REPORT No. 776

THE THEORY OF PROPELLERS. II—METHOD FOR CALCULATING THE AXIAL INTERFERENCE VELOCITY

By Theodore Theodorsen

SUMMARY

A technical method is given for calculating the axial interference velocity of a propeller. The method involves the use of certain weight functions P, Q, and F. Numerical values for the weight functions are given for two-blade, three-blade, and six-blade propellers.

INTRODUCTION

It has formerly been the practice to use the Glauert-Lock simplified assumption that the interference velocity is proportional to the loading at the point considered. This assumption is obviously inadequate since the interference flow depends on the slope and curvature of the loading function as well as on the local magnitude. A method is developed herein for calculating the axial interference flow for any loading. The method is accurate to the first order and therefore gives the interference flow in ratio to the loading for small loadings. It can be shown that this accuracy is adequate for all technical applications.

The present paper is the second in a series on the theory of propellers. Part I deals with a method for obtaining the circulation function for dual-rotating propellers. (See reference 1.)

SYMBOLS

v_1	axial interference velocity at x_1 $[v_x(x_1)]$
w	rearward displacement velocity of helical vortex surface (at infinity)
V	advance velocity of propeller
p	number of blades
n	order number of blade $(0 \leqq n \leqq p-1)$
ω	angular velocity of propeller
Γ	circulation at radius x
K	circulation coefficient to first order $\left(\frac{p\Gamma\omega}{2\pi Vw}\right)$
x	nondimensional radius in terms of tip radius
x_1	reference point at which interference velocity is calculated
θ	angular distance of vortex element from propeller
λ	advance ratio $(V/\omega R)$
R	tip radius of propeller
$P_1(x)$	function defined in equation (1)
$Q_1(x)$	function defined in equation (3)
P	used for $P_1(x)$ in tables and figures; refers to other blades $(n \neq 0)$
Q	used for $Q_1(x)$ in tables and figures; refers to blade itself $(n=0)$
r	phase angle of nth blade $\left(\frac{2n\pi}{p}\right)$
φ_1	helix angle at x_1 $\left(\tan^{-1}\frac{\lambda}{x_1}\right)$

WEIGHT FUNCTION $P_1(x)$

It can be shown that the axial interference flow is given by the expression

$$\frac{v_1}{\frac{1}{2}w}=\frac{1}{p}\sum_n\int\frac{dK}{dx}x\frac{dP_1(x)}{dx}dx$$

where the summation is over the number of blades 0 to $p-1$. The important function $P_1(x)$ is defined as

$$P_1(x)=\int_0^\infty\frac{d\frac{\theta}{2\pi}}{\sqrt{\frac{1}{4\pi^2\lambda^2}\left[x^2+x_1^2-2xx_1\cos\left(\theta+\frac{2n\pi}{p}\right)\right]+\left(\frac{\theta}{2\pi}\right)^2}}$$

where $n=0, 1, 2, \ldots p-1$, the number of the particular blade. The problem is thus essentially solved by giving the function $P_1(x)$ for each point along the radius.

It is convenient to make $P_1(x)$ finite by subtracting a quantity that is independent of x. The function $P_1(x)$ may therefore be redefined as

$$P_1(x)=\int_0^\infty\left\{\frac{1}{\sqrt{\frac{1}{4\pi^2\lambda^2}\left[x^2+x_1^2-2xx_1\cos\left(\theta+\frac{2n\pi}{p}\right)\right]+\left(\frac{\theta}{2\pi}\right)^2}}-\frac{1}{\sqrt{1+\left(\frac{\theta}{2\pi}\right)^2}}\right\}d\frac{\theta}{2\pi} \tag{1}$$

It is noticed that, in the integral $P_1(x)$, the integrand changes from $+\infty$ to $-\infty$ at $x=x_1$ for $\theta=0$. This difficulty, which occurs only for $n=0$ (that is, for the blade itself), is overcome in the following manner: The expression

$$\int_0^\infty\left[\frac{1}{\sqrt{\frac{1}{4\pi^2\lambda^2}(x-x_1)^2+\left(\frac{xx_1}{\lambda^2}+1\right)\left(\frac{\theta}{2\pi}\right)^2}}-\frac{1}{\sqrt{\frac{1}{4\pi^2\lambda^2}+\left(\frac{xx_1}{\lambda^2}+1\right)\left(\frac{\theta}{2\pi}\right)^2}}\right]d\frac{\theta}{2\pi} \tag{2}$$

which is integrable and equal to

$$-\sqrt{\frac{\lambda^2}{\lambda^2+xx_1}}\log|x-x_1|$$

may be subtracted from $P_1(x)$ to yield a finite and smooth integrand. Thus, by subtraction, a quantity

$$Q_1(x)=\int_0^\infty\left[\frac{1}{\sqrt{\frac{1}{4\pi^2\lambda^2}(x^2+x_1^2-2xx_1\cos\theta)+\left(\frac{\theta}{2\pi}\right)^2}}-\frac{1}{\sqrt{1+\left(\frac{\theta}{2\pi}\right)^2}}-\frac{1}{\sqrt{\frac{1}{4\pi^2\lambda^2}(x-x_1)^2+\left(\frac{xx_1}{\lambda^2}+1\right)\left(\frac{\theta}{2\pi}\right)^2}}+\frac{1}{\sqrt{\frac{1}{4\pi^2\lambda^2}+\left(\frac{xx_1}{\lambda^2}+1\right)\left(\frac{\theta}{2\pi}\right)^2}}\right]d\frac{\theta}{2\pi} \qquad (3)$$

is obtained. Finally, for the blade itself ($n=0$),

$$P_1(x)=Q_1(x)+F$$

where

$$F=-\sqrt{\frac{\lambda^2}{\lambda^2+xx_1}}\log|x-x_1|$$

The integral $Q_1(x)$ is convenient for graphical integration and is, in fact, small in comparison with the function F.

