 10^1 \pm 2.1 \times 10^{-1}$

Figure 3. Time evolution of LP turbine torque y_1

Figure 4. Time evolution of HP feedwater heater exit temperature y_5

The values of component characteristics ω_i for the actual simulated fault are compared with those estimated under the component states c_3 and c_6 in Table II. Each estimate for the 9 component characteristics corresponds to the mean of a normal distribution with associated standard deviation obtained with 10 batches of 7.6×10^5 histories each. For the two correct component sequences, we estimate the likelihood $p(c_3|y) = 65.36\% \pm 0.15\%$ and $p(c_6|y) = 9.65\% \pm 0.03\%$, with the likelihood for other 8 sequences ranging from $<10^{-5}\%$ to 18.44%. We note that the sequence c_6 , with a smaller likelihood $p(c|y)$, yields a better quantitative agreement in component characteristics ω , with the correct value contained within one standard deviation of each estimate. Although the agreement in ω is poorer for the sequence c_3 , this component state represents smaller deviations from nominal state, yielding $p(c_3|y) > p(c_6|y)$.

It is noteworthy that our statistical diagnosis algorithm, comprising multiple adaptive filters and hypothesis testing procedure, yields correct and meaningful diagnosis of relatively small component faults, which are almost unnoticeable in simulated raw measurement data plotted in Figures 3 and 4. On the other hand, the optimal estimate from a standard Kalman filter run, with nominal component and system states, merely distributes the cause of the observed model deficit over the entire set of components and cannot

identify either of the component faults. The use of point estimates, i.e., mean values, for all uncertain variables also yields significantly different estimates for the likelihood $p(c|y)$ than obtained through the Monte Carlo sampling of actual distributions $p(x|y)$ and $p(x,c,t)$. The likelihood for several key sequences including c_6 are grossly underestimated, indicating the need to account for uncertainties or statistical distributions in reliability data and system estimates.

5. Transient Management and Control Algorithm (Task 3)

With the emphasis in our project placed on the development of efficient diagnostics and surveillance tools for detection and identification of component failures and degradations covered under Tasks 1 and 2, our effort has been rather limited in regard to the development of control and transient management algorithms. Our overall objective in Task 3 was to use power plant simulation models as an integral part of accurate and efficient algorithms for managing routine and off-normal transient events encountered in nuclear power plants. We envisioned using both detailed and lumped-parameter simulation models in this effort, with the lumped-parameter model either separately developed or estimated online through an implicit Kalman filter algorithm.¹⁰ We also considered a combined, synergistic use of deterministic and stochastic control algorithms, that could make full use of power plant simulation models of varying complexity. With limited effort in this task, we studied the feasibility of using a Monte Carlo optimization algorithm to obtain solutions to well-known optimal control problems described by simple ordinary differential equations.

The Metropolis algorithm²³ is a biased Monte Carlo scheme originally developed for statistical physics analysis but has been recently utilized as a general optimization algorithm in the context of a simulated annealing (SA) process.²⁴ Through utilization of a Boltzmann weighting factor to retain some control strategies that may not look highly promising initially, the SA approach avoids the possibility of getting trapped in a local extremum point. For every feasible state x corresponding to control u randomly selected, we evaluate the change ΔJ in the objective function J and accept u with

(i) Probability = 1, if $\Delta J < 0$,

(ii) Probability = $\exp(-\Delta J / T)$, if $\Delta J > 0$,

where T is a parameter that plays the role of temperature in statistical mechanics. Thus, any solution obtained through the algorithm is always feasible, i.e., satisfies the state constraints, and can be incrementally improved as computing time allows.

