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Comments on the Kinetic Dynamo

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Abstract

It is conjectured that transport by parallel mass flow in a braided magnetic field, rather than hyper-resistivity, drives the "dynamo" effect after stochasticity is established. In this paper we do not attempt a rigorous proof of this conjecture, which requires showing that braiding introduces correlations analogous to those giving rise to the neoclassical bootstrap current. We do offer plausible arguments for the conjecture and show that it leads to interesting consequences if true. Namely, magnetic fluctuations would then scale with the magnetic Reynolds number S like $\tilde{B}/B \sim S^{-1/2}$ and the Rechester-Rosenbluth thermal diffusivity like $\chi_e \propto S^{-1}$. This scaling would explain the highest temperatures obtained in the CTX spheromak. It also suggests that a "fully-bootstrapped" current drive experiment could be carried out on the DIII-D tokamak.

1.0 Introduction

An earlier attempt to explain temperatures achieved in the CTX spheromak raised an intriguing question concerning the scaling of magnetic fluctuations, $\tilde{B}$, with the magnetic Reynolds number, S .^{1,2} On the one hand, evidence from RFP's seems to link transport to tearing modes,³ for which Strauss finds $\tilde{B}/B \sim S^{-1/3}$ in the non-linear hyper-resistive regime.⁴ On the other hand, there is evidence for $\tilde{B}/B = 1.87 S^{-1/2}$ in RFP's and assuming this same value for spheromaks gives a much more reasonable fit to CTX than does the hyper-resistive scaling.⁵

Alfven waves, rather than tearing modes, would give $\tilde{B}/B \simeq S^{-1/2}$ as shown in Ref. 1, and tearing modes can cascade to Alfven-like modes of shorter wavelength.^{6,7} However, the evidence from Ref. 3 for the correlation of transport with tearing modes rather than the cascade causes us to seek a different explanation. Resolving this issue would increase our confidence in predicting spheromak behavior. Moreover, $S^{-1/2}$ scaling is more favorable to achieving ohmic ignition in spheromaks, as discussed in Ref. 2, and it favors more rapid penetration of bootstrap current to the magnetic axis in tokamaks.

2.0 Parallel Transport

Perhaps the best theoretical model for transport by low-frequency magnetic turbulence is that of Rechester and Rosenbluth,⁸ who find an electron thermal diffusivity:

$$\chi_e = v_e L_c \frac{\tilde{B}^2}{B^2} \quad (1)$$

where v_e is the electron thermal speed, L_c is a correlation length, and $\tilde{B}$ and B are the magnetic fluctuation and average field, respectively. The fitting to CTX discussed above consists of inserting Eq. (1) into the heat balance,

$$-\nabla \cdot n \chi_e \nabla T = \eta j^2 \quad (2)$$

with the result

$$\beta \frac{\tilde{B}^2}{B^2} \cong \frac{v_A}{v_e} \frac{R}{L_c} S^{-1} \quad (3)$$

Here β is the usual pressure parameter, v_A is the Alfvén speed, and R is the flux conserver radius. Since L_c is unknown, Eq. (3) is used to calculate L_c/R from experimental parameters, taking either $\tilde{B}/B = S^{-1/3}$ (the hyper-resistive estimate) or $\tilde{B}/B = S^{-1/2}$. By far the more reasonable value of L_c is obtained for $\tilde{B}/B = S^{-1/2}$, which fits two available cases ($R = .6m$, $T = 100$ eV and $R = .3m$, $T = 400$ eV) with a single value $L_c/R = 0.3$.⁵ (Both of these cases are collisionless, with $S = 10^5$ to 10^6 , in which case we expect $L_c/R \simeq \pi$,⁸ whereas forcing a fit with $\tilde{B}/B = S^{-1/3}$ implies $L_c/R \leq .01$.) The actual value used in Ref. 5 is $\tilde{B}/B = 1.87 S^{-1/2}$ obtained from RFP data.⁹

