FEHMN 1.0: Finite Element Heat and Mass Transfer Code

Los Alamos

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.
Prepared by Kay Coen, Group EES-5

This work was supported by the Yucca Mountain Project Office as part of the Civilian Radioactive Waste Management Program. This Project is managed by the US Department of Energy, Nevada Operations Office.

An Affirmative Action/Equal Opportunity Employer

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither The Regents of the University of California, the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by The Regents of the University of California, the United States Government, or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of The Regents of the University of California, the United States Government, or any agency thereof.
FEHMN 1.0: Finite Element Heat and Mass Transfer Code

George Zyvoloski
Zora Dash
Sharad Kelkar
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, make any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.
# Contents

**FIGURES** .......................................................... vi

**TABLES** ........................................................... vii

**ABSTRACT** ......................................................... 1

**I. MODEL DESCRIPTION** ........................................... 1

A. Nature and Purpose ............................................. 1

B. Mathematical Formulation ..................................... 2

C. Discretization of the Governing Equations .................. 14

D. Boundary Conditions ........................................... 17

E. Solution Methods ............................................... 17

F. Dual Porosity Formulation .................................... 21

**II. CODE DESCRIPTION** ........................................... 25

A. Subroutines .................................................... 25

B. Common Block and Parameter Statements ..................... 33

C. Subroutine Structure for FEHMN ............................. 47

**III. USER'S MANUAL** ............................................. 51

A. Automatic Mesh Generation ................................... 51

B. Macro Command Input for FEHMN ............................. 56

C. Graphics Postprocessing ...................................... 76

D. Example Input Files and Output Files ...................... 79

**ACKNOWLEDGMENTS** .............................................. 111

**REFERENCES** .................................................... 111
FIGURES

1. Comparison of nodal connections for conventional and Lobatto integrations. ... 16
2. Matrix band structures showing second-order fill-in.
   x - original matrix positions.
   o - fill-in with order 2 factorization. ....................... 20
3. Computational volume element showing dual porosity parameters .......... 22
4. Structure for subroutine calls for FEHMN. ........................... 48
5. Elements available with FEHMN in 2-D and 3-D problems showing
   numbering convention. ................................... 52
6. Nodal ordering for defining 2-D and 3-D domain blocks for GENMSH Input. 53
7. Terminal output for a FEHMN run on a CRAY computer. ..................... 59
8. Schematic diagram of the 3-D heat conduction problem. .......................... 84
9. Input for preprocessor GENMESH for 3-D heat conduction example. .... 84
10. Grid for 4 x 4 x 4 heat conduction problem. ................................... 85
11. Input file for FEHMN for 3-D heat conduction example. ..................... 86
12. Computer output for the 3-D heat conduction example. ..................... 90
13. Instruction file (HIS.INS) for postprocessor FEHPLTR for the heat
    conduction problem. ...................................... 92
14. History plot for the heat conduction problem. ............................... 93
15. Solution domain and results for the Toronyi example. .......................... 94
16. Input for GENMSH for the Toronyi example. .................................... 94
17. Input file for FEHMN for Toronyi example. ................................... 95
18. Output for the Toronyi example. ..................................... 98
19. Schematic diagram of the geometry and boundary conditions for
    the DOE code comparison project problem. ............................ 100
20. Comparison of FEHMN production well temperatures with results
    from other codes. ........................................ 100
21. Comparison of FEHMN production well pressure with results from other codes. 101
22. Comparison of FEHMN observation well pressure with results from other codes. 101
23. Input for GENMSH for the DOE example. ................................... 102
24. Input for FEHMN for the DOE example. ................................... 102
25. Output from FEHMN for the DOE example. .................................... 108
26. Instruction file (CON.INS) for the postprocessor FECPLTR for
    the DOE problem. ......................................... 109
27. Contour plot of temperature for the DOE problem. ............................. 110
# TABLES

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Notation</td>
<td>3</td>
</tr>
<tr>
<td>II</td>
<td>Sorption Isotherm Models</td>
<td>10</td>
</tr>
<tr>
<td>III</td>
<td>Subroutine Calls Within FEHMN</td>
<td>25</td>
</tr>
<tr>
<td>IV</td>
<td>Capabilities of FEHMN with Macro Command References</td>
<td>57</td>
</tr>
<tr>
<td>V</td>
<td>FEHMN Files</td>
<td>57</td>
</tr>
<tr>
<td>VI</td>
<td>Macro Control Statements for FEHMN</td>
<td>60</td>
</tr>
<tr>
<td>VII</td>
<td>Input Description for FEHMN</td>
<td>62</td>
</tr>
<tr>
<td>VIII</td>
<td>Input Parameters for the 3-D Heat Conduction Problem</td>
<td>80</td>
</tr>
<tr>
<td>IX</td>
<td>Sample Problems for 3-D Heat Conduction</td>
<td>81</td>
</tr>
<tr>
<td>X</td>
<td>Comparison of Analytical and Model Results for 3-D Heat Conduction</td>
<td>81</td>
</tr>
<tr>
<td>XI</td>
<td>Input Parameters for Toronyi Example</td>
<td>82</td>
</tr>
<tr>
<td>XII</td>
<td>Input Parameters for the DOE Code Comparison Project</td>
<td>83</td>
</tr>
</tbody>
</table>
FEHMN 1.0:
FINITE ELEMENT HEAT AND MASS TRANSFER CODE

by
George Zyvoloski, Zora Dash, and Sharad Kelkar

ABSTRACT

A computer code is described which can simulate non-isothermal multiphase multicomponent flow in porous media. It is applicable to natural-state studies of geothermal systems and ground-water flow. The equations of heat and mass transfer for multiphase flow in porous and permeable media are solved using the finite element method. The permeability and porosity of the medium are allowed to depend on pressure and temperature. The code also has provisions for movable air and water phases and noncoupled tracers; that is, tracer solutions that do not affect the heat and mass transfer solutions. The tracers can be passive or reactive. The code can simulate two-dimensional, two-dimensional radial, or three-dimensional geometries. A summary of the equations in the model and the numerical solution procedure are provided in this report. A user’s guide and sample problems are also included.

I. MODEL DESCRIPTION

A. Nature and Purpose

The FEHMN (Finite Element Heat and Mass Nuclear) code, described in this report, is a version of FEHM (Finite Element Heat and Mass, Zyvoloski et al., 1988) developed for the Yucca Mountain Project (YMP) and documented as required by NUREG-0856. This report satisfies the document requirements for model description and user’s manual as presented in the LANL YMP Computer Software Control Quality Procedure (SQAP-3.7, R0). The verification and validation reports are produced as separate documents. The main use of FEHMN will be to assist in the understanding of flow fields in the saturated zone below the proposed Yucca Mountain Repository. This is referred to as the C-Wells project (YMP-LANL-SP-8.3.1.2.3.1.7). Also in regards to that project, the code will be used to design tracer tests (reactive and non-reactive) to characterize the flow field below Yucca Mountain. In addition, FEHMN will be used to study coupled processes (multicomponent and natural convection) in the unsaturated zone (YMP-LANL-SCP-8.3.1.3.7.1). We note here that the model requirements may be found in the above mentioned study plans.
Yucca Mountain is extremely complex both hydrologically and geologically. The computer codes that are used to model flow must be able to describe that complexity. For example, the flow at Yucca Mountain, in both the saturated and unsaturated zones is dominated by fracture and fault flow in many areas. With permeation to and from faults and fractures, the flow is inherently three-dimensional (3-D). Birdsell et al. (1990) recently presented calculations showing the importance of 3-D flow at Yucca Mountain. Coupled heat and mass transport occurs in both the unsaturated and saturated zones. In the far field unsaturated zone, Weeks (1987) has described natural convection that occurs through Yucca Mountain due to seasonal temperature changes. Heat and mass transfer are important in the saturated zone modeling of temperature logs and pressure tests.

The governing equations and solution techniques used in the FEHMN code are described below. In addition, a user's manual with example problems is provided. Future changes to the code and documentation will be made in accordance with quality assurance procedures being established for the Yucca Mountain Project.

B. Mathematical Formulation

1. Governing equations. Detailed derivations of the governing equations for two-phase flow including heat transfer have been presented by several investigators (Mercer et al., 1974, Mercer and Faust, 1975, and Brownell et al., 1975, for example), and therefore only a brief development will be presented here. The notation used in this report is given in Table I.

Conservation of mass is expressed by the equation

$$\frac{\partial A_m}{\partial t} + \nabla \cdot \bar{f}_m + q_m = 0 \quad ,$$

where the mass per unit volume, $A_m$, is given by

$$A_m = \phi (S_v \rho_v + S_\ell \rho_\ell) \quad ,$$

and the mass flux, $\bar{f}_m$, is given by

$$\bar{f}_m = \rho_v \bar{V}_v + \rho_\ell \bar{V}_\ell \quad .$$

Here $\phi$ is the porosity of the matrix, $S_v$ and $S_\ell$ are saturations, $\rho_v$ and $\rho_\ell$ are densities, and $\bar{V}_v$ and $\bar{V}_\ell$ are velocities with the subscripts $v$ and $\ell$ indicating quantities for the vapor phase and the liquid phase, respectively. Source and sink terms (such as bores, reinjection wells, or groundwater recharge) are represented by the term $q_m$.

Conservation of fluid-rock energy is expressed by the equation

$$\frac{\partial A_e}{\partial t} + \nabla \cdot \bar{f}_e + q_e = 0 \quad .$$
<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>parameter used in nonlinear adsorption model (Freundlich, modified Freundlich)</td>
</tr>
<tr>
<td>$[A]$</td>
<td>solution matrix for system of nonlinear equations</td>
</tr>
<tr>
<td>$A_c$</td>
<td>accumulation term for tracer ($M/L^3$)</td>
</tr>
<tr>
<td>$A_e$</td>
<td>energy accumulation term ($M^2/L$)</td>
</tr>
<tr>
<td>$A_m$</td>
<td>mass accumulation term ($M/L^3$)</td>
</tr>
<tr>
<td>$\tilde{A}$</td>
<td>approximation of matrix $[A]$</td>
</tr>
<tr>
<td>$a$</td>
<td>weighting factor for time discretization</td>
</tr>
<tr>
<td>$a_1, a_2, a_3, a_4$</td>
<td>coefficients used in reaction rate model</td>
</tr>
<tr>
<td>${b}$</td>
<td>right hand side (forcing function) for system of linear equations</td>
</tr>
<tr>
<td>$C_t$</td>
<td>tracer concentration in liquid</td>
</tr>
<tr>
<td>$C_R$</td>
<td>tracer concentration adsorbed on rock</td>
</tr>
<tr>
<td>$C_v$</td>
<td>tracer concentration in vapor</td>
</tr>
<tr>
<td>$C_{max}$</td>
<td>parameter used in nonlinear adsorption model (modified Freundlich)</td>
</tr>
<tr>
<td>$[C]$</td>
<td>capacitance matrix</td>
</tr>
<tr>
<td>$C_{up}^{ij}$</td>
<td>upwind value of tracer concentration</td>
</tr>
<tr>
<td>$c_g$</td>
<td>parameter used in linear porosity model</td>
</tr>
<tr>
<td>$c_p$</td>
<td>heat capacity of air $L^2/\theta^2 T$</td>
</tr>
<tr>
<td>$c_r$</td>
<td>parameter used in linear porosity model</td>
</tr>
<tr>
<td>$c_{p1}, c_{p2}$</td>
<td>parameters used in the capillary pressure models</td>
</tr>
<tr>
<td>$c_{p3}, c_{p4}$</td>
<td>parameters used in the capillary pressure models</td>
</tr>
<tr>
<td>$D_c$</td>
<td>dispersion coefficient for tracer ($L^2/\theta$)</td>
</tr>
<tr>
<td>$[D_c]$</td>
<td>finite element coefficients for tracer dispersion term</td>
</tr>
<tr>
<td>$D_{el}$</td>
<td>energy transmissibility term for liquid ($L^2/\theta$)</td>
</tr>
<tr>
<td>$D_{e,v}$</td>
<td>energy transmissibility term for vapor ($L^2/\theta$)</td>
</tr>
<tr>
<td>$D_{mt}$</td>
<td>mass transmissibility term for liquid ($\theta$)</td>
</tr>
<tr>
<td>$D_{m,v}$</td>
<td>mass transmissibility term for vapor ($\theta$)</td>
</tr>
<tr>
<td>$D_{up}$</td>
<td>upwind energy transmissibility term ($L^2/\theta$)</td>
</tr>
<tr>
<td>$D_{m}^{up}$</td>
<td>upwind mass transmissibility term ($\theta$)</td>
</tr>
<tr>
<td>${e}$</td>
<td>unit vector</td>
</tr>
<tr>
<td>$E$</td>
<td>Young's modulus ($ML/\theta^2$)</td>
</tr>
<tr>
<td>$F$</td>
<td>function representation</td>
</tr>
<tr>
<td>$|F|$</td>
<td>$L^2$ norm of residuals</td>
</tr>
<tr>
<td>$[F_c]$</td>
<td>residual for tracer equation</td>
</tr>
<tr>
<td>$F_e$</td>
<td>flux vector for energy equation ($M/\theta^3$)</td>
</tr>
<tr>
<td>$F_m$</td>
<td>flux vector for mass equation ($M/L^2$)</td>
</tr>
<tr>
<td>$\overline{F(x)}$</td>
<td>vector of equation residuals</td>
</tr>
<tr>
<td>${F_e}$</td>
<td>residual for energy equations</td>
</tr>
<tr>
<td>${F_m}$</td>
<td>residual for mass equation</td>
</tr>
<tr>
<td>$f(y)$</td>
<td>intermediate calculation in GMRES acceleration</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration of gravity ($L/\theta^2$)</td>
</tr>
<tr>
<td>$[H]^m$</td>
<td>intermediate calculation in GMRES acceleration</td>
</tr>
</tbody>
</table>
TABLE I. Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>generalized energy variable ($T$ or $L^2/\theta^2$)</td>
</tr>
<tr>
<td>$h_a$</td>
<td>enthalpy of air ($L^2/\theta^2$)</td>
</tr>
<tr>
<td>$h_m$</td>
<td>parameter in GMRES acceleration</td>
</tr>
<tr>
<td>$h_v$</td>
<td>mass fraction of air in the vapor phase</td>
</tr>
<tr>
<td>$h_v, w$</td>
<td>vapor enthalpy ($L^2/\theta^2$)</td>
</tr>
<tr>
<td>$h_l$</td>
<td>mass fraction of air in the liquid phase</td>
</tr>
<tr>
<td>$h_l, w$</td>
<td>liquid enthalpy ($L^2/\theta^2$)</td>
</tr>
<tr>
<td>$I_i$</td>
<td>impedance at node $i$ ($\theta/L^2$)</td>
</tr>
<tr>
<td>$I_e, i$</td>
<td>heat flow impedance at node $i$ ($M/L^3\theta T$)</td>
</tr>
<tr>
<td>$[K]$</td>
<td>finite element coefficient for heat conduction term</td>
</tr>
<tr>
<td>$K_D$</td>
<td>retardation coefficient (linear adsorption)</td>
</tr>
<tr>
<td>$K_{h,v, w}$</td>
<td>thermal conductivity ($ML/T\theta^3$)</td>
</tr>
<tr>
<td>$k$</td>
<td>intrinsic rock permeability ($L^2$)</td>
</tr>
<tr>
<td>$L_f, L_0$</td>
<td>length scales used in dual porosity problem</td>
</tr>
<tr>
<td>$L_{f1}, L_{f2}$</td>
<td>length scales used in dual porosity problem</td>
</tr>
<tr>
<td>$[\tilde{L}]$</td>
<td>approximate lower factorization of matrix</td>
</tr>
<tr>
<td>$m$</td>
<td>exponent used in Gangi stress model</td>
</tr>
<tr>
<td>$[N]$</td>
<td>finite element shape function</td>
</tr>
<tr>
<td>$Q_m$</td>
<td>source term for mass equation ($M/\theta L^3$)</td>
</tr>
<tr>
<td>$q_c$</td>
<td>source term for tracer ($M/\theta L^3$)</td>
</tr>
<tr>
<td>$q_e$</td>
<td>source term for energy equation ($M/L^3\theta$)</td>
</tr>
<tr>
<td>$q_i$</td>
<td>mass source term at node $i$ ($M/\theta L^3$)</td>
</tr>
<tr>
<td>$q_e, i$</td>
<td>energy source term at node $i$ ($M/L^3\theta$)</td>
</tr>
<tr>
<td>${q_e}$</td>
<td>coefficient for energy source terms</td>
</tr>
<tr>
<td>${q_m}$</td>
<td>coefficient for mass source terms</td>
</tr>
<tr>
<td>$P_c$</td>
<td>closure stress for use in Gangi stress model ($ML$)</td>
</tr>
<tr>
<td>$P_o$</td>
<td>initial pressure ($ML/\theta^2$)</td>
</tr>
<tr>
<td>$P_l$</td>
<td>liquid pressure ($ML/\theta^2$)</td>
</tr>
<tr>
<td>$P_v$</td>
<td>vapor pressure ($ML/\theta^2$)</td>
</tr>
<tr>
<td>$P_{cap}$</td>
<td>capillary pressure ($ML/\theta^2$)</td>
</tr>
<tr>
<td>$P_{flow}$</td>
<td>flowing pressure at node $i$ ($ML/\theta^2$)</td>
</tr>
<tr>
<td>$[R_e]$</td>
<td>coefficients for tracer gravity term</td>
</tr>
<tr>
<td>$R_l$</td>
<td>liquid relative permeability</td>
</tr>
<tr>
<td>${R_e}$</td>
<td>gravity term coefficient for energy</td>
</tr>
<tr>
<td>${R_m}$</td>
<td>gravity term coefficient for mass</td>
</tr>
</tbody>
</table>
Table I. Notation

- \( R_v \): vapor relative permeability
- \( \{r\} \): residual of linear system of equations
- \( r, r_b \): parameters used in nonlinear adsorption model (Langmuir)
- \( r_{p1}, r_{p2} \): parameters used in relative permeability models
- \( r_{p3}, r_{p4} \): parameters used in relative permeability models
- \( S \): normalized liquid saturation
- \( S_t \): liquid saturation
- \( \hat{S}_t \): normalized liquid saturation
- \( S_{tr} \): residual liquid saturation
- \( S_{ts} \): maximum liquid saturation
- \( S_v \): vapor saturation
- \( T \): temperature (\( T \))
- \( T^* \): absolute temperature (\( T \))
- \( [T_c(P, h)] \): stiffness matrix for energy equation
- \( [T_c(C)] \): stiffness matrix for concentration equation
- \( [T_m(P, h)] \): stiffness matrix for mass equation
- \( T_{flow} \): flowing temperature at node \( i \) (\( T \))
- \( T_{ff}, T_{ff1}, T_{ff2} \): transfer terms in dual porosity solution
- \( t \): time (\( \theta \))
- \( u_l \): liquid internal energy (\( L^2/\theta^2 \))
- \( u_v \): vapor internal energy (\( L^2/\theta^2 \))
- \( u_r \): rock internal energy (\( L^2/\theta^2 \))
- \( V_f \): volume fraction for fractures in a dual porosity problem
- \( V_{f0}, V_{f1} \): volume fractions used in a dual porosity problem
- \( V_{f2} \): volume fractions used in a dual porosity problem
- \( V_l \): liquid velocity (\( L/\theta \))
- \( V_v \): vapor velocity (\( L/\theta \))
- \( [V]^m \): intermediate calculation in GMRES acceleration
- \( [\hat{V}] \): approximate upper factorization of matrix
- \( \{v\} \): intermediate calculation of GMRES acceleration
- \( \{w\} \): parameter in GMRES acceleration
- \( \{y\} \): intermediate calculations in GMRES acceleration
- \( \overline{x^i} \): vector of corrections at iteration \( i \)
- \( \alpha \): coefficient of thermal expansion (1/\( T \))
- \( \alpha_1, \alpha_2 \): coefficients used in sorption models
Table I. Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$</td>
<td>tolerance taken for solution scheme</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>parameter used in defining the van Genutchen capillary pressure model</td>
</tr>
<tr>
<td>$\mu_a$</td>
<td>air viscosity (M/L$\theta$)</td>
</tr>
<tr>
<td>$\mu_\ell$</td>
<td>liquid phase viscosity (M/L$\theta$)</td>
</tr>
<tr>
<td>$\mu_{\ell, w}$</td>
<td>liquid water viscosity (M/L$\theta$)</td>
</tr>
<tr>
<td>$\mu_v$</td>
<td>vapor phase viscosity (M/L$\theta$)</td>
</tr>
<tr>
<td>$\mu_{v, w}$</td>
<td>steam viscosity (M/L$\theta$)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>porosity</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>initial porosity</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>fractional vapor flow parameter</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>in situ stress (ML/$\theta^2$)</td>
</tr>
<tr>
<td>$\left( \frac{\partial F}{\partial x} \right)$</td>
<td>Jacobian matrix for nonlinear system</td>
</tr>
</tbody>
</table>

* Units given in ML$\theta$T system of dimensions:
  Mass [M], length [L], time [$\theta$], temperature [T]

where the energy per unit volume, $A_e$, is given by

$$A_e = (1 - \phi)\rho_r u_r + \phi(S_v \rho_v u_v + S_\ell \rho_\ell u_\ell)$$ (5)

and the energy flux, $\overline{f}_e$, is given by

$$\overline{f}_e = \rho_v h_v \overline{V}_v + \rho_\ell h_\ell \overline{V}_\ell - K \nabla T$$ . (6)

Here the subscript $r$ refers to the rock matrix; $u_r$, $u_v$, and $u_\ell$ are specific internal energies; $h_v$ and $h_\ell$ are specific enthalpies; $K$ is an effective thermal conductivity; $T$ is the temperature; and $q_\ell$ is the energy contributed from sources and sinks.

To complete the governing equations it is assumed that Darcy’s Law applies to the movement of each phase:

$$\overline{V}_v = -\frac{k R_v}{\mu_v} \left( \nabla p_v - \rho_v g \right)$$ (7)

and

$$\overline{V}_\ell = -\frac{k R_\ell}{\mu_\ell} \left( \nabla p_\ell - \rho_\ell g \right)$$ . (8)
Here \( k \) is the permeability, \( R_v \) and \( R_t \) are the relative permeabilities, \( \mu_v \) and \( \mu_t \) are viscosities, \( p_t \) and \( p_v \) the phase pressures, and \( g \) represents the acceleration due to gravity (the phase pressures are related by \( p_v = p_t + p_{cap} \)). For simplicity, the equations are shown for an isotropic medium, though this restriction does not exist in the computer code.

Using Darcy’s Law the basic conservation equations (1) and (4) can be rewritten

\[
-\nabla \cdot (D_{mt}\nabla p_t) - \nabla \cdot (D_{mt}\nabla p_v) + q_m + \frac{\partial}{\partial z}g(D_{mt}p_t + D_{mv}p_v) + \frac{\partial A_m}{\partial t} = 0
\]

and

\[
-\nabla \cdot (D_{ct}\nabla p_t) - \nabla \cdot (D_{ct}\nabla p_v) + \nabla \cdot (K\nabla T) + q_e + \frac{\partial}{\partial z}g(D_{ct}p_t + D_{cv}p_v) + \frac{\partial A_e}{\partial t} = 0,
\]

where \( z \) is oriented in the direction of gravity. Here the transmissibilities are given by

\[
D_{cv} = h_v D_{mv}, \quad D_{ct} = h_t D_{mt}
\]

and

\[
D_{mt} = \frac{k R_t \rho_t}{\mu_t}, \quad D_{mv} = \frac{k R_v \rho_v}{\mu_v}.
\]

The source and sink terms in equations (1) and (4) arise from bores, and if the total mass withdrawal, \( q_m \), for each bore is specified, then the energy withdrawal, \( q_e \), is determined as follows:

\[
q_e = q_v h_v + q_t h_t
\]

where

\[
q_v = \sigma q_m, \quad q_t = (1 - \sigma) q_m
\]

and

\[
\sigma = \frac{1}{\left(1 + \frac{\rho_t R_t \mu_v}{\rho_v R_v \mu_t}\right)}.
\]

The form of equation (15) shows how important the relative permeability ratio \( R_t/R_v \) is in controlling the discharge composition. The relative permeability and capillary pressure functions are summarized in the next section, Constitutive Relationships.
In addition to the flow of heat and mass, FEHMN is also capable of simulating noncondensible gas flow (usually air) and passive tracer flow. The noncondensible gas conservation equation is

\[ -\nabla \cdot (C_t D_{mt} \nabla p_t) - \nabla \cdot (C_v D_{mv} \nabla p_v) + \frac{\partial}{\partial t} (C_t D_{mt} + C_v D_{mv} \rho_v) + \frac{\partial A_c}{\partial t} + q_c = 0 \]  

(16)

Here \( C \) is the concentration of the noncondensible gas and is expressed as a fraction of total mass. The term \( \overline{f}_c \) is the mass flux, \( q_c \) is the source (or sink) strength, and \( A_c \) is the accumulation term:

\[ \overline{f}_c = C_v \rho_v \overline{V}_v + \rho_t \overline{V}_t C_t \]  

(17)

\[ q_c = C_v q_v + C_t q_t \]  

(18)

\[ A_c = \phi (S_v \rho_v C_v + S_t \rho_t C_t) \]  

(19)

The passive tracer equation is not directly coupled to the pressure field but merely uses the pressure field obtained by the heat and mass transfer solution:

\[ -\nabla \cdot (C_t D_{mt} \nabla p_t) - \nabla \cdot (C_v D_{mv} \nabla p_v) + \nabla \cdot (D_c \nabla C_t) + g \frac{\partial}{\partial t} (C_t D_{mt} + C_v D_{mv} C_v) + \frac{\partial A_c}{\partial t} + q_c = 0 \]  

(20)

(where the terms are defined analogously to the condensible gas equation terms) the additional term \( \nabla \cdot (D_c \nabla C_t) \) is the dispersion term. At present the code allows for up to 10 tracers.

2. Constitutive Relationships. In the equations described above the porosity, permeability, density, enthalpy, and viscosity can be strong functions of pressure and temperature. These functions make the code very nonlinear. In addition the relative permeabilities and capillary pressure can be strong functions of saturation.

The pressure and temperature dependent behavior of the density, enthalpy, and viscosity are represented by rational polynomials. Using a technique developed at the University of Auckland, New Zealand, accurate fits to the National Bureau of Standards (NBS) steam table data were obtained over the ranges:

\[ 0.001MPa \leq P \leq 110MPa \]  

(21)

\[ 0.001^\circ C \leq T \leq 360^\circ C \]
The maximum error was 0.3% for all the functions. The rational polynomial representation proved to be both accurate and fast. For details the reader is referred to Zyvoloski and Dash (1991). In addition a low pressure thermodynamics set is provided with the ranges $0.001 MPa \leq P \leq 20 MPa$, $0.5^\circ C \leq T \leq 360^\circ C$. The maximum error for this set is also 0.3%.

