Continuous Contour Phase Plates for Tailoring the Focal Plane Irradiance Profile

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Continuous contour phase plates for tailoring the focal plane irradiance profile

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ABSTRACT

We present fully continuous phase screens for producing super-Gaussian focal-plane irradiance profiles. Such phase screens are constructed with the assumption of either circular symmetric near-field and far-field profiles or a separable phase screen in Cartesian co-ordinates. In each case, the phase screen is only a few waves deep. Under illumination by coherent light, such phase screens produce high order super-Gaussian profiles in the focal plane with high energy content. Effects of beam aberrations on the focal plane profiles and their energy content are also discussed.

Keywords: Kinoform phase plates, diffractive optics, computer generated holograms

2. INTRODUCTION

It is now widely recognized that spatial beam smoothing (homogenization) is essential for efficiently coupling the laser energy through the laser entrance hole of a hohlraum in indirect drive Inertial Confinement Fusion (ICF) and for uniform target illumination in the case of direct drive ICF. Although the uniformity requirements and optimum envelopes are different in the two cases, both approaches require an efficient redistribution of the laser energy into a controlled spot in the focal plane. For indirect drive ICF, the desired focal plane irradiance profile is a high-order super-Gaussian containing a very high fraction (greater than 95%) of the incident energy. Binary random phase plates (RPPs) have been extensively used in ICF laser facilities worldwide for producing spatially homogenized far-fields. Although easy to fabricate and use, their utility in producing super-Gaussian distributions is limited since the RPPs produce essentially an Airy function envelope containing only 84% (upper limit) of the energy inside the central spot. Approaches using the lenslet arrays (refractive or diffractive) have limited use since they operate in the quasi-far-field and as such have short depth of focus.

To overcome these limitations, we introduced the concept of continuously varying phase screens called kinoform phase plates (KPPs). An iterative algorithm, similar to that used in phase retrieval, is used to design phase screens which can produce arbitrary far-field distributions with high (>90%) efficiency. An initial phase screen is systematically improved by repeatedly transforming between the near-field and the far-field planes (by a Fourier transform) and applying appropriate constraints in each plane. Typically, the near-field constraint consists of replacing the intensity obtained during the transformation of the far-field amplitude by the given incident intensity (flat-top). Typical far-field constraint consists of multiplying the far-field intensity pattern by a suitable envelope (a flat-top super-Gaussian for example). A major portion of the convergence is obtained within a few iterations. However, if the iterative process is launched starting from a random initial phase, it is observed that the resulting phase screen has many open-ended $2\pi$ discontinuities (called '2$\pi$ lines') in addition to the closed looped $2\pi$ discontinuities (called '2$\pi$ loops'). The ends of the $2\pi$ lines correspond to phase singularities and the $2\pi$ line is a branch cut. Figure 1 displays a section of such a phase screen containing several $2\pi$ line discontinuities. The algorithm stagnates at the ends of such $2\pi$ lines and further iterations are unable to remove these $2\pi$ jumps or convert them into $2\pi$ loops. A phase screen containing such $2\pi$ lines cannot be unwrapped into a continuous phase screen. In addition, the rapid variation of the phase around the ends of the $2\pi$ loops leads to large modulations in the diffracted intensity following the phase plate. The phase singularities also lead to some energy scattering outside the desired region in the far-field. Finite widths of the $2\pi$ jumps (lines as well as...
Figure 1. Section of a kinoform phase plate that contains several $2\pi$ branch cuts. Grey scale corresponds to a phase variation from 0 (black) to $2\pi$ (white). The boundaries between black and white correspond to $2\pi$ phase jumps. Open ended $2\pi$ phase jumps represent the branch cuts.

loops) during the fabrication process lead to additional energy losses due to large angle scattering. This last source of energy loss could be minimized by minimizing the width of these jumps.

Continuously varying phase screens offer several advantages over those containing $2\pi$ line or loop discontinuities. One important advantage is that the propagated field past the KPP exhibits a low level of intensity modulation. This should minimize the damage threat to the downstream optics. Since the phase appears continuous for the fundamental and harmonic wavelengths as well, the intensity modulations remain small at these wavelengths and hence the unconverted laser light also does not experience any significantly increased level of intensity modulation because of the KPP. The absence of $2\pi$ jumps also eliminates the large angle scattering losses from these edges and increases the energy concentration inside the central spot.

