PERTURBATIVE RESUMMATION OF GLUON RADIATION
FOR TOP-QUARK PRODUCTION

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ABSTRACT

We present a calculation of the total cross section for top quark production based on
a new perturbative resummation of gluon radiative corrections to the basic QCD
subprocesses. We use Principal Value Resummation to calculate all relevant large
threshold corrections. Advantages of this method include its independence from arbitrary
infrared cutoffs and specification of the perturbative regime of applicability. For
$p\bar{p}$ collisions at center-of-mass energy $\sqrt{s} = 1.8$ TeV and a top mass of 175 GeV,
we compute $\sigma(t\bar{t}) = 5.52^{+0.07}_{-0.06}$ pb.

1. Introduction

The quest for the top quark $t$ reached fruition recently with the publication of results by
two collaborations of experimenters studying $t\bar{t}$ pair production in proton-antiproton
collisions at the Fermilab Tevatron$^1$: $p + \bar{p} \rightarrow t + \bar{t} + X$. Among theoretical
questions deserving attention is the quantitative reliability of cross sections based on the
main production mechanism in perturbative quantum chromodynamics (pQCD), namely
$t\bar{t}$ pair creation.

At lowest order (tree-level), the two partonic subprocesses are quark-antiquark
annihilation:

\[ q + \bar{q} \rightarrow t + \bar{t} \]  \hspace{1cm} (1)

and gluon-gluon fusion:

\[ g + g \rightarrow t + \bar{t} \] \hspace{1cm} (2)

These subprocesses are of order $\alpha_s^2$ in the strong coupling strength. The calculated
top quark cross sections depend on these subprocess cross sections and on the parton
density distributions that specify the probability densities of the quarks, antiquarks,
and gluons of the incident $p$ and $\bar{p}$. Both $O(\alpha_s^2)$ and the next-to-leading $O(\alpha_s^3)$
contributions have been investigated thoroughly. One observation is that the size of the
$O(\alpha_s^2)$ terms in the partonic cross sections is particularly large near the $t\bar{t}$ production
threshold, raising questions about the reliability of perturbation theory. This region of
phase space is important for top quark production at the Tevatron owing to the large
mass of the top quark. These enhancements motivate a complete study of the large

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logarithmic corrections at the partonic level, to all orders in perturbation theory. In this report we summarize a recently published\textsuperscript{4} resummation of soft gluon corrections to the $t\bar{t}$ cross section employing the Principal Value Resummation (PVR) technique.\textsuperscript{5}

As in other hard-scattering processes, where large threshold contributions are present, resummation of these corrections to all orders in $\alpha_s$ is important both for theoretical understanding of the perturbative process and for numerical control of the resulting predictions. One method for resummation has been implemented previously for $t\bar{t}$ pair production.\textsuperscript{6} An inherent uncertainty and limitation of the resummation method of [6] is its dependence on an undetermined infrared (IR) cutoff. This cutoff enters when the exponentiated resummation function is integrated over phase space. In the formulation of [6], it cuts off the Landau pole of the QCD running coupling constant and the corresponding threshold region. Since the function is exponentiated, the dependence of the resummed cross section on this cutoff is important numerically.

The main advantage of PVR is that it does not depend on arbitrary IR cutoffs, as all Landau-pole singularities are by-passed by a Cauchy principal-value prescription. Due to the absence of extra undetermined scales, it allows for an evaluation of the perturbative regime, i.e., the region of the radiation phase space where perturbation theory should be valid. Further, PVR has been tested successfully in $Z\bar{Z}$ production where its definition of resummed pQCD is in excellent agreement with experiment,\textsuperscript{7} even at relatively low values of the hard scale (in the region of 5-10 GeV).

2. Perturbative content of PVR

In the remainder of this report, we present our results for the physical inclusive total cross section for $t\bar{t}$ production in PVR for a top-mass range $m \in \{150, 250\}$ GeV, including a discussion of the remaining theoretical uncertainties. We present our predictions in the $\overline{MS}$ factorization scheme, in which the $q, \bar{q}$ and $g$ densities and the next-to-leading order partonic cross sections are defined unambiguously.

It has been observed that the functional form of the leading threshold corrections, order-by-order in perturbation theory, appears to be universal, i.e., independent of the hard-scattering process, except for differences in multiplicative color factors and process-specific kinematics. An example is the “universality” between dilepton- ($ll$) and $t\bar{t}$ production, emphasized in [6].