The tracer module of FEHMN has provisions for reactive tracers. At present four adsorption models have been included. These are the linear, Langmuir (Satter et al., 1980), Freundlich, and modified Freundlich isotherm models. The discussion follows the description given by Robinson (1988). With adsorption the equations describing the conservation of species, Eq. (16) is modified by the addition of the term

\[ \rho_r \frac{\partial C_R}{\partial t} \]

where $\rho_r$ is the rock density and $C_R$ represents the adsorption of species onto the reservoir rock and is given by

\[ C_R = \frac{\alpha_1 C_I^\beta}{1 + \alpha_2 C_I^\beta} . \tag{22} \]

The terms in Eq. (22) are given in Table II where $K_D$, $A$, $\beta$, $C_{max}$, $r_b$, and $r$ are parameters associated with the sorption models. The adsorption term is included in the tracer solution by replacing the $A_c$ term, Eq. (19), with the new term

\[ A_c = \phi(S_v \rho_v C_v + S_I \rho_I C_I) + \rho_r C_R . \tag{23} \]

Chemical reactions are modeled by modifying the source term, $q_c$ in Eq. (18), with the addition of the term $q_cR$ given below:

\[ q_cR = \frac{a_1 C_I^{2a}}{a_3 + a_4 C_I} . \tag{24} \]
TABLE II. Sorption Isotherm Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Expression</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>$C_R = K_D C_t$</td>
<td>$K_D$</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Freundlich</td>
<td>$C_R = A C_t^b$</td>
<td>$A$</td>
<td>0</td>
<td>$0 &lt; b &lt; 1$</td>
</tr>
<tr>
<td>Modified</td>
<td>$C_R = A C_t^b$</td>
<td>$A$</td>
<td>0</td>
<td>$0 &lt; b &lt; 1$</td>
</tr>
<tr>
<td>Freundlich</td>
<td>$C_R = A C_t^b$</td>
<td>$A$</td>
<td>0</td>
<td>$0 &lt; b &lt; 1$</td>
</tr>
<tr>
<td>Langmuir</td>
<td>$C_R = \frac{r_1 C_t}{1 + r C_t}$</td>
<td>$r_b$</td>
<td>$r$</td>
<td>1</td>
</tr>
</tbody>
</table>

where the quantities $a_1$, $a_2$, $a_3$, and $a_4$ are user inputted parameters. The general case given in Eq. (24) can sometimes be replaced by a simpler expression $q_{CR} = a_1(C_t - a_2)$. Both of the models are addressed in the input section.

Several well known relative permeability functions are available to the user. They are the simple linear functions, the Corey (1954) relationships, and the Sandia functions (van Genuchten, 1980).

The linear functions are given by

\[
\begin{align*}
R_t &= 0, \quad S_t < \rho_{p1} \\
R_t &= (S_t - \rho_{p1})/(\rho_{p3} - \rho_{p1}), \quad \rho_{p1} \leq S_t \leq \rho_{p3} \\
R_t &= 1, \quad S_t \geq \rho_{p3} \\
R_v &= 0, \quad S_v \leq \rho_{p2} \\
R_v &= (S_v - \rho_{p2})/(\rho_{p4} - \rho_{p2}), \quad \rho_{p2} \leq S_v \leq \rho_{p4} \\
R_v &= 1, \quad S_v \geq \rho_{p4}
\end{align*}
\]

where $\rho_{p1}$, $\rho_{p2}$, $\rho_{p3}$ and $\rho_{p4}$ are user supplied quantities.

The Corey relative permeability functions are given by

\[
\begin{align*}
R_t &= \hat{S}_t^4 \\
R_v &= (1 - \hat{S}_t)^2(1 - \hat{S}_t^2)
\end{align*}
\]

where $\hat{S}_t = (S_t - \rho_{p1} - \rho_{p2})/(1 - \rho_{p1} - \rho_{p2})$ and where the input quantities $\rho_{p1}$ and $\rho_{p2}$ are the residual liquid and vapor saturations respectively.
The van Genuchten (van Genuchten, 1980) functions are described by the following formulae:

\[
\begin{align*}
\hat{S} &= \frac{(S_t - r_{p2})}{(r_{p3} - r_{p2})}, \\
R_t &= \sqrt{\hat{S}[1.0 - (1.0 - (\hat{S})^{1/\lambda})^{\lambda}]^2}, \quad \hat{S}_t < r_{p3} \\
R_v &= 1.0 - R_t
\end{align*}
\]

where \( \lambda = r_{p1} \) and \( r_{p1}, r_{p2} \) and \( r_{p3} \) are parameters supplied by the user.

Composite relative permeability curves, as described by Klavetter and Peters (1985), are also available. See Section III B, macro-command \textit{RLP} for an input description.

\( R_t \) and \( R_v \) are restricted by the relationships

\[
0 \leq R_v \leq 1.0 \\
0 \leq R_t \leq 1.0
\]

and the relative permeability functions are truncated to the appropriate value if the above conditions are violated.

The capillary functions included with FEHMN are the linear function and the Sandia capillary pressure model. Our terminology follows that of Pruess (1985).

The linear capillary function model is given by the following equations

\[
\begin{align*}
P_{\text{cap}} &= c_{p1}, \quad S_t \leq c_{p2} \\
P_{\text{cap}} &= 0.0, \quad S_t \geq c_{p3} \\
P_{\text{cap}} &= c_{p1} \frac{c_{p3} - \hat{S}_t}{c_{p3} - c_{p1}}, \quad c_{p2} < S_t < c_{p3}
\end{align*}
\]

where \( c_{p1}, c_{p2}, \) and \( c_{p3} \) are parameters supplied by the user. The restriction \( c_{p3} > c_{p2} \) is also necessary.

The van Genuchten functions (van Genuchten, 1980) are described by the following equations

\[
\begin{align*}
\hat{S} &= \frac{S_t - S_{tr}}{S_{ts} - S_{tr}} \\
P_{\text{cap}} &= 0, \quad S_t \geq c_{p5} \\
P_{\text{cap}1} &= P_o \left( (\hat{S})^{-1/\lambda} - 1.0 \right)^{1.0 - \lambda} \\
P_{\text{cap}} &= P_{\text{max}}, P_{\text{cap}1} \geq P_{\text{max}} \\
P_{\text{cap}} &= P_{\text{cap}1}, P_{\text{cap}1} < P_{\text{max}}
\end{align*}
\]

where \( \lambda = c_{p1}, P_o = 1.0/c_{p3}, P_{\text{max}} = c_{p4}, S_{ts} = c_{p5}, S_{tr} = c_{p2} \).
The capillary pressure curves approach an infinite value as $S_\ell$ approaches 0. This was observed by Nitao (1989) who suggested using Eq. (30) only to a value of one-half the maximum suction observed in the rock ($P_{max}$) and linearly extrapolating the capillary pressure to the maximum capillary pressure at $S_\ell = 0$. We have also employed this technique in FEHMN.

The van Genuchten relative permeability curves and capillary pressure functions are the primary ones used for Yucca Mountain calculations.

If the flow of air is also modeled, appropriate thermodynamic information for air must also be provided. The density of air is assumed to obey the ideal gas law, using atmospheric conditions as our reference state we have

$$\rho_a = 1.292864 \left( \frac{273.0}{T + 273.0} \right) \left( \frac{P}{0.101325} \right)$$

(31)

where $\rho_a$ has the units $kg/m^3$, $T$ is in °C, and $P$ is in MPa. The mixture density is given by

$$\rho_v = \rho_{v,w} + \rho_a$$

(32)

where $\rho_{v,w}$ is the density of the water vapor. The density of the liquid phase is assumed to be unaffected by the amount of dissolved air it contains.

The enthalpy of air is a function of temperature only

$$C_{pa} = 6122.0 - 11.76(T^*) + 0.0177(T^*)^2$$

$$h_a = C_{pa}(T \cdot 10^{-6})$$

(33)

where $h_a$ is the enthalpy of air $MJ/kg$ ($MJ/kg^o C$) is the heat capacity of air, and $T^* = T + 273$. The mixture enthalpy for the vapor phase is

$$h_v = h_{v,w}(1 - \eta_v) + h_a \eta_v$$

(34)

where $h_{v,w}$ is the enthalpy of steam and $\eta_v$ is the fraction by mass of air in the vapor phase. The enthalpy of the liquid phase is given by the enthalpy of the dissolved air, the heat of solution, and the enthalpy of the water. At present, we neglect the heat of solution and use the enthalpy of air described above:

$$h_\ell = h_{\ell,w}(1 - \eta_\ell) + h_a \eta_\ell$$

(35)
where $h_{l,w}$ is the enthalpy of liquid water and $\eta_\ell$ is the mass fraction of $a$ in the liquid phase.

Assuming ideal gas behavior, the mass fraction of air in the vapor phase may be expressed as

$$\eta_v = \frac{\rho_a}{\rho_v}$$

the mass fraction of air in the liquid phase is assumed to obey Henry's law or

$$\eta_e = \alpha P_a$$

where $\alpha$ is the Henry's law constant ($\alpha = 1.611 \times 10^{-4} P_a^{-1}$) for air and $P_a$ is the partial pressure of air.

The viscosity of the vapor phase is assumed to be a linear combination of the air viscosity and the water viscosity

$$\mu_v = \mu_{v,w}(1 - \eta_v) + \mu_a \eta_v \quad (36)$$

where $\mu_{v,w}$ is the steam viscosity and is obtained from steam data. The viscosity of air is assumed constant

$$\mu_a = 1.82 \times 10^{-5} \text{cp} \quad (37)$$

The liquid phase viscosity is assumed to be independent of the amount of dissolved air and is obtained from steam table data:

$$\mu_\ell = \mu_{\ell,w} \quad (38)$$

Often it is necessary to accommodate changes in the rock porosity and permeability due to changes in effective stress caused by temperature and pore fluid pressure changes. A linear and nonlinear model are currently incorporated into the code for this purpose.

The linear pore pressure model for porosity is given by

$$\phi = \phi_o + (1 - \phi_o)(c_r - c_g)(P - P_o) \quad (39)$$

where $\phi$ is the porosity at pressure $P$, $\phi_o$ is the initial porosity at pressure $P_o$, $c_r$ is the pore volume compressibility of the rock and $c_g$ is the compressibility of the matrix grain material.

The nonlinear model for porosity (Gangi, 1978) currently used in the code is given by
\[ \phi = \phi_o \left[ 1 - \left( \frac{P_c}{P_o} \right)^m \right] \]  

(40)

and

\[ P_c = \sigma - P_o - \alpha E \Delta T \]  

(41)

where \( P_c \) is the closure stress, \( \sigma \) is the \textit{in situ} stress (assumed isotropic), \( \alpha \) is the coefficient of thermal expansion of the rock, \( E \) is Young's modulus, \( \Delta T \) is the temperature change and \( P_o \) and \( m \) are parameters in the model.

For either case the effect of stress and temperature changes on permeability are modeled with

\[ k = k_o \left( \frac{\phi}{\phi_o} \right)^3 \]  

(42)

where \( k \) is the permeability at porosity \( \phi \).

C. Discretization of the Governing Equations

The time derivatives in the Eqs. (1-42) are discretized using the standard first order method given by

\[ f(t^{n+1}) = f(t^n) + \Delta t \left( a f'(t^{n+1}) + (1 - a) f'(t^n) \right) \]  

(43)

where \( f(t^{n+1}) \) is the desired function at time \( t^{n+1} \), \( f(t^n) \) is the known value of \( f \) at time \( t^n \), \( \Delta t \) is the time step, \( f' \) is the derivative of \( f \) with respect to time and \( a \) is a weighting factor. For \( a = 1 \), the scheme is fully implicit (backward Euler) and for \( a = 0 \), the scheme is fully explicit (forward Euler).

The space derivatives in the governing equations are discretized using the finite element formulation. The finite element equations are generated using the Galerkin formulations. For a detailed presentation of the finite element method the reader is referred to Zienkiewicz (1977). In this method the flow domain, \( \Omega \), is assumed to be divided into finite elements; and variables \( P, h \) and \( T \), along with the accumulation terms \( A_m \) and \( A_e \), are interpolated in each element:

\[
\begin{align*}
P &= [N]\{P\}, & h &= [N]\{h\}, & T &= [N]\{T\}, & A_m &= [N]\{A_m\}, & A_e &= [N]\{A_e\}
\end{align*}
\]

where \([N]\) is the shape function. Here \( h \) is used to represent the generalized energy variable; \( T \) in single phase water states, \( S \) in 2-phase water states.
These approximations are introduced in Eqs. (9), (10), and (20), and the Galerkin formulation (described by Zienkiewicz and Parekh, 1973) is applied. The following equations are derived:

\[ [T_m(P, h)]\{P\} + \hat{C}\frac{\partial A_m}{\partial t} + \{q_m\} - g\{R_m\} = \{F_m\} \quad (44) \]

and

\[ [T_e(P, h)]\{P\} + [K]\{T\} + \hat{C}\frac{\partial A_e}{\partial t} + \{q_e\} - g\{R_e\} = \{F_e\} \quad (45) \]

where

\[ T_{mi} = \int \nabla N_i \cdot D_m^{UP} \nabla N_j dV \quad , \quad (46) \]
\[ T_{ei} = \int \nabla N_i \cdot D_e^{UP} \nabla N_j dV \quad , \quad (47) \]
\[ K_{ij} = \int \nabla N_i \cdot \nabla N_j dV \quad , \quad (48) \]
\[ \hat{C}_{ij} = \int \Omega_j N_i N_j dV \quad , \quad (49) \]
\[ R_{mi} = \int \Omega_j \frac{\partial N_i}{\partial y} N_j D_m^{UP} \rho_m dV \quad , \quad (50) \]
\[ R_{ei} = \int \Omega_j \frac{\partial N_i}{\partial y} N_j D_e^{UP} \rho_e dV \quad . \quad (51) \]

Equations (50) and (51) need some comment. The term \( D_e^{UP} \) indicates an upstream-weighted transmissibility (Dalen, 1974). This technique has worked well in the low-order elements (3-node triangle, 4-node quadrilateral), where the schemes resemble difference techniques. The upstream weighting is determined by comparing the velocities at the nodes \( i \) and \( j \). The shape function coefficients are generated in a unique way that requires the integrations in Eqs. (50) and (51) to be performed only once and the nonlinear coefficients to be separated from this integration. The reader is referred to Zyvoloski (1983) for more details.

The integration scheme used in this report is similar to that described by Young (1981). His implementation differs from common methods in that it uses Lobatto instead of Gauss integration. The net effect is that, while retaining the same order of integration accuracy (at least for linear and quadratic elements), there are considerably fewer nonzero terms in the resulting matrix equations. Figure 1 shows a comparison of the nodal connections for Lobatto and Gauss integration methods. It should be noted that these results hold on an orthogonal grid only. If a nonorthogonal grid were introduced, then additional
nonzero terms would appear in the Lobatto quadrature method. Note also that the linear elements yield the standard 5- or 7-point difference scheme. The reader is referred to Young’s paper for more details.

A similar approach is used to solve the transport equation. Following the discussion above the species concentration, $C$, and the species accumulation term, $A_c$, are interpolated in each element.

$$C = [N]^T\{C\}, \quad A_c = [N]^T\{A_c\}$$

Using these approximators and using a Galerkin approach, we obtain the following equation

$$[T_c(C)]\{P\} + [D_c]\{C\} + \dot{c} \frac{\partial A_c}{\partial t} + \{f_c\} + \{R_c\} = \{F_c\} \quad (52)$$

when
where \( C_{ij} \) was defined previously and \( C_{ij}^{UP} \) is an upstream weighted species concentration. This approach is similar to the finite difference method for solving the transport equations.

**D. Boundary Conditions**

Two mass flow boundary conditions are allowed in FEHMN. They are no flow and specified variable values. The no flow conditions are automatically satisfied by the finite element mesh. Specified variable quantities are obtained in the following manner. A pressure dependent flow term is used:

\[
q_i = I_i(P_{flow,i} - P_i)
\]

where \( P_i \) is the pressure at node \( i \), \( q_i \) is the flow rate, \( I_i \) is the impedance, and \( P_{flow,i} \) is the specified following pressure at node \( i \). By specifying a large \( I_i \), we can force the pressure to be equal to \( P_{flow,i} \). The energy (or temperature) specified at node \( i \) refers only to the incoming fluid value, if fluid flows out of the reservoir, stability dictates that the energy of the in-place fluid be used in the calculations. Species concentration is handled in a manner analogous to the energy variable. Only the incoming species concentration can be specified; if the flow is out of the reservoir the in-place species concentration is used.

In addition to the mass flow boundary conditions, heat flow boundary conditions are also provided. A specified heat flow can be input or a specified temperature obtained

\[
q_{e,i} = I_{e,i}(T_{flow,i} - T_i)
\]

where \( T_i \) is the nodal temperature, \( T_{flow,i} \) is the specified flowing temperature at node \( i \), \( I_{e,i} \) is the impedance to heat flow at node \( i \), and \( q_{e,i} \) is the heat flow at node \( i \). This heat flow is superimposed on any existing heat flow from the other boundary conditions or source terms.

**E. Solution Methods**

The application of the discretization methods to the governing partial differential equations yields a system of nonlinear algebraic equations. To solve these equations, the Newton-Raphson iterative procedure is used. This is an iterative procedure that makes use of the derivative information to obtain an updated solution from an initial guess. Let the set equations to be solved be given by

\[
T_{cij} = \int_{\Omega} \nabla N_i \cdot D_m C_{ij}^{UP} \nabla N_j dv
\]

\[
D_{cij} = \int_{\Omega} \nabla N_i \cdot D_c \nabla N_j dv
\]

\[
R_{cij} = \int_{\Omega} \frac{\partial N_i}{\partial y} \cdot D_m C_{ij}^{UP} \nabla N_j dv
\]
\[ F(\bar{x}) = 0 \]  
\[ (57) \]

where \( \bar{x} \) is the vector of unknown values of the variables that satisfy the above equation. The procedure is started by making an initial guess at the solution, say \( \bar{x}^0 \). This is usually taken as the solution from the previous time step. Denoting the value of \( \bar{x} \) at the \( k \text{th} \) iteration by \( \bar{x}^k \), the updating procedure is given by

\[
\bar{x}^{(k+1)} = \bar{x}^k - F(x^k) \left( \frac{\partial F}{\partial x^k} \right)^{-1} .
\]
\[ (58) \]

At each step, the residuals \( F(x^k) \) are compared with a prescribed error tolerance. The prescribed error tolerance, \( \epsilon \), is an input parameter and an \( \ell^2 \) norm is used:

\[
\|F\|_k = \left( \sum_i F_i^2 \right)^{1/2} .
\]

Convergence is achieved when

\[
\|F\|_k \leq \epsilon \|F\|_0 .
\]

\( \epsilon \) is usually in the range \( 10^{-4} - 10^{-7} \). Semiautomatic timestep control is designed based on the convergence of the Newton iterations. If the code is unable to find a solution \( \bar{x}^k \) such that the residuals become less than the tolerance within a given number of iterations, the time step is reduced and the procedure repeated. On the other hand, if convergence is rapid, the timestep is increased by multiplying with a user supplied factor, thus allowing for large timesteps when possible.

The matrix equation to be solved at each Newton-Raphson iteration has the form

\[
[A] \{x\} = \{b\} ,
\]
\[ (59) \]

where \( [A] \) is the Jacobian matrix, \( \{x\} \) is the solution vector, and \( \{b\} \) is the residual. The \( [A] \) matrix is sparse and banded. For computation, \( \{x\} \) is replaced by \( \{\delta x\} = \{x\}^{n+1} - \{x\}^n \), and \( \{b\} \) is replaced by \( \{r\} = \{b\} - [A]\{x\}^n \), where \( n \) is the iteration number. (Note that this is different from the overall Newton iteration.) Thus, only changes in \( \{x\} \) are calculated.

Complete Gaussian elimination or factorization on Eq. (59) would result in fill-in of all the bands between the lowest subdiagonal and the highest superdiagonal bands. Incomplete factorization involves finding only approximate upper and lower triangular matrices \( \bar{L} \) and \( \bar{U} \), requiring very little fill-in. The computing effort for obtaining the incomplete factors is much less than for the complete factors. However, an iterative procedure is now
required. The presentation of the theory below follows that of Saad and Schultz (1986). The matrix $[A]$ is partially factored into upper and lower triangular form

$$[A] = [\mathcal{L}][\mathcal{U}],$$

where $[A] \neq [A]$ because, in general, $[\mathcal{L}]$ and $[\mathcal{U}]$ will contain only some of the steps of the elimination process. To incompletely factor a matrix, a prior knowledge of the fill-in pattern is required. A symbolic factorization is performed (just once) to establish where the fill-in occurs and to determine the order of each term. Figure 2 shows a matrix and its first level of fill (0's) resulting from factorization of the original matrix elements ($x$'s). We could continue the process to get higher-order terms, but usually the lower-order terms are all that are necessary. The amount of factorization is limited by computer storage considerations. Once factored, the solution is easy to carry out as it consists of a forward and a backward substitution:

$$\{v\} = [\mathcal{U}]^{-1}\left\{[\mathcal{L}]^{-1}\{r\}\right\}$$

where $\{v\}$ is the approximate change in $\{x\}$. The following recursion scheme is used to refine the approximation:

$$\{v\}^{n+1} = [A]^{-1}\{r\}^n = [\mathcal{U}]^{-1}\left\{[\mathcal{L}]^{-1}\{r\}^n\right\},$$

$$\{x\}^{n+1} = \{x\}^n + \{v\}^{n+1},$$

$$\{r\}^{n+1} = \{r\}^n - [A]\{v\}^{n+1},$$

where $n$ is the iteration index and $\{x\}^0 = 0$ since the original matrix equation is written in residual form [see paragraph after Eq. (59)]. The solution scheme, comprising Eqs. (61)-(64), may be accelerated by over-relaxation procedures, but it is better to use orthogonalization-minimization. If the Jacobian matrix were symmetric, conjugate gradient could be used. For nonsymmetric matrices, the GMRES (Generalized Minimal Residual Equation Solver) procedure (Saad and Schultz 1986) is used.

For comparison purposes, the simple over-relaxation procedure is discussed first. In the recursion scheme [Eqs. (62)-(64)], Eq. (63) is replaced by $\{x\}^{n+1} = \{x\}^n + w\{v\}^{n+1}$, and Eq. (64) is replaced by $\{r\}^{n+1} = \{r\}^n - w[A]\{v\}^{n+1}$. Here $w$ is the over-relaxation parameter and satisfies the relation $1 < w < 2$. After Eq. (64) is calculated, the procedure returns to Eq. (61) and continues until $\{r\}^{n+1}$ is reduced to some predetermined value. For many reservoir problems, convergence is extremely slow, and the ortho-minimization procedures provide a valuable alternative.
The GMRES algorithm is
1. Set $\{r\}^0 = \{b\}$ and set $\{v\}^1 = \{r\}^0 / \|\{r\}\|$ where $\|\{r\}\|$ is the sum squared norm ($\|\{r\}\| = \{\sum_{i=1}^{n} r_i\}^{1/2}$) of $\{r\}$ and $n$ is the number of unknowns. We also assume that $\{x\}^0 = \{0\}$.
2. Perform orthogonalization calculation, $m = 1, \text{NORTH}$

$$\{w\}^{m+1} = [A]\{v\}^m - \sum_{i=1}^{m} ([A]\{v\}^i, \{v\}^i)\{v\}^i$$

$$h_m^{m+1} = \|\{w\}^{m+1}\|$$

$$\{v\}^{m+1} = \{w\}^{m+1} / h_m^{m+1}$$

Where $\text{NORTH}$ is the maximum number of orthogonalizations allowed.
3. Form approximate solution ($n = \text{NORTH}$)

$$\{x\}^m = \{x\}^0 + [V]^m\{y\}^m$$
where $[V]^m$ is an $n$ by $m$ matrix which contains the $\{v\}^i$ ($i = 1, m$) and $\{y\}^m$ is an $n$ dimensional vector which minimizes the function.

$$f(y) = \|B\{e\}^1 - [H]^m\{y\}\|.$$  

(69)

Here $B = \|r^o\|$, $\{e\}^1$ is the unit vector, and $[H]^m$ is a $k + 1 (=n)$ by $k$ matrix whose only nonzero entries are the $h_{i+1}$ calculated in Eq. (66). This minimization is economically done using $Q-R$ factorization. (See Saad and Shultz, 1986 for details.)

4. Calculate $\|r_m\|$ where

$$\{r\}^m = \{b\} - [A]\{x\}^m $$  

(70)

If $\|r_m\| \leq \epsilon$, some specified tolerance, then the process is complete. Otherwise we set $\{x\}^o = \{x\}^m$, $\{v\}^1 = \{r\}^m/\|r_m\|$ and proceed from Eq. (65) of the GMRES algorithm. There is an intimate relationship between the outer iteration (Newton-Raphson) and the inner iteration (solution of linear equations). If the outer iteration is far from convergence, we can solve the linear system of equations with a rather large tolerance. As the outer iterations converge we must likewise have a finer tolerance for the inner iteration. The method used here is one developed by Diaz, Jines, and Steihaug (1985). Regions of linear and quadratic convergence are defined, and multipliers for the square of the residuals and the residuals are specified. Comparing the magnitude of the terms determines the largest possible tolerance for the inner iteration.

A mathematical theory similar to that for conjugate gradient methods (for example, showing a reduction in condition number with each iteration) is not available. In basic terms the $h_{i+1}^m$ in Eqs. (66) and (67) ensure that the $\{v\}^i$ are orthogonal and that the $\{y\}^m$ in Eqs. (68) and (69) are such that the square of the $\{r\}^n+1$ is minimized. For more details the reader is referred to Meijerink and Van der Vorst (1981), Behie and Forsythe (1984), and Behie and Vinsome (1982). To minimize the storage required, the procedure is restarted after every NORTH iteration. The more orthogonalizations before restarting, the faster the convergence, but this benefit must be weighed against the greater storage and longer running time required for the additional orthogonalizations.

F. Dual Porosity Formulation

Many problems are dominated by fracture flow. In these cases the fracture permeability controls the pressure communication in the reservoir even though local storage around the fracture may be dominated by the porous rock which communicates only with the closest fractures. Moench (1984) has studied several wells in the saturated zone beneath Yucca Mountain and found the results could be understood if dual porosity methods were used. Figure 3 depicts the dual porosity concept. The computational volume in the figure
consists of a fracture which communicates with fractures in other computational cells, and matrix material which only communicates with the fracture in its computational cells. This behavior of the matrix material is both a physical limitation and a computational tool. The physical limitation results from the model's inability to allow the matrix materials in different cells to communicate directly. This yields only minor errors in saturated zone calculations, but could pose larger errors in the unsaturated zone where capillary pressures would force significant flow to occur in the matrix material. The computational advantages will be addressed at the end of this section.

