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AUTHORS:
Wojciech Zurek, T-6
Juan Pablo Paz, Ciudad Universitaria

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WHY WE DON’T NEED QUANTUM PLANETARY DYNAMICS, OR
ON DECOHERENCE AND THE CORRESPONDENCE PRINCIPLE FOR CHAOTIC SYSTEMS.

WOJCIECH HUBERT ZUREK(1) AND JUAN PABLO PAZ(1,2)

(1): Theoretical Astrophysics, Los Alamos National Laboratory, Los Alamos, NM 87545, USA
(2): Departamento de Física, Facultad de Ciencias Exactas y Naturales, Pabellón 1, Ciudad Universitaria, 1428 Buenos Aires, Argentina

ABSTRACT: Violation of correspondence principle may occur for very macroscopic byt isolated quantum systems on rather short timescales as illustrated by the case of Hyperion, the chaotically tumbling moon of Saturn, for which quantum and classical predictions are expected to diverge on a timescale of approximately 20 years. Motivated by Hyperion, we review salient features of “quantum chaos” and show that decoherence is the essential ingredient of the classical limit, as it enables one to solve the apparent paradox caused by the breakdown of the correspondence principle for classically chaotic systems.

1. Introduction

Is the correspondence principle valid for quantum systems whose classical counterparts are chaotic? This question has been at the center of a debate that has taken place in recent years within the community of scientists interested in quantum chaos [1, 2, 3]. In this paper we will argue that the apparent failure of the correspondence principle is cured by decoherence, which is an essential ingredient to properly define a classical limit. We shall begin by schematically presenting the problem. Subsequently, we shall sketch the solution provided by decoherence.

There is no unique way to state the correspondence principle. Indeed, various approaches can be found in the literature. All of them predict failure of
the quantum-classical correspondence when applied to quantum systems which are classically chaotic. What most authors seem to understand by correspondence is the rough idea that quantum mechanics, when applied to macroscopic systems must agree with the predictions of classical Newtonian dynamics. For Bohr and Heisenberg – and most quantum mechanics textbooks – the correspondence principle is expected to be valid in the limit of large quantum numbers, $\hbar \to 0$, $1/n$ or the like. Another way of looking at this issue, based on Ehrenfest theorem, is to note that for a sharply peaked wave packet, characterized by large occupation numbers, the expectation values $< x >$ and $< p >$ follow classical trajectories satisfying Newton's laws.

As mentioned above, in any of its forms, correspondence principle seems to be in trouble when applied to systems which are classically chaotic. To clearly state the problem [4] it is convenient to use the phase space formulation of quantum mechanics based on the Wigner function $W(x, p)$ whose evolution equation (entirely equivalent to Schrödinger equation) reads [5]:

$$\dot{W} = \{H, W\}_MB$$
$$= \{H, W\}_PB + \sum_n \frac{\hbar^{2n}(-1)^n}{(2n + 1)!2^{2n}} \partial_x^{(2n+1)}V \partial_p^{(2n+1)}W.$$ (1)

The operator in the right hand side of (1) is known as the Moyal bracket. When the potential $V$ is analytic, Moyal bracket can be expanded to yield Liouville equation with quantum corrections, as it is illustrated above. The first term in that expansion is the ordinary Poisson bracket, which generates the Liouville flow in the phase space according to which a classical distribution function evolves. The sum in the second term contains all the quantum mechanical effects. Therefore, Liouville flow in phase space (and consequently, classical dynamics) is obtained from the basic quantum picture as long as the quantum corrections appearing in (1) are negligible.

Consider now an initial state that corresponds to a Gaussian packet which is round and smooth over scales much larger than $\hbar$ (i.e, $\Delta x_0 \Delta p_0 \gg \hbar$). For such a state the sum in (1) is negligible since it involves derivatives of a smooth function. Indeed one can see that the $n$–th order term in the sum is proportional to $(\hbar/\chi \sigma_p)^{2n}$ where $\sigma_p$ is the scale over which the Wigner function varies along the momentum direction and $\chi$ is the scale over which the potential is nonlinear (e. g., $\chi \simeq \sqrt{\frac{\partial_x V}{\partial_{xxx} V}}$) within the range where it is influencing the evolution of the state.
Therefore, a smooth initial state will start evolving with negligible quantum corrections. Each point in phase space will start following its corresponding classical trajectory. However, this state of affairs cannot last forever: After some time \( t_h \), the Wigner function that evolves according to equation (1) will start looking different from a classical distribution function which has originated from the same initial condition but which has evolved according to Liouville equation. From that time, the difference between the quantum expectation values \( \langle x^k(t) \rangle, \langle p^k(t) \rangle \), calculated from the Wigner function, and their classical counterparts obtained from the classical distribution function will tend to increase.