In spite of the advantages of continuous phase screens, the problem of designing such phase screens has eluded us for some time. Typically, smoothly varying phase screens lead to Gaussian envelopes in the far-field whose size is related to the correlation length and the variance of the phase profile. Researchers at the Laboratory at Laser Energetics at Rochester have attempted to flatten these Gaussian far-field profiles by imposing an additional periodic phase modulation in the near-field\textsuperscript{11}. This leads to multiple, overlapping Gaussian spots in the focal plane resulting in a super-Gaussian-like envelope. This approach has limited flexibility in its ability to produce high order super-Gaussians and also complex profiles. In a most recent approach\textsuperscript{12} to improve on the iterative algorithm, the authors used a continuous phase as a starting point in an iterative algorithm and concluded that the $2\pi$ line discontinuities crept in after only two iterations. This hints at a potential instability in the iteration process as more and more $2\pi$ line discontinuities would be expected to be introduced with additional iterations.
We have designed continuous phase plates by restricting the problem to situations where the phase screen is can be unwrapped. Since the phase is always unwrappable in one-dimension, a continuous phase screen can be designed in cases where the problem can be reduced to a one-dimensional one. Examples where this is possible are situations where the near- and the far-field profiles are constrained to be circularly symmetric and, where a separable approximation in Cartesian coordinates is imposed. In the following sections we present results of numerical calculations for continuous phase screens generated under these assumptions.

3. CIRCULAR SYMMETRIC KINOFORM PHASE SCREENS

Let us consider designing a phase screen which produces a circularly symmetric far-field profile from a circularly symmetric near-field profile. The symmetry of the problem implies that the phase screen will also be circularly symmetric. We shall call such circular symmetric phase plates as kinoform zone plates (KZPs). If \( E_n(r) \) and \( E_f(r) \) denote the complex near-field and the far-field amplitudes, these are related to each other through a Hankel transform

\[
E_f(r) = \frac{2\pi i}{\lambda f} \int_0^\infty E_n(r') J_0 \left( \frac{2\pi r r'}{\lambda f} \right) r' \, dr'
\]

where \( \lambda \) denotes the laser wavelength and \( f \) the lens focal length and \( J_0 \) the zeroth order Bessel function.

![Figure 2a. FoCAL plane radial profile generated by a circularly symmetric kinoform zone plate (shown in figure 2b). \( \lambda = 351 \text{ nm} \) and \( f = 7 \text{ m} \) were assumed in the numerical calculations. The superimposed smooth curve corresponds to a 12th power super-Gaussian (see equation 3 for definition) with a 1/e radius of 245 \( \mu \text{m} \).]
The inverse transform has a similar form; i.e.

\[
E_n(r') = -\frac{2\pi i}{\lambda f} \int_0^\infty E_r(r) J_0(\frac{2\pi r'}{\lambda f}) r \, dr.
\]  

(2)

An iterative procedure can be applied to equations 1 and 2 to design the required circularly symmetric zone plate. The procedure begins by constructing \(E_n(r')\) using the near-field intensity profile and a set of random phases \(\phi(r')\). Propagation of this complex field to the focal plane using equation 1 leads to an \(E_f(r)\) that has both amplitude and phase fluctuations. At this point the amplitude is replaced by the desired super-Gaussian profile and the phase is left unchanged. We call this the replacement constraint. The modified field is then transformed back to the near-field using equation 2. A replacement constraint similar to that for the far-field is applied to the near-field amplitude. The iterative loop is repeated until a satisfactory convergence is achieved.

We have applied such an iterative algorithm to construct a KZP for producing a 250 \(\mu\)m radius super-Gaussian profile from a 40-cm diameter flat-top near-field. \(\lambda = 351\) nm and \(f = 7\) m were assumed in these calculations. Equations 1 and 2 were directly integrated with a near-field grid spacing of 1 mm and a far-field spacing of 2 \(\mu\)m. We found that the algorithm converges after only a few iterations. We have examined hundreds of different samples for the initial random phase profile and found that some of the converged phase screens were only a few waves deep. The far-field profiles were similar for most of these samplings. One such far-field profile is shown in figure 2a and the corresponding KZP phase screen after phase unwrapping is shown in figure 2b. This phase screen has a depth of approximately 3 waves. The calculated far-field profile is seen to agree well with a 12th power super-Gaussian of the form.
\[ e^{-\left(\frac{r}{r_0}\right)^{12}} \]  

(3)

with \( r_0 = 245 \, \mu m \). A radial integration of the far-field indicates that greater than 95% of the energy is contained inside a circle of 250 \( \mu m \) radius.