The resummed partonic cross sections, including all large threshold corrections according to PVR, can be written as

$$\sigma_{ij}^{PVR}(\eta, m^2) = \int_{1-4(1+\eta)+4\sqrt{1+\eta}}^1 dz \left[ 1 + \mathcal{H}_{ij}(z, \alpha) \right] \sigma_{ij}(\eta, m^2, z).$$

In Eq. (3),

$$\mathcal{H}_{ij}(z, \alpha) = \int_0^{\ln(1/(1-z))} dz^* e^{B_{ij}(z^*, \alpha)} \sum_{j=0}^{\infty} Q_j(z, \alpha),$$

$$\sigma_{ij}^{PVR}(\eta, m^2, z) = \frac{d(\sigma_{ij}^{(0)}(\eta, m^2, z))}{dz}, \text{ and } \sigma_{ij}^{(0)}$$

is the tree-level partonic cross section expressed in terms of inelastic kinematic variables\textsuperscript{8} to account for the emitted radiation. The variable $\eta = s/4m^2 - 1$ is the distance from the partonic production threshold, $\alpha \equiv \alpha_s(m)/\pi$, and $ij \in \{q\bar{q}, gg\}$ denotes the initial parton channel. The integration
in Eq. (3) is over the phase space of the radiated gluons, parametrized through the dimensionless variable \( z \). In \( \bar{t} t \) production \( z = Q^2 / \beta \), where \( Q \) is the invariant dilepton mass, and \( z = 1 \) corresponds to zero gluon momentum.

Equation (4) is the main content of PVR. The *leading* large threshold corrections are contained in the exponent \( E_{ij}(x, \alpha) \), which is a calculable polynomial in \( x \). \( \{Q_j(x, \alpha)\} \) are calculable functions produced by the analytical inversion of the Mellin transform from moment space (where the argument of \( E_{ij} \) corresponds to \( \alpha \rightarrow \ln \alpha \)) to the physically relevant momentum space expressed in Eq. (4). These functions are produced by the resummation and are expressed in terms of successive derivatives of \( E \): \( P_k(x, \alpha) \equiv \delta^k E(x, \alpha) / k! \delta^k x \). Given that \( E \) contains at most one more power of \( x \) than of \( \alpha \), \( P_k \) contains at least \( k - 1 \) fewer powers of \( x \) than of \( \alpha \), and \( Q_j \) contributes the *resummed version* of terms containing \( j \) fewer powers of \( x \) than of \( \alpha \) in the integrand of Eq. (4). For example \( Q_0 \) contains all powers \( P \), \( Q_1 \) all powers \( P \), etc.

The main object of the resummation, \( E_{ij} \), was studied extensively in the context of \( \bar{t} t \) production.\(^7\) Embodying the universality discussed earlier, its functional form for \( \bar{t} t \) production is identical to that of \( \bar{t} t \) production, except for the identification of the two separate channels, denoted by the subscript \( ij \). However, there are significant differences between the two processes which affect the way we implement PVR practically for the process at hand. These can be described as follows:

First, only the *leading* threshold corrections are universal. Therefore, from all structures \( \{Q_j\} \) in Eq. (4), the very leading one should be considered universal. This is the linear term in \( P_1 \), which turns out to be \( P_1 \) itself. Hence, Eq. (4) can be integrated explicitly, and Eq. (3) may be written as

\[
\sigma_{ij}^{PV}(\eta, m^2) = \int_{1-4/(1+\eta)+4\sqrt{1+\eta}}^1 dx E_{ij}(\ln(1-1/x), \alpha) \sigma_{ij}'(\eta, m^2, z).
\] (5)

The second issue has to do with the perturbative regime in momentum space. To characterize a region in momentum space as "higher-twist", however, one must first convert to momentum space through inversion of the Mellin transform, Eq. (4). Specification of the boundary, mentioned already in,\(^5\) is realized by the constraint that all \( \{Q_j\}, j \geq 1 \) be small compared to \( Q_0 \) which provides the leading integrand in Eq. (4), according to the previous power counting. This constraint can be shown quite generally to correspond to

\[
P_1 \left( \ln \left( \frac{1}{1-z} \right), \alpha \right) < 1.
\] (6)