To see if this obvious property of quantum evolution poses a problem for the correspondence principle, the relevant question is: "How long is the correspondence breakdown time \( t_h \)?". The answer to this question is dramatically different depending on the nature of the evolution – that is, on whether the system is classically chaotic or integrable. For a classically chaotic system, an initially smooth phase space patch will be exponentially stretched in the directions corresponding to positive Lyapunov exponents. As the volume in phase space is preserved by the Liouville flow, \( W(x,p) \), will tend to shrink in other directions. Consequently, derivatives of the Wigner function will grow exponentially fast generating the growth of the "quantum corrections". The time after which the initially small quantum corrections become comparable with the Liouville term is [4, 6]:

\[
 t_h \simeq \frac{1}{\lambda} \ln \left( \frac{\chi \sigma_p}{\hbar} \right),
\]  

where \( \lambda \) is the Lyapunov exponent while \( \chi \) and \( \sigma_p \) are defined above. A similar estimate,

\[
 t_r \simeq \frac{1}{\lambda} \ln \left( \frac{A_0}{\hbar} \right),
\]  

was obtained earlier on the basis of a rather different argument by Berman and Zaslavsky [7]. Above, \( A_0 \) is some characteristic action which – for macroscopic systems – is presumably very large compared with the Planck constant. Moreover, typical \( A_0 \) is large (and often very large) compared with the volume in the phase space \( \chi \sigma_p \) associated with the initial conditions. Thus, \( t_r \geq t_h \) is likely to be satisfied.

By contrast, for integrable systems, analogous correspondence breakdown occurs only at:

\[
 t_h^{(\text{int})} \simeq \frac{1}{\Omega} \left( \frac{A_0}{\hbar} \right)^\alpha,
\]  

where \( \Omega \) is some characteristic volume, and \( \alpha \) is a positive constant.
where $\Omega$ is some dynamical frequency, $A_0$ is a characteristic action (that plays the role of the product $\chi\sigma_p$ in (2)) and $\alpha$ is some positive power. The difference between the behavior displayed in equations (2) and (3) on the one hand and (4) on the other is quite dramatic: Quantized counterparts of classically chaotic systems depart from classical behavior much sooner than classically integrable systems – on an uncomfortably short timescale $t_\hbar$ which increases only logarithmically with the decrease of the Planck constant.

2. For how long will Hyperion be classical?

After taking a superficial look at equations (2), (3), and (4) one may be tempted to conclude that there is no problem at all with the correspondence principle: Taking the $\hbar \to 0$ limit in both equations one obtains $t_r \to \infty$. However, this is not enough. Thus, classicality simply does not follow "as $\hbar \to 0$" in most physically interesting cases (including chaos). Planck constant is $\hbar = 1.05459 \times 10^{-27}$ [erg s] and – licentia mathematica to vary it notwithstanding – it is a constant. The right question is: "What is the value of $t_r$ (or $t\hbar$ for macroscopic quantum systems)?". And this is precisely where the true problem with the correspondence principle shows up since one easily discovers that (2) is simply too short, even for systems where classical behavior is expected and observed.

