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Population Dynamics of Minimally Cognitive Individuals

Part I: Introducing Knowledge into the Dynamics

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POPULATION DYNAMICS OF MINIMALLY COGNITIVE INDIVIDUALS

Part I: Introducing Knowledge into the Dynamics

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ABSTRACT

We present a new approach for modeling the dynamics of collections of objects with internal structure. Based on the fact that the behavior of an individual in a population is modified by its knowledge of other individuals, a procedure for accounting for knowledge in a population of interacting objects is presented. It is assumed that each object has partial (or complete) knowledge of some (or all) other objects in the population. The dynamical equations for the objects are then modified to include the effects of this pairwise knowledge. This procedure has the effect of projecting out what the population *will* do from the much larger space of what it *could* do, i.e., filtering or smoothing the dynamics by replacing the complex detailed physical model with an effective model that produces the behavior of interest. The procedure therefore provides a minimalist approach for obtaining emergent collective behavior. The use of knowledge as a dynamical quantity, and its relationship to statistical mechanics, thermodynamics, information theory, and cognition microstructure are discussed.

I. INTRODUCTION

A general problem that appears in many disciplines is that of modeling the behavior of collections of objects. If the objects are nearly identical, the collection is termed a population, and the challenge is to derive the emergent collective behavior of such populations from the dynamics of the individual objects. A common approach in such systems is to specify the dynamics of the individuals, and track each individual forward in time, extracting population measures (such as mean size of individuals) whenever desired (Lomnicki, 1988). The goal in such investigations is to discover ordered states, regions of stability or instability, critical parameter values, phase transitions, the effects of perturbations, and similar phenomena.

This paper presents a new approach to population dynamics based on the idea that each individual has some *knowledge* of other individuals and that its behavior is modified by that knowledge. We have a general concept of "knowing," and could say that each individual in a population in some sense "knows" about other individuals. Clearly, individual behavior in a population depends strongly on what (or who) that individual knows, and how that knowledge is used. We quantify this idea by postulating a procedure for modifying the normal dynamical equations of motion to include the amount and distribution of pairwise knowledge among the individuals, and how that knowledge is altered by individual interactions and other processes. Thus, "knowledge" is regarded as a dynamical quantity, and its evolution in time can be described by a set of equations. Introduction of these equations into the dynamical system introduces dynamical *constraints*. These constraints allow prediction of what *will* happen from the much larger range of what *could* happen.

There is currently considerable effort on this subject that appears under several titles: individual-based ecosystems (DeAngelis and Gross, 1992), artificial life (Langton, 1989, 1992), cellular automata (Gutowitz, 1991), and connectionism (Farmer, 1990), among others. These problems are sometimes described as information-driven: the size of a tractable problem and the speed of the computation are determined by the amount of information that must be processed (Brillouin, 1962). Usually concomitant to the approach to such problems is a search for ways to reduce the amount of information that must be carried along.

While knowledge and information are similar, they are distinct: information is *data in motion*; knowledge is *data at rest*. Information is involved in the *construction* of a data structure; knowledge is involved in the *use* of that data structure. Information implies the ability to correctly *establish* a representation; knowledge implies the ability to *examine* that representation. Information is used in correctly *storing* the answer to a question; knowledge is used in correctly *recovering* the answer to the question.

What we propose in this work is a hybrid dynamics specifically designed for populations of objects that have sufficient internal complexity to have some knowledge of other objects and to take actions based at least partially on that knowledge. It is most appropriately applied to simple behavior such as schooling or flocking, in which the full complexity of the real individual is not necessary to produce the collective behavior. A perfect exemplary system is a population of simple robots with communicating internal microprocessors: although the devices themselves are complex, their actions are determined by a set of simple rules modified by small data set (stored knowledge) that can be altered by interactions among individuals. This approach should be useful for simple animals, robots, globally coupled relaxation oscillators, arrays of point vortices, computer networks, vehicular traffic, and a wide variety other individual-based systems, including a massively parallel computer.

Although the dynamical principle presented here will have applications to biological, chemical, sociological, and many other systems, our approach to the subject is via physics: We seek consistent and useful ways to modify an existing physical model to include the effects of pairwise knowledge. We will not track the knowledge itself (e.g., the names of books); rather, we will track a quantity giving the amount and distribution of that knowledge. This quantity, a matrix **K**, describes what each individual "knows" about every other individual. **K** is precisely defined in terms of physically determined probabilities, hence provides a link to statistical mechanics, thermodynamics, and information theory. Because the matrix **K** reasonably approximates the basic dynamics of knowledge in real systems, it should be an extremely efficient means for generating very complex emergent behavior that reasonably simulates reality.

In this paper, we define the terminology and describe general properties of minimally cognitive population dynamics. Throughout this paper, **K** is assumed to be constant in time. In a companion paper, we investigate some fundamental effects that arise when **K** is not constant.

II. INCLUDING KNOWLEDGE IN INDIVIDUAL INTERACTIONS

We begin by postulating that individuals in the population can be represented as carrying along with them a quantity of "knowledge," and that this knowledge influences the individuals' behavior. While we assume the individuals are capable of storing and processing knowledge, we do not necessarily require them to be sufficiently talented to use it in any manner resembling humans might associate with intelligence. The individuals may be nothing more than simple devices capable of only a small number of reflexive actions, or they might be extremely complex organisms operating on a limited number of alternatives. We will sometimes refer to these individuals as having *cognitive ability*, and we will have most interest in populations of *minimally cognitive individuals*.

The simplest kind of interaction between individuals is *pairwise*. If the objects are simple, i.e., have no internal structure, they interact directly, and the interaction obeys Newton's Third Law. If, however, the objects have internal structure, the interaction is mediated by an *agent*, and the interaction need not obey Newton's Third Law (cf., Figure 1). An example is pursuit-and-flight: one object may experience an attraction to another object, while the second feels a repulsion. Such dynamics are enabled by the agent, which acts break the symmetry between the objects.

It is implied in this picture that there are two or more kinds of objects in the population. Indeed, if all the objects are truly identical, there could be no breaking of the symmetry, and Newton's Third Law must be satisfied. However, we are interested in modeling populations of complex objects, and therefore we can assume the individual objects have minor differences that enable the Agent to distinguish them. In this sense, the set of individuals is not a true population (precisely identical individuals). But of course, neither is a real population in Nature: every ant, no matter how imperceptibly, is different from every other ant. It is conventional to refer to a group of individuals that are alike in most (but not all) respects as a "population."

The detailed nature of the agent is left unspecified: we only assume the effective force law, or its equivalent, between any pair of objects. This is really the central approximation, and the source of advantage, in the approach taken here: we ignore the details of the agent, and use only the effective interactions.

III. EFFECTS OF KNOWLEDGE ON POPULATION DYNAMICS

Next we postulate that the collective emergent behavior of the population will be determined, at least in part, by the individuals' pairwise knowledge. We emphasize that *qualitatively* new behavior results if the individuals have cognitive ability [Kampis, 1991].

Figure 2 presents a graphical representation of the knowledge links in a population of 10 individuals. The widths of the connecting lines are proportional to the amount of knowledge each individual has of other individuals. While the positions of the individuals are arbitrary, they can represent actual positions, which would then vary according to the system dynamics.

As an example, consider flocking in birds. The detailed physics of bird flight allows each individual bird to fly wherever it might want, and no flocking results. In order to produce flocking, we introduce constraints and additional dynamics. These additional factors serve to constrain the collective motion to a subset of all possible motions, i.e., produce flocking. The conventional approach to introducing these constraints (Kshatriya and Blake, 1992) is to attempt to model all the elementary *physical* factors that determine individual dynamics: two birds cannot approach closer than a minimum distance, the power needed to maintain flight is minimum for a certain interbird distance, individual birds experience a bias in flight direction according to migration pattern, etc.

What we propose here is that these factors can be more efficiently related to *knowledge* within the individual: each bird has a representation of all other birds, and takes its action accordingly. By phrasing these relations as pairwise knowledge, we obtain the minimal set of constraints necessary to produce the observed emergent collective behavior. We do not attempt to model all of the detailed physical processes, but rather to capture their net result through a paradigm of pairwise knowledge.

We emphasize the simplicity of the knowledge necessary for reasonable precision. Consider a circling hawk that spots a small animal on the ground. How much must the hawk know about its potential prey for it to decide to attack? Surely the hawk must know the position and size of the animal, whether it is alive, whether it is moving sufficiently slowly that there would be time to capture it, etc. But it is quite unnecessary for the hawk to know the species, sex, age, stomach contents, and state of parasitism of its victim. In fact, the amount of knowledge that the hawk must have is *spectacularly small*. We encounter this fact in almost every social situation: in general, actions are taken based on an extremely small amount of very specific knowledge, regardless of the complexity of the real individual.

The major motivation for using knowledge in a dynamical model is therefore its efficiency. Modeling all the elementary physical processes is not wrong—ultimately it may even yield reasonable emergent behavior—but it becomes extremely complicated and is not guaranteed to yield anything meaningful (the list of physics effects may be inadequate, the system may be numerically intractable, the introduction of many parameters may introduce unacceptable uncertainties, the correct procedure for extracting meaningful emergent behavior may not be clear, etc.). In contrast, lumping much of the detailed physics into a few parameters resembling pairwise knowledge which we postulate from the observed behavior of the population will easily produce the appropriate emergent behavior—and nothing else. Of course, such lumped model simulations will have less detailed predictive ability. The goal here is to find an efficient way to produce the gross behavior we want and avoid having to consider detail we don't want. In fact, we seek the *minimal* description of the system that produces the desired emergent behavior.

We will be most interested in systems in which the interactions between individuals vanish if the pairwise knowledge is zero. When this happens, all individuals act independently; there is no collective dynamics and the population is nothing more than a collection of individuals behaving independently. If the pairwise knowledge in such systems is low—most individuals know little or nothing about other individuals—a change in one individual will produce a small change in other individuals. If the pairwise knowledge is high—most individuals know a great deal about most other individuals—the population is strongly interacting, and exhibits collective behavior. In the limit that the pairwise knowledge is total (everyone knows everything about everyone else), the population is fully connected. In this case an effect on one individual is fully felt by all members of the population. This is the normal case of physics, in which the individual interactions are fully felt.

IV. PHYSICAL DEFINITION OF KNOWLEDGE

In this section we define the knowledge matrix **K** in terms of physically well-defined probabilities. This grounds the present development within physics, and provides links to information theory, thermodynamics, and statistical mechanics.

Knowledge as probability

Knowledge, in the traditional use of the word, implies a test: we "know" something if we can give the correct answer to a question about that something. We imagine repeatedly asking individual {i} to identify the state of individual {j}. Two extreme cases are immediately obvious:

(1) If {i} has *complete knowledge* of {j}, then {i} always (probability=1) correctly identifies the state of {j};
(2) If {i} has *no knowledge* of {j}, {i} has random probability p_r of giving the correct answer).

We now interpolate between these extremes with the following definitions:

(3) If {i} has *partial knowledge* of {j}, {i} has some probability p_{ij} in the range $p_r < p_{ij} < 1$ of correctly identifying the state of {j};
(4) If {i} has *incorrect knowledge* of {j}, {i} has some probability p_{ij} in the range $0 \le p_{ij} < p_r$ of correctly identifying the state of {j}.

Thus, we can associate various common expressions about knowledge with probabilities (cf., Figure 3).

In more modern phraseology, we "know" something if we have an internal representation of that something. Thus, in order for us to "know" the number 3.141596, somewhere inside our brain must be a representation of this number. The representation is similar to a reference: it can be consulted by the individual in order to determine appropriate action. This definition avoids having to model the process of hearing and answering the question. We will assume in this paper that the process of converting stored knowledge into an answer to a question is perfect. Thus, the following statements will be taken as equivalent:

$$p_{ij} = \text{the probability that} \begin{cases} \{i\} \text{ correctly identifies the state of } \{j\} \\ \{i\} \text{ has a true representation of } \{j\} \\ \{i\} \text{ "knows" the state of } \{j\} \end{cases}$$

We postulate that the probability matrix **p** is derivable from physics. It is a well-defined dynamical quantity that follows from the structure and evolution of the individual. Whether we can do this in practice for an arbitrary system is immaterial to the present discussion.

