Region-of-Interest Cone-Beam Computed Tomography

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REGION-OF-INTEREST CONE-BEAM COMPUTED TOMOGRAPHY

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ABSTRACT
A methodology for solving the general cone-beam region-of-interest (ROI) problem on a circular trajectory is presented using the mathematical framework described by Grangeat [1]. The algorithm, called Radon-ROI, takes scans at two different resolutions—low resolution covering the entire object and high resolution covering only the ROI—and combines the scans in both projection and Radon spaces so that the ROI is reconstructed at high resolution without artifacts from missing-data, under-sampling, or cone-beam errors. A circular source trajectory is assumed and the object must have low spatial frequencies outside the ROI. Simulated and experimental results of the Radon-ROI code show marked improvement on resolution within the ROI.

1. INTRODUCTION
The region of interest problem in computed tomography (CT), where only a small region of the patient is irradiated and imaged, has been extensively studied for the two-dimensional case. Although a complete set of projections is theoretically required for 2D CT reconstruction, the ROI problem has been solved in several ways. Some of the best ROI reconstruction methods involve the use of models or extra measurements for regeneration of the missing ray-sums. One can use a model of the object to complete the missing data [2, 3], or use assumed models to estimate a best fit to the missing data [4, 5]. The difficulty of these methods is in the choice and alignment of the model to the data; errors in these steps cause reconstruction errors and no way of quantifying the artifacts. A more heuristic method is to acquire a second scan that completely covers the object, but at low resolution (LR), and to combine this data with the high resolution (HR) ROI projections or reconstructions [6] using interpolation. When a complete set of ray-sums is estimated, by whatever method, reconstruction of the ROI image is straightforward using filtered backprojection or other known 2D technique.

However, extending the above methods to the 3D cone-beam ROI problem is not straightforward. The model-based methods could be extended with the use of 3D models to describe the object, but now the modeling and alignment problems are even more difficult. The heuristic method of combining two scans with differing resolution is not directly possible, in general, since the LR scan will not contain the missing ray-sums needed by the HR scan. Only those ray-sums in the mid-plane (the plane containing the source trajectory) can be interpolated directly from the LR scan. Normally, no ray-sums off of the mid-plane are colinear between the two scans, and hence cannot be combined in the same sense.

Still, the main advantage of cone-beam 3D CT over parallel-beam lies in the ability to achieve higher resolution of the object through geometric magnification. At relatively low cone-angles (less than ±12 degrees) [7], algorithms exist that perform reasonably well for circular source trajectories in spite of the so-called “shadow zone” of missing data.

In this paper, we develop a new algorithm for the cone-beam ROI algorithm and test it in several cases. We will first describe the method, then follow with some results.
2. GENERAL APPROACH

Extending the 2D heuristic approach above to 3D is one possible approach for cone-beam ROI CT we will call reconstruction/reprojection. The LR scan could be reconstructed for a full but crude view of the object, then mathematically reprojected at small sample spacing to generate missing data from the ROI scan. This can be time-consuming and still not address other missing data in the high-resolution scan (i.e., the shadow zones above and below the source trajectory plane in Radon space).

We propose a similar method, but using the methodology described by Grangeat [1] for cone-beam reconstruction. In this way, we have an intermediate space—the 3D Radon domain or VR-space—that can be used for combining LR and HR data. We will show how the reprojected ray-sums can be found more rapidly through an intermediate space (Radon space), and how performing a second combination process in this intermediate space will reduce cone-beam artifacts. Therefore, two new features of this technique, increased speed and reduced artifacts, make it more desirable than the simple reconstruction/reprojection method.

3. CONE-BEAM RECONSTRUCTION

The reconstruction process proposed by Grangeat [1], called the Radon algorithm, starts with the x-ray transform of the 3D volume \( f() \), which is the mathematical representation of the cone-beam data acquisition process:

\[
\mathcal{X}f(S, A) = \int_0^\infty f \left( S + a \frac{SA}{\|SA\|} \right) da 
\]

for all points \( A \) on the detection plane and all positions \( S \) on the source trajectory.

The first step in reconstruction involves computing the 3D Radon transform of the object function defined as follows:

\[
\mathcal{R}f(\rho, \bar{n}) = \int_{(\bar{0}M, \bar{n})=\rho} f(M) dM, 
\]

where \( \bar{n} \) is a 3D unitary direction vector from the object origin, given in standard spherical coordinates \((\rho, \theta, \phi)\).