No discontinuities arise in the P functions for the other blades ($n\neq 0$). The P functions are therefore used directly in the calculation for the other blades. It should be noted that the functions P, Q, and F are all symmetrical in x and x_1. The use of the subscript, which has been used to indicate reference to the point x_1, is therefore discontinued. In the following discussion, the functions Q and F refer to the blade itself and P refers to the other blades.

Since the weight function is needed in the form $x\frac{dP}{dx}$, it is written as

$$x\frac{dP}{dx}=x\frac{dQ}{dx}+x\frac{dF}{dx}$$

It is to be noted that by far the largest contribution comes from the logarithmic function F since it really represents the entire field in the neighborhood of the point considered. In developed form,

$$x\frac{dF}{dx}=-\frac{1}{\sqrt{1+\frac{xx_1}{\lambda^2}}}\frac{x}{x-x_1}+\frac{1}{2}\frac{\frac{xx_1}{\lambda^2}}{\sqrt{\left(1+\frac{xx_1}{\lambda^2}\right)^3}}\log|x-x_1| \qquad (4)$$

NUMERICAL EVALUATION OF WEIGHT FUNCTIONS Q, F, AND P

The weight functions Q, F, and P are shown in a series of tables and figures. The first step of integrating against the angle θ is omitted for simplicity. The functions $\frac{dQ}{dx}$ and $\frac{dP}{dx}$ have been obtained by graphical differentiation of the Q and P functions with actual calculation at the end points $x=0$ and 1 for accuracy. It should be noted that these functions and their derivatives are continuous and smooth. The results are given in the following order:

(1) Table I and figure 1: Q against x ($0\leqq x\leqq 1.00$; $0.1564\leqq x_1\leqq 1.00$; $\lambda=\frac{1}{2}$, 1, and 2), obtained from equation (3)

(2) Table II and figure 2: $\frac{dQ}{dx}$ against x ($0\leqq x\leqq 1.00$; $0.1564\leqq x_1\leqq 1.00$; $\lambda=\frac{1}{2}$, 1, and 2), where $\frac{dQ}{dx}$ is obtained by graphical differentiation of Q except for $x=0$ and 1, for which $\frac{dQ}{dx}$ is obtained analytically

(3) Table III and figure 3: $-x\frac{dQ}{dx}$ against x ($0\leqq x\leqq 1.00$; $0.1564\leqq x_1\leqq 1.00$; $\lambda=\frac{1}{2}$, 1, and 2), obtained by multiplying values in table II by $-x$

(4) Table IV: $x\frac{dF}{dx}$ against x ($0\leqq x\leqq 1.00$; $0\leqq x_1\leqq 1.00$; $\lambda=\frac{1}{2}$, 1, and 2), obtained from equation (4)

(5a) Table V: P against x for $\tau=60°$ ($0\leqq x\leqq 1.00$; $0.1564\leqq x_1\leqq 1.00$; $\lambda=\frac{1}{2}$, 1, and 2), obtained from equation (1)

(5b) Figure 4: $P(x)-P(1)$ against x for $\tau=60°$ ($0\leqq x\leqq 1.00$; $0.1564\leqq x_1\leqq 1.00$; $\lambda=\frac{1}{2}$, 1, and 2)

(6a) Table VI: same as table V for $\tau=120°$

(6b) Figure 5: same as figure 4 for $\tau=120°$

(7a) Table VII: same as table V for $\tau=180°$

(7b) Figure 6: same as figure 4 for $\tau=180°$

(8a) Table VIII: same as table V for $\tau=240°$

(8b) Figure 7: same as figure 4 for $\tau=240°$

(9a) Table IX: same as table V for $\tau=300°$

(9b) Figure 8: same as figure 4 for $\tau=300°$

(10) Table X: $\frac{dP}{dx}$ against τ for $\lambda=\frac{1}{2}$ ($\tau=60°$, 120°, 180°, 240°, and 300°; $x=0$ and 1.00; $0.1564\leqq x_1\leqq 1.00$), obtained analytically

(11) Table XI: same as table X for $\lambda=1$

(12) Table XII: same as table X for $\lambda=2$

(13) Table XIII and figure 9: $-x\frac{dP}{dx}$ against x for $\lambda=\frac{1}{2}$ ($\tau=60°$, 120°, 180°, 240°, and 300°; $0.1564\leqq x\leqq 1.00$; $0.1564\leqq x_1\leqq 1.00$), obtained by multiplying values in table X by $-x$

