Active Structural Control by Fuzzy Logic Rules: An Introduction

by Y. Tang

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by

Yu Tang

Reactor Engineering Division

July 1995
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ACTIVE STRUCTURAL CONTROL BY FUZZY LOGIC RULES: 
AN INTRODUCTION

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ABSTRACT

An introduction to fuzzy logic control applied to the active structural control to reduce the dynamic response of structures subjected to earthquake excitations is presented. It is hoped that this presentation will increase the attractiveness of the methodology to structural engineers in research as well as in practice. The basic concept of the fuzzy logic control are explained by examples and by diagrams with a minimum of mathematics. The effectiveness and simplicity of the fuzzy logic control is demonstrated by a numerical example in which the response of a single-degree-of-freedom system subjected to earthquake excitations is controlled by making use of the fuzzy logic controller. In the example, the fuzzy rules are first learned from the results obtained from linear control theory; then they are fine tuned to improve their performance. It is shown that the performance of fuzzy logic control surpasses that of the linear control theory. The paper shows that linear control theory provides experience for fuzzy logic control, and fuzzy logic control can provide better performance; therefore, two controllers complement each other.
I. INTRODUCTION

The active control theory applied to civil structures subjected to environmental loadings has been the topic of numerous studies in the past 20 years since the concept proposed in a paper by Yao in 1972 [1]. Both linear and nonlinear control laws have been proposed and studied [2,3,4]. A good review of recent progress and developments of this topic can be found in Refs. 5 and 6. However, despite all these developments, more research is still needed in this area before it is implemented to real structures [7]. References 7, 8 and 6 have laid out the future research directions. The paper by Housner, Soong and Masri also identified the unique requirements imposed by the civil engineering structural control and indicated that to meet some of these challenges, a different approach from the conventional control theory may be needed. In the past ten years, an unconventional approach in the field of control has emerged and is becoming very active. It is fuzzy logic control (FLC). FLC is known to be a simple but robust and fault tolerant control, easy to implement; also, FLC does not require structural modeling, and it can handle the structural nonlinear behaviors and uncertainties with great ease.

Although FLC has enjoyed its popularity in electric and mechanical engineering fields for many years, it is still relatively new for civil engineering in the active structural control area. Recently FLC has been applied to civil engineering structural control; however, papers on this topic are still rare, and most of these papers are aimed at researchers already familiar with the topic, not researchers in general; for example, Refs. 9, 10, 11, 12, and 13. The basic concept of FLC is therefore not stated clearly in these papers. So, there is a need to have a paper that presents FLC in a fundamental manner and uses simple examples to illustrate the procedure to introduce FLC to the general readers. This paper is intended to be responsive to this need.

The aim of this paper is to provide the reader with fundamental background knowledge about FLC, and hope that it will increase the attractiveness of the methodology to structural engineers. In this paper a simple example is used to demonstrate the effectiveness, robustness and simplicity of FLC.

Fuzzy logic control is based on the fuzzy set theory. A fuzzy set is a special type of set that allows its members to be described in degrees; therefore, it is very powerful in dealing with the vagueness or fuzziness of verbal ideas such as old, young, nice, etc. The name "fuzzy set" was formally introduced by Lofti Zadeh in 1965 [14]; however the concept of the fuzzy set theory can be traced back to the 1600s under another name "vagueness set" [15]. FLC has attained increased attention during the last seven years since the Japanese industry provided a few ground-breaking applications such as the Sendai subway, anti-lock brakes and auto transmission of automobiles. FLC differs drastically from the conventional control theory. For the conventional control, the system equations and performance index are required, and usually complicated mathematical skill is needed to solve the system equations and find the control law, whereas for FLC all of these are not needed. One does not need to know much math or control theory to build and tune a fuzzy system as stated in Ref. 15.