Definition of the knowledge matrix K

We now define the knowledge matrix **K** in terms of the probability matrix **p**. The matrix element K_{ij} will be taken to mean "how much {i} knows about {j}." Note that we are not going to track *what* {i} knows about {j}; we are going to track *how much* {i} knows about {j}, suitably normalized.

The simplest convention is to define $K_{ij}=1$ if {i} has complete knowledge if {j}, and $K_{ij}=0$ if {i} has no knowledge of {j}. Assume that the population is comprised of a set of identical individuals that can be in any one of G possible discrete states. Assume next that {j} is in only one of the G states accessible to it, and that {i} makes successive (randomly chosen) guesses of *which* state. If {i} has *complete* knowledge of {j} ($K_{ij}=1$), then {i} would correctly identify the state of {j} with probability $p_{ij} = 1$. If {i} has *no* knowledge of {j} ($K_{ij}=0$), then {i} would correctly identify the state of {j} with the random probability $p_{ij} = p_r = 1/G$. These limiting cases are most simply connected by the linear relation

$$p_{ij} = \frac{1}{G} + \left(1 - \frac{1}{G}\right) K_{ij} \qquad 0 \le K_{ij} \le 1$$

With this definition, individual {i} knows between nothing and everything (inclusive) about another individual {j}. Note that $K_{ii} = 1$, which means every individual always knows everything about itself.

For $0 \le p_{ij} < 1/G$, {i} has *less* than a random probability of correctly identifying the state of {j}, i.e., it is worse than a poor guesser. For $p_{ij}=0$, the guess is *always* wrong. We would say that {i} has *negative knowledge* about {j}, and we can extend the definition of K_{ij} to include this possibility:

$$p_{ij} = \frac{1}{G}\left(1 + K_{ij}\right) \qquad -1 \le K_{ij} \le 0$$

With this definition, individual {i} has incorrect knowledge of another individual {j}. This situation arises for example when, as {i} attempts to establish an internal representation of {j}, some process systematically causes it to be done incorrectly. Such situations are well-known, e.g., dyslexia.

We can now invert the two equations above to obtain K_{ij} in terms of p_{ij}:

$$K_{ij} = \begin{cases} \dfrac{G p_{ij} - 1}{G - 1} & 1/G \le p_{ij} \le 1 \\ \\ G p_{ij} - 1 & 0 \le p_{ij} \le 1/G \end{cases}$$

which we take to be the formal definition of K_{ij}.

Figure 4 shows the relation between p_{ij} and K_{ij}. The piecewise linear relation is by no means unique; it is proposed as the simplest nontrivial relationship.

We emphasize again that we will not track knowledge itself, but only a numerical quantity K_{ij} in the interval (-1,+1) that is a plausible measure of the *amount of knowledge*. Rather than introduce a new term for **K**, we simply refer to it as the *knowledge*.

Different kinds of knowledge

The property of individual {j} known by individual {i} quantified by the matrix element K_{ij} is quite arbitrary. It can be location, size, age, sex, color, condition, or any other property. For each property we wish to track, we specify a separate matrix: **K**, **K**', ... Furthermore, we may have a different definition of "interaction" for each property. For example, *fluttering* (behavior 1) of a butterfly may *attract* (behavior 2) a distant predator, but at close range it is found from the butterfly's *coloring* (property 1) that it is *distasteful* (property 2). The different matrices **K**, **K**' ... will in general enter the dynamical equations differently.

V. LUMPED MEASURES OF THE KNOWLEDGE

Each matrix element K_{ij} represents what one individual knows about one other individual. Therefore, each row in the matrix **K** represents what one individual knows about every other individual, and each column represents what every other individual knows about one individual. The principal diagonal represents what each individual knows about itself (always =1).

Matrix norms

The sum of elements in a row, called the row-submatrix norm, represents the total knowledge of all other individuals held *by* individual {i}:

$$K_i = \Sigma_j K_{ij}$$

The sum of elements in a column (called the column-submatrix norm) represents the total knowledge *of* individual {i} held by all other individuals:

$$K^T_j = \Sigma_i K_{ij} \quad \text{or} \quad K^T_i = \Sigma_j K^T_{ij}$$

The sum of all matrix elements, called the norm, is the total knowledge in the population held by all individuals of all other individuals:

$$K = \Sigma_i K_i = \Sigma_i \Sigma_j K_{ij} \quad .$$

This quantity is conveniently normalized to unity:

$$k = K/N^2 \quad .$$

Alternatively, we can remove the self-knowledge N (= number of individuals in the population) and normalize the total pairwise knowledge to unity:

$$k_m = [K - N]/[N^2 - N].$$

This quantity is zero if there is no pairwise knowledge ($K_{ij}=\delta_{ij}$), and 1 if there is complete pairwise knowledge ($K_{ij}=1$).

Eigenvalues and eigenvectors

If all the elements of **K** are all real and positive, the matrix can be diagonalized by a similarity transformation

$$\mathbf{M}\,\mathbf{K}\,\mathbf{M}^{-1} = \aleph \qquad\qquad \aleph_{ij} = \kappa_i \delta_{ij}$$

Furthermore, if **K** is real and symmetric, all eigenvalues κ_i will be real (Horn and Johnson, 1985). Therefore, to the extent that **K** is *not* symmetric, we might expect to see imaginary components of the eigenvalues. Conversely, we might take the presence of imaginary components of the eigenvalues as indication of the asymmetry of **K**. We would say that the appearance of complex eigenvalues indicates that {i} knows more (or less) about {j} than {j} knows about {i}.

Associated with each eigenvalue κ_i is an eigenvector $\mathbf{S}_i$ that satisfies

$$\mathbf{K}\bullet\mathbf{S}_i = \kappa_i\,\mathbf{S}_i$$

If **K** is real and symmetric, all components of the eigenvectors will be real. Again, to the extent that **K** is asymmetric, we will have imaginary components in the eigenvectors.

The eigenvectors $\mathbf{S}_i$ represent pseudo-individuals that have the peculiar property of knowing nothing about any other pseudo-individual. This is obvious from the fact that the transformed matrix is diagonal, i.e., the off-diagonal elements $\aleph_{ij}$ are zero. The rather strange circumstance of being able to associate individuals into noninteracting groups will be mentioned again when we discuss dynamics of populations with knowledge.

VI. INTRODUCING KNOWLEDGE INTO THE DYNAMICS

Having defined the matrix **K**, we are ready to use it to modify a dynamical system described by a state vector **X**. We begin by enunciating a dynamical principle that will have wide, albeit not universal, applicability:

> Given a population described by a dynamical model in which interactions between individuals are specified, the effects of pairwise knowledge are included by weighting the strengths of the interactions by the magnitudes of their pairwise knowledge.

By interactions, we mean potentials, forces, or any other quantity that represents a strength of coupling between pairs of individuals. The choice of simple weighting of the interactions is in the same spirit as the relationship we defined between **K** and **p**: it is the simplest nontrivial such relationship we can imagine. The introduction of **K** into the dynamics can be considered either a weakening of existing interactions ($K_{ij}<1$), or the introduction of new interactions ($K_{ij}>0$).

The introduction of **K** into the dynamics will have two profound effects:

(1) **K** will break the symmetry between the interacting pairs;
(2) **K** will alter the relative strengths of the interactions.

Thus, {i} will experience an effect due to {j} that is not, in general, equal to the effect on {j} due to {i}. Neither effect will be the same as in the original dynamical system.

With the principle stated above, much of the body of analysis that stems from classical dynamics is amenable to modification to include the effects of pairwise knowledge. In the remainder of this section, we give several examples of the application of this principle to general categories of systems. We will first show the original model equations, and then modify them according to the principle.

Hamiltonian systems

Many dynamical systems can be described with a Hamiltonian

$$H = \sum_{i}^{N} T_i + \sum_{i}^{N} \sum_{j \neq i}^{N} V_{ij} = T + V$$

$$\mathbf{H} = \sum_{i}^{N} T_i + \sum_{i}^{N} \sum_{j \neq i}^{N} V_{ij} = \mathbf{T} + \mathbf{V}$$

The term V_{ij} represents the potential energy between $\{i\}$ and $\{j\}$. The force experienced by $\{i\}$ due to $\{j\}$ with this Hamiltonian is

$$\mathbf{F}_{ij} = - \nabla_i V_{ij}$$

In accordance with the principle stated above, we write the modified Hamiltonian as

$$\mathcal{H} = \sum_{i}^{N} T_i + \sum_{i}^{N} \sum_{j \neq i}^{N} K_{ij} V_{ij} = T + (\mathbf{K} \bullet \mathbf{V})$$

where $\mathbf{K} \bullet \mathbf{V}$ is the Hadamard product $(\mathbf{K} \bullet \mathbf{V})_{ij} \equiv K_{ij} V_{ij}$. The modified force is

$$\mathcal{F}_{ij} = - K_{ij} \nabla_i V_{ij}$$

For such systems, the simple prescription is to replace the potential matrix $\mathbf{V}$ by $(\mathbf{K} \bullet \mathbf{V})$. In the limit of full knowledge, $K_{ij}=1$, the dynamical equations return to their unmodified forms. Clearly, $\mathbf{K}$ provides a constraint on the evolution of the system; once the dynamical equation for $\mathbf{K}$ is specified, the collective behavior of the population can be determined.

Phase transitions and stability

The interesting possibility of *phase transitions* in cognitive populations can be investigated by looking for singularities in the equation of state. Normally this is done in the limit $\Omega \longrightarrow \infty$. The "phases" are behavioral patterns, seen in the collective dynamics, so to distinguish them from conventional phase space, which is defined on the configuration (e.g., coordinates and momenta), we refer to them as "behaviors." We might therefore encounter populations that suddenly shift from one behavior to another. A very good example is that of fish schooling [Huth and Wissel, 1992]. At rest, fish mill about with uncorrelated random orientation. When a weak threat is perceived, the fish move away in a highly ordered, oriented structure. If the threat becomes strong, the school will split into two or more smaller schools. If the threat is very strong, the school will disintegrate, and the motion of individuals is again uncorrelated. Because these behaviors are triggered by the introduction of new knowledge in the population, these transitions might be called "knowledge-driven behavioral transitions." While the existence of these transitions in familiar populations is common knowledge, what we have presented in this paper is a procedure for quantitatively predicting them by modifying an existing dynamical model.

Instability and *chaotic behavior* of complex systems are well-known [May, 1975], and it is well-known that it takes very little complexity to produce the possibility of instability. We therefore expect to commonly see reversible and irreversible switching between emergent behaviors in cognitive populations, triggered by a change in the knowledge. For strong coupling, i.e., rapid knowledge exchange, we might expect to see chaotic behavior switching. An unstable system might become stable after introduction of small $\mathbf{K}$, and then become unstable again for large $\mathbf{K}$. In general, we expect to be able to map regions of stability and instability, or any other general descriptor of behavior, as a function of $\mathbf{K}$. For a population in which we track several types of knowledge, $(\mathbf{K}_1 \ldots \mathbf{K}_n)$, a plot of the regions of stability in the $(\mathbf{K}_n, \mathbf{K}_m)$ plane will provide a phase portrait of the emergent behavior as a function of these two quantities.