![Diagram](image)

Fig. 3. Computational volume element showing dual porosity parameters.

Two parameters characterize a dual porosity reservoir. The first is the volume fraction, $V_f$, of the fractures in the computational cell. For the simple system shown in Fig. 3a this fraction is $a/b$. The second parameter is related to the fracture's ability to communicate with the local matrix material. In the literature this parameter takes a variety of forms. The simplest is a length scale, $L_f$. This quantifies the average distance the matrix material is from the fracture. This length scale is also shown in Fig. 3a. With just one node in the matrix material the transient behavior in the matrix material cannot be modeled. To improve this situation, two nodes are used in FEHM to represent the matrix material. Conceptually, this is the same formulation as just described with only additional fracture volume needed (it is assumed the length scale of each matrix volume is proportional to the volume fraction). This is shown in Fig. 3b. More matrix nodes could be added, but data is rarely good enough to justify the use of even two matrix nodes. We note here that the
simple slab model depicted in Fig. 3 is just one of several different geometric arrangements. Moench (1984) and Warren and Root (1962) list other reservoir types. All of them are similar in the assumption of a local one dimension connection of the matrix to the fracture.

Computationally, the volume fraction and length scale are used to create one dimensional versions of Eq. (9), Eq. (10), Eq. (16), and Eq. (20). The length scale is used to modify spacial difference terms and the volume factors are used to modify the accumulation terms [the $\tilde{C}$ matrix in Eq. (44)]. The volume fractions for FEHMN are given by

$$V_f = 1 - V_{f1} - V_{f2}, \text{ volume fraction of fractures}$$
$$V_{f1} - \text{ fraction of first matrix volume (input)}$$
$$V_{f2} - \text{ fraction of second matrix volume (input)}$$

The length scales are given by

$$L_f = L_{f0} V_f, \text{ length scale for the fracture volume}$$
$$L_{f0} - \text{ length scale (input)}$$
$$L_{f1} = L_{f0} V_{f1} \text{ length scale of first matrix volume}$$
$$L_{f2} = L_{f0} V_{f2} \text{ length scale of second matrix volume}$$

The geometric factor representing the spacial differencing of the one dimensional equation for flow between the fracture and the first matrix node [analogous to the geometric part of Eq. (46) and Eq. (47)] is given by

$$T_{ff1} = \frac{1.0}{L_{f1}} (L_{f1} + L_f) \quad (71)$$

The analogous term for the flow from the first matrix volume to the second matrix volume is given by

$$T_{f1f2} = \frac{1.0}{L_{f2}} (L_{f1} + L_{f2}) \quad (72)$$

The one dimensional nature of the equations provides a computationally efficient method to solve the algebraic equations arising from the dual porosity simulation. Equation (73) shows the matrix equation arising from a dual porosity simulation.

$$\begin{bmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \end{bmatrix} = -\begin{bmatrix} R_0 \\ R_1 \\ R_2 \end{bmatrix}$$

Here the subscript 0 refers to the fracture, 1 refers to the first matrix volume, and 2 refers to the second matrix volume.

The $p$ represents the unknown variable or variable pair. The one dimensional character of the matrix diffusion means that the second matrix node can only depend on the first
matrix node. Therefore, the submatrix $A_{20}$ is empty. The fact that matrix nodes cannot communicate with matrix nodes in other computational cells means that the submatrices $A_{21}$ and $A_{22}$ are diagonal. Therefore we may write

$$\{P_2\} = [A_{22}]^{-1}[-\{R_2\} - [A_{21}]{P_1}] \quad (74)$$

where the inversion is trivial because $[A_{22}]$ is diagonal. Substituting this expression into the equation for the first matrix node we have

$$[A_{10}]{P_0} + [A_{11}]{P_1} + [A_{12}][A_{22}]^{-1}\left\{-\{R_2\} - [A_{21}]{P_1}\right\} = -\{R_1\} \quad . \quad (75)$$

Rearranging, we have

$$[A_{10}]{P_0} + \left([A_{11}] - [A_{12}][A_{22}]^{-1}[A_{21}]\right){P_1} = -\{R_1\} + [A_{12}][A_{22}]^{-1}\{R_2\}$$

or

$$\{P_1\} = \tilde{A}_{11}^{-1}\left\{-R_1 - [A_{10}]{P_0}\right\} \quad (76)$$

where

$$\tilde{A}_{11} = [A_{11}] - [A_{12}][A_{22}]^{-1}[A_{21}]$$

and

$$\{\tilde{R}_1\} = -\{R_1\} + [A_{12}][A_{22}]^{-1}\{R_2\}$$

where the inversion and multiplications are trivial because of the diagonal nature of the matrices involved. Equation (76) may next be substituted into the equation for the fracture variables. Noting that $[A_{13}]$ is empty (the fracture can only communicate with the first matrix volume) we have

$$[A_{00}]{P_0} + [A_{01}][\tilde{A}_{11}]^{-1}\left\{-\{\tilde{R}_1\} - [A_{10}]{P_0}\right\} = -\{R_0\} \quad . \quad (77)$$

Rearranging terms we have

$$\left[[A_{00}] - [A_{01}][\tilde{A}_{11}]^{-1}[A_{10}]\right]{P_0} = -\{R_0\} + [A_{01}][\tilde{A}_{11}]^{-1}\{\tilde{R}_1\} \quad (78)$$

Equation (78) consists of an augmented fracture matrix of the same form as the original fracture matrix $[A_{00}]$. The operations carried out only add a few percent to the solution time required to solve a single porosity system. After the solution of Eq. (78) is obtained with the methods described in Section E, the solution in the fracture volume can be obtained by using Eq. (74) and Eq. (76).
II. CODE DESCRIPTION

In this section we present three important descriptions of FEHMN. First descriptions of all the subroutines are presented. Next the common block variables and calling parameters are described. Finally, the subroutine call tree structure for FEHMN is provided.

A. Subroutines

The main program of the code FEHM is GZ. This program calls subroutine DATA to initialize most of the common block variables and calls subroutine INPUT which initializes most non-array variables and directs the data input. GZ also performs one time startup calculations: scaling of some variables, directing the calculation of element coefficient arrays, and the setting up of equation solution schemes. The major time step loop is contained within GZ. In this loop the model equations are solved at each time step, solution parameters adjusted, and variables updated for the next time step. Plotting data is also outputted from this loop.

The following subroutines are called from GZ (excluding calls to intrinsic functions): ANONP, BNSWER, COEFFC, CONCEN, CONTR, CO2CTR, DATA, DATCHK, DISK, DUAL, ENTHP, INPUT, NEAR3, PEINT, PLOT, RADIUS, RARNG, RESETV, SETORD, SICE, SLVESU, STEADY, STORSX, STRESS, TIMCRL, TYMING, USER, VARCHK, VELOC, WELBOR, WRTOUT.

Table III contains a complete alphabetical listing of the FEHMN subroutines. A brief description of each routine is provided along with listings of which subroutines call the given routine and which subroutines it calls (intrinsic functions are excluded).

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Calls</th>
<th>Called by</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANONP</td>
<td>Categorize elements. Call routines to generate finite element coefficients.</td>
<td>GENCOF</td>
<td>GZ</td>
</tr>
<tr>
<td>BNSWER</td>
<td>Call routines to assemble finite element equations and solve for the Newton-Raphson equations.</td>
<td>VARCHK, OUTBND, GENSL13, GENSL14, GENSL1</td>
<td>INPUT, THRMWC, THERMW</td>
</tr>
<tr>
<td>CAPPR</td>
<td>Calculate capillary pressure functions.</td>
<td></td>
<td>GZ, THRMWC</td>
</tr>
<tr>
<td>CNSWER</td>
<td>Call routine to generate tracer transport equations. Call tracer equation of state routines.</td>
<td>THERMC, GENCON</td>
<td>CSOLVE</td>
</tr>
<tr>
<td>Name</td>
<td>Description</td>
<td>Calls</td>
<td>Called by</td>
</tr>
<tr>
<td>----------</td>
<td>------------------------------------------------------------------------------</td>
<td>--------------------------------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>COEFFC</td>
<td>Change coefficients of polynomial fits of the thermodynamic properties specified.</td>
<td>RDCON, CSOLVE, WRTCON, PLOTC, DISKC, CONTRC</td>
<td>GZ</td>
</tr>
<tr>
<td>CONCEN</td>
<td>Organize calls required to run tracer transport model.</td>
<td></td>
<td>GZ, WRTOUT, DISK, INPUT, CONTR</td>
</tr>
<tr>
<td>CONEQ1</td>
<td>Generate the equations of tracer transport.</td>
<td></td>
<td>GENCON</td>
</tr>
<tr>
<td>CONTR</td>
<td>Write out data for time history plots at particular nodes.</td>
<td>CONCEN, VELOC</td>
<td>GZ, TIMCRL</td>
</tr>
<tr>
<td>CONTRC</td>
<td>Write out tracer data for contour plots.</td>
<td>CONCEN</td>
<td></td>
</tr>
<tr>
<td>CO2CTR</td>
<td>Read input for noncondensible gas and initialize variables associated with the noncondensible gas.</td>
<td>PSAT</td>
<td>GZ, TIMCRL, WRTOUT, INPUT</td>
</tr>
<tr>
<td>CSOLVE</td>
<td>Organize tracer solution so smaller time steps can be used for the tracer solution than for the flow solution.</td>
<td>CNSWER, PLOTC1 THERMC</td>
<td>CONCEN</td>
</tr>
<tr>
<td>DATA</td>
<td>Initialize common block variables.</td>
<td></td>
<td>GZ</td>
</tr>
<tr>
<td>DATCHK</td>
<td>Initial value analysis and data check.</td>
<td></td>
<td>GZ</td>
</tr>
<tr>
<td>DISK</td>
<td>Read and/or write to files for restart purposes.</td>
<td>CONCEN, STRESS</td>
<td>GZ, TIMCRL</td>
</tr>
<tr>
<td>DISKC</td>
<td>Read and write restart files for tracer variables.</td>
<td>CONCEN</td>
<td></td>
</tr>
<tr>
<td>DUAL</td>
<td>Read dual porosity data. Initialize dual porosity variables.</td>
<td>VARCHK, DUALFA, DUALEX</td>
<td>GZ, TIMCRL, GENSL4, GENSL1, INPUT</td>
</tr>
<tr>
<td>DUALEX</td>
<td>Extract dual porosity solution from primary variable solution.</td>
<td></td>
<td>DUAL</td>
</tr>
<tr>
<td>Subroutine</td>
<td>Description</td>
<td>Prerequisites</td>
<td>Notes</td>
</tr>
<tr>
<td>-----------</td>
<td>-------------</td>
<td>---------------</td>
<td>-------</td>
</tr>
<tr>
<td>DUALFA</td>
<td>Assemble and load dual porosity solution and modify primary variables solution to account for dual porosity effects.</td>
<td>VARCHK, GENEQ1, GENEQ3 DUAL</td>
<td>DUAL</td>
</tr>
<tr>
<td>ENTHP</td>
<td>Calculate the temperature for a given input enthalpy.</td>
<td>GZ, STEADY</td>
<td></td>
</tr>
<tr>
<td>GENCOF</td>
<td>Call routines to generate finite element coefficients. Perform the numerical integration of the elements.</td>
<td>GNCF3, GNCF2 ANONP</td>
<td></td>
</tr>
<tr>
<td>GENCON</td>
<td>Call routines to generate tracer equations. Call solver to obtain Newton-Raphson equations for tracer variables.</td>
<td>CONEQ1, SOLVE, RD1DOF CNSWER</td>
<td>CNSWER</td>
</tr>
<tr>
<td>GENDAT</td>
<td>Generate coordinates and element information in simple geometric problems.</td>
<td>INPUT, ZONE</td>
<td></td>
</tr>
<tr>
<td>GENEQC</td>
<td>Generate equations for Newton-Raphson corrections for the water and non-condensible gas flow.</td>
<td>GENSLS4</td>
<td></td>
</tr>
<tr>
<td>GENEQ1</td>
<td>Generate equations for Newton-Raphson corrections for water/vapor flow with no noncondensible gas.</td>
<td>DUALFA, GENSLS1</td>
<td></td>
</tr>
<tr>
<td>GENEQ3</td>
<td>Generate equations for Newton-Raphson corrections for heat conduction only (i.e., permeability).</td>
<td>DUALFA, GERSLS4, GENSLS1, GENSLS3</td>
<td></td>
</tr>
<tr>
<td>GENSLS1</td>
<td>Call routines to generate the Newton-Raphson equations for water only problems. Call equation solver subroutine.</td>
<td>GENEQ1, GENEQ3, GENEQ3, DUAL, NORMAL, SOLVE2, RD2DOF</td>
<td>BNSWER</td>
</tr>
<tr>
<td>GENSLS3</td>
<td>Call routines to generate the Newton-Raphson equations for heat conduction only. Call equation solver routine.</td>
<td>GENEQ3, SOLVE, RD1DOF</td>
<td>BNSWER</td>
</tr>
<tr>
<td>GENSLS4</td>
<td>Calls routines to generate equations for water/noncondensible gas problems and calls equation solver.</td>
<td>GENEQC, GENEQ3, DUAL, NORMAL, SOLVE3, RD3DOF</td>
<td>BNSWER</td>
</tr>
<tr>
<td>Name</td>
<td>Description</td>
<td>Calls</td>
<td>Called by</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>---------------------------</td>
<td>------------------------------------</td>
</tr>
<tr>
<td>GEOIN</td>
<td>Reads element and coordinate information from MARC preprocessor.</td>
<td>INPUT,ZONE</td>
<td></td>
</tr>
<tr>
<td>GNCF3</td>
<td>Generate 3-D finite element coefficients.</td>
<td>SHAP3R, SHAP3P</td>
<td>GENCOF</td>
</tr>
<tr>
<td>INPUT</td>
<td>Initialize variables, read input, write out some input information.</td>
<td></td>
<td>CAPPR,CONCEN CO2CTR,DUAL, GENDAT,GEOIN, IOFILE, POROSI, RLPERM, SICE, STHER, STRESS, USER, WELBOR, ZONE</td>
</tr>
<tr>
<td>IOFILE</td>
<td>Open up initial pressure distribution when gravity is present (enabled).</td>
<td></td>
<td>INPUT</td>
</tr>
<tr>
<td>LUDBKS B</td>
<td>Performs forward and rock substitution for (N) degree of freedom matrix elements.</td>
<td>SOLVEN</td>
<td></td>
</tr>
<tr>
<td>LUDCOMP</td>
<td>Performs Gauss elimination on (N) degree of freedom matrix elements.</td>
<td>SOLVEN</td>
<td></td>
</tr>
<tr>
<td>NEAR3</td>
<td>Finds nearest node to a set of coordinates.</td>
<td>GZ</td>
<td></td>
</tr>
<tr>
<td>NELMFL</td>
<td>Load nodal connection array.</td>
<td></td>
<td>GENCOF</td>
</tr>
<tr>
<td>NOPCRNR</td>
<td>Identify form of incomplete LU decomposition matrix for the Newton-Raphson solution matrix with reordering of the node numbering.</td>
<td>SLVESU</td>
<td></td>
</tr>
<tr>
<td>NOPCNV</td>
<td>Identify form of LU decomposition matrix for the Newton-Raphson solution matrix with no recording of the node numbers.</td>
<td>SLVESU</td>
<td></td>
</tr>
<tr>
<td>NORMAL</td>
<td>Normalize Newton-Raphson equations and calculates sum-squared sum of residuals.</td>
<td></td>
<td>GNSL4, GENSL1</td>
</tr>
<tr>
<td>Name</td>
<td>Description</td>
<td>Calls</td>
<td>Called by</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>----------------------------------------------------------------------</td>
<td>----------------------------</td>
</tr>
<tr>
<td>OUTBND</td>
<td>Test the dependent variables to determine if they are within the bounds set by the thermodynamics properties.</td>
<td></td>
<td>BNSWER</td>
</tr>
<tr>
<td>PEINT</td>
<td>Set up initial temperature gradients where gradient information is user specified.</td>
<td></td>
<td>GZ</td>
</tr>
<tr>
<td>PLOT</td>
<td>Write out data for time history plots at particular nodes.</td>
<td></td>
<td>GZ, TIMCRL</td>
</tr>
<tr>
<td>PLOTC1</td>
<td>Write out tracer data for time history plots. Print out at flow time steps.</td>
<td></td>
<td>CONCEN, RDCON</td>
</tr>
<tr>
<td>POROSI</td>
<td>Read in data for pressure dependent porosity and permeability models. Calculate porosity and permeability functions.</td>
<td>ROCK, INPUT, WRTOUT,THERMW</td>
<td></td>
</tr>
<tr>
<td>PSAT</td>
<td>Calculate the saturation pressure of water for a given temperature.</td>
<td></td>
<td>VARCHK, CO2CTR, THERMW</td>
</tr>
<tr>
<td>RADIUS</td>
<td>Modify finite element coefficients to obtain a radial model.</td>
<td></td>
<td>GZ</td>
</tr>
<tr>
<td>RARNG</td>
<td>Rearrange 3-D coordinates to obtain 2-D problems when enabled.</td>
<td></td>
<td>GZ</td>
</tr>
<tr>
<td>RDCON</td>
<td>Read in tracer data. Initialize tracer variables.</td>
<td>THERMC, PLOTC1, SOLVE</td>
<td>CONCEN</td>
</tr>
<tr>
<td>RDIDOF</td>
<td>Solves the equations generated for heat conduction by a reduced degree of freedom method.</td>
<td>SOLVE</td>
<td>GENSL3, GENCON</td>
</tr>
<tr>
<td>RD2DOF</td>
<td>Solve the equations generated for the water only problems by the reduced degree of freedom method.</td>
<td>SOLVE</td>
<td>GENSL1</td>
</tr>
<tr>
<td>RD3DOF</td>
<td>Solve the equations generated for water/noncondensible gas problems by the reduced degree of freedom method.</td>
<td>SOLVE2, SOLVE</td>
<td>GENSL4</td>
</tr>
<tr>
<td>Name</td>
<td>Description</td>
<td>Calls</td>
<td>Called by</td>
</tr>
<tr>
<td>----------</td>
<td>------------------------------------------------------------------------------</td>
<td>------------------------------</td>
<td>--------------------</td>
</tr>
<tr>
<td>RESETV</td>
<td>Reset the dependent variables to the last time step value. Used when iteration limits are exceeded and a particular time step is restarted.</td>
<td></td>
<td>GZ</td>
</tr>
<tr>
<td>RLPERM</td>
<td>Calculate relative permeability functions.</td>
<td></td>
<td>GZ, THRWMWC, THERMW</td>
</tr>
<tr>
<td>SFN2R</td>
<td>Interpolation routine used by ZONE (2-D).</td>
<td></td>
<td>ZONE</td>
</tr>
<tr>
<td>SFN3R</td>
<td>Interpolation routine used by ZONE (3-D).</td>
<td></td>
<td>ZONE</td>
</tr>
<tr>
<td>SHAP2R</td>
<td>Evaluate 2-D finite element shape functions at quadrature points.</td>
<td></td>
<td>GNCF2</td>
</tr>
<tr>
<td>SHAP3P</td>
<td>Evaluate 3-D prism elements at quadrature points.</td>
<td></td>
<td>GNCF3</td>
</tr>
<tr>
<td>SHAP3R</td>
<td>Evaluate 3-D finite elements shape functions at quadrature points.</td>
<td></td>
<td>GNCF3</td>
</tr>
<tr>
<td>SICE</td>
<td>Reads in data for simulation with ice present.</td>
<td>STHER</td>
<td>GZ, WRTOUT</td>
</tr>
<tr>
<td>SLVESU</td>
<td>Set up equation solver by identifying fill-in positions in the Newton-Raphson matrix.</td>
<td>NOPCNV, NOPCNR, STORAG</td>
<td>GZ</td>
</tr>
<tr>
<td>SOLVE</td>
<td>Solve the one degree of freedom linear system of equation.</td>
<td></td>
<td>RD3DOF, RD2DOF, GENSL3, RD1DOF, GENCON</td>
</tr>
<tr>
<td>SOLVEN</td>
<td>Solve the N degree of freedom linear system of equations.</td>
<td>LUBKSB, LUDCMP</td>
<td></td>
</tr>
<tr>
<td>SOLVE2</td>
<td>Solve the two degree of freedom linear system of equations.</td>
<td></td>
<td>RD3DOF, GENSL1</td>
</tr>
</tbody>
</table>
TABLE III. Subroutine Calls Within FEHMN

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Calls</th>
<th>Called by</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOLVE3</td>
<td>Solve the three degree of freedom linear system of equations.</td>
<td>GNSTR3, GENSL4, RD3DOF</td>
<td></td>
</tr>
<tr>
<td>STEADY</td>
<td>Set up initial pressure distribution when gravity is present (enabled).</td>
<td>BNSWER, ENTHP</td>
<td>GZ</td>
</tr>
<tr>
<td>STORAG</td>
<td>Write out storage requirements of matrices and arrays associated with the solution of linear equations (Newton-Raphson equations).</td>
<td></td>
<td>SILVESU</td>
</tr>
<tr>
<td>STORSX</td>
<td>Manage the storage or retrieval of element coefficients from auxiliary file.</td>
<td>GZ</td>
<td></td>
</tr>
<tr>
<td>THERMC</td>
<td>Evaluate tracer equation of state information.</td>
<td>RDCON, CNSWER</td>
<td></td>
</tr>
<tr>
<td>THERMW</td>
<td>Evaluate the thermodynamic properties (density, enthalpy, and viscosity) as a function of pressure and temperature (or saturation).</td>
<td>CAPPR, POROSI, RLPERM</td>
<td>VARCHK, SOLVE</td>
</tr>
<tr>
<td>THRMWC</td>
<td>Evaluate the thermodynamic properties (density, enthalpy, and viscosity) as a function of pressure, temperature and partial pressure of noncondensible gas for water/noncondensible gas problems.</td>
<td>RLPERM, CAPPR, PSAT, VFCAL</td>
<td>VARCHK</td>
</tr>
<tr>
<td>TIMCRL</td>
<td>Control time step information and stopping criteria.</td>
<td>DUAL, DISK, CO2CTR, GZ</td>
<td></td>
</tr>
<tr>
<td>TYMING</td>
<td>Calculate CPU time for a particular computer run.</td>
<td>SYSTEM, CLOCK*</td>
<td>GZ</td>
</tr>
<tr>
<td>USER</td>
<td>A user programmed subroutine that provides for changing common block variably every time step. Disabled for YMP work.</td>
<td>GZ, INPUT</td>
<td></td>
</tr>
</tbody>
</table>

*Call is made to the system clock of the computer on which it is run.
## TABLE III. Subroutine Calls Within FEHMN

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Calls</th>
<th>Called by</th>
</tr>
</thead>
<tbody>
<tr>
<td>USERC</td>
<td>User defined subroutine for changing common variables associated with tracer transport. Disabled for YMP project.</td>
<td></td>
<td>CONCEN</td>
</tr>
<tr>
<td>VARCHK</td>
<td>Decide, based on current pressure, temperature and saturation values current phase state (fully saturated, partially saturated). Call routine THERMW or THRMWC to update thermodynamic properties of density, enthalpy, and viscosity. Add the Newton-Ralphson corrections to the dependent variables.</td>
<td>THRMWC, THERMW, PSAT</td>
<td>GZ, BNSWER, DUAL, DUALFA</td>
</tr>
<tr>
<td>VELOC</td>
<td>Calculate velocities in coordinate directions.</td>
<td></td>
<td>GZ, CONTR</td>
</tr>
<tr>
<td>VFCAL</td>
<td>Change porosity and permeability as functions of pressure (when enabled).</td>
<td></td>
<td>THRMWC, THERMW</td>
</tr>
<tr>
<td>WRTCON</td>
<td>Write output for tracer at specified intervals.</td>
<td></td>
<td>CONCEN</td>
</tr>
<tr>
<td>WRTOUT</td>
<td>Print output information at a user specified interval.</td>
<td>POROSI, CO2CTR, CONCEN, SICE,</td>
<td>GZ</td>
</tr>
<tr>
<td>ZONE</td>
<td>Divide the input problem space by allowing the user to define zones geometrically and then labeling the nodes as to which zone they belong to. These zones are then used to assign properties to the nodes.</td>
<td>GENDAT, SFN3R, SFN2R, GEOIN</td>
<td>INPUT</td>
</tr>
</tbody>
</table>
B. Common Block and Parameter Statements

The common block and parameter statements are contained in INCLUDE files. The INCLUDE files needed to compile FEHMN are COMAI, COMBI, COMCI, COMDI, COMEI, COMFI, COMGI, COMHI, COMII, COMPRI, and DAVIDI. The INCLUDE file COMDTI contain the parameter statements and will be described first. If a variable is entered, the input MACRO statement is indicated parenthetically.

INCLUDE file COMDTI (parameter statements)

Only parameters that are different from those found in COMPRI are described.