(14) Table XIV and figure 10: same as table XIII and figure 9 for $\lambda=1$

(15) Table XV and figure 11: same as table XIII and figure 9 for $\lambda=2$

(16) Table XVI and figure 12: $\sum -x\frac{dP}{dx}$ against x for three-blade and six-blade propellers ($\tau=120°$ and 240° for three-blade propeller; $\tau=60°$, 120°, 180°, 240°, and 300° for six-blade propeller; $0.1564\leqq x\leqq 1.00$; $0\leqq x_1\leqq 1.00$; $\lambda=\frac{1}{2}$, 1, and 2); it may be noted that these values for two-blade propellers are given by $-x\frac{dP}{dx}$ for $\tau=180°$ in tables XIII to XV and in figures 9 to 11

APPLICATION OF METHOD

Steps to obtain the induced velocity expressed as $\frac{v_1}{\frac{1}{2}w}$ are as follows:

(1) Plot the quantity $x\frac{dQ}{dx}$ against the circulation coefficient K and perform graphically the integration

$$\int x\frac{dQ}{dx}\,dK$$

(2a) Plot similarly the functions $x\frac{dF}{dx}$ against K and perform the integration

$$\int x\frac{dF}{dx}dK$$

Since $x\frac{dF}{dx}$ becomes infinite at $x=x_1$, it is necessary to exclude a gap from $x_1-\frac{1}{2}\Delta x$ to $x_1+\frac{1}{2}\Delta x$ and to consider this gap separately by use of a Taylor expansion.

(2b) The contribution from the gap Δx becomes

$$\Delta=-b\left[x_1K''+\left(1-\frac{1}{2}c\log\frac{\Delta x}{2}\right)K'\right]\Delta x$$

where

$$\Delta x=2|x-x_1|$$

$$b=\frac{\lambda}{\sqrt{\lambda^2+x_1^2}}=\sin\phi_1$$

$$c=\frac{x_1^2}{\lambda^2+x_1^2}=\cos^2\phi_1$$

and K' and K'' are the derivatives of K with respect to x.

(3) Finally, there is a contribution from the other blades. This contribution is obtained by plotting $x\frac{dP}{dx}$ against K for the other blades. Since the value $\sum -x\frac{dP}{dx}$ can be taken directly from the tables, this work contains only one step with a single graphical integration

$$\int \sum x\frac{dP}{dx}\,dK$$

By addition of the results of steps (1) to (3), the total interference velocity v_1 in the axial direction is obtained. The relationship between the axial interference velocity v_1 at the radius x_1 to the axial displacement velocity w of the vortex sheet may be seen from the sketch in figure 13. The relation is

$$v_1=\frac{1}{2}w\cos^2\phi_1$$

or, conversely, the displacement velocity w of the vortex sheet may be obtained from the calculated axial interference velocity v_1 by the relation

$$\frac{1}{2}w=\frac{v_1}{\cos^2\phi_1}$$

which gives the axial displacement velocity at the propeller disk. For the case of the ideal loading this axial displacement velocity must come out as a constant, thus permitting a check on the weight functions. Cases of nonideal loading are evidently of more practical concern. It is the purpose of this paper to give a method for calculation of the axial interference and displacement velocity for any (light) loading.

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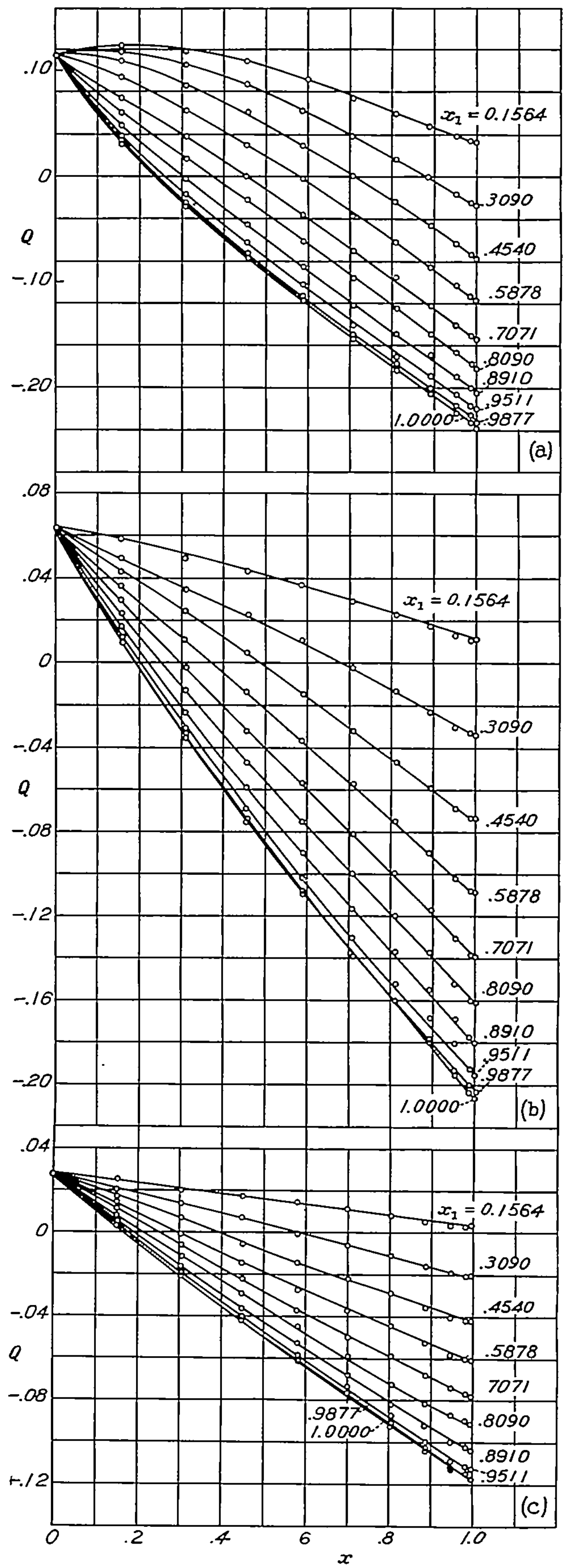
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *September 19, 1944*.