In this paper we will give a brief summary on the concept of fuzzy sets and fuzzy logic control. For more details the reader is referred to Refs. 16, 17, 18 and 19. The paper then presents an example where FLC is used to control the dynamic response of a single-degree-of-freedom system subjected to ground excitations.
II. FUZZY SET

Let U denote the symbolic name of a linguistic variable, for example, age, deformation of a structure, control force, etc., and let X be the linguistic values that U can take on, for example, X can be old, young if U denotes the age, or X can be large, medium, small if U denotes the control force. Let U be the ordinary set of all crisp values of U under consideration; for example, U may be the integers from 1 to 120 if U denotes age. The fuzzy membership function of X, denoted by m_x, is defined as a mapping whose domain is U and the range is the interval of [0, 1]. Then the fuzzy set of X, denoted by \( X \), is defined as

\[
X = \{(x, m_X(t)) | x \in U\}
\]  

(1)

The linguistic meaning of the membership function is the level of support. For example, if U denotes "age" and X denotes "old", an element (55, 0.2) of X means that the level of support for a statement that a 55-year-old person is an old person is 0.2. This concept can be depicted by a diagram which is shown in Fig. 1. In this diagram U denotes the "age" which takes two linguistic values, "not-old" and "old", i.e., X = {not old, old} and their membership functions are represented by bi-linear lines. Note that the membership function can be of any shape. Several mathematical functions have been proposed to model the membership functions: the Gaussian, triangle, trapezoidal, Z and S functions are some examples. The diagram in Fig. 1 indicates that there is 100% support for a person above 70 years of age being "old". For 55 years of age, there is a 80% support for him to be "not old", and 20% for "old". One should notice from Fig. 1 that there is a drastic difference between the ordinary sets and the fuzzy sets. For the ordinary set, the intersection of A and not-A is the empty set, whereas for the fuzzy sets, the intersection of A and not-A is usual not the empty set. This is very important since it renders the power to deal with the uncertainty in the real world. Note that the process of transforming a crisp value to elements of fuzzy sets is called fuzzification. In Fig. 1, 55 is fuzzified to elements (55, 0.8) and (55, 0.2) in fuzzy sets of "not old" and "old", respectively. 0.8 and 0.2 are called the membership degrees or weights.

III. FUZZY LOGIC CONTROLLER

The purpose of controllers is to regulate the performance of a process based on the changes in the state variables of the process. Modern control theory has been very successful in controlling systems for which a precise mathematical model can be derived. However, the modern control theory is often hand-tightened for cases where the systems are too complex or ill-defined to be precisely mathematically modeled. In 1965 Zadeh propounded the concept of fuzzy set theory that can model the vagueness in the real world [14]. Later he introduced the concept of linguistic variable whose values are words rather than numbers, and suggested that these fuzzy set theory and linguistic variables may be used for modeling and analyzing complex processes [20]. The intuitive control strategies used by the operators may be viewed as fuzzy algorithms and be described by the fuzzy sets and linguistic variables.
FLC differs substantially from conventional controller in two ways. First, FLC uses heuristic rules rather than mathematical equations to control the system. These rules are patches that cover the curve that represents the mathematical control equation that the fuzzy rules are based upon [15]; as a result, the FLC is more robust and fault tolerant. Second, FLC models the behavior of the human control operator rather than that of the system. That is, it models what the operator does and asks no questions. So FLC is modeling the human decision making process. This concept can best be understood by an analogy. A car driver controls the steering wheel without knowing the mathematical equations that model the car. FLC mimics the driver’s reaction and does not care about how to model the car. Since FLC does not involve the system modeling, there are no complicated mathematics involved. In addition, since FLC mimics the human control, it is by no means an optimal control. Then a question arises; how do we pass judgement on the effectiveness of the FLC? The answer is simple; it is judged by one’s own "satisfaction". Remember that the FLC is modeling the human behavior, and it is very unlikely that humans make optimal decisions. Humans makes decisions that satisfy themselves. For example, one buys merchandise on the basis of cost. One seldom searches all the stores in the city for the lowest price. So, the performance of a FLC depends very much on the expert used. If rules are set up to control the steering wheel based on a new student in driving school, good performance is not expected. So, the FLC may be viewed as a means of formalizing the heuristic control behavior of human operators, derived from their experience.