The principle is to compute, and then to invert, the first derivative of the Radon transform. This uses the Radon domain as a rebinning space. The 3D Radon transform, therefore, is formed by integrating the object function over planes, rather than lines. Another way to write the Radon transform is:

\[
\mathcal{R}f(\rho, \theta, \phi) = \iiint f(x, y, z) \delta(\bar{n}^T \bar{x} - \rho) dx dy dz. 
\]

where \( \bar{x} \) is the vector representing a point \( M(x, y, z) \) and \( \delta() \) is the Dirac delta function.

Grangeat found the following exact relationship for computing the 3D Radon transform from the x-ray transform:

\[
\frac{\|OS\|^2}{\|OS \times n\|^2} \frac{\partial (\mathcal{S}f)}{\partial \rho'}(S, \bar{n}) = \frac{\partial (RF)}{\partial \rho} (OS \cdot \bar{n}, \bar{n}) 
\]

where \( \rho = \|OC\|, \rho' = \|OC'\|, \) and

\[
\mathcal{S}f(S, \bar{n}) = \int_{A \in D(S, \bar{n})} \frac{\|SA\|}{\|\bar{A}\|} \mathcal{X}f(S, A) dA 
\]

where \( D(S, \bar{n}) \) is the intersection line between the detection plane and the plane passing through \( S \) and perpendicular to \( \bar{n} \). Equation 4 gives an exact method to compute some values of the first derivative of the Radon transform from the x-ray transform. For notational purposes, define \( D \) as the partial derivative operator with respect to the first variable, so that

\[
D\mathcal{R}f = \frac{\partial \mathcal{R}f}{\partial \rho}.
\]

In the rest of this paper, references to the Radon space actually denote the first derivative of the Radon transform in \( \rho, \) or VR-space.

An inversion of the 3D Radon transform, then, can be given by the following equation:

\[
f(M) = -\frac{1}{4\pi^2} \int_0^\infty \int_0^\pi \frac{\partial^2 \mathcal{R}f}{\partial \rho^2} [\bar{n}^T \bar{x}, \theta, \phi] \sin \theta d\theta d\phi. 
\]

The inverse Radon transform can be easily and efficiently implemented with existing 2D filtered backprojection algorithms.

The Radon algorithm has been shown to be accurate, efficient, and has some properties that are useful in the ROI technique.

4. CONE-BEAM GENERATION

Assume that a complete Radon-space representation of an object is available. We now generate computed cone-beam projections from any new source position, \( S \), given \( D\mathcal{R}f(\rho, \theta, \phi) \). First, define "parallel lines in \( z \)" as all the lines that lie in a fixed-\( z \) plane. Each parallel line in \( z \) can be parameterized by the polar coordinates \( r \) and \( \phi \) in that plane. Then define a special case of the x-ray transform that only represents integrals along these parallel lines in \( z \):

\[
\mathcal{X}_z f(r, \phi) = \int_{-\infty}^{\infty} f(x, y, z) \delta(x \cos \phi + y \sin \phi - r) dx dy.
\]
This is called the parallel x-ray transform at a constant value of \( z \). It is the same as if ray-sum measurements were acquired with the x-ray source at infinity so that the incoming radiation is along parallel lines.

It can be shown that \( \tilde{X}_z f \) is also an intermediate form of the 3D inverse Radon transform of Equation 7:

\[
\tilde{X}_z f(r, \phi) = \frac{1}{2\pi} \int_0^\pi \mathcal{H}R f(z \cos \theta + r \sin \theta, \theta, \phi) \, d\theta \quad (9)
\]

where \( \mathcal{H} \) is the Hilbert transform. This is a simple filtered backprojection and it can be used to compute any parallel ray-sum. Physically, in the Radon domain, the backprojection integration is along a circle in the \( \phi \)-plane (or meridian plane) defined by

\[
\rho = z \cos \theta + r \sin \theta. \quad (10)
\]

This circle passes through the origin and the point \((r, z, \phi)\) in cylindrical coordinates.

Using this result, any ray-sum in the volume can be computed by rotating the coordinates of 3D space and performing the same backprojection. Instead of backprojection integration along a circle aligned in Radon space, we can choose any circle passing through the origin. For example, with a different source trajectory (changed magnification), we pick the circles to correspond to the new rays in the acquisition coordinate system.

The above algorithm has been implemented and tested. It accepts cone-beam projections, computes their Radon transform, \( \mathcal{R} f \), and then generates new projections at any source distance. The Hilbert transform is computed in the Fourier domain and trilinear interpolation of the Radon space is used during backprojection. In comparison with the reconstruction/reprojection method, this technique is less compute intensive because only 2D projections and backprojections are needed, rather than 3D projections and backprojections.