The unwrapped phase screen consists of fairly large regions of approximately linearly varying phase. These regions act like wedges or prisms in the beam and steer that portion of it to different positions in the far-field. The amount of the far-field offset is determined by the slope of the phase. The iterative algorithm then determines the distribution of such offset profiles to produce a super-Gaussian envelope in the far-field. This intuitive interpretation, based on the geometrical optics, is corroborated by detailed diffractive calculations presented here.

We also find that the zone sizes in the KZP do not get smaller as one moves away from the center of the profile. In the conventional Fresnel lenses and zone plates and also in the more recent phase zone plates (PZP)\(^6\), the zones get progressively smaller and smaller in order to produce a diffraction limited focal spot. In the present case, however, since the desired focal spot is many times diffraction limited, one need not steer all regions of the beam towards the center. Consequently, the zone sizes do not have to decrease near the edge of the KZP. This feature should facilitate the fabrication of such KZPs.

![Figure 3](image.png)

Figure 3. One-dimensional continuous phase screens iteratively generated for constructing a two-dimensional continuous KPP within a separable approximation. The solid and the dashed lines represent two converged phase screen samples.

**4. CONTINUOUS PHASE SCREENS IN A SEPARABLE APPROXIMATION**

Another situation where the phase plate design problem simplifies is if a separable approximation is imposed in the near-field and the far-field profiles. In this case, the two-dimensional phase plate design
Figure 4. Two dimensional phase screen constructed from the one-dimensional phase profiles from figure 3. The grey scale corresponds to a phase variation from zero (black) to 25.2 radians (white).

The problem reduces to two decoupled one-dimensional KPP designs in the x- and the y-directions. The x and y portions of the near- and the far-fields are related to each other through the Fourier transform relations

\[ E_f(x) = \sqrt{\frac{1}{i \lambda f}} \int_{-\infty}^{\infty} E_n(x') \ e^{-\left[ \frac{2 \pi \lambda x'}{\lambda f} \right]} \ dx' \]  

(4)

and

\[ E_n(x') = \sqrt{\frac{1}{i \lambda f}} \int_{-\infty}^{\infty} E_f(x) \ e^{\left[ \frac{2 \pi \lambda x}{\lambda f} \right]} \ dx . \]  

(5)

The phase screens in each direction can be constructed using the conventional version of the Gerchberg-Saxton iterative algorithm. The converged phase screen can be unwrapped in each direction as well. If \( \phi(x) \) and \( \phi(y) \) denote the phase screens in x and y directions, then the separable two-dimensional phase screen can be constructed as a quadrature sum of these two phase screens, i.e.

\[ \phi(x,y) = \phi(x) + \phi(y) . \]  

(6)

We have applied such an iterative procedure using fast Fourier transform methods to construct separable KPPs for producing a 400 \( \mu \)m spot in the far-field. We used a 512 point grid over a near-field extent of 45.5 cm. The near-field intensity profile is assumed to be a rectangular 40th power super-Gaussian of 1/e width of 38 cm in each direction and \( \lambda = 351 \) nm and \( f = 7 \) m is assumed. As in the calculations described in section 3, we have tried hundreds of simulations starting with different samplings of the initial random phase and have picked the two having the smallest variations to construct the two-dimensional phase screen. The one-dimensional unwrapped phase screens are shown in figure 3. Each of these phase profiles is about two waves deep. The two-dimensional phase screen constructed using
Figure 5a. The two-dimensional far-field intensity profile generated by the phase screen shown in figure 4. The grey scale corresponds to logarithm (base 10) of the intensity between $10^{-5}$ of the peak value (black) and the peak value (white). The full range of the intensity values was thresholded at $10^{-5}$ of the peak value to enhance the visual appearance of the central profile.

equation 6 is shown in figure 4 while the far-field intensity profile generated by it is shown in figure 5. The separable nature is clearly evident in these figures. The separable nature also leads to a square focal plane intensity profile with high energy content (96% inside a 400 µm square).