The basic structure of a FLC consists of three modules as depicted in Fig. 2. The fuzzification module contains the definitions of all fuzzy sets of the controlled variables. The crisp values of the controlled variables measured from the sensors are fuzzified by calculating their membership degrees against all fuzzy sets. The fuzzy inference engine contains the fuzzy rules describing the control strategies as learned from the experts. Fuzzy rules are the heart of the FLC. There is no unique way to set up these rules. If the system cannot be modeled mathematically, one can use trial-and-error, common sense, or the test results. However, if the system equations are available, the results obtained from the modern optimal control theory can serve as an expert for one to set up and then tune the rules.

A fuzzy rule (or a fuzzy association) in the single input - single output case has the following form

\[
\text{IF } X \text{ then } \Theta
\]

where \( X \) is the input and \( \Theta \) is the output fuzzy set, respectively. Collection of the fuzzy rules for a system can be represented by a matrix form which is called the Fuzzy Associative Memory (FAM) bank.

We use diagrams in Fig. 3 to show the fuzzification and inferencing procedures of a FLC. Suppose a crisp input value of \( x_1 \) is obtained from the sensor. As shown in the diagram, \( x_1 \) is fuzzified to elements of set A and set B with degrees of 0.7 and 0.3, respectively, and two rules fire subsequently. These two rules are IF A THEN \( \Theta_1 \) and IF B THEN \( \Theta_2 \), and these two rules are fired with degrees of 0.7 and 0.3, respectively. So, \( \Theta_1 \) is shrunk or clipped by 0.7 and \( \Theta_2 \) by 0.3. The final output, \( \Theta \) is taken to be the sum of two weighted individual outputs. It is given by
\[ \Theta = 0.7 \Theta_1 + 0.3 \Theta_2 \]  

(3)

This output, \( \Theta \), is still a fuzzy set which cannot be handed to the actuator. One needs to convert this fuzzy set into a crisp value that best represents the information contained in this output fuzzy set. This process is called defuzzification and is carried out in the third module of the FLC. There are several methods of defuzzification used in the literature [15,21], e.g., Center-of-Sums (COS), Middle-of-Maximum (MOM) and Height Method (HM). A graphic representation of these three methods are shown in Fig. 4.

In order to achieve satisfactory performance of the FLC, the designer is faced with the decision of selecting many parameters, which include: input and output state variables, range or universe of discourse of each variable, number of fuzzy sets for each state variable, number of fuzzy rules, method of defuzzification. So far the trial-and-error procedure is still the major approach used in determining these parameters. However, a systematic method has been attempted [22] to alleviate this trouble.

A more general FLC can be derived from the basic structure by introducing the concepts of normalized FLC as depicted in Fig. 5. In many control applications the nature of the control methodologies are identical except for the scales of the inputs and outputs. In other words, the FLCs for those control applications will be identical in the normalized domain. The only difference between those FLCs is the scaling constants, called fuzzy gains, needed to map the state variables from their time domain to the normalized domain. In the normalized domain the fuzzy sets are defined in the normalized universe of discourse in which the domain of support of the universe is [-1,1]. The crisp inputs will be scaled, or normalized, by dividing the values to the input fuzzy gains. The defuzzified crisp outputs will also be scaled back to desired magnitudes by multiplying to the output fuzzy gains. The advantages of performing FLC in the normalized domain is that the same FLC can be applied to control applications of the same nature by simply changing the fuzzy gains without recoding the program or modifying the fuzzy sets. The normalized FLC structures also facilitate the development of the adaptive fuzzy logic controllers in which the fuzzy gains can be adjusted dynamically so that the performance of the FLC satisfies, or optimize, a desired performance index. Details of the adaptive FLC is outside the scope of this paper. Interested readers are referred to Ref. 22 for further discussions.