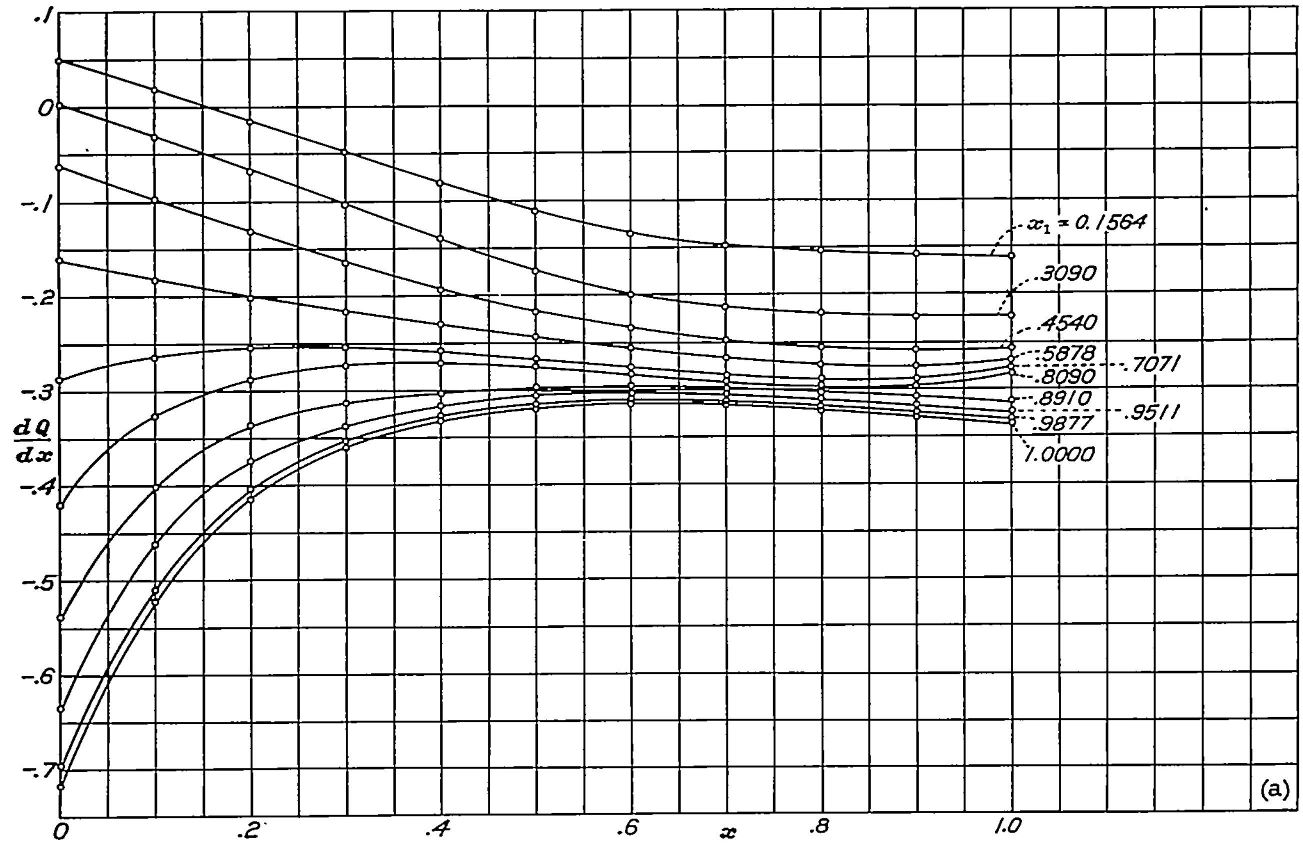
REFERENCE

1. Theodorsen, Theodore: The Theory of Propellers. I—Determination of the Circulation Function and the Mass Coefficient for Dual-Rotating Propellers. NACA Rep. No. 775 1944.