Multibody interactions

While **K** relates pairs of individuals, it should be noted that the dynamics of complex objects can be far richer and more complex than allowed by pairwise forces. For instance, we might have a system in which no action is taken unless there is an overlap ("collision") of four or more individuals. For example [Huth and Wissel, 1992], individual fish in schools are thought to interact strongly with 4 to 6 nearest neighbors and negligibly with others. Such multibody effects are generally negligible in the physics of simple objects. Here, however, the objects are potentially complex, and we seek a simplified or effective description of their behavior. One class of dynamical equations that captures such multibody effects is that of polynomials:

$$\frac{d\mathbf{X}}{dt} = \sum_n \mathbf{A}_n \bullet \mathbf{X}^n$$

The i^{th} component of the n^{th} term in this series is

$$(\mathbf{A}_n \bullet \mathbf{X}^n)_i = \sum_j \cdots \sum_k (\mathbf{A}_n)_{ij} \; \mathbf{X}_{jp} \cdots \mathbf{X}_{qk}$$

which may be considered to represent an n-body collision. By suitably defining the knowledge matrix **K** and inserting it multiplicatively, we can modify this term to weaken the effect of the collision:

$$(\mathbf{A}_n \bullet \mathbf{X}^n)_i = \sum_j \cdots \sum_k \mathbf{K}_{ij} \; (\mathbf{A}_n)_{ij} \, \mathbf{X}_{jp} \cdots \mathbf{X}_{qk}$$

When the dynamical equation for **K** is specified, the complete system evolution can be determined.

Eigen-individuals

The significance of eigenvectors of **K** was described above: they represent pseudo-individuals constructed from the individuals in such a way that the pseudo-individuals do not interact. This is only possible is the dynamical equations are linear in **K**. Let $\Phi(d/dt)$ be an arbitrary differential operator. The knowledge-modified dynamical equations are

$$\Phi(d/dt)\mathbf{X}=\mathbf{K}\bullet\mathbf{f}(\mathbf{X},t)$$

Now apply a constant matrix transformation **M** that diagonalizes **K**:

$$\Phi(d/dt)\mathbf{M}\bullet\mathbf{X}=\mathbf{M}\bullet\mathbf{K}\bullet\mathbf{M}^{-1}\bullet\mathbf{M}\bullet\mathbf{f}(\mathbf{X},t)$$

Using $\mathbf{S}=\mathbf{M}\bullet\mathbf{f}(\mathbf{X},t)$ and $\aleph=\mathbf{M}\bullet\mathbf{K}\bullet\mathbf{M}^{-1}$ (which is diagonal), we have

$$\Phi(d/dt)\mathbf{S}=\mathbf{K}\bullet\mathbf{S}$$

of which the i^{th} component is

$$\Phi(d/dt)\mathbf{S}_i=\mathbf{K}\bullet\mathbf{S}_i$$

Thus, the equations separate; the eigen-individuals behave as if they had no knowledge of any other eigen-individual.

We reiterate that this analysis is valid only for constant **K**. However, if the pseudo-individuals experience infrequent pairwise collisions (that change **K**), we can use this formalism, updating **K** at every collision, and computing its new eigen-individuals after each collision. The obvious advantage is that the separated equations will be simpler to solve, and may yield to analytic solution. Extrapolation to collision times could then be done analytically, resulting in great savings over numerical integration of the full coupled equations.

VII. NUMERICAL EXAMPLES

In this section, we present several numerical experiments that illustrate the principles developed above for including the knowledge in a dynamical system. These examples are not meant to simulate real ecosystems. We examine several systems described by 2N variables $\{x_i,y_i\}$, i=1...N. These variable can be considered Cartesian coordinates, which are conveniently plotted as trajectories on the (x,y) plane, thus exhibiting directly the emergent behavior, and its modification when the knowledge **K** is introduced. The fact that these point particles do not have sufficient internal structure to maintain and use a memory or perform cognition in anthropomorphic terms is immaterial to these examples.

Complete Knowledge

We first examine a population with complete knowledge. A generic system that serves this purpose is a population of *point chasers* described by the following equations of motion:

$$\frac{dx_i}{dt} = \sum_{j\neq i}^{N} \frac{(x_i - x_j)}{D_{ij}}$$

$$\frac{dy_i}{dt} = \sum_{j\neq i}^{N} \frac{(y_i - y_j)}{D_{ij}}$$

where

$$D_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \ .$$

These equations produce the generic action of *pursuit and flight*. Typically, the motion is unbounded; the individuals rush off to infinity, singly or in groups. We will arbitrarily enclose the individuals in a square or rectangular box and specify that individuals bounce elastically from the flat walls (we will find that clusters of individuals bounce inelastically with the wall).

The equations of motion contain only pairwise interaction terms: there are no terms that resemble a single particle in an external potential, or terms that represent kinetic energy. Therefore, by the simple dynamical principle given above, we can introduce the mutual knowledge by simply multiplying the interactions by the knowledge matrix element K_{ij}:

$$\frac{dx_i}{dt} = \sum_{j \neq i}^{N} K_{ij} \frac{(x_i - x_j)}{D_{ij}}$$

$$\frac{dy_i}{dt} = \sum_{j \neq i}^{N} K_{ij} \frac{(y_i - y_j)}{D_{ij}}$$

Individual {i} may be given complete correct knowledge of {j} by setting $K_{ij}=+1$. This condition causes individual {i} to be *repelled* by individual {j}. Any initial population with all matrix elements K_{ij} =+1 will quickly fly apart to infinity. In the confining box, the individuals rush to the corners and stay pinned there.

Individual {i} may also be given complete *incorrect* knowledge of {j} by setting $K_{ij}=-1$. This condition causes individual {i} to be *attracted* to individual {j}. Any initial population with all matrix elements K_{ij} =-1 will quickly collapse to a compact cluster. In the limit of infinitesimal step size, this cluster has vanishing size, and is stationary. If the step size is finite, the cluster appears to "swarm," and wanders randomly within the box.

More interesting behavior of the population occurs by having some pairs with $K_{ij}=+1$ and others with $K_{ij}=-1$, which produces competition between attraction and avoidance. With two possible values of K_{ij}, there will be two species of individuals in the population, and we arbitrarily define these as male (M) and female (F). Thus, every individual {i} will be either M or F, and the values of K_{ij} will be limited to $K_{ij}=\pm1$ ($i \neq j$) assigned arbitrarily for the four pairs (M,M), (M,F), (F,M), and (F,F). As always, $K_{ii}=1$.

We now examine a specific population for which we define **K** as follows:

K_{ij} =+1 (M,M)
K_{ij}=-1 (M,F), (F,M), (F,F)

Thus, males will be repelled by other males but will be attracted to females, while females will be attracted to both males and females. We would expect these relations to produce chase patterns of emergent behavior, and this is what we find. In a population of 1 male and 1 female (1M+1F), the male pursues the female (who flees) (M,M and F,F interactions are not present). Since their speeds are identical, the pair races around the available space, bouncing from the walls, dashing off in new directions. The motion is precisely straight-line, except for the wall collisions. This behavior is shown in Figure 5(a), which is a time-exposure of the trajectory over a limited time interval. Note that although the (M,F) pair races around the box like a single compact object, its internal structure causes wall collisions violate Snell's Law; the pair emerges from the wall at a different angle from which it approached.

Populations with more individuals but having the values of K_{ij} given above behave similarly. Figures 5(b)-(f) show the behavior of populations of 2M+2F, 1M+3F, 2M+4F, 3M+3F, and 3M+5F. In each of these, the entire population forms a compact cluster that chases around the box. But now the wall collisions are more disruptive to the cluster, and the cluster can spontaneously curve, kink, and form temporary local swarms. All this behavior is consistent with the fact that the individuals now feel forces from several individuals in (inverse) proportion to their separations, introducing enormous complexity into the motion. Although the motion is completely deterministic (there are no random elements anywhere), it is chaotic.

Interestingly, all the systems shown in Fig. 5 have an even number of individuals. Experimentally we found that populations with odd numbers of individuals very quickly froze into a static configuration. Even after disruption by heating, the population immediately refroze. The cause of this is unknown.

Incomplete Knowledge (Randomly Distributed)

We now ask how this system would behave if the pairwise knowledge were less than complete, i.e., $|K_{ij}|<1$. We do this by replacing the off-diagonal 1's in K_{ij} by random numbers:

$K_{ij} = +0 \leq \text{random} \leq 1$	(M,M)
$K_{ij} = -1 \leq \text{random} \leq 0$	(M,F), (F,M), (F,F).

This situation can be thought of as reducing the pairwise knowledge, i.e., weakening the pairwise interactions, while maintaining its sense. Both correct and incorrect knowledge are reduced, but we do not change incorrect knowledge into correct knowledge, and vice versa.

Figure 6 shows a set of such experiments with the population of 2 males and 4 females (2M+4F). Each panel records a short interval of the trajectories in a separate experiment. Each panel continues to evolve in similar patterns forever.

Although the order of these panels in Fig. 6 is immaterial (they were all separate independent experiments), we have ordered the panels roughly according to similar behavior:

Fig. 6(a)	Same conditions as Fig. 5(d): full knowledge ($K_{ij} = \pm 1$)
Fig. 6(b)	Increased cluster order, reduction of wall collision violence
Fig. 6(c)-(d)	A single individual chasing a cluster, and vice-versa
Fig. 6(e)-(j)	Quasi-random walking of clusters
Fig. 6(j)-(l)	Asymptotically stable patterns

Clearly, this system shows a wide range of behavior. It was impossible to predict what kind of behavior a given panel would exhibit; we simply set the **K** matrix elements and ran the experiment. Overall, there appears to be a reduction of the straight parts of the trajectories and enhanced local random curving and swarming in comparison with the case of full knowledge (Fig. 5). This is consistent with the notion that straight patterns are generated by individuals moving together along the same line, clearly an activity that requires the individuals to know a lot about each other. When that knowledge is less than complete, the correlations in their motions are lower, and their trajectories become more chaotic.

Incomplete Knowledge (Nonrandom)

What is the effect of individuals with less than full and correct knowledge in an otherwise fully knowledgeable population? To investigate this, we use the same system as in the previous two sections, but with the matrix elements K_{ij} ($i \neq j$) selected as follows:

$K_{ij} = +1$	(M,M)
$K_{ij} = -1$	(M,F), (F,M)
$K_{ij} = -1 \leq \text{variable} \leq +1$	(F,F).

Thus, we will only vary the amount of attraction or repulsion of females for other females.

Figure 7 shows typical behaviors of a population of 2 males and 4 females. As in Figs. 5 and 6, each panel is a short time-exposure taken from a separate experiment. Now, however, each panel is associated with a specific value of K_{ij}(F,F), hence the sequence is relevant.

In general, we found that the population forms a more-or-less compact cluster that moves around the box with characteristic behavior. We describe these behaviors first in some detail, then in more general terms, attempting to discern patterns and sensitivities. (Arbitrarily we extended the range of K_{ij}(F,F)<-1.0. While this violates the definition of $|K_{ij}| \leq 1$, all it means in this case is that there could have been a multiplicative constant in the equations of motion. Only the product of K_{ij} and that constant enters the equations.).

-1.27	Chaotic wandering within the box. Infrequently, the cluster moved coherently in a circular pattern, but eventually broke away and continued chaotic wandering.
-1.2	All individuals spinning in a circle as a rigid body.
-1.10	Metastable rigid circular rotating pattern, which eventually broke away, just as it did for the -1.27 case.
-1.05...-1.03	Arcing chains, attempting to form limit circles but never succeeding.
-1.01	Nearly linear trainlike motion. The legs have kinks.
-1.00	Nearly linear trainlike motion, bouncing violently against the wall. Bounces resulted in temporary disruption.

-0.98...-0.95	Kinked straight trainlike motion. Infrequently stops to form a long-lived linear pattern.
-0.9	Fixed stable linear pattern, achieved quickly.
-0.8...-0.6	Chaotic wandering. After very long times, these sometimes formed fixed, stable linear patterns.
-0.5	Fixed stable linear pattern.
-0.4	Chaotic wandering. This pattern also probably ended in a fixed stable linear pattern.
-0.3...+0.1	Stable compact rotating circular pattern.
+0.2...+0.4	Chaotic wandering.
+0.6	Spontaneously separated into subclusters that wandered chaotically. The composition (e.g., 2 males +1 female) of these clusters changed intermittently.
+0.8...+0.97	Clusters increasingly maintained fixed compositions, and followed independent trajectories.
+1.0	Population was divided into 3 (male,female) pairs that wandered chaotically and independently.