To test the feasibility in applying the SA algorithm to a control problem, we have studied a simple problem of controlling the motion of an object described by Newton's equation of motion:

$$\dot{x}_1 = x_2, \quad (27a)$$

$$\dot{x}_2 = u, \quad (27b)$$

where the variables $x_1(t)$ and $x_2(t)$ represent the displacement and speed, respectively, of the object subject to acceleration $u(t)$. With the objective function J chosen as the transit time t_f of the object and the acceleration constrained to $|u(t)| \leq 1$, the optimal solution is the well-known bang-bang control.²⁵ For the case of the initial conditions, $x_1(0) = 1$ and $x_2(0) = 0$, and the final conditions, $x_1(t_f) = 0$ and $x_2(t_f) = 0$, we illustrate the minimal-time solution in Figure 5. To account efficiently for the final conditions, we apply a penalty factor μ , for the violation of $x_2(t_f) = 0$ when the final destination $x_1(t_f) = 0$ is reached, and consider an augmented objection function :

$$J = t_f + \mu[x_2(t_f)]^2. \quad (28)$$

With 20 timesteps for the solution of system equations (27), 200 random trials were made for each temperature T and the best control, corresponding to a minimum J , is saved for the next trial as T is gradually lowered. We illustrate the SA approach to the optimal solution through plots in the phase planes, (x_1, u) and (x_1, x_2) , for two temperature points in Figure 5. At the 40th temperature point $T(40)$, the control u is far from the bang-bang solution and the (x_1, x_2) trajectory has yet to resemble the correct solution. At $T(300)$, the (x_1, x_2) trajectory has nearly attained the correct solution, although the control u still has yet to approach the step function in the vicinity of the switching point $x_1 = 0.5$. While it is perhaps easy to recognize the inherent difficulty in representing the step change in u through a discrete random variable, it is disappointing that a large number of temperature points, entailing a large number of solutions of system equations (27), is required for a reasonable solution of this simple time-optimal problem.

A few parametric studies have been performed to explore possible ways to accelerate the convergence to the optimal solution. One approach involves averaging the 20-point control distribution to 4 coarse-mesh intervals and using this coarse-mesh distribution as the starting solution for each T . In another acceleration approach, a coarse-mesh analytical solution of Eqs. (27) is used to screen out trial solutions before a fine-mesh solution is attempted. Both of these approaches help reduce the number of times the system equations have to be solved directly.

Preliminary applications of the SA algorithm have also been made to a time-optimal xenon shutdown problem,²⁶ where a bang-bang pulse of power variation is required to minimize the shutdown xenon reactivity poisoning. The problem is described by a set of

Figure 5. Stochastic optimal control solution

two nonlinear ordinary differential equations, and a direct application of the SA algorithm requires solving the system equations a large number of times, similar to Pontryagin's optimal control problem of Eqs. (27). A coarse-timestep solution to the system equations, used as a screening tool, again tends to reduce the frequency of solving the equations with a fine timestep.

6. Summary and Conclusions

The bulk of our effort during the project has been spent on developing efficient and robust algorithms for diagnosis of faults or degradations involving multiple components in nuclear power plant systems. Our diagnostics research has taken two parallel approaches using fuzzy logic and stochastic algorithms for transient diagnosis subject to uncertainties or fuzziness in system modeling and observations. Under Task 1, using the concept of fuzzy cluster center and with the help of macroscopic conservation equations, we have derived an algorithm for combining single-failure fuzzy diagnostic rules into binary-failure rules and obtained a fuzzy inference structure for determining the magnitude of failures and the associated certainty. We have developed, under Task 2, a general probabilistic framework for multiple-component diagnosis where statistical distributions associated with component failures or transition rates are explicitly and consistently accounted for. An adaptive Kalman filter algorithm yields information on the type and magnitude of feasible component transitions that are consistent with system observations, and finally we obtain the likelihood of feasible component failures or degradations that could account for off-normal system observations.

Sample calculations involving simulated transient events for the primary system of a PWR and the BOP of a BWR, respectively, for the fuzzy logic and statistical diagnosis algorithms show the validity and general usefulness of both diagnostic methods. The fuzzy logic approach of Section 3 appears, however, limited to single- and binary-failure diagnosis, with the resolution between events becoming difficult with simultaneous failures involving three components or more. In addition to macroscopic conservation equations used so far in the generation of binary-failure knowledge bases, we may need to study alternate lumped-parameter relationships which could provide robust diagnostic information for various power plant components. An ongoing doctoral dissertation of Paul J. Rank will investigate this point together with further testing of fuzzy diagnostic algorithms for PWR transients.