5. IMPROVEMENTS TO THE SEPARABLE PHASE SCREEN

In order to produce a near circular far-field profile, we used the separable phase screen shown in figure 4 as an initial guess in the two-dimensional iterative algorithm. We applied a multiplicative constraint consisting of a 400 µm diameter circle in the far-field. The iterated far-field is shown in figure 6. We observed that the phase screen remained continuous over the relevant portion of the near-field and was only slightly modified. The resultant phase screen, shown in figure 7, is seen to be quite similar to the one in figure 4. The somewhat circular nature of the profile increases the energy content inside a 400 µm circle to 99% from about 80% inside a 400 µm circle for the square far-field profile shown in figure 4.

6. EFFECTS OF BEAM ABERRATIONS ON THE KPP PERFORMANCE

All of the phase plate designs presented so-far have assumed a uniform intensity, constant phase laser beam in the near-field. However, high-power fusion laser beams such as Nova are severely aberrated. Aberrations are introduced from sources such as optical inhomogeneities, surface figure errors, and thermal aberrations in large aperture optical components. Additional aberrations are introduced when segmented optical components are used on systems such as the Nova laser. All these aberrations produce far-field profiles that are several times (20-30 times for Nova) diffraction limited. The sharpness of the far-field profile produced by the KPPs as well as the high energy content within it are degraded because of the
beam aberrations. Since the input electric field amplitude will be a product of the complex amplitude determined by the KPP phase screen and the incident laser beam amplitude, the far-field will be a convolution of the KPP far-field and the laser far-field separately. One can expect that if the KPP far-field is considerably larger than that produced by the laser without the KPP, then the convolution should not affect the overall far-field shape. The edge sharpness would be reduced. Numerical modelling as well as experiments on the Nova laser using binary IRPPs confirm this hypothesis. Realistic KPP designs must accommodate for the beam aberrations while attempting to meet the energy concentration and profile requirements.

Figure 5b. Line scan through the center of the intensity profile shown in figure 5a. $\lambda = 351\text{nm}$ and $f = 3\text{m}$ were assumed in the calculations. The superimposed smooth profile is a 12th power super-Gaussian (equation 3) with a $1/e$ radius of $200\ \mu\text{m}$.

7. SUMMARY AND CONCLUSIONS

We have presented fully continuous phase plate designs for producing super-Gaussian focal plane irradiance profiles. These phase plates were designed using an iterative algorithm under an assumption of circular symmetry in the near- and the far-fields or under a separable assumption in Cartesian coordinates. In each case the converged phase screen can be unwrapped. It is seen that such phase plates are only a few waves deep and yet produce high order super-Gaussian profiles in the far field with high energy concentration. The separable phase screen was iterated upon further to produce a near circular profile in the far field.

These continuous phase screens can be fabricated either as multiwave deep structures or, as modulo $2\pi$, one wave deep phase screen. The latter can easily be done using lithographic approaches and wet etching of fused silica. We are currently fabricating such continuous designs by both approaches. Development of design strategies for generating non-separable, two dimensional, continuous phase plates is also in progress. Results of these efforts will be reported in future publications.
8. ACKNOWLEDGEMENTS

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9. REFERENCES

Figure 6a. The far-field intensity profile generated by iterating upon the phase screen shown in figure 4. The grey scale corresponds to logarithm (base 10) of the intensity from $10^{-5}$ of the peak value (black) to the peak value (white). The full range of the intensity values was thresholded at $10^{-5}$ of the peak value to enhance the visual appearance of the central profile.

Figure 6b. Line scan through the center of the intensity profile shown in figure 6a. $\lambda = 351\text{nm}$ and $f = 3\text{m}$ were assumed in the calculations. The superimposed smooth profile is a 12th power super-Gaussian (equation 3) with a 1/e radius of 200 $\mu\text{m}$. 

\[ \text{Intensity (a. u.)} \]
\[ -600-400-200 \quad 0 \quad 200 \quad 400 \quad 600 \]
\[ \text{Far-field distance (\(\mu\text{m}\))} \]
Figure 7. Iterated phase pattern producing a near-circular super-Gaussian distribution shown in figure 6. The grey scale corresponds to phase values ranging from zero (black) to 24.8 radians (white).