Thus, we find several general asymptotic emergent behaviors, in rough order of increasing K_{ij}(female,female):

< -1.20	Rotation	Circular limit cycles
-1.2...-1.0	Pursuit	Arcing, segmented, and linear trains
-1.0...-0.8	Stasis	Fixed linear patterns
-0.8...+0.2	Stasis	Fixed circular patterns
+0.2...0.95	Branching	Separate M,F branched patterns
> +0.95	Snaking	Separate M,F sinuous trains

A very striking aspect of these experiments is the great sensitivity of the behaviors to small changes in K_{ij}(F,F) (note especially the region around -1.00). We have done other experiments in which a change of 0.001 in this quantity was enough to produce qualitatively different behavior.

It is interesting that the inventory of emergent behaviors obtained by systematically varying K_{ij}(F,F) is richer than that obtained by selecting all K_{ij} randomly (Fig. 6). We conjecture that unusual emergent behavior is produced by a matrix **K** that is far from some "equilibrium" or "balance," and that random matrices are closer to that equilibrium or balance.

Intermittancy due to Incorrect Knowledge

Populations evolving with complicated dynamics have the potential for undergoing "phase," i.e., behavioral, transitions. To investigate this possibility in the context of mutual knowledge, consider a population of *point vortices* described by a Hamiltonian (Kunin, et al., 1992):

$$H = \sum_{i \neq j} \log(D_{ij})$$

$$D_{ij} = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2}$$

where $\{x_i, y_i\}$ are the canonical coordinates and momenta, respectively. From H we find the Hamiltonian equations of motion:

$$\frac{dx_i}{dt} = \sum_{j \neq i}^{N} \frac{\left(y_i - y_j\right)}{D_{ij}}$$

$$\frac{dy_i}{dt} = -\sum_{j \neq i}^{N} \frac{\left(x_i - x_j\right)}{D_{ij}}$$

Each term in H represents the interaction of one vortex with other vortices. H produces circulating motions in which the vortices orbit around each other.

We now introduce the knowledge **K** into the dynamics. We can easily write the modified Hamiltonian as

$$\mathcal{H} = \sum_{i \neq j} K_{ij} \log(D_{ij})$$

which leads to the modified equations of motion (we use the same symbols x_i, y_i)

$$\frac{dx_i}{dt} = \sum_{j \neq i}^{N} K_{ij} \frac{\left(y_i - y_j\right)}{D_{ij}}$$

$$\frac{dy_i}{dt} = -\sum_{j \neq i}^{N} K_{ij} \frac{\left(x_i - x_j\right)}{D_{ij}}$$

For $K_{ij}=1$, the vortices have full and correct knowledge of each other. Typical behavior of such a population is shown in Figure 8(a), in which 5 vortices are confined within a square box. After a rather lengthy transient, the system achieves a rather steady pattern that rotates like a solid body. Anticipating the next paragraph, we say that the vortices are interacting with .

For $|K_{ij}|<1$, we have "less than full knowledge," and the emergent behavior of the population will be different. If $K_{ij}<0$ for *some* pairs {i,j}, we would say that those pairs have "incorrect knowledge" of each other.

Figure 8(b) shows what happens if we modify K_{ij} to introduce some incorrect knowledge. As in previous cases, we assume the population has some males (M) and some females (F). For these experiments, we assumed a population of (3M+2F). The matrix elements were set to:

$K_{ij}=+1$	(M,M), (F,M), (F,F)
$K_{ij}=-1$	(M,F)

Thus, males have fully correct knowledge of all other individuals, and females have fully correct knowledge of other females, but females have fully incorrect knowledge of males. The motion of males in this population will therefore tend to be as expected (rotating around as a rigid body), but females will tend to move oppositely when interacting with these males. In some sense this is similar to the population of chasers described above: the females run from the males, while the males chase the females.

The result is a surprisingly rich variety of new emergent behavior (Fig. 8(b)). We find that the population switches irregularly between chaotic wandering, jumbled piling against the wall, counter-rotating roughly concentric rings, and stable well-separated counter-circulating loops. The intermittancy of this system and the slow switching between attractors is a familiar expression of a system that is driven into a nonlinear regime, but not so far that it is completely chaotic.

Of course, this example is so simple that it is unnecessary to invoke cognitive ability to produce the modified emergent behavior. The point here was merely to illustrate the process of introducing knowledge into the dynamics. In a more complicated system, for instance a population of small robots, the advantage of introducing **K** is that it lumps considerable complexity into a few simple parameters which reasonably simulate reality. On the other hand, for more complicated individuals, the dynamical equations themselves may be very difficult to write.

Metaindividuals and metapopulations

A collection of individuals that in some way acts as a unit can be called a metaindividual. So long as the collection maintains some integrity and relative constancy of structure, it can exhibit its own characteristic behavior. Other individuals in the population may form into metaindividuals and behave in similar, or different, ways. The collection of metaindividuals constitutes a metapopulation. [Gilpin and Hanski, 1991]. In population biology, metapopulation dynamics is vigorously developed and debated, driven by the fact that many ecosystems in Nature appear to function as metapopulations.

Within the present paradigm of knowledge-modified dynamics, we have found numerous examples of systems that behave like metapopulations. An example is the population of male and female point vortices. Under certain conditions, the {M} and {F} subpopulations separate for long periods and move about the available space relatively autonomously, i.e., as metaindividuals.. Infrequently there is a collision of the metaindividuals, perhaps resulting in total disruption of the population.

Figure 9 shows a time sequence observed with a population of 2M and 2F vortices. The equations of motion are the same as in the previous section. The K_{ij} matrix elements were set to:

$K_{ij}=$	+1.3	(M,M)
$K_{ij}=$	-0.3	(M,F)
$K_{ij}=$	+1.3	(F,M)
$K_{ij}=$	-1.4	(M,F)

The population spontaneously separated in to two ring-like metaindividuals, one containing the 2M vortices, the other containing the 2F vortices. In each ring, the vortices circulated continuously, 2M clockwise, 2F counterclockwise. The 2F ring advanced very slowly on the 2M ring in the corner. Suddenly, the 2F ring extended a filament toward the 2M ring, initiating a major disruption of both rings. The 2M ring threw the 2F individuals backward, arching them high and back toward the wall, where eventually they reformed the 2F ring. Then the process repeated.

Knowledge-Induced Chaos

In some systems, the increase of knowledge between individuals has the effect of more tightly coupling their relative motion, hence increasing the order of the emergent behavior. Can the reverse occur, namely the reduction of order by knowledge increase? We would expect this to be possible, since increasing the number of degrees of freedom often leads to greater complexity, and even chaos. To investigate this process, we examine a population of *predators and prey.*

The Lotka-Volterra equations for a population of N prey species and M predator species are (Murray, 1993; Goel, et al., 1971):

$$\frac{dx_i}{dt} = a_i x_i - \sum_{j=1}^{N} b_{ij} x_i y_j$$

$$\frac{dy_i}{dt} = - c_i y_i + \sum_{j=1}^{M} d_{ij} y_i x_j$$

where (x_i, y_i) are the number of (prey,predator) individuals, respectively. The coefficient a_i represents prey growth rate, c_i represents predator death rate, and b_{ij} and d_{ij} describe the predation severity. In principle, these coefficients could be derived from a detailed microscopic model. These equations do not derive from a Hamiltonian.

In accordance with the general principle for introducing the effects of knowledge into the model equations, we modify those parts of the dynamical equations that represent interactions:

$$\frac{dx_i}{dt} = a_i x_i - \sum_{j=1}^{N} K_{ij} b_{ij} x_i y_j$$

$$\frac{dy_i}{dt} = - c_i y_i + \sum_{j=1}^{M} K'_{ij} d_{ij} y_i x_j$$

Note that **K**, **K'** represent two different kinds of knowledge. The matrices **K**,**K'** might evolve in time according to some externally specified rule or dynamics, or they might be strictly function of the populations (x_i, y_i). In general, **K**$\neq$**K'**. For numerical simplicity we will set $a_i = b_{ij} = c_i = d_{ij} = 1$, and **K**=**K'** Furthermore, we will take N=M, so that we have a population of N pseudo-individuals, each represented by the coordinates (x_i, y_i).

We examine a population of N=4 pseudo-individuals, divided into two subpopulations:

i=1	x=mouse	y=hawk
i=2	x=rabbit	y=fox
i=3	x=antelope	y=tiger
i=4	x=giraffe	y=lion

We also adopt the following knowledge matrix **K**:

	mouse/ hawk	rabbit/ fox	antelope/ tiger	giraffe/ lion
mouse/hawk	+1	-1	+k	+k
rabbit/fox	-1	+1	+k	+k
antelope/tiger	-k	-k	+1	+1
giraffe/lion	-k	-k	+1	+1

These relations mean that the mouse/hawk and rabbit/fox tend to be found together (-1), and the antelope/tiger and giraffe /lion tend to stay apart (+1). For k=0, the subpopulations do not interact. That is, the tiger and lion would not prey on the mouse and rabbit, and the hawk and fox would not prey on the antelope and giraffe. The cross relation K_{ij}=-k will simulate the effect of cross predation (e.g., a fox preying on giraffe). The corresponding cross relation K_{ij}=+k will simulate the effect of protection (e.g., the lion protecting the mouse). We take the magnitudes of these various effects to be the same (k) purely for simplicity in this example; we want to lump all these processes into a single parameter.

The behavior of this system is shown in Figure 10. It is well-known that the Lottka-Volterra equations are unstable (Hoppensteadt, 1982). We find this immediately for k=0 as shown in Fig. 10(a)-(d). In Fig. 10(a), all 4 predator-prey pairs very quickly approach the same familiar limit cycle that appears to be stable. However, at very long time, the 4 cycles separate, 3 shrinking toward the origin (which is in the upper left-hand corner), while the fourth grows away from the origin. In an unbounded domain, this cycle would grow without limit. In the present simulation, the system is confined within a box, and the cycle jams into the corner, eventually approaching a limit cycle (Fig. 10(d)). This system is stabilized by the box.

Now we change k from 0 to 0.1 (Fig. 10(e)). The stability is disrupted, and the limit cycles are broadened into chaotic attractors. Figures 10(f)-(h) show how increasing k to 0.2, 0.4, and 0.6, respectively, increases the disruption of the attractors. The attractors grow in size and become asymmetric as k increases. The motion of the individuals, representing numbers of each species, vary more and more chaotically as k increases. Finally, at k=0.7, the entire system collapses into two points at opposite corners. This represents populations that are essentially all predators or all prey, but not both.

Apparently what happened in this example was that the increased predation of the (hawk,fox) subpopulation on the (antelope,giraffe) subpopulation drove the latter to small numbers. Similarly, the increased protection of the (tiger,lion) subpopulation of the (mouse,rabbit) subpopulation drove the latter to high numbers. Independent of the initial numbers of various species, the final population (with $k \geq 0.7$) is mostly (mouse,rabbit,tiger,lion), with practically no (hawk,fox,antelope,giraffe).

The point of this experiment was to show that increasing a quantity associated with knowledge can drive a stable system into instability and chaos. It is not suggested that this example is in any way a reasonable model for a real ecosystem. The point of this example is to illustrate the kinds of processes that arise by including a quantity associated with knowledge into a population.