N0 - maximum number of nodes allowed
N2 - 2 * N0, storage parameter
N3 - 3 * N0, storage parameter
N4 - array storage parameter for noncondensible gas solution
N5 - array storage parameter for dual porosity solution
N6 - array storage for ice solution
N7 - array storage for tracer solution
N8 - array storage for variable porosity solution
NR - maximum space allowed for each finite element coefficient array
NBD - 180 * N0 maximum array space for incomplete LU decomposition matrix
LDN - maximum array space needed for Jacobian array matrix
NELMD - maximum array space for element connectivity array and (later) the nodal connectivity array
NNOP - maximum array space for LU decomposition connectivity array
NN - maximum number of connected elements
NQ - maximum array space for each finite element coefficient array associated with the stress solution
NE1 - array size of common block /FEBB/, NELMD + NNOP + 4N0
NE2 - array size of common /FEB/, N0 + 9NR + 6NQ
NE3 - array size of common block /FBS/, 3N0 + 336
NE5 - array size of common block /FCC/, 39N0 + 5N7
NE6 - array size of common block /FDD/, 36N0
NE7 - array size of common block /FDD1/, 14N7 + 870
NE8 - array size of common block /FDD2/, 7N8 + 4
NE9 - array size of common block /FDDI/, 5N0 + 1
NE10 - array size of common block /FHH/, 14N0
NE11 - array size of common block /CO2/, 32N4 + 1
NE12 - array size of common block /FGG/, 9NN + 2N3
NE13 - array size of common block /DUALP/, 23N5
NE14 - array size of common block /FICE/2*N6 +1
NE15 - array size of common block /ICE/N6
### INCLUDE File COMAI

**common/FAA/**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IAB</td>
<td>iteration counter in stress routines</td>
</tr>
<tr>
<td>IAC</td>
<td>counter for print-out interval</td>
</tr>
<tr>
<td>IAD</td>
<td>current iteration number in flow solution</td>
</tr>
<tr>
<td>IACCMMX</td>
<td>maximum iterations allowed for time step increase (tracer solution)</td>
</tr>
<tr>
<td>IAMM</td>
<td>maximum iterations allowed for time step increase (heat and mass solution)</td>
</tr>
<tr>
<td>IAMX</td>
<td>iteration count after which the time step will be halved</td>
</tr>
<tr>
<td>ICAPP</td>
<td>indicates capillary pressure model</td>
</tr>
<tr>
<td>ICE</td>
<td>parameter which indicates if the ICE solution is enabled</td>
</tr>
<tr>
<td>ICEN</td>
<td>parameter which indicates if the tracer solutions enabled</td>
</tr>
<tr>
<td>ICF</td>
<td>parameter indicating status of tracer solution</td>
</tr>
<tr>
<td>ICS</td>
<td>parameter indicating status of tracer solution</td>
</tr>
<tr>
<td>ICGTS</td>
<td>parameter controlling the time of solution parameter changes</td>
</tr>
<tr>
<td>ICHNG</td>
<td>number of thermodynamic region changes</td>
</tr>
<tr>
<td>ICNL</td>
<td>indicates problem dimension</td>
</tr>
<tr>
<td>ICO2</td>
<td>indicates if noncondensible gas solution is enabled (CO2I)</td>
</tr>
<tr>
<td>ICONTR</td>
<td>parameter used in contour plot management</td>
</tr>
<tr>
<td>IDOF</td>
<td>number of degrees of freedom per node for the current problem</td>
</tr>
<tr>
<td>IDUALP</td>
<td>parameter which indicates if the dual porosity solution is enabled (DUAL)</td>
</tr>
<tr>
<td>IFINSH</td>
<td>indicates if the finishing criteria for the simulation is achieved</td>
</tr>
<tr>
<td>IHF</td>
<td>parameter indicating status of flow solution (TRAC)</td>
</tr>
<tr>
<td>IHS</td>
<td>parameter indicating status of flow solution (TRAC)</td>
</tr>
<tr>
<td>III</td>
<td>print-out interval, number of time steps (TIME)</td>
</tr>
<tr>
<td>IFLAG</td>
<td>flag used in input subroutine</td>
</tr>
<tr>
<td>IINP</td>
<td>unit number input file</td>
</tr>
<tr>
<td>ILT</td>
<td>parameter used in time step control</td>
</tr>
<tr>
<td>INTG</td>
<td>indicates integration type used</td>
</tr>
<tr>
<td>IPOROS</td>
<td>indicates if deformation model is enabled (PPOR)</td>
</tr>
<tr>
<td>IPQZ</td>
<td>the number of nodes used for the contour plot output</td>
</tr>
<tr>
<td>IPSAT</td>
<td>parameter regulating call to subroutine PSAT (EOS)</td>
</tr>
<tr>
<td>IOUT</td>
<td>unit number for output file</td>
</tr>
<tr>
<td>IREAD</td>
<td>unit number for restart file (to read)</td>
</tr>
<tr>
<td>IREORD</td>
<td>parameter to reorder system of linear equations</td>
</tr>
<tr>
<td>IRPD</td>
<td>indicates relative permeability model</td>
</tr>
<tr>
<td>ISTRS</td>
<td>parameter indicating if the stress solution is enabled</td>
</tr>
<tr>
<td>ISAVE</td>
<td>unit number for restart file (to write)</td>
</tr>
<tr>
<td>ITER</td>
<td>number of iterations of last call to SOLVE(N) subroutine</td>
</tr>
<tr>
<td>ITERT</td>
<td>intermediate iteration counter</td>
</tr>
<tr>
<td>ITOTAL</td>
<td>total iteration count</td>
</tr>
<tr>
<td>ITSAT</td>
<td>parameter specifying the setting of saturation temperature</td>
</tr>
<tr>
<td>IYEAR</td>
<td>current year in simulation</td>
</tr>
</tbody>
</table>
IVFCAL - indicates if VFCAL subroutine will be called
IW - number of storage locations needed to store geometric input types
IWEILB - indicates if wellbore solution is enabled
L - current time step number
LDA - parameter which specifies if the geometric coefficients are saved (CTRL)
M - total number of nodes used for output information (NODE)
MLZ - out of bounds node
MAXIT - maximum number of iterations allowed before time step is halved (CTRL)
MINK - number of active variables
MONTH - current month in simulation
N - total number of nodes
NBND - maximum number of nonzeros per row in the incidence matrix
NCNTR - contour plot interval (CONT)
NDEM - coordinate direction perpendicular to the contour plane
NI - number of integration points per element
NICG - parameter used in time step control
NEI - total number of elements in the problem (ELEM)
NEQ - number of nodes, not including dual porosity nodes (CORD)
NEIGH - maximum number of neighbors occur in tracer solution
NEMX - number of unique (geometrically) elements
NORTH - maximum number of orthogonalizations allowing in (CTRL)
NS - number of nodes per element (ELEM)
NSAVE - indicates if a restart file will be created in the current problem
NSNK - number of source or sink terms
NSTEP - maximum number of time steps

/FAAR/
AIAA - time step multiplication factor (CTRL)
AN0 - initial tracer concentration (TRAC)
AM0 - initial mass in problem
AMASS - current mass in problem
AMP - initial energy in system problem
ASTEAM - current steam mass in problem
ASTM0 - initial stem mass in problem
AENER - current energy in problem
AW - time step weighting parameter for heat and mass solution (CTRL)
AWC - time step weighting parameter (TRAC)
AWT - value of implicitness factor
AY - time step weighting parameter for tracer (TRAC)
AYC - time step weighting parameter for tracer
CONTIM - interval (days) for contour plot output
DAY - current time step size in days
DAYCF - time at which tracer solution stops (TRAC)
DAYCMM - minimum time step size for tracer solution
DAYCMX - maximum time step size for tracer solution
DAYCS - time at which tracer solution starts (TRAC)
DAYHF - time at which flow solution stops (TRAC)
DAYHS - time at which flow solution starts (TRAC)
DAYMAX - maximum time step allowed (CTRL)
DAYMIN - minimum time step allowed (CTRL)
DAYNEW - parameter used in time step control
DAYS - current simulation time
DAYSCN - next time for contour plot
DAYSI - simulation time at last time step
DAYSP - time at next time step
DEN - intermediate value of mass accumulation term
DENE - intermediate value of energy accumulation term
DEPCNG - depth at which temperature gradient changes
DIFE - energy balance error
DIFM - mass balance error
DITND - next time step change time
DNWGT - upwind weighting parameter
DNWGTA - upwind weighting parameter in tracer solution
DTOT - current time step size in seconds
DTOTC - tracer time step size in seconds
DTOTDM - last time step size in seconds
EPC - specified solution tolerance for tracer solution
EPE - tolerance for Newton-Raphson iteration
EPS - tolerance for linear equation solver
F0 - initial time step residual
FDUM - current sum squared of residuals
GRAD2 - parameter in description of temperature gradient
GRADNT - parameter used in description of temperature gradient
GRAV - value of gravity
G1 - iteration accuracy control parameter (ITER)
G2 - iteration accuracy control parameter (ITER)
G3 - iteration accuracy control parameter (ITER)
OVERF - over relaxation factor for SOR equations (ITER)
PEIN - initial pressure of problem (INIT)
POW - power output for a given time step
QT - total outflow for time step
QTE - total energy outflow for time step
QTOT - total outflow for problem
QTOTE - total energy outflow for problem
QTOTEI - intermediate energy flow total
QTOTI - intermediate flow total
QUAD - parameter used in temperature gradient
RNMAX - maximum run time allowed
SECMA - maximum time step size in seconds
STR - multiplier for Newton-Raphson corrections
STRD - multiplier for Newton-Raphson corrections
SV - vapor saturation of a node
SW - liquid saturation of a node
TEOUTF - parameter used in calculation of energy output
TIMS - ending simulation time (TIME)
TIN - parameter used in temperature gradient
TIN0 - initial problem temperature (INIT)
TIN1 - parameter used in temperature gradient
TMCH - machine tolerance (ITER)
TOUTFL - parameter used in calculation of flow output
UPWGT - upwind weighting parameter
UPWGTA - upwind weighting factor for tracer solution (TRAC)
UPWGTB - upwind weighting factor for tracer solution
VLMAX - maximum liquid phase velocity
VTOT - total volume in problem
VVMAX - maximum vapor phase velocity

common/FAAC/

SSSOL - indicates if initial steady state solution is needed
VEQNO - contains version number of FEHMN code used

common/FAAC1/

IDATE - contains the date (mm/dd/yr)

INCLUDE file COMBI

common/FBB/

NELM(NELMD) - initially information about nodes in each element, later nodal
connectivity information
NOP(NNOP) - matrix sparsity structure for LU decomposition
NAR(N0) - array containing gauss elimination order for each node
ISTRW(N0) - starting positions in sx(nr,9) array of finite element coefficients for
each node
KA(N0) - contains boundary type information for each node
NELMDG(N0) - contains position of (i,i) element in connectivity array
common/FBC/

SX1(N0) - contains volume associated with each node
SX(NR,9) - contains finite element geometric coefficients necessary for heat and mass transfer simulation
SXS(NQ,6) - contains more finite element geometric coefficients (ie., those necessary for the stress module)

common/FBS/

CORD(N0,3) - contains the coordinates of each node
XD(8),YD(8),ZD(8) - global coordinates of the nodes in a finite element
SI(8),ETA(8),EXCI(8) - local coordinates in a finite element of the numerical integration points
XT(8),YT(8),ZT(8) - parameters needed in element calculations
W(8,8) - finite element shape functions
WX(8,8) - derivative of shape functions with respect to x
WY(8,8) - derivative of shape functions with respect to y
WZ(8,8) - derivative of shape functions with respect to z
DR(8) - contains weights for integration points (bricks, rectangles)
DP(6) - contains weights for integration points (prisms, triangles)
WR(8,8) - finite element shape functions (rectangles)
WXR(8,8) - derivative of shape functions with respect to x (rectangles)
WYR(8,8) - derivative of shape functions with respect to y (rectangles)
WZR(8,8) - derivative of shape functions with respect to z (rectangles)
WP(8,8) - finite element shape functions (prisms)
WXP(8,8) - derivative of shape functions with respect to x (prisms)
WYP(8,8) - derivative of shape functions with respect to y (prisms)
WZP(8,8) - derivative of shape functions with respect to z (prisms)

INCLUDE File COMCI

common/FCC/

DANL(N7) - derivative of liquid phase concentration with respect to total concentration
DANV(N7) - derivative of vapor phase concentration with respect to total concentration
AKC(N7) - tracer accumulation term derivative with respect to total concentration
DRC(N7) - tracer source term derivative with respect to total concentration
DEEF(N0) - derivative of energy accumulation with respect to energy variable
DEPF(N0) - derivative of energy accumulation term with respect to pressure
DMPF(N0) - derivative of mass accumulation term with respect to pressure
DMEF(N0) - derivative of mass accumulation term with respect to energy variable
DQ(N0) - derivative of mass source term with respect to pressure
DQH(N0) - derivative of energy source term with respect to pressure
DEQH(N0) - derivative of energy source term with respect to energy variable
DTPA(N0) - derivative of temperature with respect to energy variable
DTPAE(N0) - derivative of temperature with respect to energy variable
DEVEF(N0) - derivative of enthalpy with respect to energy variable
DEVF(N0) - derivative of enthalpy with respect to pressure
DRVPF(N0) - derivative of vapor relative permeability with respect to pressure
DRVEF(N0) - derivative of vapor relative permeability with respect to energy variable
DRLEF(N0) - derivative of liquid relative permeability with respect to energy variable
DILE(N0) - derivative of liquid transmissibility with respect to energy variable
DILP(N0) - derivative of liquid transmissibility with respect to pressure
ENLF(N0) - liquid enthalpy
ENVF(N0) - vapor enthalpy
DSTM(N0) - steam mass
DENI(N0) - mass accumulation term
DENEI(N0) - energy accumulation term
DIL(N0) - liquid transmissibility
DIV(N0) - vapor transmissibility
ROLF(N0) - liquid density
ROVF(N0) - vapor density
RLF(N0) - liquid phase relative permeability
RVF(N0) - vapor phase relative permeability
DENCI(N7) - tracer accumulation term
GL(N0) - liquid phase gravity term
GV(N0) - vapor phase gravity term
DIVE(N0) - derivative of vapor transmissibility with respect to energy variable
DIVP(N0) - derivative of vapor transmissibility with respect to pressure
DGVP(N0) - derivative of vapor mass gravity term with respect to pressure
DGVE(N0) - derivative of vapor mass gravity term with respect to energy variable
DPCEF(N0) - derivative of capillary pressure with respect to the energy variable

INCLUDE file COMDI

common/FDD/

THX(N0) - thermal conductivity x-direction
THY(N0) - thermal conductivity y-direction
THZ(N0) - thermal conductivity z-direction
VOLUME(N0) - volume associated at each node
SK(N0) - source strength of each node
ESK(N0) - inlet enthalpy associated with a source
QFLUX(N0) - heat flux at each node
QFLXM(N0) - heat flux impedance at each node
PNX(N3) - permeability in the x-direction, liquid velocity in the x-direction, vapor velocity in the x-direction
PNY(N3) - permeability in the y-direction, liquid velocity in the y-direction, vapor velocity in the y-direction
PNZ(N3) - permeability in the z-direction, liquid velocity in the z-direction, vapor velocity in the z-direction
PS(N0) - porosity at each node
EFLOW(N0) - energy flow at each source node
PHI(N0) - pressure at each node
PHO(N0) - last time step pressure at each node
T(N0) - temperature at each node
TO(N0) - last time step pressure at each node
VF(N0) - volume factor at each node
PFLOW(N0) - flowing pressure at each source node
DENEH(N0) - last time step energy accumulation term at each node
QH(N0) - energy source term at each node
S(N0) - liquid saturation at each node
SO(N0) - last time step saturation at each node
WELLIM(N0) - well impedance at each source node
DENH(N0) - last time step mass accumulation term at each node
DENJ(N0) - last time step mass accumulation time derivative at each node
DENEJ(N0) - last time step energy accumulation time derivative at each node
DENR(N0) - rock density at each node
CPR(N0) - rock specific heat at each node
PCP(N0) - capillary pressure at each node

common/FDDI/

NSKW(N0) - contains nodes for print-out

common/FDD1/

AN(N7) - total tracer concentration at each node
ANL(N7) - liquid tracer concentration at each node
ANV(N7) - vapor tracer concentration at each node
CNSK(N7) - tracer concentration source term at each node
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1SK(N7)</td>
<td>time when the tracer source term is activated at each node</td>
</tr>
<tr>
<td>T2SK(N7)</td>
<td>time when the tracer source term is terminated at each node</td>
</tr>
<tr>
<td>RC(N7)</td>
<td>tracer source term at each node</td>
</tr>
<tr>
<td>DENCH(N7)</td>
<td>last time step tracer accumulation term at each node</td>
</tr>
<tr>
<td>DENCJ(N7)</td>
<td>last time step tracer accumulation derivative term at each node</td>
</tr>
<tr>
<td>ANLO(N7)</td>
<td>last time step total tracer concentration at each node</td>
</tr>
<tr>
<td>FC(N7)</td>
<td>tracer equation residual at each node</td>
</tr>
<tr>
<td>TCX(N7)</td>
<td>tracer diffusion term in the x-direction</td>
</tr>
<tr>
<td>TCY(N7)</td>
<td>tracer diffusion term in the y-direction</td>
</tr>
<tr>
<td>TCZ(N7)</td>
<td>tracer diffusion term in the z-direction</td>
</tr>
<tr>
<td>CM(10)</td>
<td>total tracer mass for each specie</td>
</tr>
<tr>
<td>CM0(10)</td>
<td>initial total tracer mass for each specie</td>
</tr>
<tr>
<td>QOUT(10)</td>
<td>total injected tracer mass for each specie</td>
</tr>
<tr>
<td>QCIN(10)</td>
<td>total produced tracer mass for each specie</td>
</tr>
<tr>
<td>A1ADF(10)</td>
<td>alpha1 parameter for each specie (nonlinear adsorption)</td>
</tr>
<tr>
<td>A2ADF(10)</td>
<td>alpha2 parameter for each specie (nonlinear adsorption)</td>
</tr>
<tr>
<td>BETADF(10)</td>
<td>beta parameter for each specie (nonlinear adsorption)</td>
</tr>
<tr>
<td>A1R</td>
<td>parameter in reaction rate model</td>
</tr>
<tr>
<td>A2R</td>
<td>parameter in reaction rate model</td>
</tr>
<tr>
<td>A3R</td>
<td>parameter in reaction rate model</td>
</tr>
<tr>
<td>A4R</td>
<td>parameter in reaction rate model</td>
</tr>
<tr>
<td>RP1F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>RP2F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>RP3F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>RP4F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>RP5F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>RP6F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>RP7F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>RP8F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>RP9F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>RP10F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>RP11F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>RP12F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>RP13F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>RP14F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>RP15F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>RP16F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>RP17F(25)</td>
<td>parameter in relative permeability model</td>
</tr>
<tr>
<td>CP1F(10)</td>
<td>parameter in capillary pressure model</td>
</tr>
<tr>
<td>CP2F(10)</td>
<td>parameter in capillary pressure model</td>
</tr>
<tr>
<td>CP3F(10)</td>
<td>parameter in capillary pressure model</td>
</tr>
<tr>
<td>DIT(300)</td>
<td>array containing time step changes</td>
</tr>
</tbody>
</table>
common/FDD1I/

NSPECI - number of species for tracer solution
NSP - current specie number
NPN - parameter used in storing tracer data
NPT(10) - storage parameter used in tracer solution
IADD(10) - iteration count for tracer solution
IADDT(10) - iteration count used for linear equation solver in tracer solution
IADSF(10) - adsorption model used in tracer solution
ICAPT(10) - capillary pressure model
IRLPT(10) - relative permeability model
ICNS(10) - liquid or gas phase tracer model
ICRATE - reaction rate model
ITC(100) - array containing information used in time step changes

common/FDD2/

PHINI(N8) - initial pressure at each node
PSINI(N8) - initial porosity at each node
DPORPN8) - derivative of porosity with respect to pressure at each node
DPORTH(N8) - derivative of porosity with respect to temperature at each node
TINI(N8) - initial temperature at each node
SIGINI - parameter used in rock deformation model
PHYDRO - parameter used in rock deformation model
THEXP - parameter used in rock deformation model
YOUNG - parameter used in rock deformation model
AMGANG(N8) - parameter used in gangi model at each node
PGANGI(N8) - parameter used in gangi model at each node

common/FICE/

SII(N6) - ice saturation at each node
SIO(N6) - last time step value of ice saturation
TMELT - freezing temperature of water
common/IICE/

ICES(N6) - state of ice at each node

IIEOS(N0) - thermodynamics set at each node
IEOS(N0) - phase state of fluid at each node
IEOSC - phase change parameter
IPORF(N0) - deformation model at each node
IRLP(N0) - relative permeability model at each node
ICAP(N0) - capillary pressure model at each node

INCLUDE file COMEI

common/FEE/

A(LDN) - array containing the Jacobian matrix

common/FFF/

B(NBD) - array containing the incomplete LU decomposition of the Jacobian matrix

common/FHH/

IRB(N0) - array containing the reordered node numbers
IIRB(N0) - inverse of irb
NOPT(N0) - array indicating active variables
NPVT(N0) - pivot information for the LU decomposition matrix
IG(N0) - variable order LU decomposition information
PIV(N0,9) - the pivot elements of the LU decomposition

INCLUDE file COMFI

DMC(N4) - derivative of mass accumulation term with respect to gas at each node
DEC(N4) - derivative of energy accumulation term with respect to gas at each node
DCP(N4) - derivative of gas accumulation term with respect to pressure at each node
DCE(N4) - derivative of gas accumulation term with respect to energy at each node
DQC(N4) - derivative of mass source term with respect to gas
DEQC(N4) - derivative of energy source term with respect to gas
DCQC(N4) - derivative of gas source term with respect to gas
DENPCI(N4) - gas accumulation term at each node
DENPCH(N4) - last time step value of the mass accumulation term at each node
DIVC(N4) - derivative of vapor transmissibility with respect to gas
DGVC(N4) - derivative of vapor gravity term with respect to gas
DCVCF(N4) - derivative of gas concentration with respect to gas
DCVEF(N4) - derivative of gas concentration with respect to energy
DCVF(N4) - derivative of gas concentration with respect to pressure
DEVCF(N4) - derivative of energy with respect to gas
CNVF(N4) - concentration of gas in the vapor phase
DILC(N4) - derivative of liquid transmissibility with respect to gas
DGLC(N4) - derivative of liquid gravity term with respect to gas
ESKC(N4) - source term for gas equation
DCLCF(N4) - derivative of liquid concentration with respect to gas
DCLCF(N4) - derivative of liquid concentration with respect to energy
DCLF(N4) - derivative of liquid concentration with respect to pressure
DELCF(N4) - derivative of liquid energy with respect to gas
CNLF(N4) - gas concentration in the liquid phase
QC(N4) - source term for the gas equation
DCC(N4) - derivative of gas accumulation term with respect to gas
DTPAC(N4) - derivative of temperature with respect to gas
DENPCJ(N4) - last time step accumulation term of gas equation
DCQH(N4) - derivative of gas source term with respect to energy
PCI(N4) - gas pressure
PCIO(N4) - last time step gas pressure
DIFC - noncondensible gas mass balance error

INCLUDE file COMGI

common/FGG/

DFMP(NN) - derivative of mass accumulation term with respect to pressure for neighbor nodes
DFME(NN) - derivative of mass accumulation term with respect to energy for neighbor nodes
DFEP(NN) - derivative of energy accumulation term with respect to pressure for neighbor nodes
DFEE(NN) - derivative of energy accumulation term with respect to energy for neighbor nodes
DFMPC(NN) - derivative of mass accumulation term with respect to gas for neighbor nodes
DFEPC(NN) - derivative of energy accumulation term with respect to gas for neighbor nodes
DFPCP(NN) - derivative of gas accumulation term with respect to pressure for neighbor nodes
DFPCE(NN) - derivative of gas accumulation term with respect to energy for neighbor nodes
neighbor nodes

DFPCPC(NN) - derivative of gas accumulation term with respect to gas for neighbor nodes

BP(N3) - array of Newton-Raphson residuals, after solution, an array of Newton-Raphson corrections

BPC(N3) - array of Newton-Raphson residuals for gas, after solution, an array of Newton-Raphson corrections

INCLUDE file COMHI

common/DUALP/

VOLF1(N5) - volume fraction at each node for the first matrix layer
VOLF2(N5) - volume fraction at each node for the second matrix layer
APUV1(N5) - area per unit volume for the first matrix layer
WB11(N5) - array needed to store intermediate dual porosity results
WB12(N5) - array needed to store intermediate dual porosity results
WB21(N5) - array needed to store intermediate dual porosity results
WB22(N5) - array needed to store intermediate dual porosity results
TB11(N5) - array needed to store intermediate dual porosity results
TB12(N5) - array needed to store intermediate dual porosity results
TB21(N5) - array needed to store intermediate dual porosity results
TB22(N5) - array needed to store intermediate dual porosity results
A21MPF(N5) - array needed to store intermediate dual porosity results
A21MEF(N5) - array needed to store intermediate dual porosity results
A21EPF(N5) - array needed to store intermediate dual porosity results
A21EEF(N5) - array needed to store intermediate dual porosity results
A32MPF(N5) - array needed to store intermediate dual porosity results
A32MEF(N5) - array needed to store intermediate dual porosity results
A32EPF(N5) - array needed to store intermediate dual porosity results
A32EEF(N5) - array needed to store intermediate dual porosity results
RB2MF(N5) - array needed to store intermediate dual porosity results
RB2EF(N5) - array needed to store intermediate dual porosity results
RB3MF(N5) - array needed to store intermediate dual porosity results
RB3EF(N5) - array needed to store intermediate dual porosity results

INCLUDE file COMII

common/COEFF/

CEL(20,2) - polynomial coefficients for liquid water enthalpy equations
CRL(20,2) - polynomial coefficients for liquid water density equations
CEV(20,2) - polynomial coefficients for vapor water enthalpy equations
CRV(20,2) - polynomial coefficients for vapor water density equations
CVL(20,2) - polynomial coefficients for liquid water viscosity equations
CVV(20,2) - polynomial coefficients for vapor water viscosity equations
TSA0 - polynomial coefficient for saturation temperature equation
TSPA1 - polynomial coefficient for saturation temperature equation
TSPA2 - polynomial coefficient for saturation temperature equation
TSPA3 - polynomial coefficient for saturation temperature equation
TSPA4 - polynomial coefficient for saturation temperature equation
TSPB0 - polynomial coefficient for saturation temperature equation
TSPB1 - polynomial coefficient for saturation temperature equation
TSPB2 - polynomial coefficient for saturation temperature equation
TSPB3 - polynomial coefficient for saturation temperature equation
TSPB4 - polynomial coefficient for saturation temperature equation
PSA0 - polynomial coefficient for saturation pressure equation
PSTA1 - polynomial coefficient for saturation pressure equation
PSTA2 - polynomial coefficient for saturation pressure equation
PSTA3 - polynomial coefficient for saturation pressure equation
PSTA4 - polynomial coefficient for saturation pressure equation
PSB0 - polynomial coefficient for saturation pressure equation
PSTB1 - polynomial coefficient for saturation pressure equation
PSTB2 - polynomial coefficient for saturation pressure equation
PSTB3 - polynomial coefficient for saturation pressure equation
PSTB4 - polynomial coefficient for saturation pressure equation

common/COEFF1/

P MAX(3) - maximum pressure allowed for each coefficient set
P MIN(3) - minimum pressure allowed for each coefficient set
T MAX(3) - maximum temperature allowed for each coefficient set
T MIN(3) - minimum temperature allowed for each coefficient set

common/COEFF2/

EW1 - coefficient used in simplifying thermodynamics relations
EW2 - coefficient used in simplifying thermodynamics relations
EW3 - coefficient used in simplifying thermodynamics relations
EW4 - coefficient used in simplifying thermodynamics relations
EW5 - coefficient used in simplifying thermodynamics relations
EW6 - coefficient used in simplifying thermodynamics relations
EW7 - coefficient used in simplifying thermodynamics relations
EW8 - coefficient used in simplifying thermodynamics relations
EW9 - coefficient used in simplifying thermodynamics relations
EW10 - coefficient used in simplifying thermodynamics relations
EW11 - coefficient used in simplifying thermodynamics relations
EV1 - coefficient used in simplifying thermodynamics relations
EV2 - coefficient used in simplifying thermodynamics relations
EV3 - coefficient used in simplifying thermodynamics relations
EV4 - coefficient used in simplifying thermodynamics relations
EV5 - coefficient used in simplifying thermodynamics relations
EV6 - coefficient used in simplifying thermodynamics relations
EV7 - coefficient used in simplifying thermodynamics relations
EV8 - coefficient used in simplifying thermodynamics relations
EV9 - coefficient used in simplifying thermodynamics relations
EV10 - coefficient used in simplifying thermodynamics relations
EV11 - coefficient used in simplifying thermodynamics relations

INCLUDE file DAVIDI

common/DAVIDI/

IRDOF - reduced degree of freedom model used
ISLORD - parameter used in the reduced degree of freedom model
IBACK - LU factorization save parameter
ICOUPL - number of SOR iterations
ITEST - parameter used in the reduced degree of freedom model

common/DAVID2/

NMAT(9) - array used in the reduced degree of freedom method
NRHS(3) - array used in the reduced degree of freedom method
NMAT32(4) - array used in the reduced degree of freedom method

C. Subroutine Structure for FEHMN

Figure 4 presents the general subroutine call tree structure for the code FEHMN. Due to its complexity, the entire structure could not fit within a page, hence it is broken into several branches. As an example of the tree's use we trace the call structure from GZ (the main program), which calls INPUT (0003), which calls CONCEN(0021), which calls CSOLVE (0026), which calls CNSWER (0028) which calls THERMC (0028). At subroutine THERMC, the tree is directed to line 0024 where the calls from THERMC have been previously explained. We see that THERMC calls DABS. This was only one path of many possible; all paths may be traversed in a way analogous to above.
Fig. 4. Structure for subroutine calls for FEHMN.
Fig. 4. (Continued)
Fig. 4. (Continued)
III. USER'S MANUAL

This section provides the instructions necessary for using the code FEHMN. Section A describes an automatic mesh generation code, GENMSH, that can be used to generate the finite element mesh input required by FEHMN. Section B describes in detail the input macro-commands for FEHMN. A graphics postprocessor is described in Section C and several example problems are given in Section D.