IV. A NUMERICAL EXAMPLE

The single-degree-of-freedom (SDF) structural system used herein to illustrate the FLC is the same one demonstrated experimentally for the active seismic structural control in Ref. 23. The natural frequency and damping factor of the structure are 3.47 Hz and 0.0124, respectively. Note that, as stated above, the system properties as well as the system equation are not needed to design a fuzzy controller; however, they are needed for the numerical simulation to examine the performance of the controller. The input linguistic variables chosen for this example are "DISPLACEMENT" and "VELOCITY" which are the relative displacement and velocity of the SDF structure, and the output linguistic variable is chosen to be "CONTROL FORCE". We use the same 11 fuzzy sets for each input and output linguistic variable, and they are identified by
The membership function for each fuzzy set is assumed to have a triangular shape. The graphical representation of DISPLACEMENT, VELOCITY and CONTROL FORCE are shown in Fig. 6. Note that 0.2 inch and 4 in./sec in DISPLACEMENT and VELOCITY shown in Fig. 6 are determined from the response of the uncontrolled structure response, and 800 lb in CONTROL FORCE is taken from Fig. 8 of Ref. 23 (it could be the limitation of the actuator). Other engineers may have different opinions on how many fuzzy sets and what shape membership function should be used. It is perfectly alright to try your own design because the design shown in this example may not be better than yours. As a matter of fact, allowing for different opinions to be tried for the same system is one of the significant characteristics of the fuzzy concepts of the linguistic variables that make FLC so successful in industrial applications [24].

The fuzzy control rules (or FAM) used herein are constructed to mimic the behavior of the controller obtained by the linear control theory presented in Ref. 23 for the case of $\beta = 1$. The matrix representation of the FAM bank is shown in Table I. The flow chart of the FLC is depicted in Fig. 7.

To examine the performance of the FLC, the same input motion used in Ref. 23, which is the 0.25% of the El Centro accelerogram, is also used herein. The quantities examined include the maximum relative displacement, maximum pseudoacceleration, and the maximum control force. Three defuzzification methods, COS, MOM and HM, are used herein. The results of the simulations for FLC are summarized in Table II under the columns identified by RULE 1. The results obtained from the linear control theory presented in Ref. 23 and those for the non-control system are also listed in the table. They are identified by LCT and NO CONTROL, respectively. Examining Table II, one can see that the results of FLC are comparable to those of the LCT; the displacement and pseudoacceleration for FLC are a little less than and the control forces are a little larger than their counterparts obtained by LCT. Assume that we are not satisfied with this performance. Making use of the trial-and-error procedure, we substitute the following rules in the FAM bank shown in Table I.

\[
\begin{align*}
\text{IF DISP. is NML AND VEL. is NML THEN C. F. is NML} \\
\text{IF DISP. is NML AND VEL. is NS THEN C. F. is NL} \\
\text{IF DISP. is NS AND VEL. is NML THEN C. F. is NVL} \\
\text{IF DISP. is NS AND VEL. is NS THEN C. F. is NML} \\
\text{IF DISP. is NVS AND VEL. is PS THEN C. F. is NVS}
\end{align*}
\]
IF DISP. is PVS AND VEL. is PML THEN C. F. is NML
IF DISP. is PS AND VEL. is PML THEN C. F. is NVS
IF DISP. is PML AND VEL. is PML THEN C. F. is PS

This new FAM bank is identified as RULE 2, and the performance of RULE 2 is also listed in Table II for comparison. One can see clearly from Table II that the performance of RULE 2 is much better and surpasses that of the LCT. In the following calculation we will use RULE 2 and HM defuzzification to control the system because from Table II, it shows that the HM method requires the least control force. The time histories of the relative displacement, pseudoacceleration and the control force obtained by this combination are plotted in Figs. 8, 9 and 10, respectively. Realizing that the fuzzy rules are learned from the behavior of LCT, One should not be surprised by the resemblance between these time histories and their counterparts in Ref. 23.