In previous examples (vortices, pursuit and flight), the knowledge **K** entered the dynamical equations multiplicatively. This allowed us to use $|K_{ij}| \geq 1$, which just implies a scaling of an overall coupling. In this example, the functional form is more complicated. Hence, the behavior is nontrivially sensitive to the special cases $a_i = b_{ij} = c_i = d_{ij} = 1$ and $\mathbf{K} = \mathbf{K}'$ To do this properly, we would first specify the physical model *without* the knowledge **K**, then insert **K** into the terms representing interactions, maintaining the requirement that $|K_{ij}| \leq 1$.

VIII. RELATION TO OTHER WORK

In knowledge engineering (Harmon and King, 1985), a piece of knowledge is accompanied by a confidence factor that is the probability that the knowledge is correct. The matrix element K_{ij} can be identified as essentially this confidence factor. What we have added in this work is the postulate that the matrix **K** has a dynamics of its own, and can therefore be considered a dynamical quantity.

Many workers are producing surprisingly realistic simulations of collective behavior of real animals. Typical is the work of Huth and Wissel (1992) on fish schools and Millonas (1992) on ant colonies. Phrasing such simulations in terms of pairwise knowledge could be useful. However, this approach is relevant only if the individuals actually interact, which is not always the case in these simulations.

This work is also closely related to "agent-oriented programming" [Shoham, 1993]. Shoham defines an agent as an "entity whose state is viewed as consisting of mental components such as beliefs, capabilities, choices, and commitments." Multiple agents in a population interact to produce emergent behavior. Although these words sound far more general than the kind of object we imagine in this work, Shoham in fact describes them as being precisely defined, hence these words are merely symbols for relatively simple mathematical relationships.

This work is very closely related to parallel distributed processing (PDP) models of cognition (McClelland and Rumelhart, 1989), which are built on a paradigm of activation of knowledge atoms and their assembly into context-sensitive schemata. The evolution of such systems forward in time is described by relations such as

$$a_j(t+1) = F[\Sigma_j W_{ij} \Gamma(a_j(t))] \quad \Rightarrow \Sigma_j W_{ij}\, a_j(t)$$

in which $a_j(t)$ represents the state of the j^{th} knowledge atom, F is a transfer function, Γ is a threshold function, and **W** is a matrix of weights that are the strengths of connections between different atoms. The linear limit of this relation (on the right) emphasizes that the internal knowledge in the system is entirely contained in the weight matrix **W**. Our knowledge matrix **K** corresponds to **W**.

Additional similarity to the PDP paradigm is found in *harmony theory* (Smolensky, 1989). A function H ("harmony") is defined that is a quantitative measure of the self-consistency of a possible state of the system. The probability p that the system is actually in a particular state described by H is

$$p \propto e^{H/T}$$

where T is a "computational temperature" that must be obtained from more fundamental theory, or else treated as a phenomenological parameter. Thus, we find similarity between the PDP expression $p_j(t) = p_j(0)\exp(H_j/T)$ for the probability of $\{j\}$ *being* in a particular state, and our expression $p_{ij}(t) = p_{ij}(0)$ $\exp(I_{ij}/c)$ for the probability of $\{i\}$ *knowing* which state it is.

IX. CONCLUSIONS

In this work we have proposed a dynamical principle for determining the behavior of a collection of objects that have internal structure. The principle derives from an analogy with biological populations in which each individual has partial knowledge of every other individual. The individual dynamics is then altered according to this knowledge, and the emergent collective behavior is likewise altered.

In its simplest form, the prescription is to start from an existing dynamical model, and weight the strength of pairwise interactions by the relative amount of knowledge (normalized to the interval (-1,1)). The pairwise knowledge itself evolves according to a postulated dynamics, which may or may not depend on the system configuration. The formalism provides a means for exactly calculating the emergent collective behavior of the population.

The principle is sufficiently broad that it provides a wide latitude of generalizations, some of which have analogs in Nature. The principle was deliberately constructed to provide a *minimalist* description of the emergent behavior. It will enable vastly larger systems to be modeled than would be possible by attempting to include all physical effects from first principles. It is most appropriate for relatively small populations of minimally cognitive individuals, e.g., a collection of interacting microprocessors or robots, but it can be applied to large populations as well. Because even a minute amount of knowledge is sufficient to produce extremely complex and interesting emergent behavior of the population, the procedure will easily simulate very complex collective behavior. The main motivation for this approach is calculational efficiency: a fast algorithm that produces approximately correct gross collective behavior could allow study of perturbations that will be useful. While the fast algorithm may miss on details, the differential effects of perturbations might be insensitive to such details.

It is emphasized that the principle presented here is not so much a means to account for reality in Nature. Rather, it is a means to mathematically generate emergent behavior based on a physically reasonable model of the individual interaction, namely mutual knowledge. The approach should, however, be useful in the same sense of any model: to predict behavior and then look to see how well it did. In that regard, it may properly be called a model.

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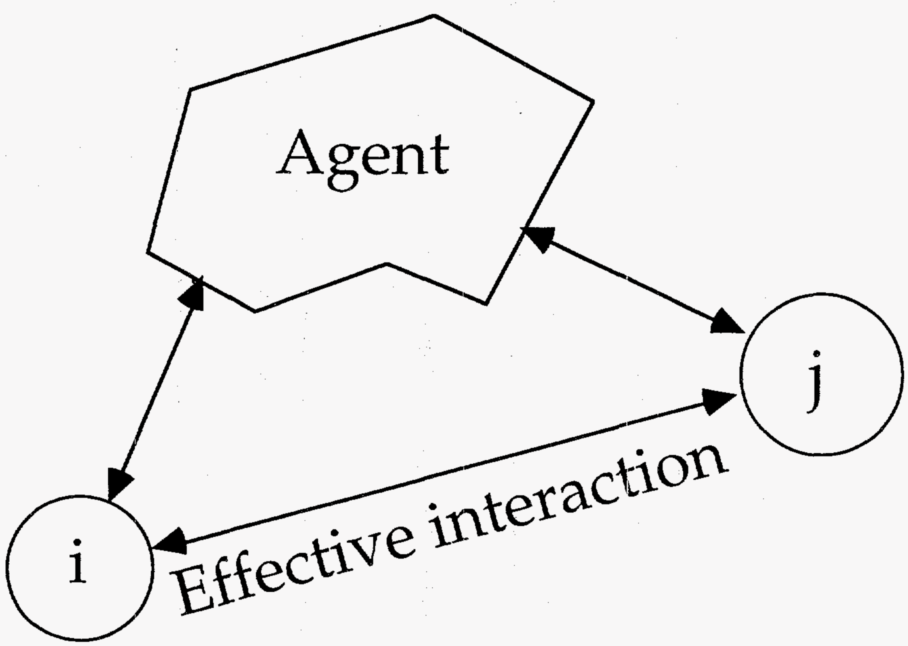


Figure 1 - Agent-mediated effective pairwise interactions.

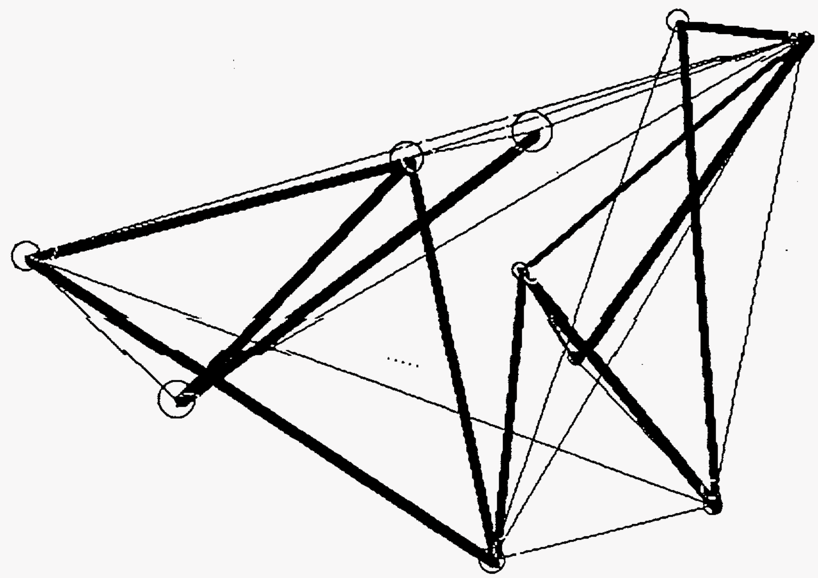


Figure 2 - A connection diagram representation of the mutual knowledge in a population of 10 individuals. The thickness of the lines represents how much each individual knows about other individuals. The agents that produce this knowledge are not specified. The positions of the individuals are arbitrary, but they can easily represent real spatial positions of the individuals.

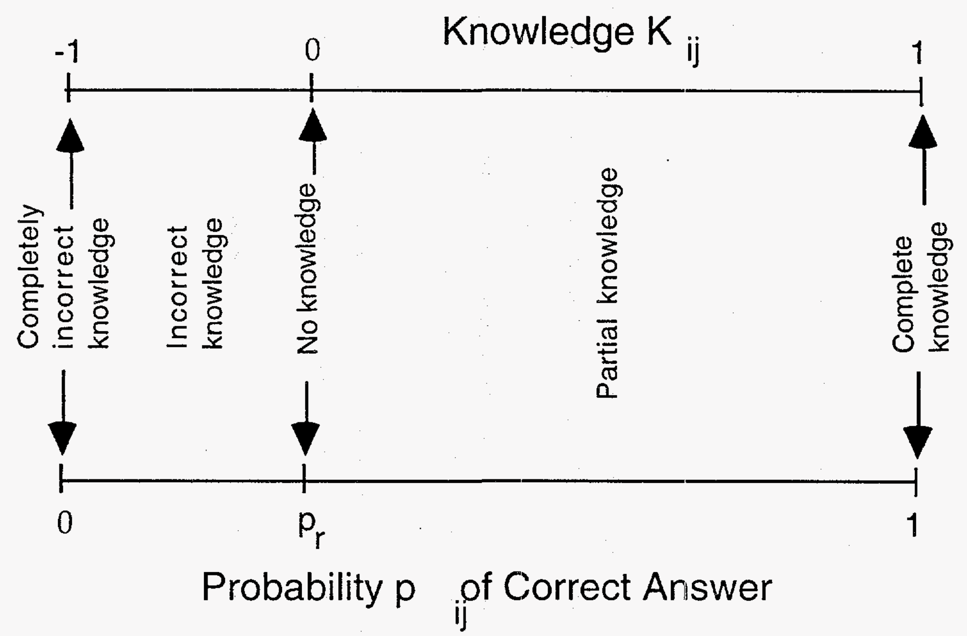


Figure 3 - Correspondence of categories of knowledge with the probability of correctly identifying the state of an individual. The quantity p_r is the random probability. For individuals that can be in any of G discrete states, $p_r = 1/G$.

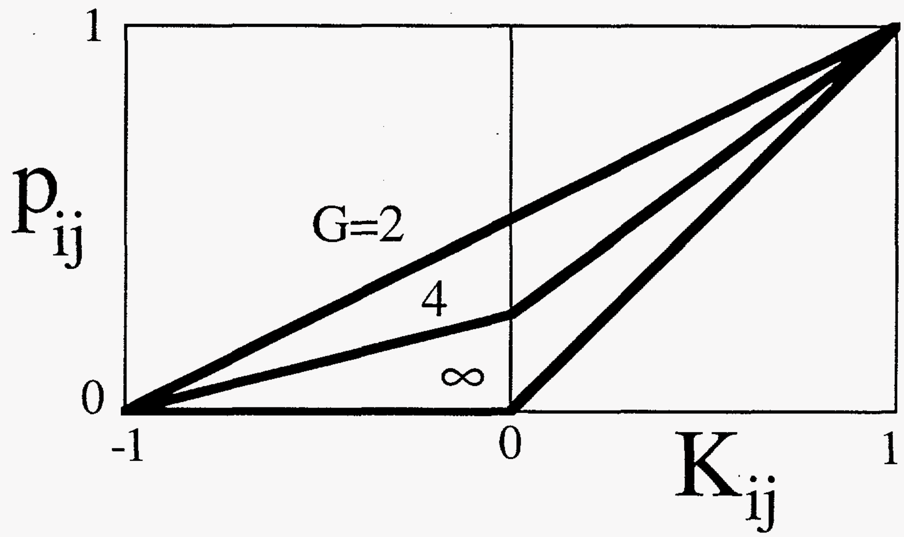


Figure 4 - Functional relationship between the probability p_{ij} that {i} can correctly identify the state of {j}, and the pairwise knowledge K_{ij}.. G is the number of states accessible to {j}.