A. Automatic Mesh Generation

The code GENMSH, which is designed to be used in conjunction with the finite element code FEHMN, operates by dividing a user-defined solution space into a prescribed number of finite elements. The solution space may be either two or three dimensional. In order to simplify geometric considerations, the solution space is broken up into a number of "blocks." Each block is then further divided into "elements." A given block uses the same "type" of element throughout, although the elements need not be of the same size. Different blocks may use different types of elements. The choice of elements available in two dimensions for output to FEHMN are 4-noded quadrilaterals and 3-noded triangles. In three dimensions, the choice is 8-noded quadrilateral polyhedrons or 3-noded triangular prisms, shown schematically in Fig. 5.

The solution space, and each block, is defined by giving the corner and/or midpoint nodes. This amounts to specifying 4 or 8 nodes per block in 2-D problems and 8 or 20 nodes per block in 3-D problems as shown in Fig. 6. The same block definition is needed for both quadrilateral and triangular elements. After discretizing the solution space and the blocks, it is necessary to specify the desired division of each block into elements. This is done by specifying the relative weights of each subdivision in each direction for each block. The reader is referred to *An Introduction to Finite Element Computations* by Hinton and Owen (pp. 328-346) for the details of the procedure and the algorithm. Note that material properties are specified at each node in FEHMN, unlike conventional finite element procedure.

A detailed description of the input to the code follows. The input must be placed in a file called MDAT. Examples of the MDAT file are given in Section D. The output is found on a file called GEOM.DAT. The output consists of a list of nodal coordinates and elements in a suitable format for the code FEHMN. Free format is used for the input. In addition, a zone definition of the input blocks is provided in file ZONE. The files GEOM.DAT and
Fig. 5. Elements available with FEHMN in 2-D and 3-D problems showing numbering convention.
Fig. 6. Nodal ordering for defining 2-D and 3-D domain blocks for input to GENMSH.
ZONE may be used directly in the input file for FEHMN as they already include the appropriate macro command headings.

Group 1
Title, up to 80 characters long

Group 2
NPOIN, NELEM, LNODE, IT, NDIME, KF, NRN
NPOIN - total number of points used to define the solution space
NELEM - number of blocks used to define the solution domain
LNODE - number of nodes used to define the solution domain
IT - element triangulation parameter, if IT=0, then no element splitting occurs, if IT≠ 0, then triangle elements are generated in 3-D.
NDIMB - dimension of problem (2 or 3)
KF - determines the vertical axis direction in 2-D. If KF < 0 then the y-axis is positive downward. If KF ≥0, the y-axis is positive upward in 3-D, the z-axis is always positive upward. See note on plotting below.
NRN - renumber parameter. If NRN ≠ 0 the code will renumber to minimize the band width. If NRN = 0 then no renumbering will occur.

Group 3 (NUMEL, Block Times)
NUMEL, (LNODS(NUMEL,INOD), INODE = 1, LNODE)
NUMEL - Block number
LNODS (NUMEL, INOD) - LNODE nodes defining the block NUMEL

Group 4
CORDX (I), I = 1, NPOIN
CORDX (I) - x-coordinates of points used to define the solution space

Group 5
CORDY (I), I = 1, NPOIN
CORDY (I) - y-coordinates of points used to define the solution space

Group 6 (only if NDIME = 3)
CORDZ (I), I = 1, NPOIN
CORDZ (I) - z-coordinates of points used to define the solution space
NOTE: Groups 7-10 are repeated for each block.
Group 7
KBLOC, NDIVX, NDIVY, NDIVZ
KBLOC - block number
NDIVX - number of divisions in x-direction
NDIVY - number of divisions in y-direction
NDIVZ - number of divisions in z-direction (only if NDIME = 3).
NOTE: The number of nodes in a direction = number of divisions + 1.

Group 8
WEITX (I), I = 1, NDIVX
WEITX (I) - weighting of the Ith division in the x-direction in block KBLOC

Group 9
WEITY (I), I = 1, NDIVY
WEITY (I) - weighting of the Ith division in the y-direction in block KBLOC

Group 10
WEITZ (I), I = 1, NDIVZ (only if NDIME = 3)
WEITZ (I) - weighting of the Ith division in the z-direction in block KBLOC

Group 11 (only for 3-D problem)
XV, YV, ZV
XV - x-coordinate of viewing point for 3-D plot
YV - y-coordinate of viewing point for 3-D plot
ZV - z-coordinate of viewing point for 3-D plot
NOTE: XV = 300, YV = 300, ZV = 50 gives a good viewing angle for many problems.

Group 12 (only for 3-D problem)
XMIN, XMAX, YMIN, YMAX, ZMIN, ZMAX
XMIN - minimum x-coordinate for plot of mesh
XMAX - maximum x-coordinate for plot of mesh
If XMIN = XMAX then the limits will be determined from the generated mesh
YMIN - minimum y-coordinate for plot of mesh
YMAX - maximum y-coordinate for plot of mesh
If YMIN = YMAX then the limits will be determined from the generated mesh
ZMIN - minimum z-coordinate for plot of mesh
ZMAX - maximum z-coordinate for plot of mesh
If ZMIN = ZMAX then the limits will be determined from the generated mesh
Group 13
If 2-D problem (NDIME=2):
NREF, XR, YR, IRFN
NREF - reference node for refinement
XR    - x-coordinate of reference node
YR    - y-coordinate of reference node
IRFN  - refinement level for reference node

If NREF < 0 then the node closest to the reference coordinates is used as the reference node. If IRFN < 0 then the node closest to the reference coordinates is used as the reference node and the coordinates set to the reference coordinates. The refinement level is ABS (IRFN). If the refinement level is 1 then the elements surrounding the reference node are subdivided once and new elements created. A refinement level of 2 means this is done twice, and so on.

Group 14
If 3-D problem (NDIME=3)
NREF, XR, YR, ZR, IRFN
NREF - reference node for refinement
XR    - x-coordinate of reference node
YR    - y-coordinate of reference node
ZR    - z-coordinate of reference node
IRFN  - refinement level for reference node

Note on plotting. If KF # 0 in Group 2 then a plot of the mesh is provided.

B. Macro Command Input For FEHMN

The finite element heat and mass transfer code (FEHMN) contains a macro control structure for data input that offers added flexibility to the input process. Blocks of data can be entered in any order, and any blocks unnecessary to a particular problem can be disregarded. All entries are free format, which adds flexibility, but requires that values be entered for all input variables (no assumed null values). As an aid to the user, the capabilities of FEHMN are summarized in Table IV with reference to the macro commands.

Values of parameters are entered either by node number or a geometric description (macro command ZONE). The user is encouraged to read the macro ZONE description.
TABLE IV. Capabilities of FEHMN with Macro Command References

I. Mass, energy balances in porous media
   A. Variable rock properties (ROCK)
   B. Variable permeability (PERM)
   C. Variable thermal conductivity (COND)
   D. Variable fracture properties, dual porosity (DUAL)

II. Multiple components available
    A. Air water mixture available, fully coupled to heat and mass transfer (NCON)
    B. Up to 10 passive tracers available (TRAC)
    C. Several different capillary pressure models (CAP)
    D. Several different relative permeability models (RLP)

III. Equation of state flexibility inherent in code (EOS)

IV. Psuedo-stress models available
    A. Linear porosity deformation (PPOR)
    B. Gangi stress model (PPOR)

V. Numerics
    A. Finite element with multiple element capabilities (ELEM)
    B. Short form input methods available (COOR, ELEM)
    C. Flexible properties assignment (ZONE)
    D. Flexible solution methods
       1. Upwinding, implicit solution available (CTRL)
       2. Iteration control adaptive strategy (ITER)

VI. Flexible time step and stability control (TIME)

TABLE V. FEHMN Files

<table>
<thead>
<tr>
<th>TAPEn1(1 ≤ n1 &lt; 10)</th>
<th>input file (user supplied)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TAPEn2(11 ≤ n2 ≤ 20)</td>
<td>output file (code generated)</td>
</tr>
<tr>
<td>TAPEn3(21 ≤ n3 ≤ 30)</td>
<td>initial value tape (read a restart, user supplied)</td>
</tr>
<tr>
<td>TAPEn4(31 ≤ n4 ≤ 40)</td>
<td>final value tape (write a restart, code generated)</td>
</tr>
<tr>
<td>FE.HIS</td>
<td>time history of variables (code generated)</td>
</tr>
<tr>
<td>FE.CON</td>
<td>contour plot data (code generated)</td>
</tr>
<tr>
<td>FE.MIS</td>
<td>contains program information of the code crashes (code generated)</td>
</tr>
<tr>
<td>FE.TRRC</td>
<td>tracer time history data (code generated)</td>
</tr>
<tr>
<td>STOR.FE</td>
<td>storage of finite element coefficients (user supplied or code generated)</td>
</tr>
<tr>
<td>FE.CHK</td>
<td>contains input analysis</td>
</tr>
</tbody>
</table>
Although most input data are contained in the input deck, some information must first be entered from the **terminal**. Most of the information required before the input deck can be read pertains to input and output files and is prompted for from within the program. Figure 7 shows the output for a typical FEHMN run on a CRAY computer. All files are of the form TAPEn, where $n$ is the tape (unit) number. Table V contains a description of FEHMN files. The terminal input follows:

*(free format) INPT, IOUT, IREAD, ISAVE, IDC*

**INPT** - unit number of the input file

**IOUT** - unit number of the file to which nodal information (pressures, enthalpies, etc.) is printed

**IREAD** - unit number of file on which initial values of pressure and enthalpy are found.

(If $\text{IREAD} = 0$, initial values are generated from input data. See macro control statement **PRES**)

**ISAVE** - unit number of file on which final time nodal information is printed for restart purposes (if ISAVE = 0, no restart file is created).

**IDC** - user subroutine parameter

- if IDC = 0 the code makes no USER subroutine calls
- if IDC ≠ 0 the code calls subroutine USER (user defined) at each time step.

The form is CALL USER (IDC).

In Fig. 8, the user entered data is after the “?”.

Here INPT = 1, IOUT = 11, IREAD = 0, ISAVE = 0, and IDC = 0. The remainder of the data is read from unit INPT (which was defined above).

The macro command structure makes use of a set of control statements recognized by the input module of the program. When a macro control statement is encountered in the input deck, a certain set of data is expected and read from the input deck. In this way, the input is divided into separate, unordered blocks of data. The input deck is therefore a collection of macro control statements, each followed by its associated data block, as the example input files (Section D) show. Note that, although the input is free format, macro control statements must appear in the first four columns of a line. Table VI lists the macro control statements with brief descriptions of the data associated with each. The macro control statements may be in any order, although the data associated with each macro control statement must follow with the prescribed format. Some statements are necessary and others are optional, as indicated in Table VI.

A detailed list of the macro control statements and the input variables associated with each is given in Table VII. In Section D example input files are given.
** program files must obey the following conventions **

input files: tape1-10
output files: tape11-20
read files (if they exist): tape21-30
write files (if they exist): tape31-40

** note **

history plot file is fe.his
contour plot file is fe.con
tracer plot file is fe.trc
input check file is fe.chk
fe.mis is used for error recovery

** and **

tape59 (terminal) is available for any use

input tape, output file, rd tape, wt tape, user call/step
? 1 11 0 0 0
input title: node
input title: rlp
input title: sol
input title: init
input title: rock
input title: cond
input title: perm
input title: flow
input title: time
input title: ctrl
input title: coor
input title: elem
input title: stop
stop
xfehmz ctss time 41.483 seconds
cpu= 18.305 i/o= 2.519 mem= 20.660

** Fig. 7. Terminal output for a FEHMN run on a CRAY computer. **

59
### TABLE VI. Macro Control Statements for FEHMN

<table>
<thead>
<tr>
<th>Control Statement</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAP</td>
<td>capillary pressure data, optional.</td>
</tr>
<tr>
<td>COOR</td>
<td>node coordinate data, <strong>required</strong>.</td>
</tr>
<tr>
<td>COND</td>
<td>thermal conductivity data, <strong>required</strong>.</td>
</tr>
<tr>
<td>CONT</td>
<td>contour plot data, optional.</td>
</tr>
<tr>
<td>CTRL</td>
<td>program control parameters, <strong>required</strong>.</td>
</tr>
<tr>
<td>DUAL</td>
<td>input for dual porosity solution, optional.</td>
</tr>
<tr>
<td>ELEM</td>
<td>element node data, <strong>required</strong>.</td>
</tr>
<tr>
<td>EOS</td>
<td>equation of state data, optional.</td>
</tr>
<tr>
<td>FLOW</td>
<td>flow data, optional.</td>
</tr>
<tr>
<td>HFLX</td>
<td>heat flux data, optional.</td>
</tr>
<tr>
<td>INIT*</td>
<td>initial value data, optional.</td>
</tr>
<tr>
<td>ITER</td>
<td>iteration parameters, optional.</td>
</tr>
<tr>
<td>NGAS</td>
<td>noncondensible gas (air) data, optional.</td>
</tr>
<tr>
<td>NODE</td>
<td>node numbers for output and time histories, <strong>required</strong>.</td>
</tr>
<tr>
<td>PERM</td>
<td>permeability and velocity data, <strong>required</strong>.</td>
</tr>
<tr>
<td>PPOR</td>
<td>pressure and temperature dependent porosity and permeability, optional.</td>
</tr>
<tr>
<td>PRES</td>
<td>initial pressure and enthalpy data, optional.</td>
</tr>
<tr>
<td>RLP</td>
<td>relative permeability data, optional (<strong>required</strong> for 2-phase problem).</td>
</tr>
<tr>
<td>ROCK</td>
<td>rock density, specific heat, and porosity data, <strong>required</strong>.</td>
</tr>
<tr>
<td>STEA</td>
<td>solution specifications, <strong>required</strong>.</td>
</tr>
<tr>
<td>STOP</td>
<td>steady state solution generated for initial variable field, optional.</td>
</tr>
<tr>
<td>TEXT</td>
<td>text input, optional.</td>
</tr>
<tr>
<td>TIME</td>
<td>time step and time of simulation data, <strong>required</strong>.</td>
</tr>
<tr>
<td>TRAC</td>
<td>tracer data, optional.</td>
</tr>
<tr>
<td>ZONE</td>
<td>geometric definition of grid for input definition, optional.</td>
</tr>
</tbody>
</table>

*One or both INIT and PRES macros are necessary.

The user is encouraged to look at the file FE.CHK. This file contains information on where the maximum and minimum input parameters occur and suggestions on decreasing the storage requirements for the run; it also summarizes initial mass and energy values.

Many input parameters such as porosity or permeability vary throughout the grid and need to be assigned different values on different nodes. This is accomplished in two ways. The first is a nodal loop-type definition:

\[ JA, JB, JC, PROP1, PROP2, \ldots \]

where

60
JA - first node to be assigned with the properties PROP1, PROP2 ...
JB - last node to be assigned with the properties PROP1, PROP2 ...
JC - loop increment for assigning properties PROP1, PROP2, ...
- properties to be assigned to the nodes. In the
input blocks to follow one or more properties are manually entered
in the above structure.

The nodal definition above is useful in simple geometries where the node numbers
are easily found. The boundary nodes, in this case come at regular node intervals and the
increment counter JC can be adjusted so the boundary conditions are easily entered.

In more complicated geometries, such as 3-D grids, the node numbers are often
difficult to determine. Here a geometric description is preferred. To enable the geometric
description the control statement ZONE (p. 74) is used in the input file before the other
property macro statements occur. The input macro ZONE requires the specification of
the coordinates of 4-node parallelograms for 2-D problems and the 8-node polyhedrons in
3-D. In one usage of the control statement ZONE all the nodes are placed in geometric
zones and assigned an identifying number. This number is then addressed in the property
input macro commands by specifying a JA < 0 in the definition of the loop parameters
given above. For example if JA = -1, the properties defined on the input line would be
assigned to the nodes defined as belonging to geometric Group 1. The control statement
ZONE may be called more than once to redefined geometric groupings.
TABLE VII. Input Description for FEHMN

Control Statement CAP (optional)
Capillary pressure models [see Eqs. (27)-(28)]
Group 1 - ICAP(i), cp1(i), cp2(i), cp3(i),
  ICAP(i) - Relative permeability model type
  ICAP(i) = 1, linear capillary pressure
  cp1(i) - maximum capillary pressure
  cp2(i) - maximum liquid saturation for capillary pressure calculation
  cp3(i) - not used
Note: Only ICAP(1) = 1 is allowed. The parameter i is incremented each time a Group 1 line is read. Group 2 lines will refer to this parameter.

Group 2 - JA, JB, JC, ICAPT
  JA, JB, JC - defined on page 60
  ICAPT - reference counter for model defined in Group 1

Control statement COOR (required)
Group 1 - N
  N - number of nodes in the grid

Group 2 - MB, CORD1, CORD2, CORD3
  MB - node number. If MB < 0 then the difference between the absolute value of MB and the previously read absolute value of MB is used to generate intermediate values by interpolation.
  CORD1 - x-coordinate (m) of node MB
  CORD2 - y-coordinate (m) of node MB
  CORD3 - z-coordinate (m) of node MB
NOTE: To end the control section a line with MB = 0 is entered.

Control statement COND (required)
Assign thermal conductivities of the rock [see Section I-B, Eq. (6)]
Group 1 - JA, JB, JC, THXD, THYD, THZD
  JA, JB, JC - defined on page 60
  THXD - thermal conductivity (W/mK) in the x-direction
  THYD - thermal conductivity (W/mK) in the y-direction
  THZD - thermal conductivity (W/mK) in the z-direction
Control statement **CONT** (optional)

Group 1 - NCNTR, CONTIM

NCNTR  - time step interval for contour plots  
CONTIM  - time (days) interval for contour plots

NOTE: the contour data will be output whenever either of the above criteria is satisfied

Control statement **CTRL** (required)

Assign various control parameters needed for equation solvers and matrix solver routines (see Section I-D)

Group 1 - MAXIT, EPM, NORTH [see Eqs. (55)-(68)]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAXIT</td>
<td>maximum number of iterations allowed in either the overall Newton cycle or the inner cycle to solve for the corrections at each iteration [see Eqs. (55)-(57) and Eqs. (63)-(68)]</td>
</tr>
<tr>
<td>EPM</td>
<td>tolerance for Newton cycle (nonlinear equation tolerance)</td>
</tr>
<tr>
<td>NORTH</td>
<td>number of orthogonalizations in the linear equation solver [see Eqs. (63)-(65)]</td>
</tr>
</tbody>
</table>

Group 2 - JA, JB, JC, IGAUS [see Eqs. (58)-(62)]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>JA, JB, JC</td>
<td>defined on page 60</td>
</tr>
<tr>
<td>IGAUS</td>
<td>the order of partial Gauss elimination</td>
</tr>
</tbody>
</table>

Group 3 - AS, GRAV, UPWGT

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS</td>
<td>implicitness factor [see Eq. (40)]</td>
</tr>
<tr>
<td></td>
<td>if AS ≤ 1, a standard pure implicit formulation will be used</td>
</tr>
<tr>
<td></td>
<td>if AS ≥ 1, a second-order implicit method will be used</td>
</tr>
<tr>
<td>GRAV</td>
<td>direction of gravity 1 = x-direction, 2 = y-direction, 3 = z-direction. If GRAV &gt; 3, GRAV is set equal to 3.</td>
</tr>
<tr>
<td></td>
<td>If GRAV = 0, then no gravity is used. If GRAV ≠ 0 then a value of gravity of 9.8 m/s² is used in the code.</td>
</tr>
<tr>
<td>UPWGT</td>
<td>value of upstream weighting (0.5 ≤ UPWGT ≤ 1.0) [see Sec. I-C, and the paragraph following Eq. (49)]</td>
</tr>
<tr>
<td></td>
<td>If UPWGT &lt; 0.5 UPWGT is set to 0.5</td>
</tr>
<tr>
<td></td>
<td>If UPWGT &gt; 1.0 UPWGT is set to 1.0</td>
</tr>
</tbody>
</table>

Group 4 - IAMM, AIAA, DAYMIN, DAYMAX

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IAMM</td>
<td>maximum number of iterations for which the code will multiply the time step size</td>
</tr>
</tbody>
</table>

Group 4 - IAMM, AIAA, DAYMIN, DAYMAX

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IAMM</td>
<td>maximum number of iterations for which the code will multiply the time step size</td>
</tr>
</tbody>
</table>

63
AIAA - time step multiplier (see Sec. I-D)
DAYMIN - minimum time step size (days)
DAYMAX - maximum time step size (days)

Group 5 - ICNL, LDA
ICNL - parameter that specifies the geometry
  if ICNL = 0 three-dimensional
  if ICNL = 1 X - Y slice
  if ICNL = 2 X - Y radial slice
LDA - parameter that specifies the external storage of
  geometric coefficients
  LDA = +1, element coefficients are read in from file STOR.FE
  and no coefficients are calculated in the code (note: STOR.FE
  must be in local file space)
  LDA = 0, element coefficients are calculated in the code
  and not saved
  LDA = -1, element coefficients are calculated in the code and
  saved on file STOR.FE

Control statement DUAL (optional, see Section F)
Group 1 - JA, JB, JC, VOLFD1
  JA, JB, JC - defined on page 60
  VOLFD1 - volume fraction for first matrix node

Group 2 - JA, JB, JC, VOLFD2
  JA, JB, JC - defined on page 60
  VOLFD2 - volume fraction for second matrix node

Group 3 - JA, JB, JC, APUVD
  JA, JB, JC - defined on page 60
  APUVD - area per unit volume (length scale) for the matrix nodes

The quantities VOLFD1 and VOLFD2 are volume fractions and related to the total volume by

VOLFD1 + VOLFD2 + VOLFF = 1.0

where VOLFF is the volume fraction of the fractures. The above relation must be satisfied at all nodes.
Control statement **ELEM** (required)

Group 1 - NS, NEI,

- NS - number of nodes per element
- NEI - number of elements

Group 2 - MB, NELM (1), NELM (2)... NELM (NS)

- MB - node number. If MB < 0 then the difference between the absolute value of MB and the previous absolute value of MB is used to generate intermediate values by interpolation in the code.
- NELM (1) - first node of element MB
- NELM (2) - second node of element MB
- NELM (NS) - last node of element MB

NOTE: To end the control section a line with MB = 0 is entered.

Control Statement **EOS** (optional)

Equation of state, see pages 8-9)

Group 1 - IIEOSD, IPSAT, ITSAT

- IIEOSD - equation of state reference number. IIEOSD = 1 or 2 refer to high and low pressure data sets respectively in FEHMN. For these values the input in Group 2 and Group 3 will be ignored after it is entered.
- IPSAT - parameter to set vapor pressure to zero. If IPSAT ≠ 0 the vapor pressure is set to zero, otherwise the vapor pressure is calculated in the code.
- ITSAT - parameter to adjust the saturation temperature. If ITSAT < 0, the saturation temperature is set to -1000°C. If ITSAT > 0, the saturation temperature is set to 1000°C.

Group 2 - EW1, EW2, EW3, EW4, EW5, EW6, EW7, EW8, EW9, EW10, EW11

- EW1 - liquid reference pressure
- EW2 - liquid reference temperature
- EW3 - liquid reference density
- EW4 - derivative of liquid density with respect to pressure at reference conditions
- EW5 - derivative of liquid density with respect to temperature at reference conditions
- EW6 - liquid reference enthalpy
EW7 - derivative of liquid enthalpy with respect to pressure at reference conditions
EW8 - derivative of liquid enthalpy with respect to temperature at reference conditions
EW9 - liquid reference viscosity
EW10 - derivative of liquid viscosity with respect to pressure at reference conditions
EW11 - derivative of liquid viscosity with respect to temperature at reference conditions

Group 3 - EV1, EV2, EV3, EV4, EV5, EV6,
EV7, EV8, EV9, EV10, EV11
EV1 - vapor reference pressure
EV2 - vapor reference temperature
EV3 - vapor reference density
EV4 - derivative of vapor density with respect to pressure at reference conditions
EV5 - derivative of vapor density with respect to temperature at reference conditions
EV6 - vapor reference enthalpy
EV7 - derivative of vapor enthalpy with respect to pressure at reference conditions
EV8 - derivative of vapor enthalpy with respect to temperature at reference conditions
EV9 - vapor reference viscosity
EV10 - derivative of vapor viscosity with respect to pressure at reference conditions
EV11 - derivative of vapor viscosity with respect to temperature at reference conditions

Control statement FLOW (optional)
[see Eq. (54)]
Group 1 - JA, JB, JC, SKD, EFLOW, AIPED
JA, JB, JC - defined on page 60
SKD - heat and mass source strength (kg/s), heat only (MJ/s).
   Negative value indicates injection into the rock mass.
EFLOW - enthalpy (MJ/kg) of fluid injected. If the fluid is flowing from the reservoir, then the in-place enthalpy is used. If EFLOW < 0, then ABS(EFLOW) is interpreted as a temperature and the enthal
calculated accordingly. In heat only problems with $\text{EFLOW} < 0$, the node is in contact with a large heat pipe that supplies heat to the node through an impedance $\text{AIPED}$ so as to maintain its temperature near $\text{ABS (EFLOW)}$. Large values (approximately 1000) of $\text{AIPED}$ are recommended.