Next, we would like to show the robustness of the FLC. Two additional natural frequencies of the system, 2.5 and 5.0 Hz, are examined, and the 29-second longitudinal component of the September 16, 1978 Tabas earthquake in Iran is used as another input excitation. Note that the Tabas earthquake has been scaled down to 10% of its intensity to have the comparable peak acceleration with the 25% of the El Centro record. The results of the simulations are tabulated in Tables III and IV for the changing of frequency and input excitation, respectively. Also, listed in the tables are the results obtained from the linear control theory (LCT) and those of the no control (N.C.). Examining these two tables, one can see that except for the case of \( f = 2.5 \) Hz in Table III, the performance of FLC is better than that of the LCT. Suppose, again, we do not satisfy the result for the case of \( f = 2.5 \) Hz. We can modify the FAM bank again. This time, only four rules are changed. They are

\[
\begin{align*}
\text{IF DISP. is PVL AND VEL. is NVS THEN C. F. is NL} \\
\text{IF DISP. is PVL AND VEL. is ZE THEN C. F. is NL} \\
\text{IF DISP. is PVL AND VEL. is PVS THEN C. F. is NL} \\
\text{IF DISP. is PS AND VEL. is PS THEN C. F. is NL}
\end{align*}
\]  

The new FAM bank is identified as RULE 3, and its performance is listed in Table V. One can see the improvement for \( f = 2.5 \) Hz case. Also observed in Table V, the results for the cases of \( f = 3.47 \) and 5.0 Hz are identical to those presented in Table III. This seems to indicate that the response of the different systems is controlled by different part entries of the FAM bank. One more thing worthwhile mentioning, is that for the case of \( f = 2.5 \) Hz, FLC achieves with less control force a better performance than that of the LCT.

V. CONCLUDING REMARKS

The basic concept of the FLC is introduced and demonstrated in this paper. It is shown by an example that FLC is an effective, simple and robust nonlinear control. The results of the example also show that the performance of FLC is better than that of LCT from which the fuzzy rules are learned. This is not a surprise. It has been argued that FLC always has a better performance than that of the expert whose knowledge it used [25,26].
Since fuzzy logic control mimics the human operator behavior in control not the system, the uncertainty in the system modeling, the time-delay of the control therefore pose no difficulty for FLC. The performance of a fuzzy controller is judged by the human satisfaction; therefore, FLC does not "look for" an optimal solution. To design a FLC one is faced with the decision of making the number of input and output fuzzy sets and their degrees of overlapping, membership functions and the defuzzification method, and the decisions are made based on the your own satisfaction. The fuzzy rules can be set up by inquiring the experience from the results obtained by the conventional control theory. Therefore, FLC and the convention controls can complement each other.

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The author wishes to thank Dr. Y. W. Chang of Argonne National Laboratory and Dr. C. K. Wu of the Mechanical and Industrial Engineering Department of the University of Texas, El Paso for helpful discussions during the performance of this work. This work was performed in the Engineering Mechanics Program of the Reactor Engineering Division of Argonne National Laboratory under the auspices of the U.S. Department of Energy under contract W-31-109-Eng-38.

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Table I. Fuzzy Associative Memory Bank

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<td>PL</td>
</tr>
</tbody>
</table>

VELOCITY
Table II. Performance Comparison with Different Defuzzification Methods and FAM Rules
\( f = 3.47 \text{ Hz, } \zeta = 0.0124, \text{ El Centro Record} \)

<table>
<thead>
<tr>
<th></th>
<th>CoS</th>
<th>MoM</th>
<th>HM</th>
<th>LCT</th>
<th>No Control</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rule 1</td>
<td>Rule 2</td>
<td>Rule 1</td>
<td>Rule 2</td>
<td>Rule 1</td>
</tr>
<tr>
<td>Maximum Relative Displacement, in.</td>
<td>0.105 0.056</td>
<td>0.105 0.061</td>
<td>0.103 0.062</td>
<td></td>
<td>0.109</td>
</tr>
<tr>
<td>Maximum Pseudoacceleration in g's</td>
<td>0.129 0.069</td>
<td>0.129 0.075</td>
<td>0.127 0.076</td>
<td></td>
<td>0.134</td>
</tr>
<tr>
<td>Control Force, lb</td>
<td>724 738</td>
<td>800 800</td>
<td>722 592</td>
<td>682</td>
<td>0</td>
</tr>
</tbody>
</table>
Table III. Performance Comparison of Different Control Laws for Changing of Natural Frequency, $\zeta = 0.0124$, El Centro Record