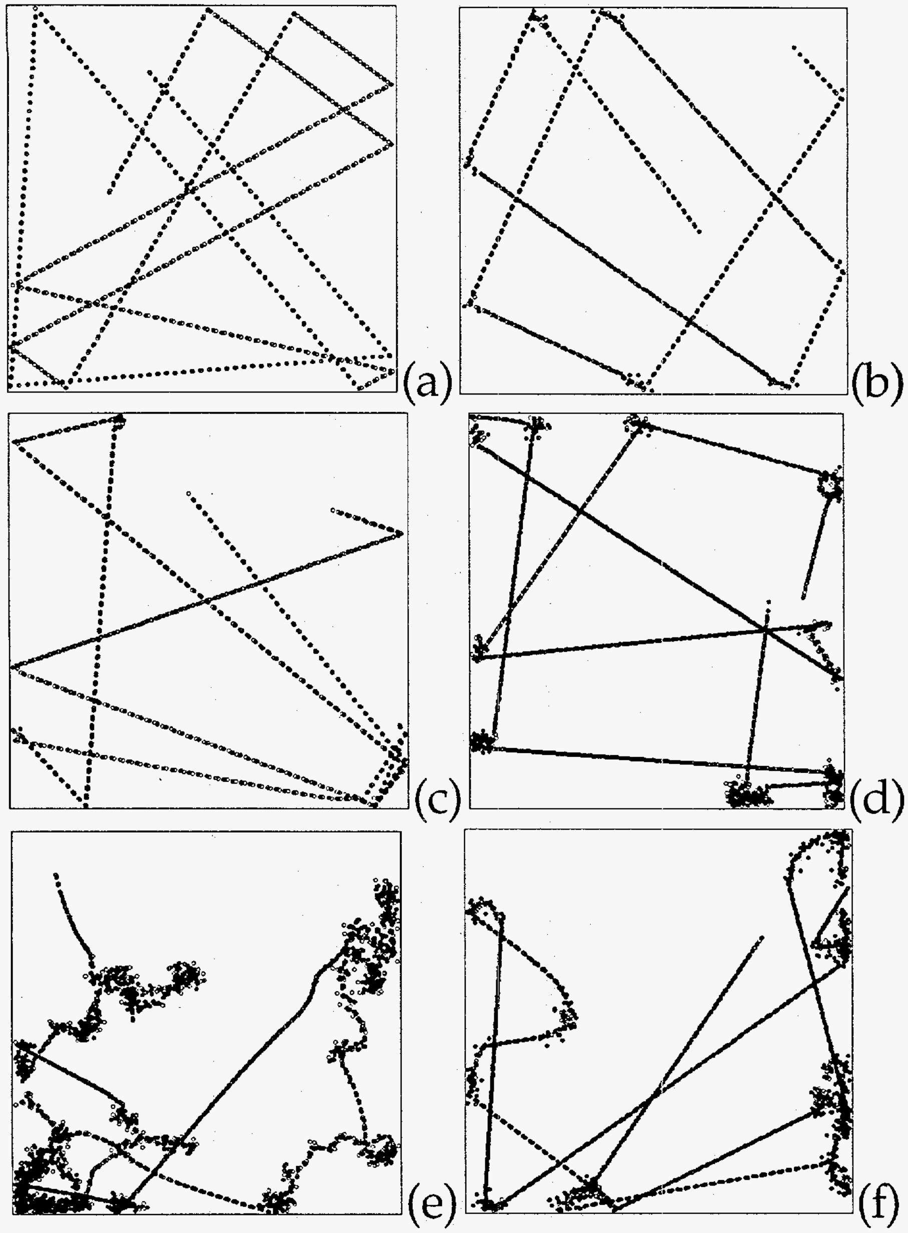


Figure 5 - Behavior of several populations for fully correct knowledge between pairs of males and fully incorrect knowledge for all other pairs. (a) 1M+1F; (b) 2M+2F; (c) 1M+3F; (d) 2M+4F; (e) 3M+3F; (f) 3M+5F. The behavior is essentially chasing around the box, with inelastic wall collisions.

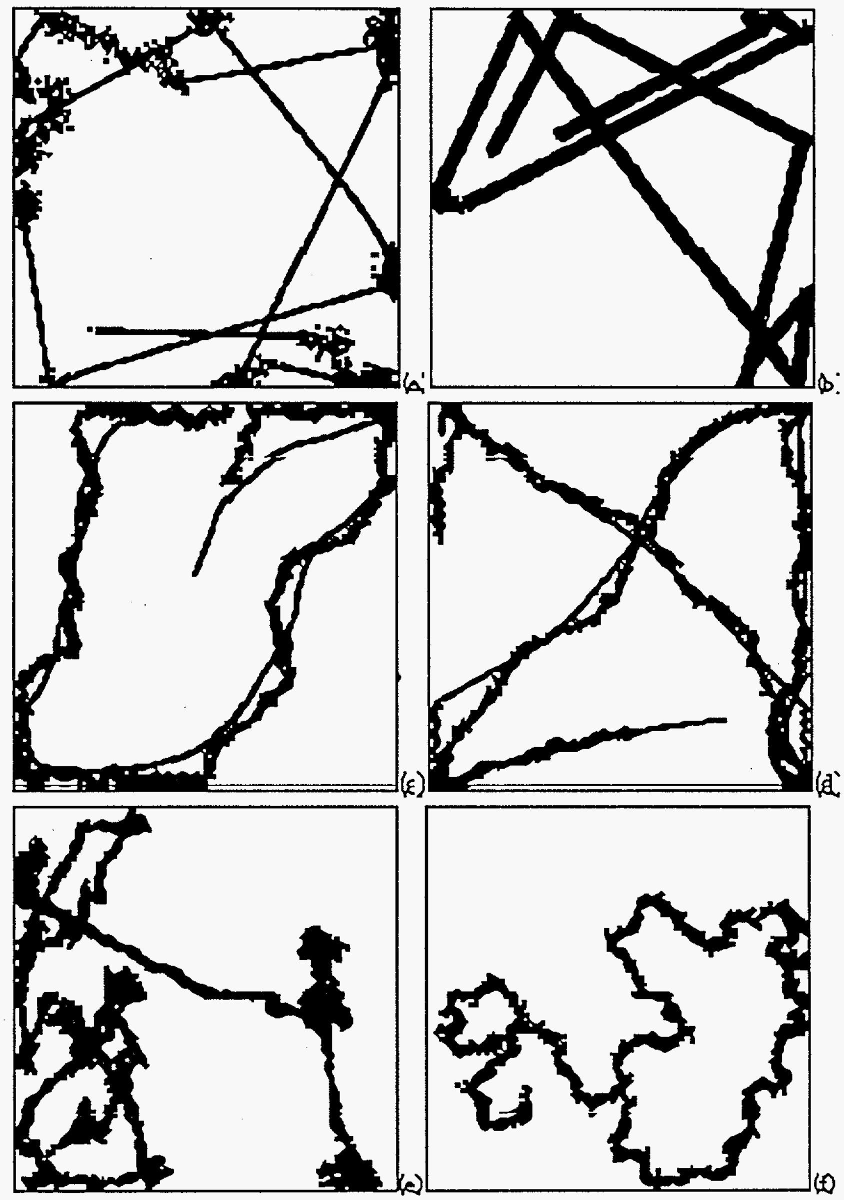


Figure 6 - Motions of 2 male and 4 female individuals in a box.
(a) Complete knowledge.
(b)-(l) Randomly incomplete knowledge.
For each panel, the mutual knowledge K_{ij} was set randomly within (0,1). The motion in each panel continued in the same pattern forever; unlike the vortex system of Fig. 4, it did not switch between several quasi-stationary states. In (c) a single individual pursued the cluster of 5, whereas in (d), the cluster of 5 pursues a single individual.

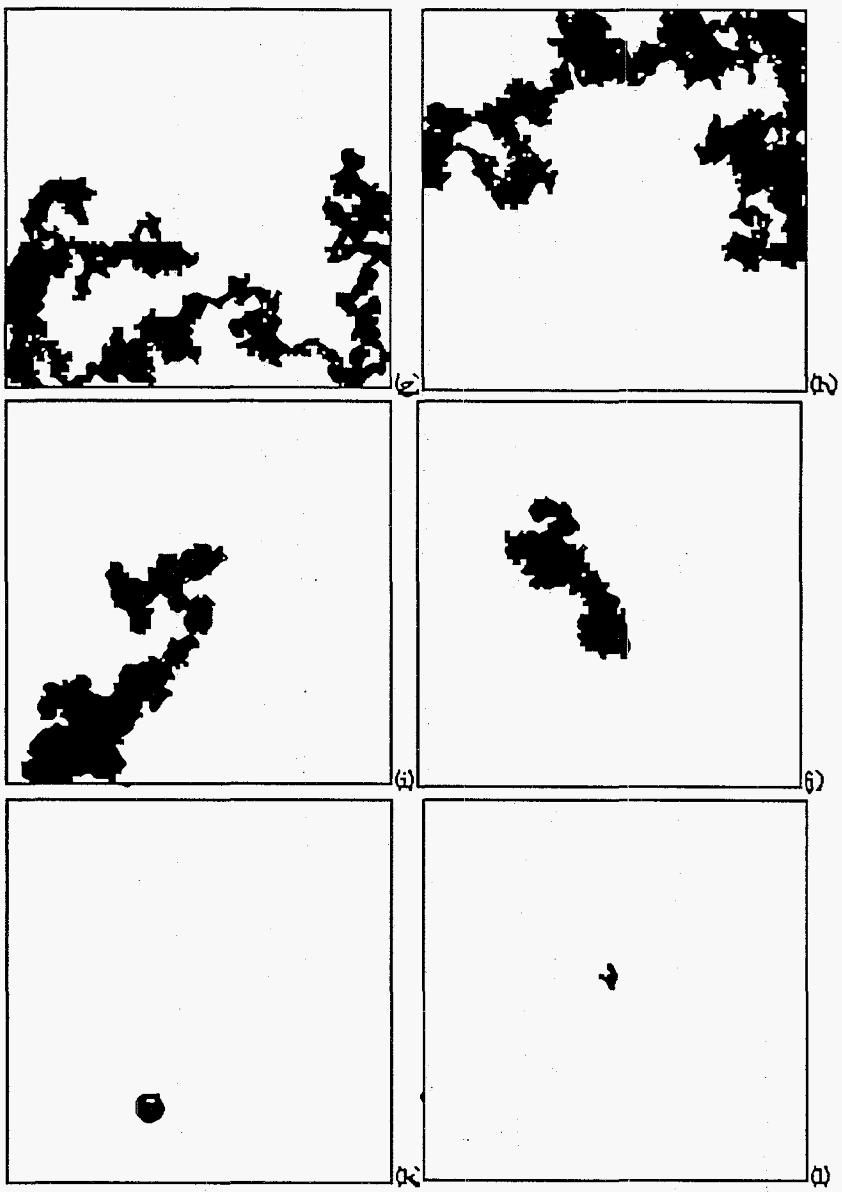


Figure 6 (con't) - In (j) the pattern achieved complete stability; the pattern shows the approach to that final state. In (k) all 6 individuals rotated stably around a single point, presenting an unchanging time-integrated pattern. In (l) the 6 individuals immediately formed a fixed cluster. When dispersed, the cluster immediately reformed into a new fixed pattern.