$\text{AIPED}$ - impedance parameter. If $\text{AIPED}$ is nonzero, the code interprets $\text{SKD}$ as a flowing wellbore pressure (MPa) with an impedance $\text{ABS(AIPED)}$. If $\text{AIPED} < 0$, flow is only allowed out of the well. For heat only, $\text{AIPED}$ is the thermal resistance. If $\text{AIPED} = 0$, $\text{SKD}$ is flow rate.

NOTE: If the porosity of the node is zero, then there is only a temperature solution, and the code forms a source proportional to the enthalpy difference $E-\text{EFLOW}$, where $E$ is the in-place enthalpy and $\text{EFLOW}$ is a specified enthalpy. The source term is given by

$$Q = \text{AIPED} (E-\text{EFLOW}).$$

Control Statement $\text{HFLX}$ (optional)

Group 1 - $\text{JA, JB, JC, FLUX, QFLXM}$

$\text{JA, JB, JC}$ - defined on page 60

$\text{QFLUX}$ - If $\text{QFLXM} = 0$, then $\text{AFLUX}$ is the heat flux (MW) if $\text{QFLXM} \neq 0$, then $\text{QFLUX}$ is a temperature and the heat flux is calculated according to the formula: $Q_H = QFLXM(TL-QFLUX)$

$\text{QFLXM}$ - multiplier for heat flux equation given in $\text{QFLUX}$ description ($\text{MW/°C}$)

Control Statement $\text{INIT}$ (optional)

Set initial pressure and temperature at all nodes

Group 1 - $\text{PEIN, TIN, TIN1, GRAD1, DEPTH, TIN2, GRAD2, QUAD}$

$\text{PEIN}$ - initial value of pressure (MPa). If initial values are read from $\text{IREAD}$ (see terminal input), then this value is ignored. If gravity is present, this is the value of the pressure at node 1, and the other nodal pressures are adjusted by considering the hydraulic head.

$\text{TIN}$ - initial value of temperature (°C). If $\text{TIN} \leq 0$, then the initial temperatures are calculated using the pressure and temperature gradient.

NOTE: The initial temperatures are set according to the following formulas only if $\text{TIN} \leq 0$. Otherwise, the initial temperatures are determined from $\text{PEIN}$ and $\text{TIN}$. 

67
\[ 0 \leq Z \leq \text{DEPTH} \quad T = T_{\text{IN}1} + \text{GRAD}1 \times Z \]
\[ Z > \text{DEPTH} \quad T = T_{\text{IN}2} + \text{GRAD}2 \times Z + \text{QUAD} \times Z^2 \]

\text{TIN1} - defined in formulas above (°C)
\text{GRAD1} - defined in formulas above (°C/m)
\text{DEPTH} - defined in formulas above (m)
\text{TIN2} - defined in formulas above (°C)
\text{GRAD2} - defined in formulas above (°C/m)
\text{QUAD} - defined in formulas above (°C/m^2)

**Control statement ITER** (optional, see defaults below)

NOTE: If the user is not familiar with the linear equation solver routines in FEHMN please leave out control statement \text{ITER} and associated lines (see Section I-D).

**Group 1** - \text{G1}, \text{G2}, \text{G3}, \text{TMCH}, \text{OVER}

\text{G1} - multiplier for the linear convergence region of the Newton-Raphson iteration
\text{G2} - multiplier for the quadratic convergence region of the Newton-Raphson iteration
\text{G3} - tolerance for the adaptive implicit method (multiplying factor for Newton-Raphson tolerance)
\text{TMCH} - machine tolerance. If satisfied by the residual norm, the Newton iteration is complete
\text{OVERF} - over relaxation factor for passive nodes in adaptive implicit method

**Group 2** - \text{IRDOF}, \text{ISLORD}, \text{IBACK}, \text{ICOUPL}, \text{RNMAX}

\text{IRDOF} - enables the reduced degree of freedom method
\text{ISLORD} - reordering parameter. The ordering can be understood by labeling the mass equation as 1, the heat equation as 2, and the CO\(_2\) equation (if it exists) as 3. The value of \text{ISLORD} and the corresponding equation order is given below.

<table>
<thead>
<tr>
<th>ISLORD</th>
<th>Mass, Heat</th>
<th>Mass, Heat, CO(_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1, 2</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>1</td>
<td>2, 1</td>
<td>1, 3, 2</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>2, 1, 3</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>2, 3, 1</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>3, 1, 2</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>3, 2, 1</td>
</tr>
</tbody>
</table>
the ordering has an effect on the speed of convergence of several solution algorithms, but will not affect most users.

IBACK - back substitution parameter
ICOUPL - number of SOR iteration used in reduced degree of freedom methods
RNMAX - maximum running time for problem before the solution is stopped

If control statement ITER is not present the following default values are used: $G_1 = 0.001$, $G_2 = 0.0001$, $G_3 = 0.0001$, $TMCH = 10^{-8}$, $OVERF = 0.0$, $IRDOF = 0$.

Control statement NGAS (optional)
Noncondensable gas transport

Group 1 - ICO2D
ICO2D - solution descriptor for noncondensible gas transport
ICO2D - 0 data from control group NGAS is read but not used
ICO2D - 1, 2, reduced degree of freedom solver technology is used
ICO2D - 3, full 3 degrees of freedom solution water/noncondensible gas solution

Group 2 - JA, JB, JC, PC02
JA, JB, JC - defined as in control statement ROCK
PC02 - initial partial pressure of non-condensible gas. If $PCO2 < 0$ then ABS ($PCO2$) is interpreted as a temperature and the partial pressure of the noncondensible gas is calculated according to the formula: $PCO2 = PT - P_{SAT}(T)$ where $PT$ is the total pressure and $P_{SAT}(T)$ is the water saturation pressure and is a function of temperature only.

Group 3 - JA, JB, JC, CPNK
JA, JB, JC - defined on page 60
CPNK - injection concentration of noncondensible gas for injection mode $(SK(i) < 0)$. If it is a production node $(SK(i) > 0)$ then the in place concentration is used.

Note: For compatibility with older versions the user may substitute "CO2I" for "NGAS" in the input file.

Control statement NODE (required)
Specify the node numbers for which detailed printed output is desired. The plotting post-processor FEHPLTR will also use these nodes.

Group 1 - M
M - number of nodes for which information will be printed on IOUT
(see terminal input). If \( M < 0 \), pressure and temperature will be written for all nodes.

**Group 2 - MN (1), MN (2) ... MN (M)**
- MN - node numbers for which information will be printed on IOUT, \( M \) nodes (see Group 1).
- If \( NN(1) < 0 \), then coordinates are used to define the print-out node. The coordinate sets \((x, y, z)\) for each \( NM(I) < 0 \) are added after Group 2. For 2-D problems put \( z = 0 \).

**Control statement PERM** (required)
Assign absolute permeabilities of the rock [see section I-B, Eq. (7)-(8)]

**Group 1 - JA, JB, JC, PNXD, PNYD, PNZD**
- JA, JB, JC - defined on page 60
- PNXD - permeability in the x-direction \((m^2)\)
- PNYD - permeability in the y-direction \((m^2)\)
- PNZD - permeability in the z-direction \((m^2)\)

**Control Statement PPOR** (Optional, default is IPOROS=0)
For variable porosity/permeability problems, define model type (see Section I-B)

**Group 1 - IPOROS, R1, R2, R3**
- IPOROS - Porosity/permeability type [see Eqs. (36)-(39)]
  - = 0 constant porosity
  - = 1 simple linear model
  - = Gangi stress model
- R1, R2, R3 - parameters used in the various models

**Group 2 - JA, JB, JC, R4, R5**
- JA, JB, JC - defined on page 60
- R4 - variable parameter used in porosity/permeability models
- R5 - variable parameter used in porosity/permeability models

**Control statement PRES** (optional)
Assign nonuniform initial pressure and temperature values

**Group 1 - JA, JB, JC, PHRD, TIND, IEOSD**
- JA, JB, JC - defined on page 60
- PHRD - initial pressure \((MPa)\)
TIND - initial temperature (If IEOSD = 1 or 3) or initial
saturation (if IEOSD = 2)
IEOSD - thermodynamic region parameter if IEOSD = 1, then
in compressed liquid region, if IEOSD = 2, the
saturation region, if IEOSD = 3, the superheated region.

NOTE: The initial values defined in control statement PRES supersede all others.

Control statement RLP (optional)
Relative permeability models [see Eqs. (24)-(26)]
Group 1 - IRLP(i), RP1, RP2, RP3, RP4
IRLP - Relative permeability model type
IRLP(i) = 1 linear type
RP1 - irreducible liquid saturation
RP2 - irreducible vapor saturation
RP3 - maximum liquid saturation
RP4 - maximum vapor saturation
IRLP(i) = 2 Corey type
RP1 - irreducible liquid saturation
RP2 - irreducible vapor saturation
RP3 - not used
RP4 - not used
IRLP(i) = 3 van Genuchten (note different input)
NOTE: With a van Genuchten formation (IRLP=3, IRLP=4), both the relative permeability and capillary information is input. Therefore there is no capillary pressure. Information should be input in the capillary pressure section for those nodes.

RP1, RP2, RP4, RP5, RP6
RP1 - residual liquid saturation
RP2 - maximum liquid saturation
RP3 - $\alpha$, parameter for model
RP4 - $\beta$ parameter for model
RP5 - maximum capillary pressure
RP6 - fractional difference in pressure at maximum and cutoff saturation

IRLP(i) = 4 combined van Genuchten model. All the values for IRLP = 3 are read in plus the following RP7, RP8, RP9, RP10, RP11, RP12, RP13, RP14, RP15
RP7 - residual liquid saturation for fracture
RP8 - maximum liquid saturation for fracture
RP9 - $\alpha$, for fractures
RP10 - $\beta$, for fractures
RP11 - maximum capillary pressure for fracture
RP12 - fractional difference in pressure at maximum and cutoff saturation
RP13 - fracture intrinsic permeability (m$^2$)
RP14 - matrix intrinsic permeability (m$^2$)
RP15 - fracture porosity

Group 2 - JA, JB, JC, I
JA, JB, JC - defined on page 60
I - number referring to the sequence of models read in Group 1

NOTE: The parameter $i$ is incremented each time a Group 1 line is read. Group 2 lines will refer to this parameter.

**Control statement ROCK** (required)

Assign rock density, specific heat and porosity [see Section I-B, Eqs. (1)-(10)]

Group 1 - JA, JB, JC, DENRD, CPRD, PSD
JA, JB, JC - defined on page 60
DENRD - rock density (kg/m$^3$)
CPRD - rock specific heat (MJ/kg/$^\circ$C). If CPRD $\leq 1$ the code will assume the units are J/kg/$^\circ$C and multiply by $10^{-6}$
PSD - porosity

If JA $< 0$ see macro ZONE for geometric description of input. The code proceeds to the next control statement when a line is encountered with JA $= 0$.

**Control Statement SOL** (required)

Group 1 - NTT, INTG
NTT - parameter that defines the type of solution required
  if NTT $\geq 0$ heat and mass transfer solution
  if NTT $< 0$ heat transfer solution
INTG - parameter that defines element integration type
  (refer to Section I-B)
  if INTG $\leq 0$ Lobatto quadrature is used, recommended for heat and mass problems without stress.
  if INTG $> 0$ Gauss quadrature is used, recommended for problems requiring a stress solution.
Control Statement STEA
No input is associated with this macro statement. This statement enables a 1-D solution in the y-direction (2-D) or z-direction (3-D) when gravity is present to generate an initial steady state solution.

Control Statement STOP (required)
NOTE: No input is associated with this control statement. It signals the end of input, and as such it always appears as the last line of an input deck.

Control Statement TEXT
Following the control statement, text is input until a blank line is inserted to signal the end of the control statement. This text is printed on the output file.

Control statement TIME (required)
Group 1 - DAY, TIMS, NSTEP, II, YEAR, MONTH
   DAY       - initial time step size (days)
   TIMS      - simulation maximum time (days)
   NSTEP     - maximum number of time steps allowed
   II        - print-out interval for nodal information (pressure, enthalpy etc.),
               as set up under control statement node.
   YEAR      - year that simulation starts
   MONTH     - month that simulation starts

Group 2 - DIT1, DIT2, DIT3, ITC
   DIT1      - time (days) for time step change
   DIT2      - new time step size (days). If DIT2 < 0 then ABS (DIT2) is the
               new time step multiplier
   DIT3      - implicitness factor for new time step (use 1.0 backward Euler,
               1.5 for second-order implicit scheme).
   ITC       - new print-out interval

NOTE: A contour plot will be drawn at each DIT1. A restart file will be written at each DIT1.

Control statement TRAC (optional)
[see Eqs. (16)-(19) and (22)-(24) and Table II]
Group 1 - ANO, AWC, EPC, UPWGTA
   ANO        - initial tracer concentration
   AWC        - Implicitness factor for tracers. AWC > 1.0 gives 2nd order
               solution; AWC ≤ 1.0 gives 1st order solution
EPC - equation tolerance for tracer solution
UPWGTA - upstream weighting term for the tracer solution

If UPWGTA < 0.5 UPWGTA is set to 0.5
If UPWGTA > 1.0 UPWGTA is set to 1.0

Group 2 - DAYCS, DAYCF, DAYHF, DAYHS
DAYCS - time which the tracer solution is enabled
DAYCF - time which the tracer solution is disabled
DAYHF - time which the heat and mass transfer solution is disabled
DAYHS - time which the heat and mass transfer solution is enabled

Group 3 - IACCMMX, DAYCM, DAYCMM, DAYCMX
IACCMS - maximum number of iterations allowed in tracer solution if
time step multiplier is enabled
DAYCM - time step multiplier for tracer solution
DAYCMM - minimum time step for tracer solution
DAYCMX - maximum time step for tracer solution

Group 4 - NSPECIES
NSPECIES - number of different tracers

NOTE: TRAC groups 5, 6, 7, and 8 are entered as a unit for each different tracer.

Group 5 - ICNS, IADSF, A1ADSF, A2ADSF, BETAD
ICNS - phase designation. IF ICNS ≤ 0 a gas phase tracer is used.
      If ICNS > 0, then a liquid phase tracer is used.
IADSF - designates tracer type [see Eq. (23)-(24) and Table II]
        = 0, refers to the conservative tracer
        = 1, refers to the linear model
        = 2, refers to the Freundlich model
        = 3, refers to the modified Freundlich model
        = 4, refers to the Langmuir model
A1ADSF - \( \alpha_1 \) parameter
A2ADSF - \( \alpha_2 \) parameter
BETAD - \( \beta \) parameter

ICRATE - designates reaction model type see Eq. (24).
A1R - parameter in rate model
A2R - parameter in rate model
A3R - parameter in rate model
A4R - parameter in rate model

ICRATE = 1 refers to the simple model (p. 9) and ICRATE ≥ 2 refers to the more complicated model (p. 9).

Group 7 - JA, JB, JC, TCXD, TCYD, TCZD
JA, JB, JC - defined on page 60
TCXD - constant dispersion coefficient in the x-direction (m²/s)
TCYD - constant dispersion coefficient in the y-direction (m²/s)
TCZD - constant dispersion coefficient in the z-direction (m²/s)

Group 8 - JA, JB, JC, ANQO
JA, JB, JC - defined on page 60
ANQO - initial concentration of tracer, which will supersede the value given by ANO in Group 1. Note that if initial values of pressure and enthalpy are read from a file, then the code will also read initial values for the tracer from the file.

Group 9 - JA, JB, JC, CNSK, T1SK, T2SK
JA, JB, JC - defined on page 60
CNSK - injection concentration of injection node. If it is a production node, then the in-place concentration is used
T1SK - time (days) when tracer injection begins
T2SK - time (days) when tracer injection ends

NOTE: Injection nodes must be specified in control statement flow.

Control statement ZONE (Optional, default is input by nodes)
Group 1 - IZONE
IZONE - identification number for geometric input

Group 2 - X1, X2, X3, X4, Y1, Y2, Y3, Y4 (for 2-D problems)
X1-X4 - X coordinates for zone IZONE
Y1-X4 - Y coordinates for zone IZONE

X1-X8, Y1-Y8, Z1-Z8 (for 3-D problems)
X1-X8 - X coordinates for zone IZONE
Y1-Y8 - Y coordinates for zone IZONE
Z1-Z8 - Z coordinates for zone IZONE
NOTE: If the first four characters in Group 2 are 'LIST' then the code reads a list of x, y, z-coordinates, one set per line until a blank line is encountered. The nodes corresponding to these coordinates make up the zone. The geometric zone description is implemented by defining geometric regions. The coordinates given in Group 2 are defined in Fig. 5. All properties defined by node (JA, JB, JC) may be defined by ZONE. In the previous macro descriptions if JA < 0, then the zone IZONE = ABS (JA) is referenced. The macro ZONE must precede the usage of a ZONE reference. ZONE can be called more than once.

C. Graphics Postprocessing

Two graphics postprocessing codes are described in this section. They are written using the DISSPLA graphics package. Except for the plot area specification only the primitive line drawing commands are used. Therefore the routines should be easily convertible to other systems. A capability for time history plots for variables (FEHPLTR) and contour plotting (FECPLTR) are provided. Though these codes have not been verified, users may use them at their convenience.

1. FEHPLTR is a postprocessor program for FEHMN. It uses output information from FEHMN in the file FE.HIS. The program organizes information found in the file FE.HIS for plotting time histories of the variables temperature, pressure, enthalpy, flow rate, concentration, and capillary pressure. The user input is found in file HIS.INS.

Group 1 - HEADER - 80 character title

Group 2 - NNODE - number of input nodes. This must correspond to the number of print-out nodes specified in the input file for FEHMN.

Group 3 - ITYPE, IPILOT(I), II=1, NNODE

ITYPE - variable designator
= 1, enthalpy plot
= 2, flow rate plot
= 3, temperature plot
= 4, pressure plot
= 5, capillary pressure plot
= 6, concentration plot

IPILOT(I) - designates if the Ith node is to be plotted on the current plot.
IPILOT(I) ≠ 0 then the Ith node will be plotted.
IPILOT(I) = 0 then the Ith node will not be plotted.
Group 4 - A - parameter to scale the plot variable according to the formula 
\[ P^* = A(P - B) \]
B - parameter to scale the plot variable according to the formula given in the last description
C - parameter to scale the time variable according to the formula 
\[ t^* = C(T - D) \]
D - parameter to scale the time variable according to the formula given in this last description
Note: If 0,0,0,0 is entered, no action is taken (i.e., P = p)

Group 5 - XLEN, YLEN
XLEN - physical plot horizontal dimension in inches
YLEN - physical plot vertical dimension in inches

Group 6 - AXIS
AXIS - plot type
AXIS < 0 time axis is a log axis and the variable axis is linear
AXIS = 0, both axes are linear
AXIS > 0, both axes are log scales

Group 7 - DAYMIN, DAYMAX, DAYTIC, NTIC
DAYMIN - minimum limit for time axis
DAYMAX - maximum limit for time axis
DAYTIC - increment for time axis
NTIC - tic marks per increment for the time axis

Group 8 - VMIN, VMAX, VTIC, NIY
VMIN - minimum limit for variable axis
VMAX - maximum limit for variable axis
VTIC - increment value for variable axis
NIY - tic marks per interval for the variable axis

Group 9 - TITLE
TITLE - 30 character (max) heading for top of plot

Group 10 - TITLX
TITLX - 30 character (max) heading for time axis
Group 11 - TITLY
   TITLY - 30 character (max) heading for the variable axis

IF ITYPE = 6 is chosen then a concentration plot is generated. Since several (up to 10) concentration profiles can be plotted, the species number to be plotted must be specified. Group 12 provides this information.

Group 12 - [ISPT (II) II=1, NSPECI]
   ISPT(II) - plot designator for the IIth specie
   ≠ 0 then that specie is plotted
   = 0 then that specie is not plotted

NOTE: the variable NSPECI is obtained by the code from the file FE.HIS.

2. FECPPLTR. The computer code FECPPLTR is the postprocessor code for FEHMN which makes contour plots. Its input file is the unformatted file FE.CON generated by FEHMN. The plot instruction commands are given in the file CON.INS. The input lines are described below and are in free format except where noted.

Group 1 - ICHOS
   ICHOS - dimensionality parameter
   = 2, 2 dimensional contour plots are generated
   ≠ 2, 3-dimensional contour plot information is provided

NOTE: the form of the input that follows is not implemented as yet for the value of ICHOS = 3.

Group 2 - HEADER
   Header - title (80 characters maximum)

Group 3 - NCI, IGRID
   NCI - contour label interval NCI = 0, no labels are drawn on contours, contours are given different line types (dash, solid, etc.) and identified on the side of the plot NCI ≠ 0, the contours are labeled on the contour line
   IGRID - grid parameter
   = 0, no grid drawn with contours
   ≠ 0, grid is drawn with contours

Group 4 - AX, BX, AY, BY
   AX - minimum x coordinate for problem (m)
   BX - maximum x coordinate for problem (m)
AY - minimum y coordinate for problem (m)
BY - maximum y coordinate for problem (m)

Group 5- VLENM, VRMAX
VLENM - length of maximum vector (in grid dimensions)
VRMAX - size of maximum vector (m/s)

Group 6- XWIN, YWIN
XWIN - physical size of the x direction of plot window (in)
YWIN - physical size of the y direction of plot window (in)

Group 7- IVLP, CNTRT, CNTRP, CNTRC, SCALE
IVLP - plot phase parameter IVLP < 0, vapor phase only plots will be drawn. IVLP > 0, liquid phase only plots will be drawn
CNTRT - contour interval for temperature (liquid region) or saturation (vapor region)
CNTRP - phase pressure contour interval
CNTRC - phase contour interval for species transport
SCALE - scaling parameter for velocity vectors. No vectors will be plotted for vectors of magnitude less than VRMAX * SCALE (see Group 5).

Note: If CNTRT, CNTRP, CNTRC are < 0, then the contour values are provided in a user supplied list. (Group 8 is used.) If they are > 0 then Group 8 is not used.

Group 8 - NV, (CVAL(I), I = 1, NV)

NV - number of contour values
CVAL(I) - contour value (can be temperature, saturation pressure, or concentration

If multiple contour plots are desired, the user may provide additional data sets for each plot (i.e., Groups 1-7). Alternatively, the user may specify the same parameter set for each plot by not providing additional sets in the data file CON.INS. This way all the plot sets output by FEHMN and in FE.CON will be plotted.

D. Example Input Files And Output Files

This section contains three example problem runs with FEHMN. Input files and output files are provided. Other examples may be found in the FEHMN software verification report (Zyvoloski and Dash, 1991).

1. Heat Conduction. This 3-D problem demonstrates the code performance on a purely conductive problem. It is useful in showing the numerical accuracy of the finite
TABLE VIII. Input Parameters for the 3-D Heat Conduction Problem

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock thermal diffusivity</td>
<td>( \kappa )</td>
<td>2.7 W/m K</td>
</tr>
<tr>
<td>Rock thermal conductivity</td>
<td>( \kappa_r )</td>
<td>2700 kg/m³</td>
</tr>
<tr>
<td>Rock density</td>
<td>( \rho_r )</td>
<td>1000 J/kgK</td>
</tr>
<tr>
<td>Rock specific heat</td>
<td>( C_r )</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Width</td>
<td>a</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Length</td>
<td>b</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Depth</td>
<td>c</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Initial temperature</td>
<td>( T_0 )</td>
<td>200°C</td>
</tr>
<tr>
<td>Surface temperature</td>
<td>( T_s )</td>
<td>100°C</td>
</tr>
</tbody>
</table>

\( \kappa = \frac{\kappa_r}{\rho_r C_r} \)

elements and providing the user with an example of 3-D input. While we give only the 4 x 4 x 4 mesh input and output, accuracy results are presented for the 4 x 4 x 4, 8 x 8 x 8, and 12 x 12 x 12 meshes. Table VIII shows the parameters used in the computer runs. Figure 8 depicts the problem geometry (due to symmetry only a quarter of the cube is depicted). Basically the cube is initially at 200°C and a boundary condition of 100°C is applied on all sides. Table IX gives the coordinate positions for the various error sampling points as well as defining the runs. As can be seen from Table X even the crudest grid produced errors of less than 9%. Other grids produced much better results. For the 4 x 4 x 4 grid, the input to the preprocessor GENMSH is shown in Fig. 9. The plot produced by GENMSH is given in Fig. 10. The input file for FEHMN is provided in Fig. 11. It is worth noting here that the part of the file from the macro command COOR to the end was produced by GENMSH (file GEOM.DAT) and appended to the other macro commands with a text editor. Figure 12 gives the output for the 4 x 4 x 4 run. The user instruction file for the postprocessor FEHPLTR is shown in Fig. 13. The plot produced by FEHPLTR is given in Fig. 14.