<table>
<thead>
<tr>
<th></th>
<th>$f = 2.5$ Hz</th>
<th></th>
<th>$f = 5.0$ Hz</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FLC</td>
<td>LCT</td>
<td>NO</td>
<td>FLC</td>
</tr>
<tr>
<td>Maximum Relative Displacement, in.</td>
<td>0.177</td>
<td>0.152</td>
<td>0.363</td>
<td>0.035</td>
</tr>
<tr>
<td>Pseudoacceleration in g's</td>
<td>0.113</td>
<td>0.097</td>
<td>0.232</td>
<td>0.0898</td>
</tr>
<tr>
<td>Control Force, lb</td>
<td>676</td>
<td>728</td>
<td>0</td>
<td>467</td>
</tr>
</tbody>
</table>
Table IV. Performance Comparison of Control Laws for Tabas Record

<table>
<thead>
<tr>
<th></th>
<th>f = 2.5 Hz</th>
<th></th>
<th>f = 3.47 Hz</th>
<th></th>
<th>f = 5.0 Hz</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FLC</td>
<td>LCT</td>
<td>NO</td>
<td>FLC</td>
<td>LCT</td>
<td>NO</td>
</tr>
<tr>
<td>Maximum Relative Displacement, in.</td>
<td>0.140</td>
<td>0.169</td>
<td>0.383</td>
<td>0.067</td>
<td>0.081</td>
<td>0.225</td>
</tr>
<tr>
<td>Maximum Pseudoacceleration in g's</td>
<td>0.089</td>
<td>0.108</td>
<td>0.244</td>
<td>0.083</td>
<td>0.099</td>
<td>0.277</td>
</tr>
<tr>
<td>Control Force, lb</td>
<td>792</td>
<td>750</td>
<td>0</td>
<td>502</td>
<td>457</td>
<td>0</td>
</tr>
</tbody>
</table>
Table V. Performance of Rule 3, El Centro Record

<table>
<thead>
<tr>
<th></th>
<th>$f = 2.5$ Hz</th>
<th>$f = 3.47$ Hz</th>
<th>$f = 5.0$ Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Relative Displacement, in.</td>
<td>0.136</td>
<td>0.062</td>
<td>0.035</td>
</tr>
<tr>
<td>Maximum Pseudoacceleration in g's</td>
<td>0.0866</td>
<td>0.076</td>
<td>0.089</td>
</tr>
<tr>
<td>Control Force, lb</td>
<td>596</td>
<td>592</td>
<td>467</td>
</tr>
</tbody>
</table>
Fig. 1. Fuzzy Sets
Fig. 2. Basic Structures of a FLC
IF A THEN $\theta_1$

IF B THEN $\theta_2$

(c) OUTPUT, $\theta = 0.7 \theta_1 + 0.3 \theta_2$

Fig. 3. Fuzzy Logic Control Procedure
(a) CENTER OF SUM (COS)

\[ \theta = \text{center of the area} \]

(b) MIDDLE OF MAXIMUM (MOM)

\[ \theta = \frac{\text{left max.} + \text{right max.}}{2} \]

(c) HEIGHT METHOD (HM)

\[ \theta = \frac{h_1 \theta_1 + h_2 \theta_2}{h_1 + h_2} \]

Fig. 4. Defuzzification Methods
Fig. 5. Normalized FLC
Fig. 6. Fuzzy Sets for Inputs and Output of the Example Problem
Fig. 7. Flow Chart of FLC for the Example Problem
Fig. 8. Time History of the Relative Displacement of the Example Problem
Pseudoacceleration

TMAX, AMAX, Tmin, AMIN = 2.58 0.2086 2.44 -0.1725

(a) Without Control

Pseudoacceleration

TMAX, AMAX, Tmin, AMIN = 1.86 0.0762 2.14 -0.0692

(b) With Control

Fig. 9. Time History of Pseudoacceleration of the Example Problem
Fig. 10. Time History of Control Force for the Example Problem