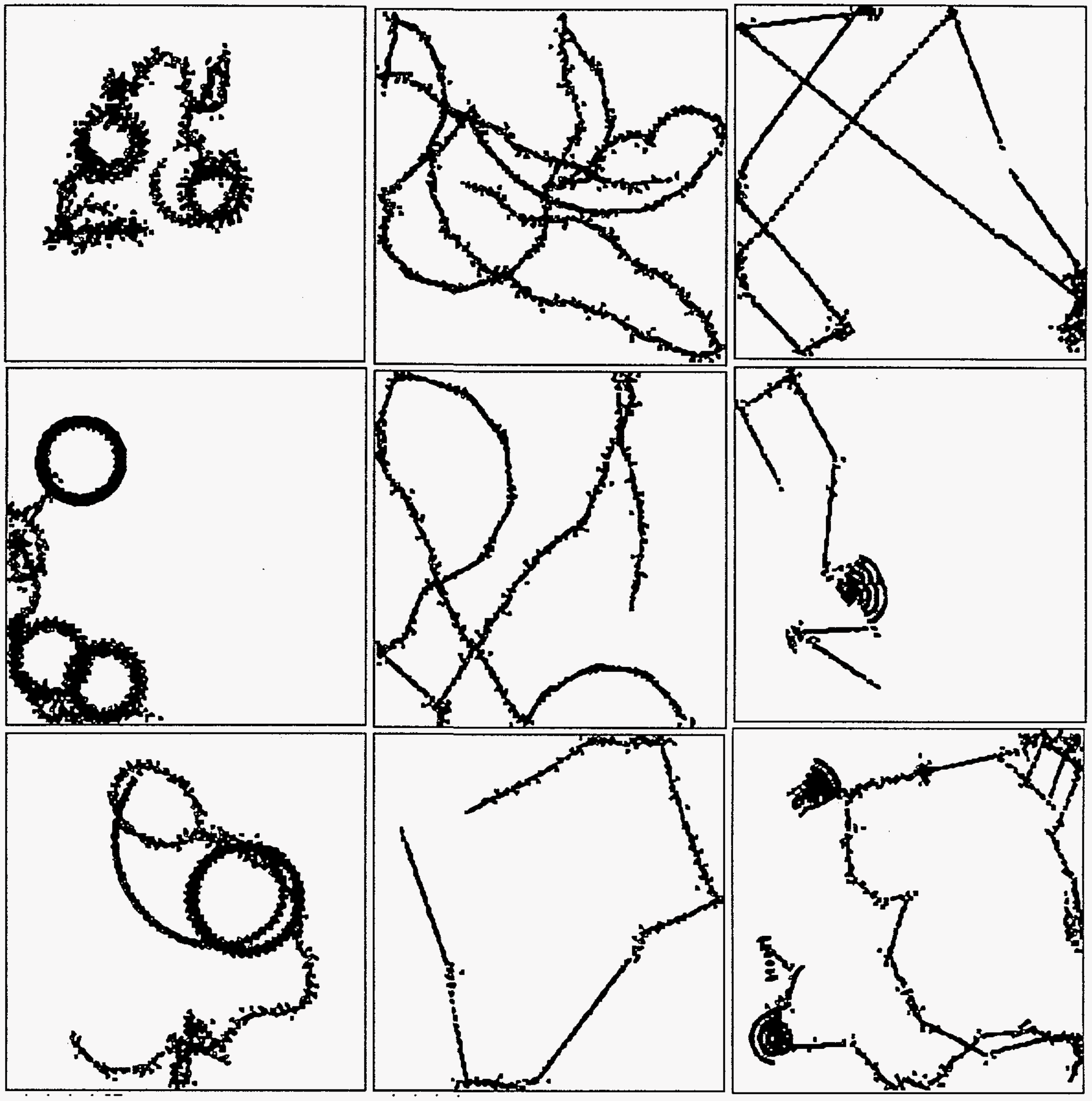


Figure 7 - Motions of 2M and 4F point chasers in a box. For each panel, the mutual knowledge was $K_{ij}=1$ for (M,M) pairs, $K_{ij}=-1$ for (M,F) and (F,M) pairs. For the 9 panels above, the values of K_{ij} for (F,F) pairs were:

-1.27	-1.05	-1.00
-1.20	-1.08	-0.98
-1.10	-1.01	-0.95

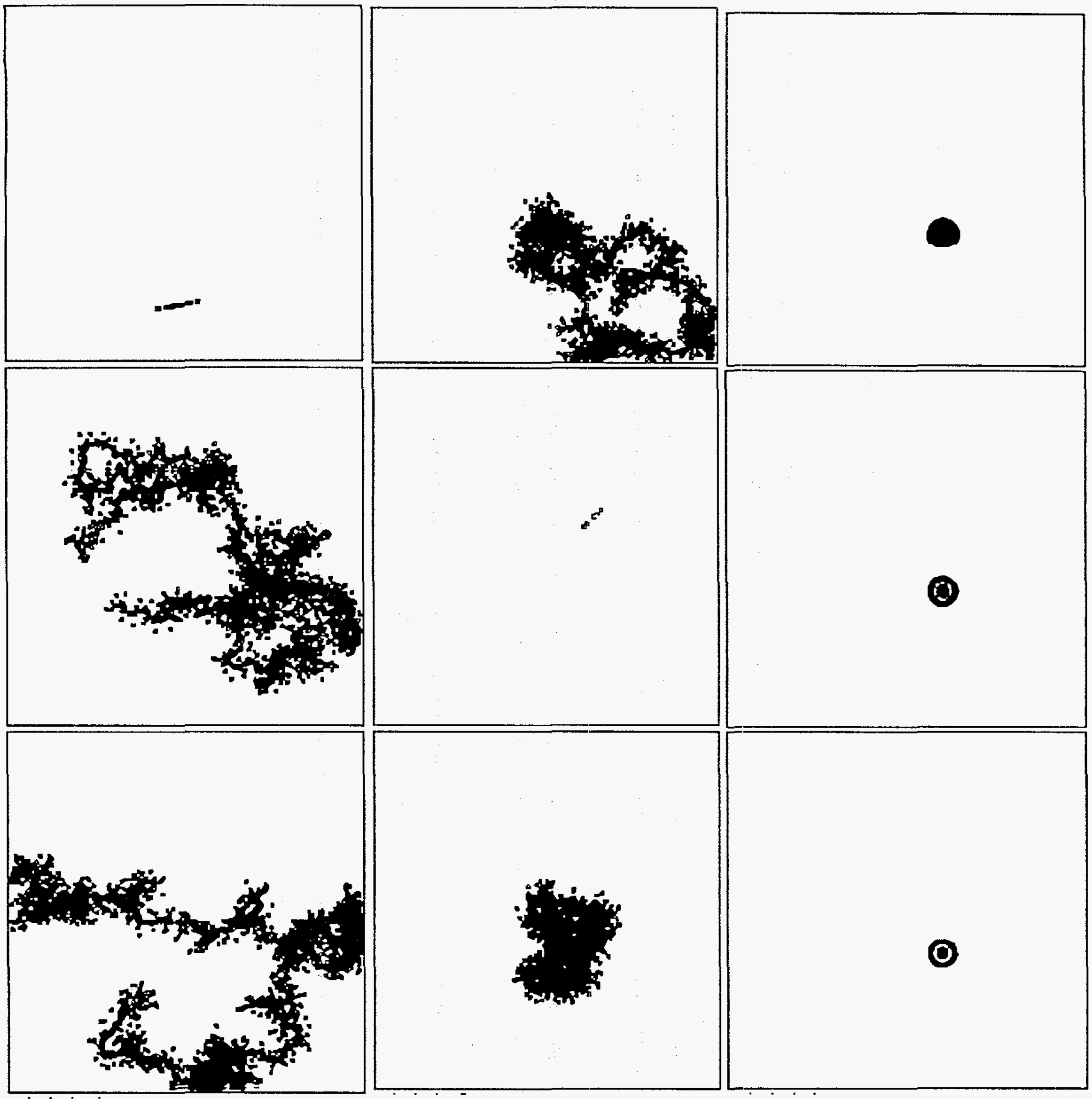


Figure 7 - (con't) For the 9 panels above, the values of K_{ij} for (F,F) pairs were:

-0.90	-0.60	-0.30
-0.80	-0.50	-0.10
-0.70	-0.40	0.00

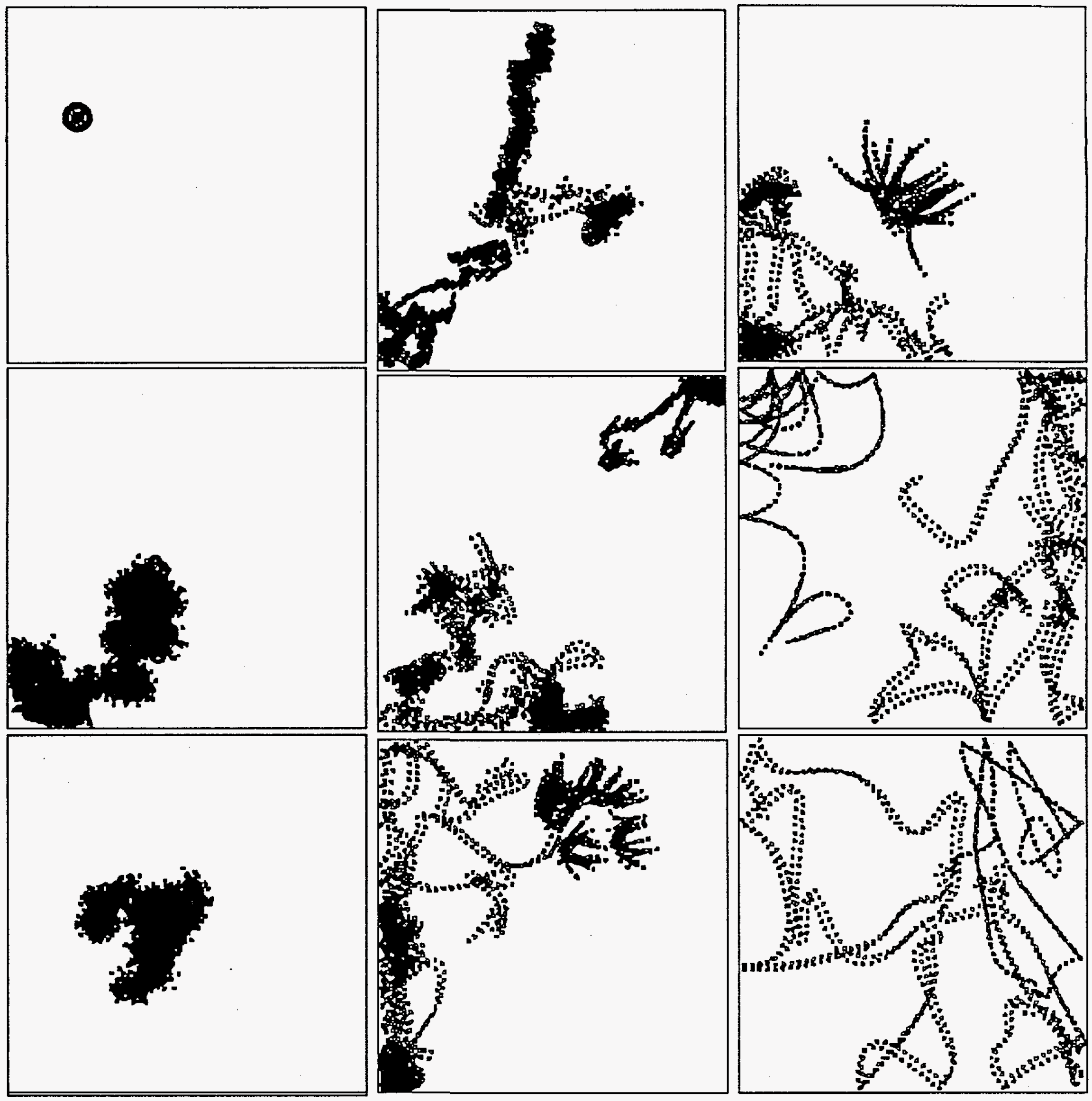
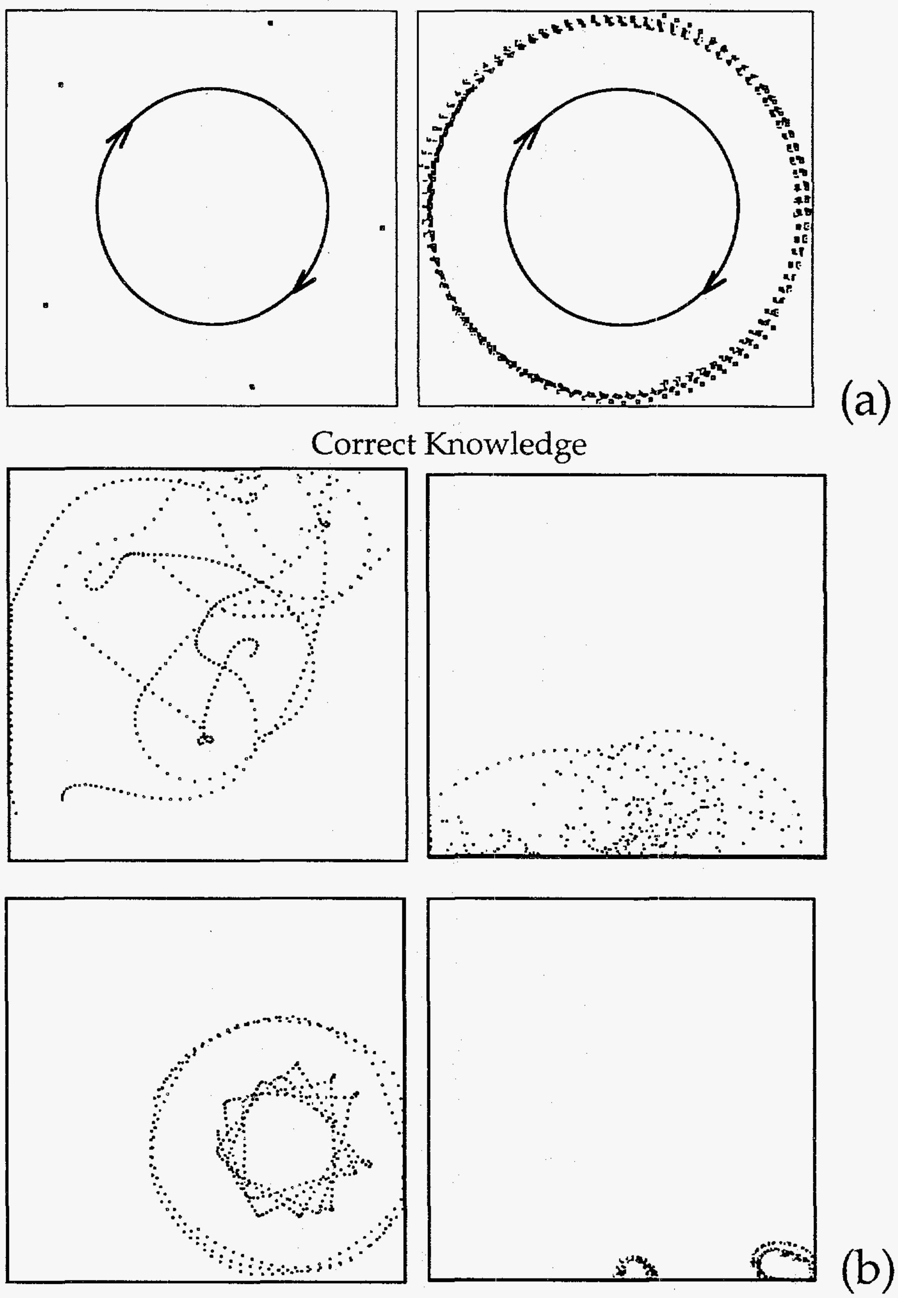


Figure 7 - (con't) For the 9 panels above, the values of K_{ij} for (F,F) pairs were:

+0.10	+0.60	+0.95
+0.20	+0.80	+0.97
+0.40	+0.90	+1.0



Incorrect Knowledge

Figure 8 - Motions of 3M+2F vortices confined in a box.

(a) Correct knowledge, using the unmodified Hamiltonian. The vortices rotate uniformly continuously counterclockwise within the box.

(b) Incorrect knowledge, using the modified Hamiltonian. The motion switches intermitantly between chaotic wandering, jumbled piling, counter-rotating rings, and well-separated counter-spinning rings.

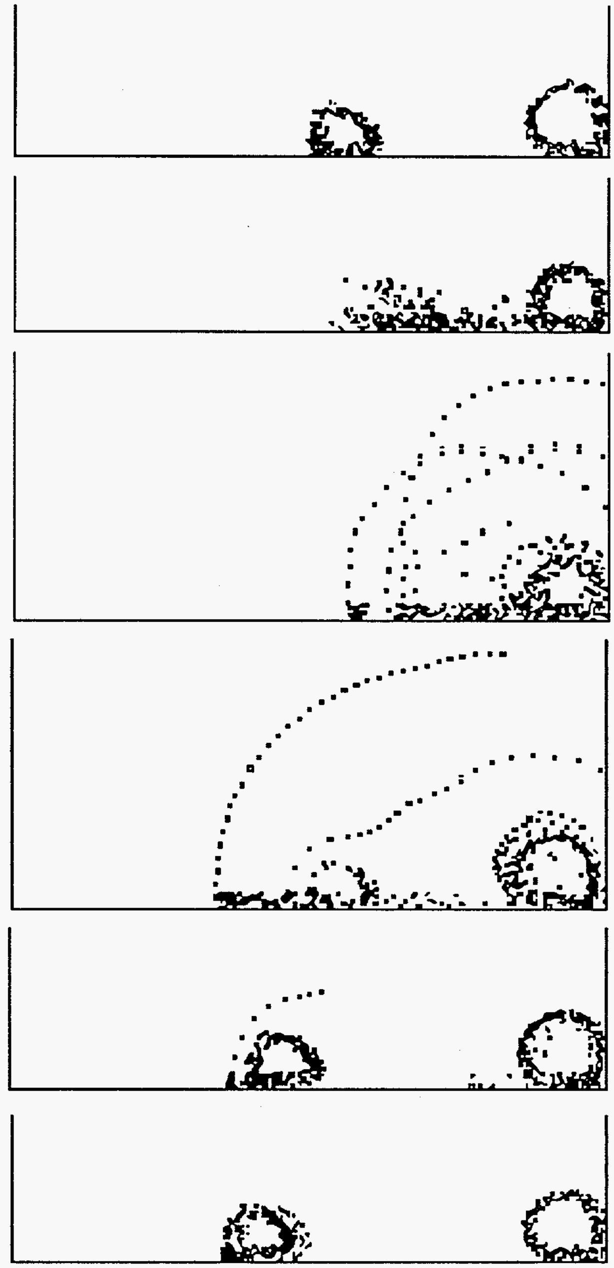


Figure 9 - Cyclic motion of 2M+2F vortices confined in a box (top to bottom). The 2M vortices are in the right ring stabilized in the corner, and the 2F vortices are in the left one. The 2F ring advances slowly on the 2M ring, extends a filament that disrupts both rings, and then reforms again to repeat the process.

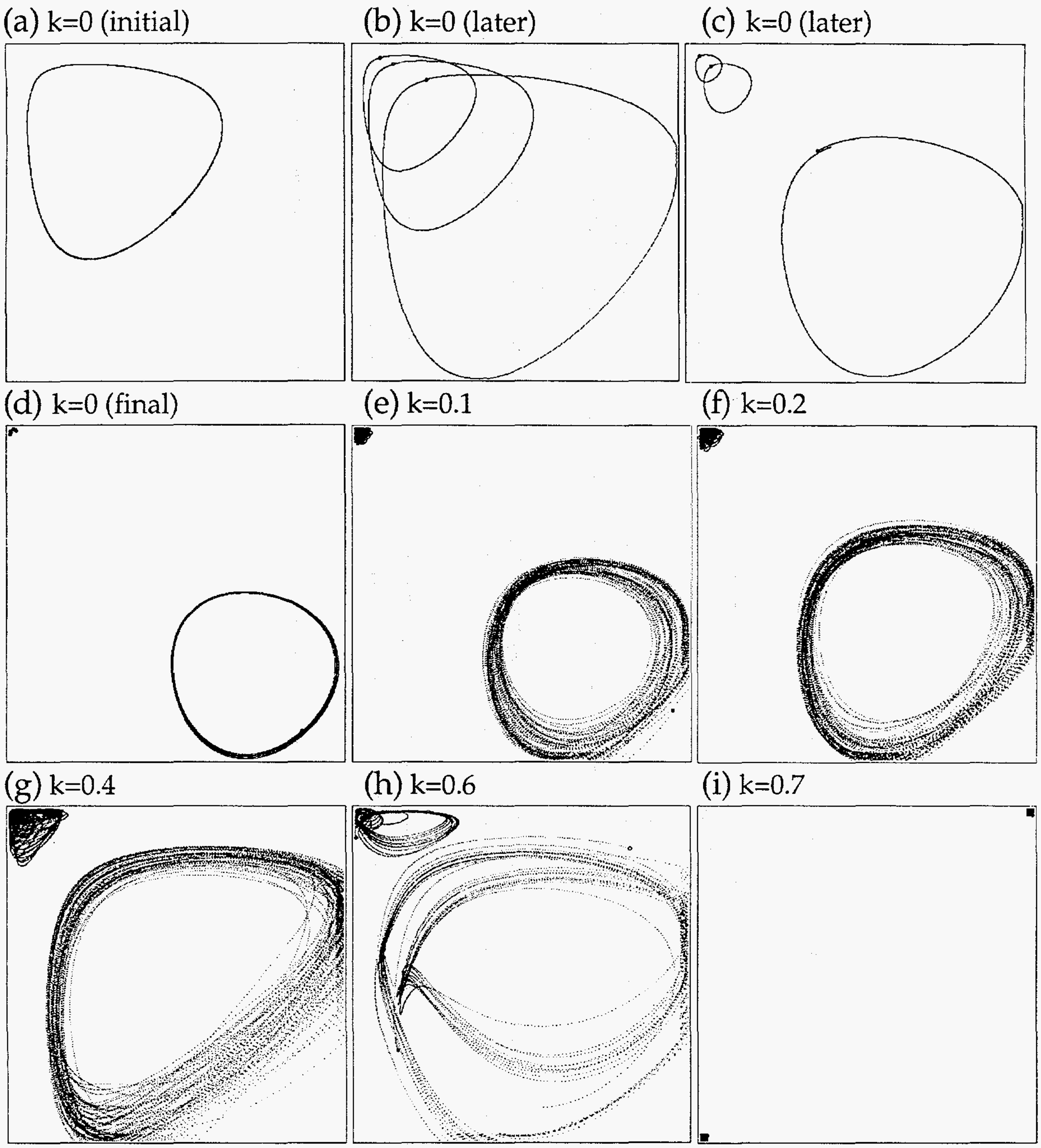


Figure 10 - Effect of knowledge on a community of 4 predator and 4 prey species. The number of predators is plotted vertically (down) against the number of prey, with the origin in the upper left-hand corner.

(a)-(c) k=0. Transient population limit cycles.
(d) k=0. Wall-stabilized limit cycles.
(e)-(g) k=0.1, 0.2, 0.4. Knowledge-induced chaos.
(h) k=0.6. Instability induced by high knowledge.
(i) k=0.7. Collapse to fixed points.

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