The stochastic algorithms discussed in Section 4 can handle multiplicity of component failures beyond ternary failures we have considered in our sample BOP diagnosis. The adaptive Kalman filter allows us to obtain the magnitude of each hypothesized fault in a

continuum fashion. This circumvents the dimensionality problem encountered when discretized fault magnitudes are to be evaluated. The computational requirements, associated with multiple Kalman filters and Monte Carlo integration of various pdf's, could still become significant for a large number of process variables monitored and for a complex system model. Thus, our statistical diagnostics algorithm, in the current form, may be applicable primarily as an off-line tool. This concern with computational requirements should be further studied together with the potential problems associated with multiple diagnostic time steps, which could require the evaluation of a large number of feasible component state trajectories.

Further study will also be required to combine the two alternate diagnostics algorithms that we have developed in this project. The fuzzy logic approach yields diagnostics information in the form of possibility and can account for diversity of reliability and system performance data. In our application of fuzzy diagnostics algorithms, we have yet to incorporate a number of different knowledge types into one coherent structure. The statistical diagnostics algorithm makes an explicit use of statistical data on component reliability and generate diagnostics information in the form of probability or likelihood of individual component degradations. The results may turn out to be in conflict with the diagnostic results from the fuzzy logic method. A systematic study is recommended to resolve potential conflicts between the two diverse types of diagnostic results and obtain robust diagnostic information on system malfunctions. Further efforts will also be required to implement both the fuzzy logic and statistical diagnostic algorithms in an integrated diagnostics guidance system, as an extension of the Rx code¹⁰ which combines model- and rule-based algorithms to perform diagnosis of off-normal events.

Our preliminary studies to date on applying the SA algorithm to time-domain control problems indicate the need to develop more efficient and general acceleration algorithms, including the use of an optimal annealing or cooling schedule. In addition, application of a pattern recognition algorithm, e.g., the entropy minimax method or artificial neural network, to identify promising control strategies should be considered. Another approach could involve the mean field theory of statistical mechanics that has been successfully²⁷ applied in some optimization problems. A combined use of deterministic and stochastic solution techniques should also be considered for tractable solution of complex system models.

Publications Resulting from the Project

1. S. E. Aumeier : "A Monte Carlo Technique for System Diagnostics, Monitoring, and Surveillance," *Trans. Am. Nucl. Soc.*, 69, 235 (1993).
2. P. J. Rank and J. C. Lee, "A Fuzzy Logic Approach for Transient Diagnostics in Nuclear Power Plants," *Trans. Am. Nucl. Soc.*, 70, 104 (1994).
3. S. E. Aumeier and J. C. Lee, "Probabilistic System Diagnostics and Surveillance," *Proc. ARS '94 - Int. Topical Meeting on Advanced Reactor Safety*, Pittsburgh, PA (1994).
4. S. E. Aumeier, "Probabilistic Techniques for Multi-Component System Diagnostics and Surveillance," Ph.D. Dissertation, University of Michigan (1994).
5. S. E. Aumeier, J. C. Lee, and A. Z. Akcasu, "Probabilistic Techniques Using Monte Carlo Sampling for Multi-Component System Diagnostics," *Proc. Int. Conf. Mathematics and Computations, Reactor Physics, and Environmental Analysis*, Portland, OR (1995).
6. S. E. Aumeier, J. C. Lee, and A. Z. Akcasu, "Probabilistic Fault Detection and Diagnosis I: A Stochastic Analysis of component Fault Manifestation," to be submitted to *Nucl. Sci. Eng.*
7. S. E. Aumeier and J. C. Lee, "Probabilistic Fault Detection and Diagnosis II: Detection and Diagnosis of Multiple Component Performance Degradation," to be submitted to *Nucl. Sci. Eng.*