2. Toronyi Two-Phase Example. This problem is a model of a two-phase, highly permeable geothermal reservoir originally proposed by Toronyi and Farouq Ali (1977) and solved by a number of authors (Mercer and Faust 1975, Thomas and Pierson 1978). The model reservoir is shown in Fig. 15. The input for the problem is presented in Table XI. As
### TABLE IX. Sample Problems for 3-D Heat Conduction

<table>
<thead>
<tr>
<th>Problem</th>
<th>3-D Mesh Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4 x 4 x 4 brick</td>
</tr>
<tr>
<td>2</td>
<td>8 x 8 x 8 brick</td>
</tr>
<tr>
<td>3</td>
<td>12 x 12 x 12 brick</td>
</tr>
<tr>
<td>4</td>
<td>8 x 8 x 8 prism</td>
</tr>
<tr>
<td>5</td>
<td>8 x 8 x 8 mixed brick and prism</td>
</tr>
</tbody>
</table>

Comparison coordinate positions:
- $x = 0.000$ $y = 0.000$ $z = 0.000$
- $x = 0.000$ $y = 0.125$ $z = 0.250$
- $x = 0.125$ $y = 0.250$ $z = 0.375$
- $x = 0.375$ $y = 0.375$ $z = 0.375$

### TABLE X. Comparison of Analytical and Model Results for 3-D Heat Conduction

<table>
<thead>
<tr>
<th>Coordinate position</th>
<th>Maximum Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$x = 0.000$ $y = 0.000$ $z = 0.000$</td>
<td>2.139</td>
</tr>
<tr>
<td>$x = 0.000$ $y = 0.125$ $z = 0.250$</td>
<td>1.784</td>
</tr>
<tr>
<td>$x = 0.125$ $y = 0.250$ $z = 0.375$</td>
<td>3.546</td>
</tr>
<tr>
<td>$x = 0.375$ $y = 0.375$ $z = 0.375$</td>
<td>8.517</td>
</tr>
</tbody>
</table>
with Thomas and Pierson, time steps of 10 days with an initial time step of 8.3 days were used. The final state corresponds to 19% of the original water mass removed. Figure 15 also shows the results obtained in this study using quadrilateral elements compared with the results obtained by Thomas and Pierson. Good agreement is evident. This is somewhat surprising considering the elements have a length/width ratio of 10. The input for the GENMSH preprocessor is shown in Fig. 16. The input file is given in Fig. 17. The part of the input file from the macro COOR was generated by GENMSH and inserted in the input file using a text editor. Output for the first computer run is shown in Fig. 18.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir permeability</td>
<td>k</td>
<td>$9.869 \times 10^{-13} \text{ m}^2$</td>
</tr>
<tr>
<td>Reservoir porosity</td>
<td>$\phi$</td>
<td>0.05</td>
</tr>
<tr>
<td>Rock thermal conductivity</td>
<td>$\kappa_r$</td>
<td>1.73 \text{ W/m} \cdot \text{K}</td>
</tr>
<tr>
<td>Rock density</td>
<td>$\rho_r$</td>
<td>2500 \text{ kg/m}^3</td>
</tr>
<tr>
<td>Rock specific heat</td>
<td>$C_r$</td>
<td>1000 \text{ J/kg} \cdot \text{K}</td>
</tr>
<tr>
<td>Aquifer length</td>
<td></td>
<td>1828 \text{ m}</td>
</tr>
<tr>
<td>Aquifer width</td>
<td></td>
<td>182.8 \text{ m}</td>
</tr>
<tr>
<td>Initial water saturation</td>
<td>$s_{ij}^0$</td>
<td>0.2</td>
</tr>
<tr>
<td>Aquifer discharge</td>
<td>$q_m$</td>
<td>0.05 \text{ kg/m} \cdot \text{s}</td>
</tr>
<tr>
<td>Initial Pressure</td>
<td>$P_{ij}^0$</td>
<td>4.4816 \text{ MPa}</td>
</tr>
</tbody>
</table>

3. DOE Code Comparison Project, Problem 5, Case A. This problem involves multiphase flow in a 2-D horizontal reservoir. The problem is characterized by a moving two-phase region, i.e., the fluid produced at the production well is replaced by cold water recharge over one of the outer boundaries. The problem parameters are given in Table XII and the geometry and boundary conditions are shown in Fig. 19. Of particular note is the variable initial temperature field and the prescribed pressure and temperature boundary. This problem shows that two-phase coding as well as the phase change algorithm are working properly, since numerical difficulties can occur as nodes go from two-phase fluid to compressed water.
There is no analytical solution for this problem, but six researchers produced results for the DOE code comparison project (Molloy, 1980). The reader is referred to this reference for a more detailed discussion of this problem and the code comparison. Results from this problem are compared to those from the other codes, obtained from Molloy (1980) as a check on FEHMN. The results for the outlet temperature, shown in Fig. 20, are in excellent agreement with the other codes. The results for the outlet pressure, Fig. 21, and pressure at an observation well 125 m distant, Fig. 22 are also in good agreement with the other codes.

The input for the preprocessor GENMSH is given in Fig. 23. The input to FEHMN is given in Fig. 24. The printout for the computer is provided in Fig. 25. The part of the input file from the macro COOR was generated by GENMSH. A contour plot of temperature was also generated for this problem. The user generated input file CON.INS is shown in Fig. 26. The contour plot is given in Fig. 27.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir permeability</td>
<td>k</td>
<td>$2.5 \times 10^{-14}$ m²</td>
</tr>
<tr>
<td>Reservoir porosity</td>
<td>$\phi$</td>
<td>0.35</td>
</tr>
<tr>
<td>Rock thermal conductivity</td>
<td>$\kappa_r$</td>
<td>1 W/m · K</td>
</tr>
<tr>
<td>Rock density</td>
<td>$\rho_r$</td>
<td>2500 kg/m³</td>
</tr>
<tr>
<td>Rock specific heat</td>
<td>$C_r$</td>
<td>1000 J/kg · °C</td>
</tr>
<tr>
<td>Reservoir length</td>
<td>$x$</td>
<td>300 m</td>
</tr>
<tr>
<td>Reservoir thickness</td>
<td>$y$</td>
<td>200 m</td>
</tr>
<tr>
<td>Liquid residual saturation</td>
<td>$s_{lr}$</td>
<td>0.3</td>
</tr>
<tr>
<td>Gas residual saturation</td>
<td>$s_{gr}$</td>
<td>0.1</td>
</tr>
<tr>
<td>Reservoir discharge</td>
<td>$q_m$</td>
<td>0.05 kg/m · s)</td>
</tr>
<tr>
<td>Initial Pressure</td>
<td>$P_o$</td>
<td>3.6 MPa</td>
</tr>
</tbody>
</table>

Production well coordinates: $x = 62.5$ m, $y = 62.5$ m
Observation well coordinates: $x = 162.5$ m, $y = 137.5$ m

Initial temperature distribution:

$$T(x,y,0) = \begin{cases} 240 & \text{if } r \leq 100 \text{ m} \\ 240 - 160 \left[ \frac{r-100}{200} \right]^2 + 80 \left[ \frac{r-100}{200} \right]^4 & \text{if } 100 < r < 300 \text{ m} \\ 160 & \text{if } r \geq 300 \text{ m} \end{cases}$$

where $r = \sqrt{x^2 + y^2}$
\[ T_s = 100^\circ C \text{ at } t = 0 \]
for all \( x, y, z = 0.5m \)

\[ T_0 = 200^\circ C \]

Fig. 8. Schematic diagram of the 3-D heat conduction problem.

Fig. 9. Input for preprocessor GENMSH for 3-D heat conduction example.
3D Model of a Cube (4x4x4 cubes)

Fig. 10. Grid for 4 x 4 x 4 heat conduction problem.
***** 3-D Heat Conduction Model (4X4X4 cubes) *****

node
8
001 056 087 094
005 025 002 125
rip
2 0 0 0 0
0
1 125 1 1
0 0 0 0
sol
-1 -1
init
10. 0. 200. 0. 0. 200. 0. 0.
1. 0. 0. 0. 0.
rock
1 125 1 2700. 1000. 0. 1.0
0 0 0 0. 0. 0.
cond
1 125 1 2.7e-00 2.7e-00 2.7e-00
0 0 0 0. 0. 0.
perm
1 125 1 1.e-30 1.e-30 1.e-30 0. 0. 0.
0 0 0 0. 0. 0.
flow
101 125 1 10.00 -100.00 1.e03
5 125 25 10.00 -100.00 1.e03
10 125 25 10.00 -100.00 1.e03
15 125 25 10.00 -100.00 1.e03
20 125 25 10.00 -100.00 1.e03
25 125 25 10.00 -100.00 1.e03
21 125 25 10.00 -100.00 1.e03
22 125 25 10.00 -100.00 1.e03
23 125 25 10.00 -100.00 1.e03
24 125 25 10.00 -100.00 1.e03
0 0 0 0. 0. 0.
time
0.005 3.00 1000 1000 1989 04
0. 0. 0. 0.
ctrl
40 1.e-04 08
1 125 1 1
0 0 0 0
1.0 0.0 1.0
10 1.0 0.00005 0.005
0 0
coor
125
1 0.00000 0.00000 0.00000
2 0.12500 0.00000 0.00000
3 0.25000 0.00000 0.00000
4 0.37500 0.00000 0.00000
5 0.50000 0.00000 0.00000
6 0.00000 0.12500 0.00000
7 0.12500 0.12500 0.00000
8 0.25000 0.12500 0.00000
9 0.37500 0.12500 0.00000
10 0.50000 0.12500 0.00000
11 0.00000 0.25000 0.00000
12 0.12500 0.25000 0.00000
13 0.25000 0.25000 0.00000
14 0.37500 0.25000 0.00000
15 0.50000 0.25000 0.00000
16 0.00000 0.37500 0.00000
17 0.12500 0.37500 0.00000
18 0.25000 0.37500 0.00000
19 0.37500 0.37500 0.00000
20 0.50000 0.37500 0.00000

Fig. 11. Input file for FEHN for 3-D heat conduction example.
<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>0.00000</td>
<td>0.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>22</td>
<td>0.12500</td>
<td>0.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>23</td>
<td>0.25000</td>
<td>0.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>24</td>
<td>0.37500</td>
<td>0.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>25</td>
<td>0.50000</td>
<td>0.00000</td>
<td>0.12500</td>
</tr>
<tr>
<td>26</td>
<td>0.12500</td>
<td>0.00000</td>
<td>0.12500</td>
</tr>
<tr>
<td>27</td>
<td>0.25000</td>
<td>0.00000</td>
<td>0.12500</td>
</tr>
<tr>
<td>28</td>
<td>0.37500</td>
<td>0.00000</td>
<td>0.12500</td>
</tr>
<tr>
<td>29</td>
<td>0.50000</td>
<td>0.00000</td>
<td>0.12500</td>
</tr>
<tr>
<td>30</td>
<td>0.00000</td>
<td>0.12500</td>
<td>0.12500</td>
</tr>
<tr>
<td>31</td>
<td>0.12500</td>
<td>0.12500</td>
<td>0.12500</td>
</tr>
<tr>
<td>32</td>
<td>0.25000</td>
<td>0.12500</td>
<td>0.12500</td>
</tr>
<tr>
<td>33</td>
<td>0.37500</td>
<td>0.12500</td>
<td>0.12500</td>
</tr>
<tr>
<td>34</td>
<td>0.50000</td>
<td>0.12500</td>
<td>0.12500</td>
</tr>
<tr>
<td>35</td>
<td>0.00000</td>
<td>0.12500</td>
<td>0.12500</td>
</tr>
<tr>
<td>36</td>
<td>0.12500</td>
<td>0.25000</td>
<td>0.12500</td>
</tr>
<tr>
<td>37</td>
<td>0.25000</td>
<td>0.25000</td>
<td>0.12500</td>
</tr>
<tr>
<td>38</td>
<td>0.37500</td>
<td>0.25000</td>
<td>0.12500</td>
</tr>
<tr>
<td>39</td>
<td>0.50000</td>
<td>0.25000</td>
<td>0.12500</td>
</tr>
<tr>
<td>40</td>
<td>0.00000</td>
<td>0.25000</td>
<td>0.12500</td>
</tr>
<tr>
<td>41</td>
<td>0.12500</td>
<td>0.37500</td>
<td>0.12500</td>
</tr>
<tr>
<td>42</td>
<td>0.25000</td>
<td>0.37500</td>
<td>0.12500</td>
</tr>
<tr>
<td>43</td>
<td>0.37500</td>
<td>0.37500</td>
<td>0.12500</td>
</tr>
<tr>
<td>44</td>
<td>0.50000</td>
<td>0.37500</td>
<td>0.12500</td>
</tr>
<tr>
<td>45</td>
<td>0.00000</td>
<td>0.50000</td>
<td>0.12500</td>
</tr>
<tr>
<td>46</td>
<td>0.12500</td>
<td>0.50000</td>
<td>0.12500</td>
</tr>
<tr>
<td>47</td>
<td>0.25000</td>
<td>0.50000</td>
<td>0.12500</td>
</tr>
<tr>
<td>48</td>
<td>0.37500</td>
<td>0.50000</td>
<td>0.12500</td>
</tr>
<tr>
<td>49</td>
<td>0.50000</td>
<td>0.50000</td>
<td>0.12500</td>
</tr>
<tr>
<td>50</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.25000</td>
</tr>
<tr>
<td>51</td>
<td>0.12500</td>
<td>0.00000</td>
<td>0.25000</td>
</tr>
<tr>
<td>52</td>
<td>0.25000</td>
<td>0.00000</td>
<td>0.25000</td>
</tr>
<tr>
<td>53</td>
<td>0.37500</td>
<td>0.00000</td>
<td>0.25000</td>
</tr>
<tr>
<td>54</td>
<td>0.50000</td>
<td>0.00000</td>
<td>0.25000</td>
</tr>
<tr>
<td>55</td>
<td>0.00000</td>
<td>0.12500</td>
<td>0.25000</td>
</tr>
<tr>
<td>56</td>
<td>0.12500</td>
<td>0.12500</td>
<td>0.25000</td>
</tr>
<tr>
<td>57</td>
<td>0.25000</td>
<td>0.12500</td>
<td>0.25000</td>
</tr>
<tr>
<td>58</td>
<td>0.37500</td>
<td>0.12500</td>
<td>0.25000</td>
</tr>
<tr>
<td>59</td>
<td>0.50000</td>
<td>0.12500</td>
<td>0.25000</td>
</tr>
<tr>
<td>60</td>
<td>0.00000</td>
<td>0.12500</td>
<td>0.25000</td>
</tr>
<tr>
<td>61</td>
<td>0.12500</td>
<td>0.25000</td>
<td>0.25000</td>
</tr>
<tr>
<td>62</td>
<td>0.25000</td>
<td>0.25000</td>
<td>0.25000</td>
</tr>
<tr>
<td>63</td>
<td>0.37500</td>
<td>0.25000</td>
<td>0.25000</td>
</tr>
<tr>
<td>64</td>
<td>0.50000</td>
<td>0.25000</td>
<td>0.25000</td>
</tr>
<tr>
<td>65</td>
<td>0.00000</td>
<td>0.37500</td>
<td>0.25000</td>
</tr>
<tr>
<td>66</td>
<td>0.12500</td>
<td>0.37500</td>
<td>0.25000</td>
</tr>
<tr>
<td>67</td>
<td>0.25000</td>
<td>0.37500</td>
<td>0.25000</td>
</tr>
<tr>
<td>68</td>
<td>0.37500</td>
<td>0.37500</td>
<td>0.25000</td>
</tr>
<tr>
<td>69</td>
<td>0.50000</td>
<td>0.37500</td>
<td>0.25000</td>
</tr>
<tr>
<td>70</td>
<td>0.00000</td>
<td>0.37500</td>
<td>0.25000</td>
</tr>
<tr>
<td>71</td>
<td>0.12500</td>
<td>0.50000</td>
<td>0.25000</td>
</tr>
<tr>
<td>72</td>
<td>0.25000</td>
<td>0.50000</td>
<td>0.25000</td>
</tr>
<tr>
<td>73</td>
<td>0.37500</td>
<td>0.50000</td>
<td>0.25000</td>
</tr>
<tr>
<td>74</td>
<td>0.50000</td>
<td>0.50000</td>
<td>0.25000</td>
</tr>
<tr>
<td>75</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.37500</td>
</tr>
<tr>
<td>76</td>
<td>0.12500</td>
<td>0.00000</td>
<td>0.37500</td>
</tr>
<tr>
<td>77</td>
<td>0.25000</td>
<td>0.00000</td>
<td>0.37500</td>
</tr>
<tr>
<td>78</td>
<td>0.37500</td>
<td>0.00000</td>
<td>0.37500</td>
</tr>
<tr>
<td>79</td>
<td>0.50000</td>
<td>0.00000</td>
<td>0.37500</td>
</tr>
<tr>
<td>80</td>
<td>0.00000</td>
<td>0.12500</td>
<td>0.37500</td>
</tr>
<tr>
<td>81</td>
<td>0.12500</td>
<td>0.12500</td>
<td>0.37500</td>
</tr>
<tr>
<td>82</td>
<td>0.25000</td>
<td>0.12500</td>
<td>0.37500</td>
</tr>
<tr>
<td>83</td>
<td>0.37500</td>
<td>0.12500</td>
<td>0.37500</td>
</tr>
<tr>
<td>84</td>
<td>0.50000</td>
<td>0.12500</td>
<td>0.37500</td>
</tr>
<tr>
<td>85</td>
<td>0.00000</td>
<td>0.25000</td>
<td>0.37500</td>
</tr>
<tr>
<td>86</td>
<td>0.12500</td>
<td>0.25000</td>
<td>0.37500</td>
</tr>
<tr>
<td>87</td>
<td>0.25000</td>
<td>0.25000</td>
<td>0.37500</td>
</tr>
<tr>
<td>88</td>
<td>0.37500</td>
<td>0.25000</td>
<td>0.37500</td>
</tr>
<tr>
<td>89</td>
<td>0.50000</td>
<td>0.25000</td>
<td>0.37500</td>
</tr>
<tr>
<td>90</td>
<td>0.00000</td>
<td>0.25000</td>
<td>0.37500</td>
</tr>
</tbody>
</table>

Fig. 11. (Continued)
<table>
<thead>
<tr>
<th>elem</th>
<th>8, 64, O</th>
<th>88</th>
</tr>
</thead>
<tbody>
<tr>
<td>91</td>
<td>0.00000</td>
<td>0.37500</td>
</tr>
<tr>
<td>92</td>
<td>0.12500</td>
<td>0.37500</td>
</tr>
<tr>
<td>93</td>
<td>0.25000</td>
<td>0.37500</td>
</tr>
<tr>
<td>94</td>
<td>0.37500</td>
<td>0.37500</td>
</tr>
<tr>
<td>95</td>
<td>0.50000</td>
<td>0.37500</td>
</tr>
<tr>
<td>96</td>
<td>0.50000</td>
<td>0.37500</td>
</tr>
<tr>
<td>97</td>
<td>0.12500</td>
<td>0.37500</td>
</tr>
<tr>
<td>98</td>
<td>0.25000</td>
<td>0.37500</td>
</tr>
<tr>
<td>99</td>
<td>0.37500</td>
<td>0.37500</td>
</tr>
<tr>
<td>100</td>
<td>0.50000</td>
<td>0.37500</td>
</tr>
<tr>
<td>101</td>
<td>0.00000</td>
<td>0.37500</td>
</tr>
<tr>
<td>102</td>
<td>0.12500</td>
<td>0.37500</td>
</tr>
<tr>
<td>103</td>
<td>0.25000</td>
<td>0.37500</td>
</tr>
<tr>
<td>104</td>
<td>0.37500</td>
<td>0.37500</td>
</tr>
<tr>
<td>105</td>
<td>0.50000</td>
<td>0.37500</td>
</tr>
<tr>
<td>106</td>
<td>0.00000</td>
<td>0.37500</td>
</tr>
<tr>
<td>107</td>
<td>0.12500</td>
<td>0.37500</td>
</tr>
<tr>
<td>108</td>
<td>0.25000</td>
<td>0.37500</td>
</tr>
<tr>
<td>109</td>
<td>0.37500</td>
<td>0.37500</td>
</tr>
<tr>
<td>110</td>
<td>0.50000</td>
<td>0.37500</td>
</tr>
<tr>
<td>111</td>
<td>0.00000</td>
<td>0.37500</td>
</tr>
<tr>
<td>112</td>
<td>0.12500</td>
<td>0.37500</td>
</tr>
<tr>
<td>113</td>
<td>0.25000</td>
<td>0.37500</td>
</tr>
<tr>
<td>114</td>
<td>0.37500</td>
<td>0.37500</td>
</tr>
<tr>
<td>115</td>
<td>0.50000</td>
<td>0.37500</td>
</tr>
<tr>
<td>116</td>
<td>0.00000</td>
<td>0.37500</td>
</tr>
<tr>
<td>117</td>
<td>0.12500</td>
<td>0.37500</td>
</tr>
<tr>
<td>118</td>
<td>0.25000</td>
<td>0.37500</td>
</tr>
<tr>
<td>119</td>
<td>0.37500</td>
<td>0.37500</td>
</tr>
<tr>
<td>120</td>
<td>0.50000</td>
<td>0.37500</td>
</tr>
<tr>
<td>121</td>
<td>0.00000</td>
<td>0.37500</td>
</tr>
<tr>
<td>122</td>
<td>0.12500</td>
<td>0.37500</td>
</tr>
<tr>
<td>123</td>
<td>0.25000</td>
<td>0.37500</td>
</tr>
<tr>
<td>124</td>
<td>0.37500</td>
<td>0.37500</td>
</tr>
<tr>
<td>125</td>
<td>0.50000</td>
<td>0.37500</td>
</tr>
<tr>
<td>0</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

Fig. 11. Continued
\begin{verbatim}
41  61  62  67  66  86  87  92  91
42  62  63  68  67  87  88  93  92
43  63  64  69  68  88  89  94  93
44  64  65  70  69  89  90  95  94
45  66  67  72  71  91  92  97  96
46  67  68  73  72  92  93  98  97
47  68  69  74  73  93  94  99  98
48  69  70  75  74  94  95 100  99
49  76  77  82  81 101 102 107 106
50  77  78  83  82 102 103 108 107
51  78  79  84  83 103 104 109 108
52  79  80  85  84 104 105 110 109
53  81  82  87  86 106 107 112 111
54  82  83  88  87 107 108 113 112
55  83  84  89  88 108 109 114 113
56  84  85  90  89 109 110 115 114
57  86  87  92  91 111 112 117 116
58  87  88  93  92 112 113 118 117
59  88  89  94  93 113 114 119 118
60  89  90  95  94 114 115 120 119
61  91  92  97  96 116 117 122 121
62  92  93  98  97 117 118 123 122
63  93  94  99  98 118 119 124 123
64  94  95 100  99 119 120 125 124
\end{verbatim}

\textbf{Fig. 11.} Continued

\textbf{Fig. 11.} Continued
**** 3-D Heat Conduction Model (4X4X4 cubes) ****

output file, wt tape, rd tp, user call parameter
 11 44 0 0

storage for geometric coefficients 726. in common(nr) 48000
pressures and temperatures set by gradients
storage needed for ncon 851, available 80000
storage needed for nop 851, available 100000
storage needed for a matrix 725, available 300000
storage needed for b matrix 725, available 720000
storage needed for gmres 1125, available 187200

time for reading input, forming coefficients 7.49

**** analysis of input data on file fe.chk ****

time step 1
 1 2.0 125.0

years 0.137e-04 days 0.5000000e-02 ts size 5.0000e-03
cpu sec for step 0.367e-01 total time 0.731e-01
node p(mpa) e(mj) l sat temp(c) well dis dis encl r eq1 r eq2
 1 10.000 0.00 0.000 200.000 0.000e+00 0.000 0.3e-12 0.3e-03
 56 10.000 0.00 0.000 199.929 0.000e+00 0.000 0.4e-07 0.7e-01
 87 10.000 0.00 0.000 197.313 0.000e+00 0.000 0.2e-06 0.3e+01
 94 10.000 0.00 0.000 192.518 0.000e+00 0.000 -0.6e-07 0.7e+01
 5 10.000 0.00 0.000 100.000 0.000e+00 0.000 -0.2e+09 0.1e+03
 25 10.000 0.00 0.000 100.000 0.000e+00 0.000 -0.2e+09 0.1e+03
 125 10.000 0.00 0.000 100.000 0.000e+00 0.000 -0.2e+09 0.1e+03

res mass 0.000000 kg vap mass 0.000000 kg energy -55.8459 mj
net discharge 0.000000 kg net energy discharge 0.000000 mj

conservation errors: mass 0.000000 energy 0.000000

this time step discharges: mass, enthalpy, power
 0.000e+00 kg 0.000e+00 mj 0.000e+00 mw

cumulative discharges: mass, enthalpy, avg power
 0.000e+00 kg 0.000e+00 mj 0.000e+00 mw

number of region changes this time step 0

Fig. 12. Computer output for the 3-D heat conduction example.
**time step 601**

```
<table>
<thead>
<tr>
<th>node</th>
<th>p(mpa)</th>
<th>e(mJ)</th>
<th>sat temp(c)</th>
<th>well disp</th>
<th>dis ent r eq1</th>
<th>r eq2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.000</td>
<td>0.000</td>
<td>0.000</td>
<td>100.107</td>
<td>0.000e+00</td>
<td>0.000</td>
</tr>
<tr>
<td>56</td>
<td>10.000</td>
<td>0.000</td>
<td>0.000</td>
<td>100.070</td>
<td>0.000e+00</td>
<td>0.000</td>
</tr>
<tr>
<td>87</td>
<td>10.000</td>
<td>0.000</td>
<td>0.000</td>
<td>100.027</td>
<td>0.000e+00</td>
<td>0.000</td>
</tr>
<tr>
<td>94</td>
<td>10.000</td>
<td>0.000</td>
<td>0.000</td>
<td>100.006</td>
<td>0.000e+00</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>10.000</td>
<td>0.000</td>
<td>0.000</td>
<td>100.000</td>
<td>0.000e+00</td>
<td>0.000</td>
</tr>
<tr>
<td>25</td>
<td>10.000</td>
<td>0.000</td>
<td>0.000</td>
<td>100.000</td>
<td>0.000e+00</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>10.000</td>
<td>0.000</td>
<td>0.000</td>
<td>100.099</td>
<td>0.000e+00</td>
<td>0.000</td>
</tr>
<tr>
<td>125</td>
<td>10.000</td>
<td>0.000</td>
<td>0.000</td>
<td>100.000</td>
<td>0.000e+00</td>
<td>0.000</td>
</tr>
</tbody>
</table>
```

Res mass 0.00000 kg, Vap mass 0.00000 kg, Energy -33.7589 mj

**Net discharge**:
- Mass: 0.00000 kg
- Energy: 0.00000 mj

**Conservation errors**:
- Mass: 0.00000
- Energy: 0.00000

This time step discharges:
- Mass, Enthalpy, Power: 0.000e+00 kg, 0.000e+00 mj, 0.000e+00 mw

Cumulative discharges:
- Mass, Enthalpy, Avg Power: 0.000e+00 kg, 0.000e+00 mj, 0.000e+00 mw

Number of region changes this time step: 0

Simulation ended: Days 3.000e+00 Timesteps 601

Total Newton-Raphson iterations: 601

---

*Fig. 12. Continued*
Fig. 13. Instruction file (HIS.INS) for postprocessor FEHPLTR for the heat conduction problem.
Fig. 14. History plot for the heat conduction problem.
Fig. 15. Solution domain and results for the Toronyi example.