A particularly remarkable example we have found is provided by Hyperion, one of the moons of Saturn. Hyperion is a highly aspherical object whose principal radii measure $(150 \times 145 \times 114 \pm 10)$[km] (see [8]). Its irregular motion has been originally detected by monitoring changes in its luminosity and has been tracked by the recent observations carried out during the Voyager 2 mission: Hyperion is tumbling in a chaotic regime while orbiting around Saturn. The Lyapunov exponent that characterizes this chaotic motion, while not directly measured, it is believed to be of the order of two orbital periods, which are 21 days. To estimate correspondence breakdown time $t_r$ we should find out the action $A_0$ or the value of the product $\chi\sigma_p$. A generous overestimate of the $A_0$ is given by the product of Hyperion's orbital kinetic energy (which is certainly larger than the energy associated with its tumbling motion) and its 21–day period. This yields $t_r \approx 100/\lambda \approx 20$ [yrs]. Therefore, given that $t_r$ is obviously orders of magnitude less than Hyperion's age one would expect the moon to be in a very non-classical superposition, behaving
in a flagrantly quantum manner. In particular, after a time of this order the phase angle characterizing the orientation of Hyperion should become coherently spread over macroscopically distinguishable orientations – the wavefunction would be a coherent superposition over at least a radian. This is certainly not the case, Hyperion's state and its evolution seem perfectly classical. Why? The answer (which we outlined in our paper [4], as well as elsewhere [6,9]) is provided by decoherence.

3. Decoherence and Classicality

The interest in the process of decoherence did not arise in the field of quantum chaos. Its importance and the role of environment induced superselection has been first recognized in the context of quantum measurement theory [10, 11, 12, 13, 14]. As we will see, the reason why decoherence can solve the “correspondence paradox” is basically the same that makes it an essential ingredient to explain the transition from quantum to classical in other contexts.

Decoherence is the process of loss of (phase) coherence by the system caused by the interaction with the external or internal degrees of freedom which cannot be followed by the observer and are summarily called ‘the environment’. Different states in the Hilbert space of the system of interest show various degrees of susceptibility to decoherence. States which are least susceptible (i.e., take longest to decohere) form the preferred basis (also known as the pointer basis in the context of quantum measurement)[10, 12, 13, 14]. Preferred states are singled out by the interaction between the system and the environment. In this way, an environment induced superselection rules arise, which effectively outlaw arbitrary superpositions. Thus, even though the superposition principle is valid in a closed quantum system, it . . . invalidated by decoherence for systems interacting with their environments.

All of the macroscopic quantum systems we encounter in our everyday existence, as well as our own memory and information processing hardware (e.g., neurons, etc.) are macroscopic enough and sufficiently strongly coupled to the environment to be susceptible to decoherence, which will eliminate truly quantum superpositions on a very short timescale. This process is absolutely essential in the transition from quantum to classical in the context of quantum measurements (where the classical apparatus tends to be very macroscopic) although resolutions
based on decoherence may not be easily palatable to everyone (i.e., see comments on decoherence in the April 1993 issue of Physics Today and also [15]).

The timescale on which decoherence takes place can be estimated by solving a specific example: a one dimensional particle moving in a potential \( V(x) \) coupled through its position with a thermal environment – e.g. with a collection of harmonic oscillators at a temperature \( T \) [16]. Under the appropriate assumptions (Markovian regime) one can derive the following equation for the reduced Wigner function of the preferred particle:

\[
\dot{W} = \{H, W\}_M + 2\gamma \partial_p W + D \partial_{pp}^2 W \tag{5}
\]

The last two terms in this equation carry all the effects of the environment producing (respectively) relaxation and diffusion. \( D = 2m\gamma k_B T \) is the diffusion coefficient and \( \gamma \) the relaxation rate. The diffusion term is the one responsible for decoherence: Consider the Wigner function corresponding to a superposition of two localized states separated by a distance \( \Delta x \). This function is the sum of three terms, two direct contributions and an interference term. The interference term is modulated by an oscillatory function of the form \( \cos(p\Delta x / \hbar) \). Thus, when evolving under equation (5) these “interference fringes” tend to be exponentially damped by the decoherence term (which, as we mentioned, is the last one in (5) and leaves the direct terms essentially unaffected). The exponential decay of the interference takes place in a decoherence timescale [17];

\[
\tau_D = \gamma^{-1} \frac{\hbar^2}{D(\Delta x)^2} = \tau_R \left( \frac{\lambda_{dB}}{\Delta x} \right)^2 , \tag{6}
\]

where \( \lambda_{dB} = (\hbar^2 / 2mk_B T)^{1/2} \) is the thermal de Broglie wavelength and \( \tau_R = \gamma^{-1} \) is the relaxation timescale.