One could actually apply Eq. (5) all the way to \( z = 1 \) by using contour integration as suggested by PVR, beyond the perturbative regime of Eq. (6), but one would then be using a *model* for non-perturbative effects, the one suggested by PVR, far beyond the knowledge justified by perturbation theory. In this work we confine our attention to the perturbative regime, and hence our cross section is evaluated accordingly, namely

\[
\sigma_{ij}^{PV\text{pert}}(\eta, m^2) = \int_{1-4/(1+\eta)+4\sqrt{1+\eta}}^{z_{\text{max}}} dx E_{ij}(\ln(1-1/x), \alpha) \sigma_{ij}'(\eta, m^2, z)
\] (7)

The upper limit is calculated through

\[
P_1 \left( \ln \left( \frac{1}{1-z_{\text{max}}} \right), \alpha \right) = 1.
\] (8)
It turns out our final result does not rely much upon the PVR method to bypass IR renormalons and associated problems, precisely because it is restricted to the perturbative regime. In that sense, the presence of arbitrary IR cutoffs in previous resummations is superfluous, as all necessary information about IR sensitivity (i.e., the perturbative regime) can be obtained by examining the perturbative asymptotic properties of the resummation functions.

3. Comparison with experiment and conclusions

We provide our final resummed cross sections for each production channel by taking into account the complete next-to-leading calculation, $\sigma_{ij}^{(0+1)}$, through the improved prediction

$$\sigma_{ij}^{\text{final}}(\eta, m^2) = \sigma_{ij}^{\text{PVR}}(\eta, m^2) + \sigma_{ij}^{(0+1)}(\eta, m^2) - \sigma_{ij}^{(0+1)}(\eta, m^2)\bigg|_{PVR}. \quad (9)$$

The last term in Eq. (9) is the part of the next-to-leading order partonic cross section included in the resummation. In Fig.1 we plot the physical cross section, obtained by convoluting Eq. (9) with CTEQ3M parton densities and summing over both channels, as a function of the "hard" scale $\mu$ in the range $\mu/m \in \{0.5, 2\}$. Notice the mild dependence, as well as the interesting shape that peaks around the value $\mu/m \approx 1$. We consider the variation of the physical cross section the range $\mu/m \in \{0.5, 2\}$ a good measure of the theoretical perturbative uncertainty.

![Graph](image)

Fig. 1. The final resummed cross section for $t\bar{t}$ production at $m = 175$ GeV and $\sqrt{s} = 1.8$ TeV. Shown also is the next-to-leading order result (dashed curve).
Our prediction for the inclusive total $t\bar{t}$-production cross section at the Tevatron, using $CTEQ3M$ parton densities, is

$$\sigma_{\text{final}}(m = 175 \text{ GeV}) = 5.52^{+0.07}_{-0.45} \text{ pb}.$$  \hspace{1cm} (10)

The "central" value of 5.52 pb is obtained with $\mu/m = 1$, and the upper and lower limits of the uncertainty band correspond to the maximum and minimum values of the cross section in the range $\mu/m \in \{0.5, 2\}$. The cross section is insensitive to the choice of parton densities. Repeating the same analysis with the $MRS(A')$ densities, we obtain

$$\sigma_{\text{final}}(m = 175 \text{ GeV}) = 5.32^{+0.08}_{-0.41} \text{ pb}.$$  \hspace{1cm} (11)

The value in Eq. (10) is larger than that of [6], but within uncertainties, and it agrees with the present CDF and D0 measurements.

In Fig. 2 we show the top-mass dependence of the physical cross section for $p\bar{p} \rightarrow (t\bar{t})X$.

![Fig. 2. Physical cross section for $p\bar{p} \rightarrow (t\bar{t})X$ at $\sqrt{s} = 1.8$ TeV as a function of top mass. Data from the CDF and D0 collaborations are shown. Shown are entries for $\mu/m = 0.5$ (dashed), 1 (solid) and 2 (dotted). The band of perturbative uncertainty quoted in Eq. (10) is relatively narrow. On the other hand, we noted in discussing Eq. (7) that $\langle \eta \rangle < 1$, meaning that there is a reasonable range of $\eta$ near threshold in which perturbative resummation does not apply. Perturbation theory is not justified in this region. Correspondingly, further strong interaction enhancements of the $t\bar{t}$ cross section may arise from physics in this region. We know of no reliable way to estimate the size of such non-perturbative effects and, therefore, cannot include such uncertainties in the estimates of the perturbative uncertainty of Eqs. (10) and (11).](image-url)
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References


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