5. RADON-SPACE COMBINATION

Now the measured HR data containing only the ROI can be combined with the computed HR data using the above technique; the measured data is simply placed in the correct position of each 2D projection. The result of this operation is called the combined HR data. The newly combined HR projections could be directly reconstructed, as in the reprojection method, and the final reconstruction gives a better view of the ROI. However, there is another step of Radon-ROI that gives further improvement in final ROI reconstruction.

By the nature of the different cone angles involved, the shadow zone of the LR scan is smaller than that of the HR scan. The common method for filling the missing data of the shadow zone is interpolation from its edges. However for the combined HR data this zone may be very large, so direct reconstruction of the HR torus will give large cone-beam artifacts. Instead, in Radon-ROI we fill the combined HR shadow zone with interpolated data from the smaller yet sparsely sampled LR shadow zone. Mixture of the LR and HR Radon spaces should diminish the cone-beam artifacts that exist in the final reconstruction while still maintaining the high spatial resolution in the ROI.

For Radon-ROI, the mixture is carried out by simply using the HR data in the high-resolution region of Radon space, and the LR data elsewhere. This includes the shadow zone of the volume where nearest neighbor interpolation of the LR torus is used. Notice that if the LR data is from a parallel projections there will be no shadow zone. In the current version of the program, the LR Radon space is bilinearly interpolated in each \( \phi \)-plane to match the sampling of the HR Radon space.

6. EXPERIMENTAL RESULTS

We have tested the Radon-ROI algorithm on several simulated objects and found it to perform well. A simulated sphere with five spherical voids is shown in vertical cross-section at the right of Figure 1. The top images (left and right) show the LR projection and Radon reconstruction respectively, while the bottom images show HR data. Notice that the HR projection requires a much large "detector" region. The middle row shows (at left) projections used in Radon-ROI; both have the same size detector, but the HR scan only shows the ROI. The Radon-ROI reconstruction at middle right shows how high resolution was achieved in the region of interest.

Using the experimental test-bed EVA industrial scanner at LETI [9], we acquired data from a cylindrical ceramic precombustion chamber of about 30 mm diameter and 20 mm height. It has a number of small cracks both at and below the surface. EVA is particularly well-suited to this work because the source-detector distance is adjustable up to two meters and the object center-of-rotation can be placed anywhere in that range. EVA uses an IRT BOMX 161 x-ray tube source with a 10 \( \mu \)m focal spot size and a tri-field Thompson image-intensifier detector system (3 lp/mm) with a CCD camera (512\(^2\)) connected to the imaging computer. The detector system has a fixed diameter of about 200 mm, so the cone-angle (and, hence, the magnification) are defined by the source and object locations.

Figure 2 shows LR, Radon-ROI, and HR results from top to bottom. Despite of a small shift from LR to HR scans, presumably due to veiling glare in the im-
age intensifier, clear increase in resolution is visible in the Radon-ROI over the LR scan. Since the projections change with the changes of magnification, various noise sources like these can cause differences in the relative ray-sum levels of the two experimental data sets. To correct for this, we used a global multiplicative mean normalization applied over each projection. Any further minor differences in local attenuation level can be minimized by local smoothing filter near the edge points, though this was not used in our experiments.

7. SUMMARY

The algorithm presented here, Radon-ROI, gives the best possible results for the ROI cone-beam CT problem without being excessively time-consuming. Radon-ROI is potentially superior to the reconstruction/reprojection method in terms of both speed and accuracy. It contains two novel features. First is the ability to calculate from $DR_f$, without complete reconstruction, any ray-sum through an object. It is done much quicker than the reprojection method. This may be a useful component for other applications as well (e.g., cone-beam SPECT). The second feature is the ability to combine the Radon-spaces of low- and high-resolution scans to reduce the shadow-zone artifacts. Incorporation of these two features into a Radon-ROI algorithm demonstrates dramatic improvements in the reconstructed images over previous methods.

In addition, this technique can be generalized to other limited-data situations including laminography, missing angles, general truncated projections, or other source trajectories and scan geometries. One could imagine applying this idea to SPECT imaging using dual collimators that generate both parallel and cone-beam emission CT data [10].

Radon-ROI is shown to be faster than previously suggested ROI methods. It has been implemented on a Cray and tested relative to a reconstruction/reprojection method. A side benefit of the technique is a fast method of generating cone-beam projections at any orientation and any cone angle from a set of cone-beam CT measurements.

8. REFERENCES


Figure 1. Simulated Radon-ROI results. Cone-beam projections (left) and vertical slices (right) of sphere with voids. Reconstructions are using LR (top), HR (bottom), and LR/ROI data with Radon-ROI (middle).

Figure 2. Experimental Radon-ROI results. Low resolution/magnification (top) and high resolution/magnification (bottom). LR/HR reconstruction with Radon-ROI (middle).