Two remarks are in order: (i) The decoherence timescale \( \tau_D \) is much shorter than the relaxation timescale \( \tau_R \) for all macroscopic situations, as typical thermal de Broglie wavelengths of macroscopic bodies are many orders of magnitude smaller than macroscopic separations \( \Delta x \). (ii) The devastating effect of decoherence on superpositions of position can be traced back to the preferential monitoring of that observable \( x \) by the environment, which was coupled to the position of the system of interest. This also tends to be the case in general: Interaction potentials depend on position and, therefore, allow the environment to monitor \( x \)[10, 12, 18].
As a result of the action of the decoherence term, the vast majority of states which could in principle describe the system of interest would be, in practice, eliminated by the resulting environment-induced superselection. Only localized states will be able to survive. They will form a preferred basis. For they will be much more stable than their coherent superpositions (even though they will be in general still somewhat unstable under the joint action of the self-hamiltonian and the environment). For example, in an underdamped harmonic oscillator the preferred states turn out to be the familiar coherent states [19]: Oscillator dynamics rotates all of the states, which, in effect, translates spread in position into spread in momentum (and vice versa) every quarter period of the oscillation. As a result, coupling to position can be quite faithfully represented in the “rotating wave approximation” which makes the master equation symmetric in $x$ and $p$ [20]. Hence, coherent states will minimize entropy production and are therefore selected by predictability sieve as classical [13, 14]. By contrast, for superpositions of coherent states entropy production will happen on a very much shorter decoherence timescale.

Summarizing, environment induced decoherence is a natural process that prevents the stable existence of generic quantum states which are spread over a large region of phase space. At this point, one may discover that this is precisely what we need to recover the correspondence principle for classically chaotic systems. Indeed, chaotic dynamics is especially effective in transforming a smooth initial state into a highly delocalized one with a complicated Wigner function and a lot of small scale structure. Decoherence will naturally compete against this process trying to favor smooth and localized states, or mixtures thereof. The result of this competition is a very interesting balance which enables us to recover the correspondence principle.

4. Decoherence, exponential instability and correspondence.

To understand the nature of the compromise between decoherence and exponential instability it is worth studying this process under simplifying assumptions [4]. We will be interested in the regime in which the coupling to the environment is sufficiently weak so that the damping (represented by the second term in (5)) is negligible. This is the so-called “reversible classical limit” [12, 17, 18] which in integrable systems yields reversible classical trajectories for localized (i.e. gauss-
sian) states but still eliminates non-local superpositions (this limit is achieved by letting \( \gamma \) approach zero but keeping \( D \) constant so that decoherence continues to be effective). In this limit, equation (5) can be rewritten as:

\[
\dot{W} = \{H, W\}_{MB} + D \partial_{pp}^2 W.
\] (7)

Let us consider, as we did above, an initial state which is smooth. Thus, the Wigner function initially evolves under the Poisson bracket and the diffusion term. Then, in the neighbourhood of any point, equation (5) can be easily expanded along the unstable \( (\lambda_i^+ > 0) \) and stable \( (\lambda_i^- < 0) \) directions in phase space \( (\sum_i (\lambda_i^- + \lambda_i^+) = 0) \). Diffusion will have little influence on the evolution of \( W \) along the unstable directions: \( W \) will be stretched simply as a result of the dynamics, so that the gradients along these directions will tend to decay anyway, without assistance from diffusion. By contrast, squeezing which occurs along the contracting directions will tend to be opposed by the diffusion. This will lead to a steady state with the solution asymptotically approaching a Gaussian with a half-width given by the critical dispersion:

\[
\sigma_{ci}^2 = 2D_i/|\lambda_i^-|
\] (8)

where \( \lambda_i^- \) is the (negative) Lyapunov exponent along the stable direction and \( D_i \) is the diffusion coefficient along the same direction. Below, we will assume that the diffusion is isotropic (as would be the case in the rotating wave approximation). Thus, after some time (and in the absence of folding – the other aspect of chaos which we will discuss below) the Wigner function will evolve into a multidimensional "hyper-pancake," still stretching along the unstable directions but with its width limited from below in the stable directions by equation (8).