Fig. 16. Input for GENMSH for the Toronyi example.
**** toronyi example ****

node
36
10 11 12 13 14 15
18 19 20 21 22 23
26 27 28 29 30 31
34 35 36 37 38 39
42 43 44 45 46 47
50 51 52 53 54 55
sol
1 -1
init
4.3000 000. 250. 0. 250. 0. 0.
rlp
2 0.05 0.05 1.0 1.0
0
1 64 1 1
0 0 0 0 0
pres
1 64 1 4.3000 2 2
0 0 0 0 0 0
rock
1 64 1 2563. 1010. 0.0500 1.0000
0 0 0 0 0 0
cond
1 64 0 1 1.73e-00 1.73e-00 0.0e-00
0 0 0 0 0 0
perm
1 64 1 9.869e-13 9.869e-13 0.e-00 0. 0. 0.
0 0 0 0 0 0 0 0
flow
0 29 0 29 0 0.082011 -025. 0.
0 0 0 0 0 0 0 0
time
8.3 78.3 00009 10 1985 07
8.3 10. 1.0 10
0. 0. 0. 0
ctrl
40 1.e-8 08
1 64 1 1
0 0 0 0
1.0 0.0 0.75
40 1.00 0.0001 010.00
1 0
coor
64
1 0.00000 182.80000 0.00000
2 152.33333 182.80000 0.00000
3 457.00000 182.80000 0.00000
4 761.66667 182.80000 0.00000
5 1066.33333 182.80000 0.00000
6 1371.00000 182.80000 0.00000
7 1675.66667 182.80000 0.00000
8 1828.00000 182.80000 0.00000
9 0.00000 167.56667 0.00000
10 152.33333 167.56667 0.00000
11 457.00000 167.56667 0.00000
12 761.66667 167.56667 0.00000
13 1066.33333 167.56667 0.00000
14 1371.00000 167.56667 0.00000
15 1675.66667 167.56667 0.00000
16 1828.00000 167.56667 0.00000
17 0.00000 137.10000 0.00000
18 152.33333 137.10000 0.00000
19 457.00000 137.10000 0.00000
20 761.66667 137.10000 0.00000
21 1066.33333 137.10000 0.00000
22 1371.00000 137.10000 0.00000
23 1675.66667 137.10000 0.00000
24 1828.00000 137.10000 0.00000
25 0.00000 106.63333 0.00000

Fig. 17. Input file for FEHMN for Toronyi example.
26 152.33333 106.63333 0.00000  
27 457.00000 106.63333 0.00000  
28 761.66667 106.63333 0.00000  
29 1066.33333 106.63333 0.00000  
30 1371.00000 106.63333 0.00000  
31 1675.66667 106.63333 0.00000  
32 1828.00000 106.63333 0.00000  
33 0.00000 76.16667 0.00000  
34 152.33333 76.16667 0.00000  
35 457.00000 76.16667 0.00000  
36 761.66667 76.16667 0.00000  
37 1066.33333 76.16667 0.00000  
38 1371.00000 76.16667 0.00000  
39 1675.66667 76.16667 0.00000  
40 1828.00000 76.16667 0.00000  
41 0.00000 45.70000 0.00000  
42 152.33333 45.70000 0.00000  
43 457.00000 45.70000 0.00000  
44 761.66667 45.70000 0.00000  
45 1066.33333 45.70000 0.00000  
46 1371.00000 45.70000 0.00000  
47 1675.66667 45.70000 0.00000  
48 1828.00000 45.70000 0.00000  
49 0.00000 15.23333 0.00000  
50 152.33333 15.23333 0.00000  
51 457.00000 15.23333 0.00000  
52 761.66667 15.23333 0.00000  
53 1066.33333 15.23333 0.00000  
54 1371.00000 15.23333 0.00000  
55 1675.66667 15.23333 0.00000  
56 1828.00000 15.23333 0.00000  
57 0.00000 0.00000 0.00000  
58 152.33333 0.00000 0.00000  
59 457.00000 0.00000 0.00000  
60 761.66667 0.00000 0.00000  
61 1066.33333 0.00000 0.00000  
62 1371.00000 0.00000 0.00000  
63 1675.66667 0.00000 0.00000  
64 1828.00000 0.00000 0.00000  
65 0.00000 0.00000 0.00000  

```
4, 49, 0

1  9  10  2  1
2 10  11  3  2
3 11  12  4  3
4 12  13  5  4
5 13  14  6  5
6 14  15  7  6
7 15  16  8  7
8 17  18  10  9
9 18  19  11 10
10 19  20  12 11
11 20  21  13 12
12 21  22  14 13
13 22  23  15 14
14 23  24  16 15
15 25  26  18 17
16 26  27  19 18
17 27  28  20 19
18 28  29  21 20
19 29  30  22 21
20 30  31  23 22
21 31  32  24 23
22 33  34  26 25
23 34  35  27 26
24 35  36  28 27
25 36  37  29 28

Fig. 17. (Continued)

96
Fig. 17. Continued.
### Toronyi Example

#### Output File, WT Tape, RD TP, User Call Parameter

- **Time Step:** 1
- **Years:** 3
- **Res Mass:** 287727 kg
- **Vap Mass:** 290027 kg
- **Energy:** 21279990000 J
- **Cumulative Discharges:**
  - Energy: 58814 J
  - Mass: 58814 kg

#### Conservation Errors:
- **Mass:** -66302 J
- **Energy:** -66939 J

#### Storage Details:
- **Geometric Coefficients:** 218 MB
- **GMRES:** 1152 MB
- **A Matrix:** 1152 MB
- **B Matrix:** 1152 MB
- **NCON:** 353 MB

#### Analysis of Input Data
- **Pressure:** 4.300 MPa
- **Temperature:** 273.15 K
- **Energy:** 1.27 MJ

#### Storage Needed:
- **Node:** 218 MB
- **A Matrix:** 1152 MB
- **B Matrix:** 1152 MB

### Example

- **Net Discharge:** 58814 kg
- **Net Energy Discharge:** 1642246 J
- **Cumulative Discharges:**
  - Mass: 58814 kg
  - Enthalpy: 1642246 J
- **Number of Region Changes:** 0

---

**Fig. 18.** Output for the Toronyi example.
this time step discharges
simulation ended: days 7.830e+01 timesteps 9
conservation errors: mass -0.29266e-08 energy -0.43337e-08

Years 0.214e+00 days 0.7830010e-02 ts size 1.000e-04
cpu sec for step 0.5129e-01 total time 0.1539e+01

node p(mpa) e(mJ) l sat temp(c) well dis ent r eql r eq2
10 4.249 1.30 0.213 253.895 0.000e+00 0.000 0.3e-13 -0.4e-14
11 4.249 1.30 0.273 253.895 0.000e+00 0.000 -0.2e-13 -0.2e-14
12 4.209 1.32 0.153 253.336 0.000e+00 0.000 0.2e-12 -0.5e-14
13 4.146 1.37 0.120 252.441 0.000e+00 0.000 0.4e-13 -0.5e-16
14 4.203 1.32 0.149 253.242 0.000e+00 0.000 0.7e-13 -0.3e-14
15 4.232 1.31 0.164 253.650 0.000e+00 0.000 0.3e-13 -0.5e-14
16 4.267 1.29 0.183 254.151 0.000e+00 0.000 -0.7e-13 -0.9e-15
18 4.267 1.29 0.183 254.151 0.000e+00 0.000 -0.7e-13 -0.9e-15
19 4.249 1.30 0.173 253.895 0.000e+00 0.000 -0.2e-13 -0.5e-14
20 4.209 1.32 0.153 253.336 0.000e+00 0.000 -0.7e-13 -0.2e-15
21 4.145 1.37 0.119 252.421 0.000e+00 0.000 -0.9e-13 -0.5e-14
22 4.203 1.32 0.149 253.242 0.000e+00 0.000 -0.7e-13 -0.6e-14
23 4.232 1.31 0.164 253.650 0.000e+00 0.000 -0.1e-13 -0.5e-16
24 4.203 1.32 0.149 253.242 0.000e+00 0.000 -0.7e-13 -0.6e-14
25 4.267 1.29 0.183 254.151 0.000e+00 0.000 -0.5e-13 -0.5e-14
27 4.249 1.30 0.173 253.895 0.000e+00 0.000 -0.5e-13 -0.5e-14
28 4.209 1.32 0.153 253.336 0.000e+00 0.000 -0.3e-14 -0.2e-14
29 4.142 1.38 0.118 252.382 0.820e-01 2.799 -0.6e-13 -0.2e-14
30 4.203 1.32 0.149 253.242 0.000e+00 0.000 -0.7e-14 -0.6e-14
31 4.232 1.31 0.164 253.650 0.000e+00 0.000 -0.5e-13 -0.1e-13
34 4.267 1.29 0.183 254.151 0.000e+00 0.000 -0.5e-13 -0.4e-14
35 4.249 1.30 0.173 253.895 0.000e+00 0.000 -0.1e-13 -0.9e-15
36 4.210 1.32 0.153 253.337 0.000e+00 0.000 -0.1e-13 -0.7e-14
37 4.146 1.37 0.120 252.439 0.000e+00 0.000 -0.1e-13 -0.7e-14
38 4.203 1.32 0.149 253.242 0.000e+00 0.000 -0.8e-14 -0.7e-14
39 4.232 1.31 0.164 253.650 0.000e+00 0.000 -0.8e-13 -0.3e-14
40 4.267 1.29 0.183 254.151 0.000e+00 0.000 -0.3e-13 -0.4e-14
41 4.249 1.30 0.173 253.895 0.000e+00 0.000 0.1e-13 -0.6e-14
42 4.210 1.32 0.153 253.337 0.000e+00 0.000 0.1e-13 -0.6e-14
43 4.249 1.30 0.173 253.895 0.000e+00 0.000 0.1e-13 -0.6e-14
44 4.210 1.32 0.153 253.337 0.000e+00 0.000 0.1e-13 -0.6e-14
45 4.149 1.37 0.121 252.477 0.000e+00 0.000 -0.7e-13 -0.4e-15
46 4.203 1.32 0.149 253.243 0.000e+00 0.000 0.5e-13 -0.4e-14
47 4.232 1.31 0.164 253.650 0.000e+00 0.000 0.1e-13 -0.5e-14
50 4.267 1.29 0.183 254.151 0.000e+00 0.000 -0.2e-13 -0.8e-15
51 4.249 1.30 0.173 253.895 0.000e+00 0.000 0.8e-13 -0.1e-14
52 4.210 1.32 0.153 253.339 0.000e+00 0.000 0.2e-12 -0.2e-14
53 4.150 1.37 0.122 252.496 0.000e+00 0.000 0.3e-13 -0.5e-14
54 4.203 1.32 0.149 253.242 0.000e+00 0.000 -0.1e-12 -0.4e-15
55 4.232 1.31 0.164 253.650 0.000e+00 0.000 -0.7e-13 -0.7e-15

res mass 0.238127e+07 kg vap mass 298427. kg energy 0.211341e+09 mj

net discharge 0.55481e+06 kg net energy discharge 0.15518e+07 mj

conservation errors: mass -0.29266e-08 energy -0.43337e-08

this time step discharges: mass, enthalpy, power
7.086e-01 kg 1.983e+00 mj 2.295e-01 mw

cumulative discharges: mass, enthalpy, avg power
5.548e-05 kg 1.552e+06 mj 2.294e-01 mw

cumulative changes this time step

number of region changes this time step 0

simulation ended: days 7.830e+01 timesteps 9
contour plt: fe.con, history plt: fe.his

total newton-raphson iterations 19

Fig. 18. (Continued)
Fig. 19. Schematic diagram of the geometry and boundary conditions for the DOE code comparison project problem.

Fig. 20. Comparison of FEHMN production well temperatures with results from other codes.
Fig. 21. Comparison of FEHMN production well pressure with results from other codes.

Fig. 22. Comparison of FEHMN observation well pressure with results from other codes.
2D Model of a Rectangle
8 1 2 2 0 2
1 1 2 3 4 5 6 7 8
0. 150. 300. 300. 300. 150. 0. 0.
0. 0. 0. 0. 100. 200. 200. 200. 100.
1 13 9 1
.5 1 1 1 1 1 1 1 .5
.5 1 1 1 1 1 1 .5

Fig. 23. Input for GENMSH for the DOE example.

*** DOE Code Comparison Project, Problem 5, Case A ***

node
2
46 92
sol
1 1
init
3.6000 0.00 240. 0. 0. 240. 0. 0.
rlp
2 0.3 0.1 0.0 0.0
0
1 140 1 1
0 0 0 0
rock
1 140 1 2563. 1010. 0.3500
0 0 0 0 0. 0. 0.
cond
1 140 1 1.00e-00 1.00e-00 0.00e-00 0.00e-00
0 0 0 0.00e-00 0.00e-00 0.00e-00 0.00e-00
perm
1 140 1 2.50e-14 2.50e-14 0.00e-00
0 0 0 0.00e-00 0.00e-00 0.00e-00 0.00e-00
flow
46 46 1 0.050 -25. 0.
14 140 14 3.600 -160. 1.
0 0 0 0 0. 0. 0.
time
30.0 3650. 10000 1000 1989 10
1.0 -1.2 1.5 10
0. 0. 0. 0
ctrl
40 1.0e-7 8
1 140 1 1
0 0 0 0
1.0 0.0 1.00
40 1.2 0.0 60.000
1 0

Fig. 24. Input for FEHMN for the DOE example.
<table>
<thead>
<tr>
<th>coor</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>12.5000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>3</td>
<td>25.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>4</td>
<td>37.5000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>5</td>
<td>50.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>6</td>
<td>62.5000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>7</td>
<td>75.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>8</td>
<td>87.5000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>9</td>
<td>100.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>10</td>
<td>112.5000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>11</td>
<td>125.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>12</td>
<td>137.5000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>13</td>
<td>150.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>14</td>
<td>162.5000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>15</td>
<td>175.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>16</td>
<td>187.5000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>17</td>
<td>200.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>18</td>
<td>212.5000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>19</td>
<td>225.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>20</td>
<td>237.5000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>21</td>
<td>250.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>22</td>
<td>262.5000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>23</td>
<td>275.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>24</td>
<td>287.5000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>25</td>
<td>300.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>26</td>
<td>312.5000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>27</td>
<td>325.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>28</td>
<td>337.5000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>29</td>
<td>350.0000</td>
<td>37.5000</td>
<td>0.0000</td>
</tr>
<tr>
<td>30</td>
<td>362.5000</td>
<td>37.5000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Fig. 24. (continued)
<table>
<thead>
<tr>
<th></th>
<th>37.50000</th>
<th>37.50000</th>
<th>0.00000</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>32</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>33</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>34</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>35</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>36</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>37</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>38</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>39</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>40</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>41</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>42</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>43</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>44</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>45</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>46</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>47</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>48</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>49</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>50</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>51</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>52</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>53</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>54</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>55</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>56</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>57</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>58</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>59</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>60</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>61</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>62</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>63</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>64</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>65</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>66</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>67</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>68</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>69</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>70</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>71</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>72</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>73</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>74</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>75</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>76</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>77</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>78</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>79</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>80</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>81</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>82</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>83</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>84</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>85</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>86</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>87</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>88</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>89</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>90</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>91</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>92</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>93</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>94</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>95</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>96</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>97</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>98</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>99</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
<tr>
<td>100</td>
<td>62.50000</td>
<td>37.50000</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

Fig. 24. (continued)
<table>
<thead>
<tr>
<th>elem</th>
<th>4.117.0</th>
<th>4.117.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>14</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td>16</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>17</td>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>19</td>
<td>20</td>
<td>21</td>
</tr>
<tr>
<td>20</td>
<td>21</td>
<td>22</td>
</tr>
<tr>
<td>21</td>
<td>22</td>
<td>23</td>
</tr>
<tr>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>23</td>
<td>24</td>
<td>25</td>
</tr>
<tr>
<td>24</td>
<td>25</td>
<td>26</td>
</tr>
<tr>
<td>25</td>
<td>26</td>
<td>27</td>
</tr>
<tr>
<td>26</td>
<td>27</td>
<td>28</td>
</tr>
<tr>
<td>27</td>
<td>28</td>
<td>29</td>
</tr>
<tr>
<td>28</td>
<td>29</td>
<td>30</td>
</tr>
<tr>
<td>29</td>
<td>30</td>
<td>31</td>
</tr>
<tr>
<td>30</td>
<td>31</td>
<td>32</td>
</tr>
<tr>
<td>31</td>
<td>32</td>
<td>33</td>
</tr>
<tr>
<td>32</td>
<td>33</td>
<td>34</td>
</tr>
<tr>
<td>33</td>
<td>34</td>
<td>35</td>
</tr>
<tr>
<td>34</td>
<td>35</td>
<td>36</td>
</tr>
<tr>
<td>35</td>
<td>36</td>
<td>37</td>
</tr>
<tr>
<td>36</td>
<td>37</td>
<td>38</td>
</tr>
<tr>
<td>37</td>
<td>38</td>
<td>39</td>
</tr>
<tr>
<td>38</td>
<td>39</td>
<td>40</td>
</tr>
<tr>
<td>39</td>
<td>40</td>
<td>41</td>
</tr>
<tr>
<td>40</td>
<td>41</td>
<td>42</td>
</tr>
<tr>
<td>41</td>
<td>42</td>
<td>43</td>
</tr>
<tr>
<td>42</td>
<td>43</td>
<td>44</td>
</tr>
<tr>
<td>43</td>
<td>44</td>
<td>45</td>
</tr>
<tr>
<td>44</td>
<td>45</td>
<td>46</td>
</tr>
</tbody>
</table>

**Fig. 24. (continued)**
<table>
<thead>
<tr>
<th>31</th>
<th>33</th>
<th>34</th>
<th>48</th>
<th>47</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>34</td>
<td>35</td>
<td>49</td>
<td>48</td>
</tr>
<tr>
<td>33</td>
<td>35</td>
<td>36</td>
<td>50</td>
<td>49</td>
</tr>
<tr>
<td>34</td>
<td>36</td>
<td>37</td>
<td>51</td>
<td>50</td>
</tr>
<tr>
<td>35</td>
<td>37</td>
<td>38</td>
<td>52</td>
<td>51</td>
</tr>
<tr>
<td>36</td>
<td>38</td>
<td>39</td>
<td>53</td>
<td>52</td>
</tr>
<tr>
<td>37</td>
<td>39</td>
<td>40</td>
<td>54</td>
<td>53</td>
</tr>
<tr>
<td>38</td>
<td>40</td>
<td>41</td>
<td>55</td>
<td>54</td>
</tr>
<tr>
<td>39</td>
<td>41</td>
<td>42</td>
<td>56</td>
<td>55</td>
</tr>
<tr>
<td>40</td>
<td>42</td>
<td>43</td>
<td>57</td>
<td>56</td>
</tr>
<tr>
<td>41</td>
<td>43</td>
<td>44</td>
<td>58</td>
<td>57</td>
</tr>
<tr>
<td>42</td>
<td>44</td>
<td>45</td>
<td>59</td>
<td>58</td>
</tr>
<tr>
<td>43</td>
<td>45</td>
<td>46</td>
<td>60</td>
<td>59</td>
</tr>
<tr>
<td>44</td>
<td>46</td>
<td>47</td>
<td>61</td>
<td>60</td>
</tr>
<tr>
<td>45</td>
<td>47</td>
<td>48</td>
<td>62</td>
<td>61</td>
</tr>
<tr>
<td>46</td>
<td>48</td>
<td>49</td>
<td>63</td>
<td>62</td>
</tr>
<tr>
<td>47</td>
<td>49</td>
<td>50</td>
<td>64</td>
<td>63</td>
</tr>
<tr>
<td>48</td>
<td>50</td>
<td>51</td>
<td>65</td>
<td>64</td>
</tr>
<tr>
<td>49</td>
<td>51</td>
<td>52</td>
<td>66</td>
<td>65</td>
</tr>
<tr>
<td>50</td>
<td>52</td>
<td>53</td>
<td>67</td>
<td>66</td>
</tr>
<tr>
<td>51</td>
<td>53</td>
<td>54</td>
<td>68</td>
<td>67</td>
</tr>
<tr>
<td>52</td>
<td>54</td>
<td>55</td>
<td>69</td>
<td>68</td>
</tr>
<tr>
<td>53</td>
<td>55</td>
<td>56</td>
<td>70</td>
<td>69</td>
</tr>
<tr>
<td>54</td>
<td>56</td>
<td>57</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td>55</td>
<td>57</td>
<td>58</td>
<td>72</td>
<td>72</td>
</tr>
<tr>
<td>56</td>
<td>58</td>
<td>59</td>
<td>73</td>
<td>73</td>
</tr>
<tr>
<td>57</td>
<td>59</td>
<td>60</td>
<td>74</td>
<td>74</td>
</tr>
<tr>
<td>58</td>
<td>60</td>
<td>61</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>59</td>
<td>61</td>
<td>62</td>
<td>76</td>
<td>75</td>
</tr>
<tr>
<td>60</td>
<td>62</td>
<td>63</td>
<td>77</td>
<td>76</td>
</tr>
<tr>
<td>61</td>
<td>63</td>
<td>64</td>
<td>78</td>
<td>77</td>
</tr>
<tr>
<td>62</td>
<td>64</td>
<td>65</td>
<td>79</td>
<td>78</td>
</tr>
<tr>
<td>63</td>
<td>65</td>
<td>66</td>
<td>80</td>
<td>79</td>
</tr>
<tr>
<td>64</td>
<td>66</td>
<td>67</td>
<td>81</td>
<td>80</td>
</tr>
<tr>
<td>65</td>
<td>67</td>
<td>68</td>
<td>82</td>
<td>81</td>
</tr>
<tr>
<td>66</td>
<td>68</td>
<td>69</td>
<td>83</td>
<td>82</td>
</tr>
<tr>
<td>67</td>
<td>69</td>
<td>70</td>
<td>84</td>
<td>83</td>
</tr>
<tr>
<td>68</td>
<td>70</td>
<td>71</td>
<td>85</td>
<td>84</td>
</tr>
<tr>
<td>69</td>
<td>71</td>
<td>72</td>
<td>86</td>
<td>85</td>
</tr>
<tr>
<td>70</td>
<td>72</td>
<td>73</td>
<td>87</td>
<td>86</td>
</tr>
<tr>
<td>71</td>
<td>73</td>
<td>74</td>
<td>88</td>
<td>87</td>
</tr>
<tr>
<td>72</td>
<td>74</td>
<td>75</td>
<td>89</td>
<td>88</td>
</tr>
<tr>
<td>73</td>
<td>75</td>
<td>76</td>
<td>90</td>
<td>89</td>
</tr>
<tr>
<td>74</td>
<td>76</td>
<td>77</td>
<td>91</td>
<td>90</td>
</tr>
<tr>
<td>75</td>
<td>77</td>
<td>78</td>
<td>92</td>
<td>91</td>
</tr>
<tr>
<td>76</td>
<td>78</td>
<td>79</td>
<td>93</td>
<td>92</td>
</tr>
<tr>
<td>77</td>
<td>79</td>
<td>80</td>
<td>94</td>
<td>93</td>
</tr>
<tr>
<td>78</td>
<td>80</td>
<td>81</td>
<td>95</td>
<td>94</td>
</tr>
<tr>
<td>79</td>
<td>81</td>
<td>82</td>
<td>96</td>
<td>95</td>
</tr>
<tr>
<td>80</td>
<td>82</td>
<td>83</td>
<td>97</td>
<td>96</td>
</tr>
<tr>
<td>81</td>
<td>83</td>
<td>84</td>
<td>98</td>
<td>97</td>
</tr>
<tr>
<td>82</td>
<td>84</td>
<td>85</td>
<td>99</td>
<td>99</td>
</tr>
<tr>
<td>83</td>
<td>85</td>
<td>86</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>84</td>
<td>86</td>
<td>87</td>
<td>101</td>
<td>100</td>
</tr>
<tr>
<td>85</td>
<td>87</td>
<td>88</td>
<td>102</td>
<td>101</td>
</tr>
<tr>
<td>86</td>
<td>88</td>
<td>89</td>
<td>103</td>
<td>102</td>
</tr>
<tr>
<td>87</td>
<td>89</td>
<td>90</td>
<td>104</td>
<td>103</td>
</tr>
<tr>
<td>88</td>
<td>90</td>
<td>91</td>
<td>105</td>
<td>104</td>
</tr>
<tr>
<td>89</td>
<td>91</td>
<td>92</td>
<td>106</td>
<td>105</td>
</tr>
<tr>
<td>90</td>
<td>92</td>
<td>93</td>
<td>107</td>
<td>106</td>
</tr>
<tr>
<td>91</td>
<td>93</td>
<td>94</td>
<td>108</td>
<td>107</td>
</tr>
<tr>
<td>92</td>
<td>94</td>
<td>95</td>
<td>109</td>
<td>108</td>
</tr>
<tr>
<td>93</td>
<td>95</td>
<td>96</td>
<td>110</td>
<td>109</td>
</tr>
<tr>
<td>94</td>
<td>96</td>
<td>97</td>
<td>111</td>
<td>110</td>
</tr>
<tr>
<td>95</td>
<td>97</td>
<td>98</td>
<td>112</td>
<td>111</td>
</tr>
<tr>
<td>96</td>
<td>98</td>
<td>99</td>
<td>113</td>
<td>113</td>
</tr>
<tr>
<td>97</td>
<td>99</td>
<td>100</td>
<td>114</td>
<td>114</td>
</tr>
<tr>
<td>98</td>
<td>100</td>
<td>101</td>
<td>115</td>
<td>114</td>
</tr>
<tr>
<td>99</td>
<td>101</td>
<td>102</td>
<td>116</td>
<td>115</td>
</tr>
<tr>
<td>100</td>
<td>102</td>
<td>103</td>
<td>117</td>
<td>116</td>
</tr>
</tbody>
</table>

Fig. 24. (continued)
|   | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 | 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 | 121 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 | 130 | 131 | 132 | 133 | 134 | 135 | 136 | 137 | 138 | 139 | 140 | 141 |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0 | 108 | 109 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 | 130 | 131 | 132 | 133 | 134 | 135 | 136 | 137 | 138 | 139 | 140 | 141 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Fig. 24. (continued)
**Fig. 25.** Output from FEHMN for the DOE problem.
temp contours for doe problem
0 0
0 300 0 200
0 0
6.5 6.5
1 -1. 0.0 0.0 0.0

Fig. 26. Instruction file (CON.INS) for the postprocessor FECPLTR for the DOE problem.
temperatures
time = 10,000 years liquid phase temp contours for doe problem

Fig. 27. Contour plot of temperature for the DOE problem.
ACKNOWLEDGMENTS

The authors thank Kay Coen, Kay Birdsell, and Mary Ann Olson for their contributions to this document.

REFERENCES