The corresponding particle diffusion coefficient is similar to Eq. (1), with v_e replaced by v_i since electrons cannot leak faster than ions,

$$D = 2 v_i L_c \frac{\tilde{B}^2}{B^2} . \quad (4)$$

The factor 2 accounts approximately for the ambipolar linkage of ion and electron fluxes. A particle diffusion rate like Eq. (4) has been observed in RFP's.¹⁰

3.0 The Dynamo

We now turn to the dynamo effect and its relation to magnetic fluctuation levels. One dynamo-type term is $\langle \vec{u} \times \vec{B} \rangle$ obtained upon averaging ohm's law over fluctuations:

$$E_0 + \langle \vec{u} \times \vec{B} \rangle = \eta j . \quad (5)$$

Recall that $\vec{u}$ is the mass velocity,

$$\vec{u} \simeq \vec{u}_i = n^{-1} \int d\vec{v} f_i \vec{v} . \quad (6)$$

The hyper-resistive dynamo is of the order

$$\langle u \times B \rangle \simeq \langle \tilde{u}_1 \times \tilde{B}_1 \rangle \equiv \frac{\gamma}{k_\perp} \frac{\tilde{B}^2}{B} \quad (7)$$

where $\tilde{u}_1$ is the solution of

$$-\gamma A_z + u_{1x} B_{y0} \equiv 0 \quad (8)$$

with $\gamma \tau_A \equiv \tilde{B}/B$ and $\tau_A \equiv R/v_A$.^{1,4} It is the extra factor of $\tilde{B}$ through the hyper-resistive γ that gives $\tilde{B}/B \sim S^{-1/3}$ at saturation.^{1,4}

Applying this same quasi-linear analysis to mass transport, we obtain

$$\frac{\partial \rho_0}{\partial t} + \nabla \cdot \langle \rho_1 \tilde{u}_1 \rangle = 0 \quad (9)$$

where, using the linearized solutions for ρ_1 and $\tilde{u}_1$,

$$\langle \rho_1 \tilde{u}_1 \rangle = \langle \tilde{u}_1 \frac{1}{\gamma} \tilde{u}_1 \rangle \cdot \nabla \rho_0 \quad (10)$$

or

$$D = \langle \tilde{u}_1 \frac{1}{\gamma} \tilde{u}_1 \rangle \equiv \frac{\gamma}{k_\perp^2} \frac{\tilde{B}^2}{B^2} . \quad (11)$$

Again, near saturation it is the hyper-resistive $\gamma \tau_A \sim S^{-1/3}$ that applies.

For any interesting value of v_i , the Rechester-Rosenbluth D in Eq. (4) exceeds the hyper-resistive D in Eq. (11) by a large factor, since $\gamma \sim S^{-1/3}$ is very small. Indeed, the Rechester-Rosenbluth D assumes that $\tilde{B}$ is static, as if $\dot{\gamma} \sim 0$. If some kind of Onsager symmetry holds, one suspects that taking the hyper-resistive dynamo given by Eq. (7), but particle transport given by the Rechester-Rosenbluth rate, Eq. (4), is inconsistent. Particle transport like Eq. (11) implies

$$u_x = -D \left(n^{-1} \frac{\partial n}{\partial x} \right) \quad (12)$$

from which

$$u_x B_{y0} = -D \left(n^{-1} \frac{\partial n}{\partial x} \right) B_{y0} \cong \alpha \frac{2 v_i L_c}{a} \frac{\tilde{B}^2}{B^2} B_{y0} \quad (13)$$

where α is an undetermined parameter. This expression is much larger than the hyper-resistive dynamo in Eq. (2), again because $\gamma \sim S^{-1/3}$ is very small. The saturation fluctuation level is also different (actually smaller), given by

$$u_x B_{y0} = \eta j_z \quad (14)$$

or, using Eq. (13),

$$\frac{\tilde{B}}{B} = \left(\frac{v_A a}{2 v_i L_c} \right)^{1/2} S^{-1/2} \cong \beta_i^{-1/4} S^{-1/2} \quad (15)$$

This estimate applies to the typical circumstance in which ohmic current drive, or helicity injection, or ohmic decay, continuously drives the system away from the stable Taylor state over a large portion of the radial current profile. Since

typically $\beta_i \sim .05 - .1$, $\beta_i^{1/4}$ is order unity and Eq. (15) predicts $\tilde{B}/B \sim S^{-1/2}$, consistent with the RFP data of Ref. 9 and with our fitting of CTX discussed above.