The existence of this critical width, an important consequence of the interplay between decoherence and exponential instability, has remarkable consequences concerning the rate of entropy production. In fact, at this stage, entropy will be approximated by the logarithm of the effective volume of the hyper-pancake. As its extent in the stable direction is fixed by the critical width (8), its volume will tend to increase at a rate given by the positive exponents. Consequently,

\[
\dot{H} \approx \sum_i \lambda_i^+.
\] (9)

This constant rate will set in after a time larger than the decoherence timescale and after a time over which the initial Wigner distribution becomes squeezed by
the dynamics to the dimension of order of the critical dispersion \( \sigma_c \). Equation (9) will be valid until the pancake fills in the available phase space and the system reaches (approximately) uniform distribution over the accessible part of the phase space, that is after a time defined by:

\[
 t_{eq} = \frac{H_{eq}/H_0}{\dot{H}},
\]

where \( H_0 \) is the initial entropy, and \( H_{eq} \) is the entropy uniformized by the chaotic dynamics.

Astute reader will note that \( H_{eq} \) above need not be a true equilibrium entropy with the temperature given by \( T \). Rather, it will correspond to dynamical quasi-equilibrium — the approximately uniform distribution over this part of the phase space which (given specific initial conditions) is accessible to the chaotic system as a result of its dynamics. The corresponding timescale will have a similar dependence on \( \hbar \) as the timescale \( t_\hbar \) defined by (2). This is because entropy is approximately given by the logarithm of the volume of the phase space over which the probability distribution has spread in the units of Planck constant. Nevertheless, \( t_\hbar \) (or \( t_r \)) and \( t_{eq} \) depend on rather different aspects of the initial and final state, and one can expect the correspondence breakdown time to be typically a fraction of \( t_q \).

The existence of the critical width (8) is a property of classically chaotic systems. By contrast, in integrable systems stretching of the corresponding hyper-pancake in phase space will proceed only polynomially. Thus, even when it will get to the stage at which, in the contracting direction, diffusion will become important, stretching in the unstable direction will be only polynomial (rather than exponential). Consequently, the volume of the hyper-pancake will increase only as some power of time. Hence, the entropy will grow only logarithmically as the entropy production rate will fall as \( \dot{H} \propto 1/t \): It will take exponentially long to approach dynamical quasi-equilibrium. This difference in behavior between chaotic and integrable open quantum systems is striking and can be used as a defining feature of quantum chaos [6].

Let us now focus on the recovery of the correspondence principle. Decoherence limits the extent over which the wavefunction can remain coherent. This is because a finite minimal dispersion in momentum (8) corresponds to quantum coherence over distances no longer than:

\[
 l = \hbar/\sigma = \hbar(2D/\lambda)^{-1/2}.
\]
Thus, when the scale $\chi$ on which nonlinearities in the potential are significant is small compared to the extent of the wavefunction

$$\chi \ll l$$

(12)
decoherence will have essentially no effect. Evolution will remain purely quantum and will be generated by the full Moyal bracket.

By contrast, when the opposite is true, the evolution will never squeeze Wigner distribution function enough for the full Moyal bracket to be relevant. Poisson bracket will suffice to approximate the flow of probability in phase space. The inequality characterizing this case can be written in a manner reminiscent of the Heisenberg indeterminacy principle:

$$\hbar \ll \chi \sigma_c.$$  

(13)

That is, as long as decoherence keeps the state vector from becoming too narrow in momentum, Poisson bracket is all that is required to evolve the Wigner function. Therefore, inequality (13) defines the regime in which one recovers the correspondence principle.

There is one more interesting regime where the chaotic motion is dynamically reversible (that is, $\dot{H} = 0$) even if the system satisfies inequality (13). This happens when the initial patch in phase space is large (volume much larger than the Planck volume – initial entropy larger than a single bit) and regular. Then the initial stage of the evolution will proceed reversibly, in accord with the Poisson bracket generated flow. Decoherence will have little effect. This is because its influence will set in only as the dimension of the Wigner distribution in the contracting direction will approach the critical dispersion $\sigma_c$: In a simple example (see [4]) the entropy production will increase as:

$$\dot{H} = \lambda \frac{1}{1 + \left(\frac{\sigma_x^2(0)}{\sigma_c^2} - 1\right) \exp(-2\lambda t)}$$

(14)