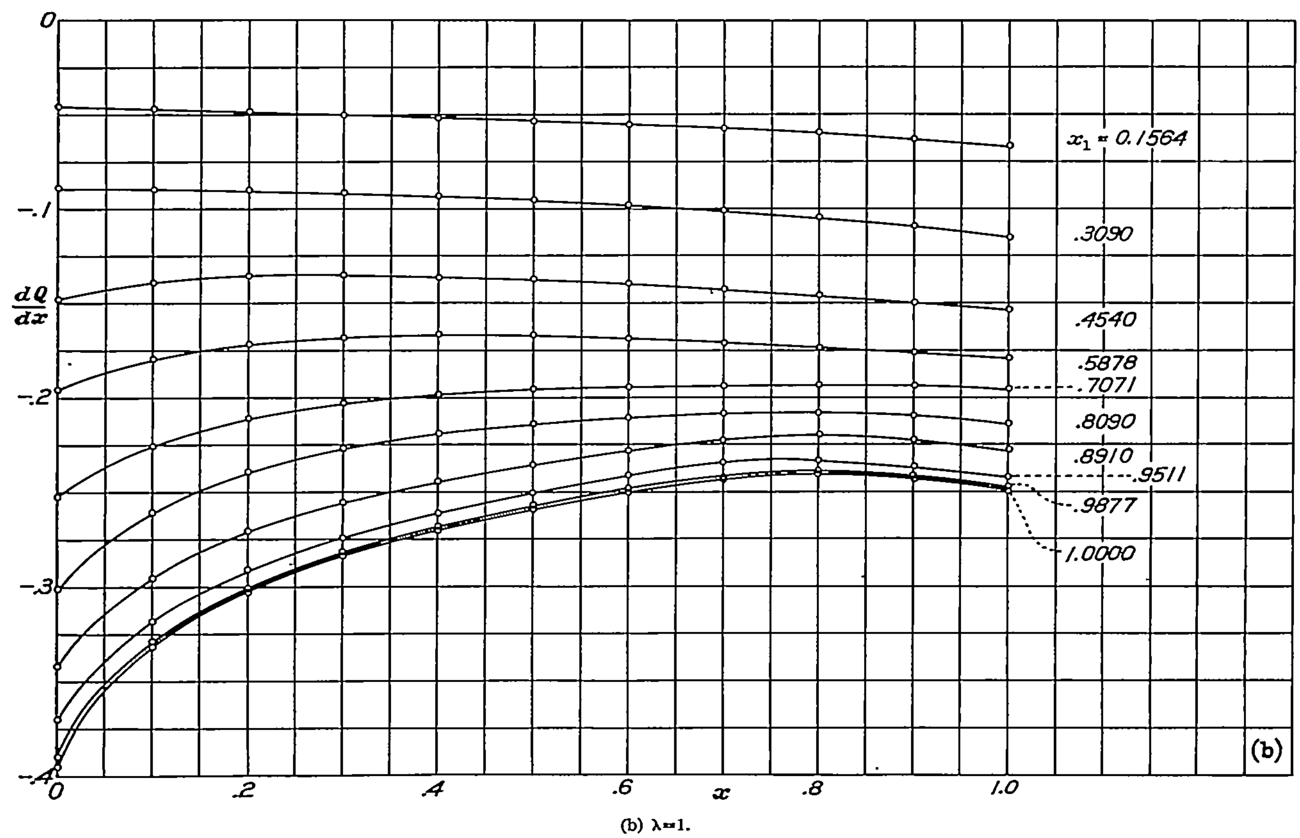


(a) $\lambda=\frac{1}{2}$. (b) $\lambda=1$. (c) $\lambda=2$.

FIGURE 1.—Function Q against x.