Note that Eq. (3) implies a relatively fixed value of β that is smaller than unity, consistent with CTX and RFP results in the (collisionless) high temperature regime to which Eq. (3) applies. Equation (3) is an ohmic heating limit on β , not a pressure-driven instability limit. That the heating limit on β is less than unity arises from the fact that χ_e in the heat transport equation is much larger (by a factor v_e/v_i) than D appearing in the current transport, Eq. (13). For theories in which $D(\text{mass}) = \chi_e$, Eq. (3) would allow $\beta \simeq 1$ in RFP and spheromak, thereby always driving the system to pressure-driven instability, contrary to experience. The hyper-resistive $\chi = D$ is such a theory.

The kinetic "dynamo" discussed above is better thought of as a fluctuation-driven bootstrap current. For the neoclassical case, Eq. (14) follows from symmetry in the z (toroidal) direction (whereas the even larger poloidal $u_x B_{0z}$ term is balanced by poloidal pressure variations appearing in the generalized ohm's law).¹¹ If symmetry also eliminates toroidal variations in pressure and electrostatic potential in the braided field, Eq. (14) would again apply with u_x given by Rechester-Rosenbluth transport.

Unlike the corresponding neo-classical bootstrap term, Eq. (13) is not proportional to η but rather it contains a variable factor $v_i \tilde{B}^2$ that can grow to large values. Thus our fluctuation-driven bootstrap current can be large even if beta-poloidal is small, as it is in spheromaks. On the other hand, being proportional to the poloidal field that vanishes at the magnetic axis, our bootstrap current, like the neo-classical bootstrap, must be supplemented, but

only very near the axis since the fluctuation bootstrap can be abnormally large off-axis.

Fortunately, Rechester-Rosenbluth transport produces yet another kinetic dynamo effect which, though weaker than the bootstrap, does persist to the axis. This is the viscous current diffusion proposed by Jacobson and Moses.⁶ Including both effects yields a toroidal (z-direction) ohm's law with the bootstrap term ($\propto D$):

$$E_{\theta z} - D \left(n^{-1} \frac{\partial n}{\partial x} \right) B_{\theta y} - \frac{4\pi}{\omega_{pe}^2} \frac{\partial}{\partial x} \chi_e \frac{\partial j_z}{\partial x} - \eta j_z = 0 \quad (16)$$

This expression would be correct for a neo-classical D .¹³

4.0 Application to Tokamaks

Boozer has proposed that the dynamo mechanism be employed to transport neo-classical bootstrap current to the magnetic axis in order to achieve a "fully-bootstrapped" tokamak.^{13,14} A critical issue in order to test this concept is the time scale for current propagation compared to the duration of experiments.

To carry out a fully-bootstrapped experiment on DIII-D requires current propagation across a dimension $a = 0.7$ m in a second or less. Roughly the transport time is, for our kinetic "bootstrap",

$$t \sim \frac{a^2}{4D} = \frac{a^2}{8v_i L_c} (\beta_i^{1/2} \text{ S})$$

where we have used Eqs. (4), (13) and (15). For DIII-D we find, crudely, $t \sim 1$ sec (for $\beta_i = .01$, $S = 10^8$, $v_i = 10^6 \text{ ms}^{-1}$, $L_c = a$). This would indicate a marginal

capability to do a fully-bootstrapped experiment in DIII-D. A measurable fraction of bootstrap current has been produced in DIII-D in earlier experiments. A more accurate estimate requires joining the Jacobson-Moses transport to the bootstrap near the magnetic axis, as discussed in Sec. 3, Eq. (16).

Incidentally, applied to tokamaks, Eqs. (3) and (15) would imply a heating limit on the poloidal $\beta \ll 1$.^{1,2} That $\beta_{\text{pol}} \simeq 1$ is actually achieved in ohmically-heated tokamaks gives evidence for good magnetic surfaces and no dynamo effect. To invoke a dynamo effect in tokamak, Boozer calls upon the hollow current profile that would arise in the absence of external current drive in order to excite tearing modes that give rise to the dynamo.