References

1. J. Reifman and J. C. Lee, "Reactor Diagnostics Rule Generation Through Pattern Recognition," *Nucl. Sci. Eng.*, 107, 291 (1991).
2. P. J. Rank and J. C. Lee, "A Fuzzy Logic Approach for Transient Diagnostics in Nuclear Power Plants," *Trans. Am. Nucl. Soc.*, 70, 104 (1994).
3. S. E. Aumeier and J. C. Lee, "Probabilistic System Diagnostics and Surveillance," *Proc. ARS '94 - Int. Topical Meeting on Advanced Reactor Safety*, Pittsburgh, PA (1994).
4. J. C. Lee, "Simulation-Based Diagnostics and Control for Nuclear Power Plants," Progress Report for the Period April 15, 1992 - April 14, 1993, DOE/ER-75712-1 (January 1993).
5. J. C. Lee, "Simulation-Based Diagnostics and Control for Nuclear Power Plants," Progress Report for the Period April 15, 1993 - April 14, 1994, DOE/ER-75712-2 (March 1994).
6. S. E. Aumeier : "A Monte Carlo Technique for System Diagnostics, Monitoring, and Surveillance," *Trans. Am. Nucl. Soc.*, 69, 235 (1993).
7. S. E. Aumeier, J. C. Lee, and A. Z. Akcasu, "Probabilistic Techniques Using Monte Carlo Sampling for Multi-Component System Diagnostics," *Proc. Int. Conf. Mathematics and Computations, Reactor Physics, and Environmental Analysis*, Portland, OR (1995).
8. S. E. Aumeier, "Probabilistic Techniques for Multi-Component System Diagnostics and Surveillance," Ph.D. Dissertation, University of Michigan (1994).
9. P. J. Rank, J. Reifman, and J. C. Lee, "Fuzzy Diagnosis of Multiple Failures in Nuclear Power Plants," *Trans. Am. Nucl. Soc.*, 64, 176 (1991).
10. J. A. Hassberger and J. C. Lee, "A Simulation-Based Expert System for Nuclear Power Plant Diagnostics," *Nucl. Sci. Eng.*, 102, 153 (1989).
11. L. A. Zadeh, "Fuzzy Sets," *Inform. Control*, 8, 338 (1965).
12. J. C. Bezdek, *Pattern Recognition with Fuzzy Objective Function Algorithms*, Plenum Press (1981).
13. H. D. Brunk, *An Introduction to Mathematical Statistics*, Xerox College Publishing (1975).
14. A. Kandel, *Fuzzy Mathematical Techniques with Applications*, Addison-Wesley (1986).
15. P. Gmytrasiewicz, J. A. Hassberger, and J. C. Lee, "Fault Tree Based Diagnostics Using Fuzzy Logic," *IEEE Trans. Pattern Analysis and Machine Intelligence*, 12, 1115 (1990).
16. M. Sugeno, "An Introductory Survey of Fuzzy Control," *Inform. Sci.*, 36, 59 (1985).

17. M. M. Methnani, "Nuclear Power Plant Simulation with a Moveable-Boundary Steam Generator Model," Ph.D. Dissertation, University of Michigan (1983).
18. C. W. Gardiner, *Handbook of Stochastic Methods*, Second Ed., Springer-Verlag (1989).
19. A. Z. Akcasu and B. Hammouda, "Motion of a Charged Particle in a Randomly Varying Magnetic Field," *Physica*, 131A, 485 (1985)
20. J. Devooght and C. Smidts, "Probabilistic Reactor Dynamics—I: The Theory of Continuous Event Trees," *Nucl. Sci. Eng.*, 111, 229 (1992).
21. A. J. Jazwinski, *Stochastic Processes and Filtering Theory*, Academic Press (1970).
22. P. V. G. Shankar, "Simulation Model of a Nuclear Reactor Turbine," *Nucl. Eng. Design*, 44, 269 (1977).
23. N. Metropolis, et al., "Equations of State Calculations by Fast Computing Machines," *J. Chem. Phys.*, 21, 1087 (1953).
24. S. Kirkpatrick, C. D. Gelatt, Jr., and M. P. Vecchi, "Optimization by Simulated Annealing," *Science*, 220, 671 (1983).
25. G. Leitman, *An Introduction to Optimal Control*, McGraw-Hill (1966).
26. Z. R. Rosztoczy and L. E. Weaver, "Optimum Reactor Shutdown Program for Minimum Xenon Buildup," *Nucl. Sci. Eng.*, 20, 318 (1964).
27. J. Engel, "Teaching Feed-Forward Neural Networks by Simulated Annealing," *Complex System*, 2, 641 (1988).