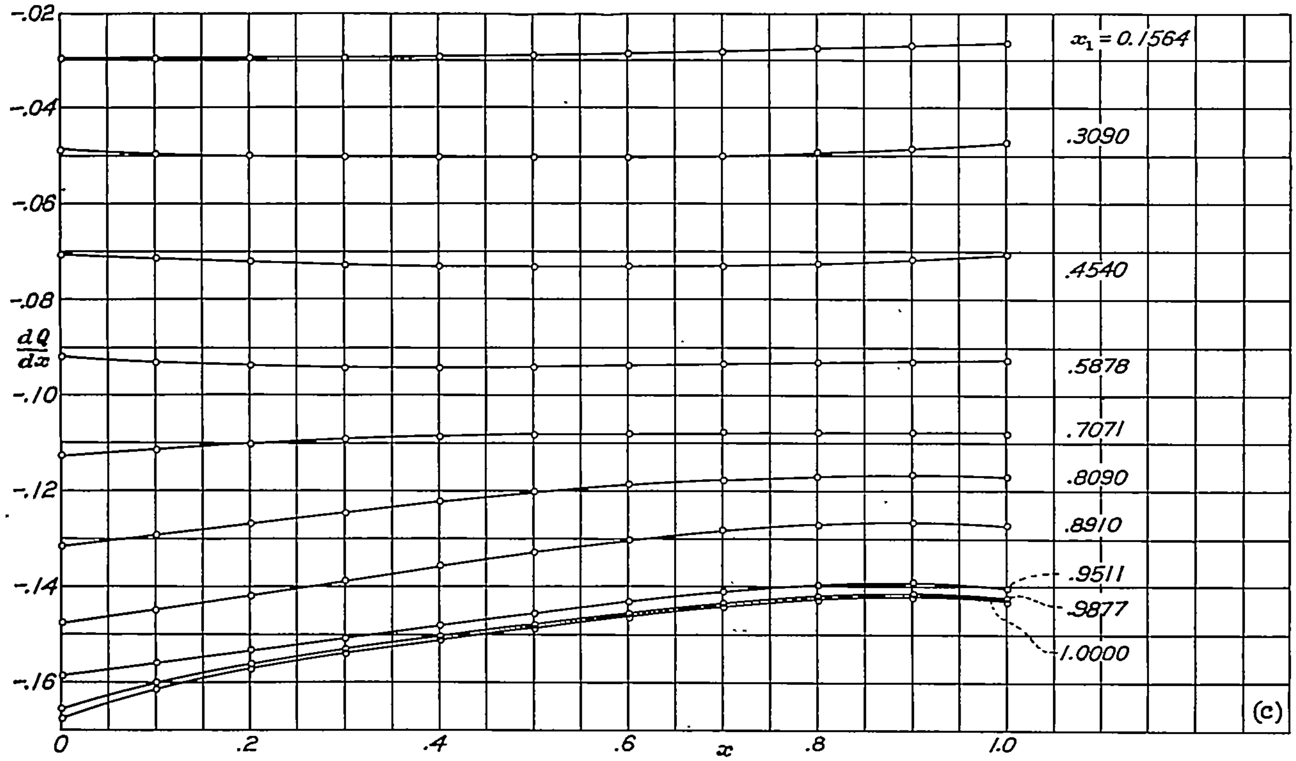


(a) $\lambda=\frac{1}{2}$.



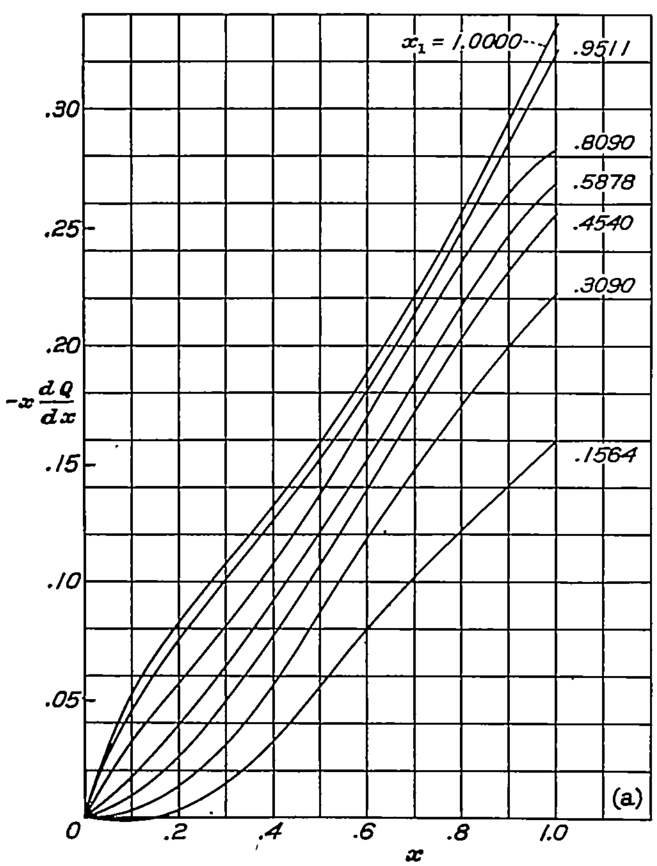
(b) $\lambda=1$.

FIGURE 2.—Function dQ/dx against x.



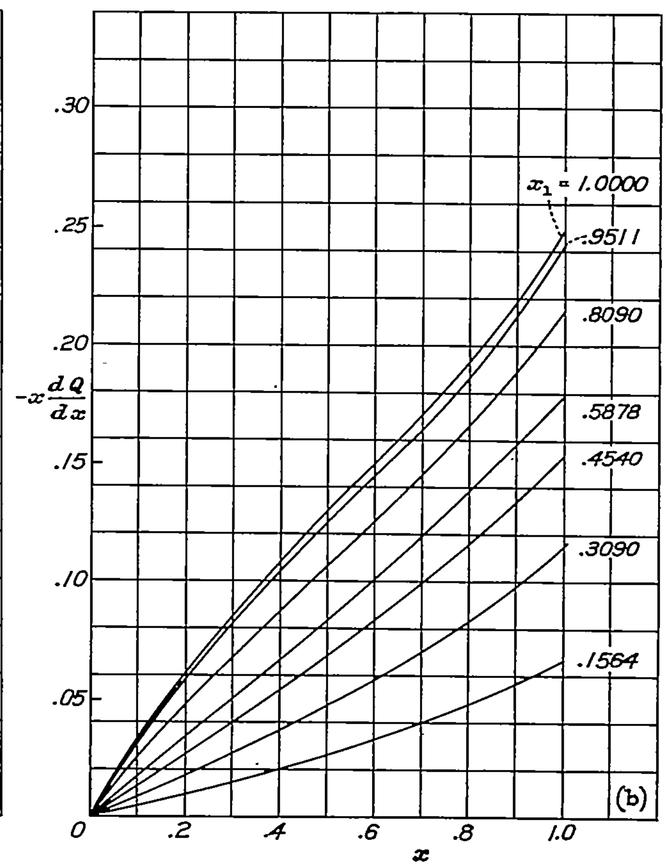
(c) λ=2.

FIGURE 2.—Concluded.