5.0 Bootstrap Formalism

The above discussion is only suggestive and requires proof to justify the existence of the Rechester-Rosenbluth bootstrap term.

The formally correct procedure is the Generalized Balescud-Lenard (gBL) theory,¹⁵ which extends Kaufman's quasi-linear theory for transport in the space of adiabatically conserved action variables $\vec{J} = (J_1, J_2, J_3)$.¹⁶ The gBL theory adds the drag term to give the appropriate Fokker-Planck equation including collisions and fluctuations simultaneously,

$$\frac{\partial f_0}{\partial t} = \frac{\partial}{\partial \vec{J}} \cdot \left(\vec{D} \cdot \frac{\partial f_0}{\partial \vec{J}} - \vec{F} f_0 \right). \quad (17)$$

The theory automatically exhibits Onsager symmetry.^{15,16,17}

With simplifying assumptions, Mynick and Duvall have successfully used the gBL theory to derive neoclassical and Rechester-Rosenbluth transport from the same formalism.¹⁷

The Mynick-Duvall results for magnetic turbulence transport do include terms that appear to be analogous to those that generate the neo-classical bootstrap current.¹⁸ Even so, since their results depend on yet another conjecture (a “pseudo-thermal” spectrum), one would desire corroboration by identifying in braided fields the microscopic physical processes that generate the current. According to Bernstein and Molvig,¹⁸ the neo-classical bootstrap current arises from the fact that, when passing particles moving in opposite directions along the field are scattered toward the trapped state, they move oppositely in radius. Thus, given a density gradient, a current can be created when an excess of (-) directed particles on an inner flux surface and fewer (+) directed particles on an outer surface scatter toward the same flux surface lying between the two original locations of the particles. The magnetic fluctuation-generated bootstrap current postulated in this paper would require that analogous correlations of scattering events be created by magnetic braiding. One such mechanism is discussed in Appendix A.

We note that, unlike the Jacobson-Moses current viscosity term, the Rechester-Rosenbluth bootstrap term may not conserve helicity. While Bhattacharjee and Hameiri have shown that, within reduced MHD theory, the time-averaged $\langle \mathbf{V} \times \mathbf{B} \rangle$ term does conserve helicity,¹⁹ and Boozer has shown that a helicity-conserving transport term should have divergence form (like the current viscosity term),²⁰ the Rechester-Rosenbluth bootstrap based on particle collisional correlations in a static braided magnetic field may not be included in these theories.²¹ The new effect, analogous to the neoclassical theory mentioned

above and missing from the MHD theory, is a pressure tensor force F_y appearing in the y (poloidal) ohm's law equation:

$$-\frac{\partial A_y}{\partial t} + F_y - u_x B_z = \eta j \quad . \quad (18)$$

With signs chosen to make $K > 0$, the helicity K is then driven by

$$\frac{dK}{dt} = \int 2 F_y B_y - \int 2 \eta \vec{j} \cdot \vec{B} \quad . \quad (19)$$

Finally, unlike the neo-classical bootstrap driven by the ever-present thermal fluctuations, the Rechester-Rosenbluth bootstrap would cease to exist in a stable Taylor state decaying classically and therefore by itself it cannot be depended on to drive a steady state, some portion of which would invariably attain a stable profile.

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Appendix A

Further light can be shed on the origin of bootstrap current by again considering the ohm's law for average field quantities:

$$-\frac{\partial \vec{A}}{\partial t} = \eta \vec{j} - \vec{F}_e - \vec{u}_e \times \vec{B} \quad (\text{A1})$$

where

$$\vec{F}_e = (ne)^{-1} (\nabla \cdot \mathbf{P}_e - ne \nabla \phi) + \langle \mathbf{u}_1 \times \mathbf{B}_1 \rangle \quad (\text{A2})$$

$$\vec{u}_e \equiv B^{-2} \left(-\frac{\partial \vec{A}}{\partial t} \times \vec{B} \right) + (-p^{-1} D \nabla p) + \vec{u}_D \quad (\text{A3})$$

$$D = D_c + D_{NC} \quad (\text{A4})$$

Here $\mathbf{P}_e$ is the electron stress tensor, p is the scalar pressure, $D_c = v_{ei} r_{Le}^2$ is the classical diffusion coefficient, D_{NC} is any non-classical part, and $\vec{u}_D$ is the diamagnetic term that balances the radial component of $\vec{F}_e$, of no further interest.