So far, we have not taken into account (or, at least, not taken into account explicitly) the other major characteristic of chaos: In addition to exponential instability, chaotic systems “fold” the phase space distribution. While this problem may require further study, we believe that the fundamentals of folding are already implicit in the above discussion: Folding will happen on the scale $\chi$ of nonlinearities
in the potential (which will typically – but not always – coincide with the size of the system, as it is defined by the range of its classical trajectory). Hence, preventing the system from maintaining coherence over distances of the order of $\chi$ will also ascertain its classical behavior in course of folding. There will simply be no coherence left between the fragments of the wavepacket which will come into proximity as a result of folding, if they had to be separated by distances larger than $l$ in the course of the preceding evolution. Thus, folding will proceed as if the system was classical, but with a proviso: After sufficiently many folds the distribution function (which in the stable direction cannot shrink to less than $\sigma_c$) will simply fill in the available phase space. This will be achieved in the previously defined equilibrium timescale $t_{eq}$. These conclusions are consistent with the studies of quantum maps corresponding to open quantum systems such as the “standard map” carried out by Graham and his coworkers\[21].

5. Summary

We have argued that decoherence is the essential ingredient that enables us to solve the apparent paradox caused by the lack of validity of the correspondence principle for classically chaotic systems. Violation of correspondence principle may occur for isolated quantum systems on a rather short timescales as illustrated by Hyperion, the chaotically tumbling moon of Saturn. Decoherence or, more precisely, the continuous monitoring by the environmental degrees of freedom and the ensuing “reduction” of the quantum state of Hyperion (or any other open quantum system) – continually forces them to be classical. This process in turn leads to environment - induced superselection as a result of which only a small subset of preferred pointer states in the Hilbert space of the system are sufficiently immune to be predictable and to belong to “classical reality”.

Decoherence gurarantees the validity of the correspondence principle by precluding the growth of gradients of the Wigner function ensuring that the quantum corrections to equation (5) remain small. This process is accompanied by the increase of entropy: The information acquired by the environment is lost to the observer. We also explained why entropy production is so different for quantum open systems which are classically regular or chaotic: In the last case, the exponential instability tends to create fine structure in the Wigner function $W$ but
this process is stopped by the diffusion induced by the environment. Thus, $W$ cannot squeeze beyond the critical width $\sigma_c$ given by (8). At this point entropy starts growing linearly in time at a rate fixed by the Lyapunov exponent. This is how most of the entropy in an open chaotic system starting from a low entropy, localized ($\sim$ classical) state will be produced. Eventually, close to equilibrium the support of $W$ will fill in the phase space available to the system at the energy shell consistent with the initial conditions, and the entropy production rate will decrease to halt at $H_{eq}$. This will occur near $t_{eq} \simeq \lambda^{-1}H_{eq}/H(0)$, where $t_{eq}$ is the timescale for reaching equilibrium. By contrast, in a regular (integrable) system trajectories diverge (or become squeezed) only with a power of time. Hence, the support of $W$ in presence of diffusion will increase only as $t^n$, so that nearly all of the entropy is gained very slowly, while $\dot{H} \sim 1/t$. While we have argued for these conclusions with the help of an exactly solvable model – unstable oscillator (which is of course not chaotic, but represents well the local instability of chaotic evolution) – we believe that our conclusions concerning $\dot{H}$ will hold for $t_h < t < t_{eq}$ for chaotic systems. Indeed, we have conjectured that entropy production rate in a slightly open system may be a good “diagnostic” to distinguish between chaotic and regular quantum systems [6].

Decoherence caused by the environment (considered unsatisfactory by some authors [15]) is not a subterfuge of a theorist, but a fact of life: Macroscopic systems are exceedingly difficult to isolate from their environments for a time comparable to their dynamical timescale. Moreover, even if their energy is almost perfectly conserved, purity of their wavepacket may not be assured: As the examples studied in our paper and elsewhere indicate, the boundary between the system and the environment may be nearly impenetrable to energy, but very “leaky” for information. This imperfect isolation is, we believe, the reason why classical behavior emerges from the quantum substrate.

Sections 3-5 of this manuscript are based in part on the paper which was also presented at a meeting on Quantum Complexity in Mesoscopic Systems, and will appear in its proceedings [6].

References

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