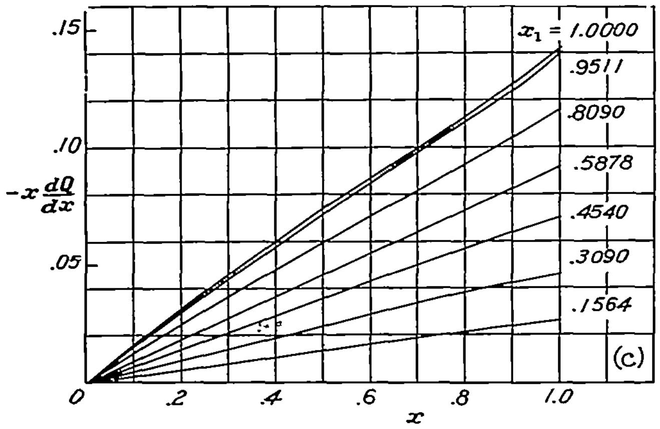
(a) $\lambda=\frac{1}{2}$.

FIGURE 3.—Function $-x\frac{dQ}{dx}$ against x.

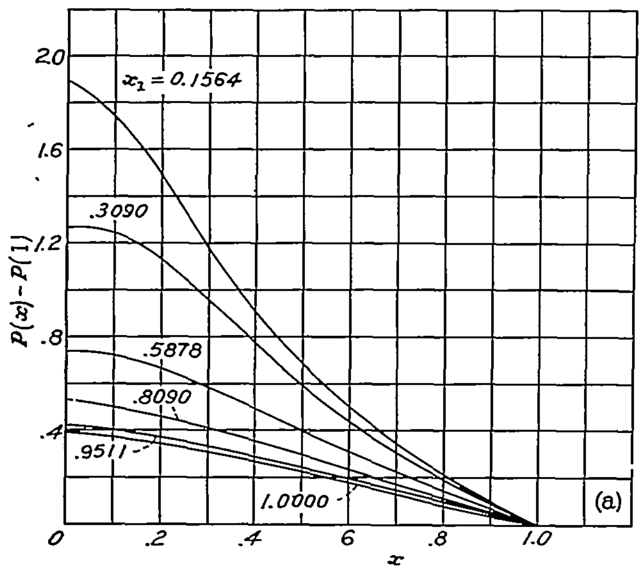


(b) λ=1.

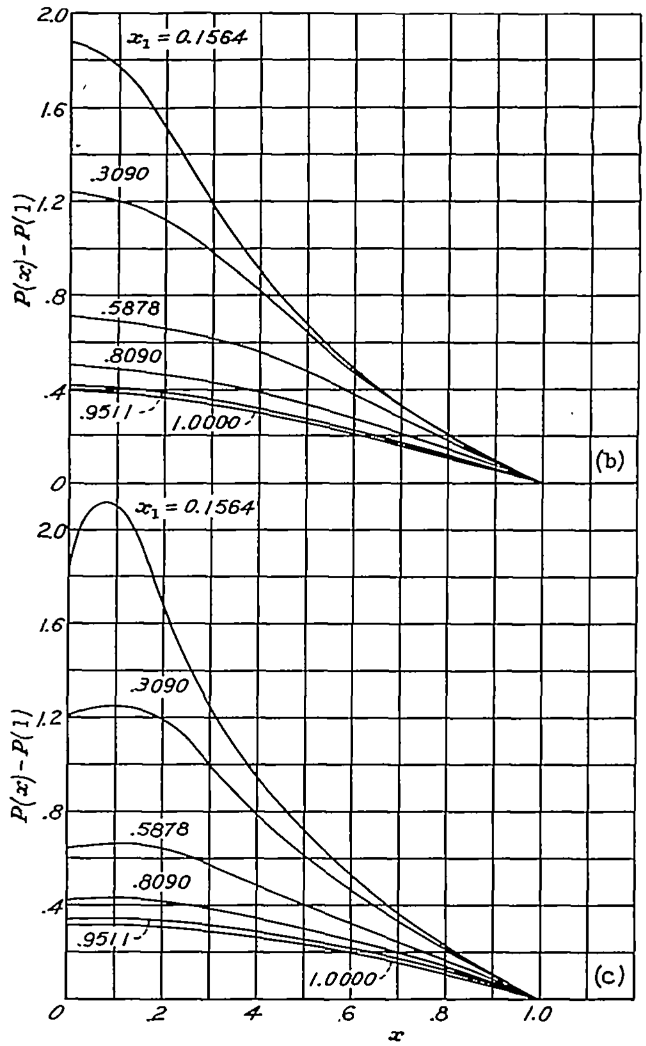
FIGURE 3.—Continued.



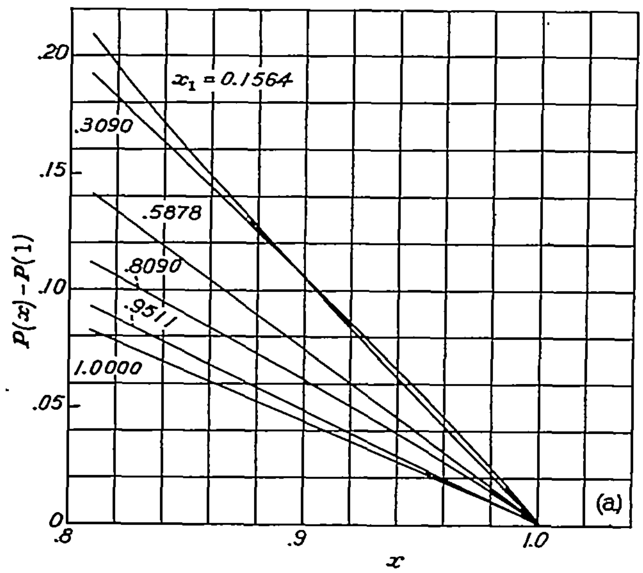
(c) $\lambda=2$.
FIGURE 3.—Concluded;