For D_{NC} , we consider only neo-classical transport,¹¹ or Rechester-Rosenbluth transport in a static braided field,⁸ in which case we may take η to be classical (or neo-classical). We have, however, retained a possible Rechester-Rosenbluth contribution to $\langle \mathbf{u}_1 \times \mathbf{B}_1 \rangle$ averaged over microscopic motion of electrons in the braided field.

For a classical η , the $j_\perp$ and D_c terms cancel in Eq. (A1), leaving

$$-\frac{\partial A_{\parallel}}{\partial t} = \eta j_{\parallel} - F_{e\parallel} \quad (\text{A5})$$

$$F_{e\perp} = D_{\text{NC}} p^{-1} (\nabla p \times \vec{B})_{\perp} \quad (\text{A6})$$

where $\perp$ means perpendicular to $\vec{B}$ but lying in the flux surface. Thus the existence of D_{NC} demands the existence of a force $F_{e\perp}$, but the bootstrap follows only if this force contributes a parallel component in Eq. (A5). For neo-classical transport, caused by toroidal bending of the plasma column, $\vec{F}_e$ is purely poloidal by symmetry, from which we find

$$F_{e\parallel} = \frac{B_{\theta}}{B_z} F_{e\perp} \equiv D_{\text{NC}} \frac{B_{\theta}}{p} \frac{dp}{dr} \quad (\text{A7})$$

(using $B_{\theta} \ll B_z$ as in tokamaks). Substituting banana diffusion,

$$D_{\text{NC}} = \sqrt{\epsilon} \rho_{\theta} e^2 v_{ei} ,$$

gives $F_{e\parallel} = \eta j_B$ where j_B is the familiar tokamak bootstrap current.¹¹ This approximate result follows solely from force balance, symmetry and the heuristic estimate of D_{NC} for neo-classical transport, without detailed knowledge of $\vec{F}_e$.

From this example we see that the conjecture in this paper requires that $\vec{F}_e$ again be purely poloidal for transport in a braided field. The neo-classical argument based on toroidal symmetry no longer applies, since magnetic braiding (due to tearing, etc.) could occur even in a straight cylinder. However, on closer inspection we see that braiding also implies poloidal distortions since braiding can be regarded as a superposition of tiny discharge filaments, each of which is twisted poloidally in the same sense in its attempt to follow the helical lines of the average field. Then we can again estimate the bootstrap current from Eq. (A7)

with D_{NC} given by Rechester-Rosenbluth mass transport, the result being equivalent to Eq. (14) of the main text.

Using Mynick's theory¹⁵ (our Eq. (17)), we find that, for Rechester-Rosenbluth transport, his D term produces the $\vec{u}_{NC} \times \vec{B}$ term of the ohm's law. This implies that his friction term $\vec{F}$ corresponds to $\vec{F}_e$ in the ohm's law, $\vec{F}_{e\perp}$ being required to cancel $\vec{u}_{NC} \times \vec{B}$. Whether or not there is a residual $\vec{F}_{e\parallel}$ to drive the bootstrap in Mynick's theory is a more subtle question not resolved at this writing. However, we note that Mynick has shown that his $\vec{F}$ term does drive an inward particle pinch,²² which Bernstein and Molvig found to be the process that is Onsager-symmetric to the bootstrap current in the neoclassical case.¹⁸ From this we tentatively infer that it is Mynick's friction — actually non-local in the sense that one must average over fluctuation radial wavelengths¹⁷ — that provides the necessary correlations to produce a current.

Dawson²² and Shiang²³ have found similar bootstrap currents driven by electric field turbulence. Ma and Dawson²² conclude that the effect is indeed due to symmetry based on general considerations which, if correct, would apply also to the magnetic turbulence bootstrap postulated here.