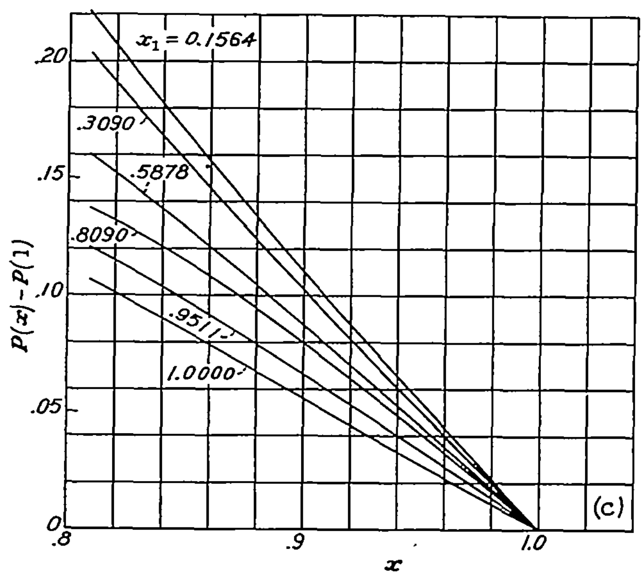
(a) $\lambda=\frac{1}{2}$.
FIGURE 4.—$P(x)-P(1)$ against x for $\tau=60°$.



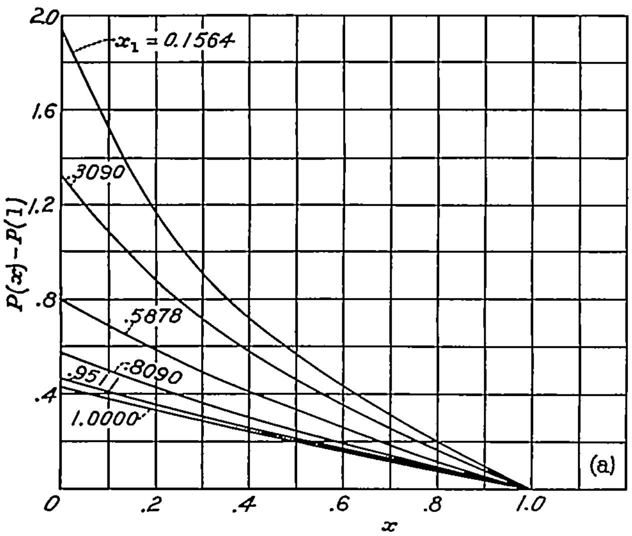
(b) $\lambda=1$. (c) $\lambda=2$.
FIGURE 4.—Continued.



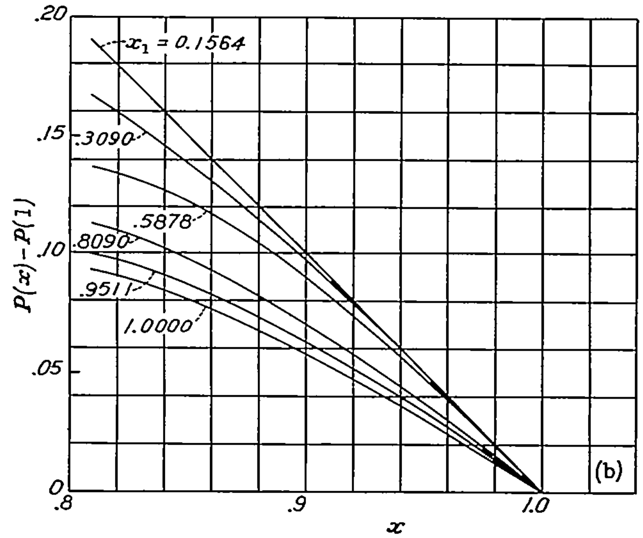
(a) $\lambda=\frac{1}{2}$—Concluded.
FIGURE 4.—Continued.



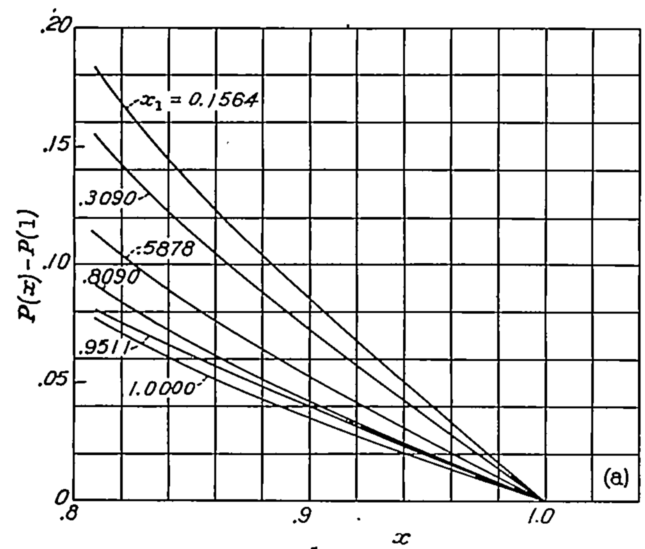
(c) $\lambda=2$—Concluded.
FIGURE 4.—Concluded.



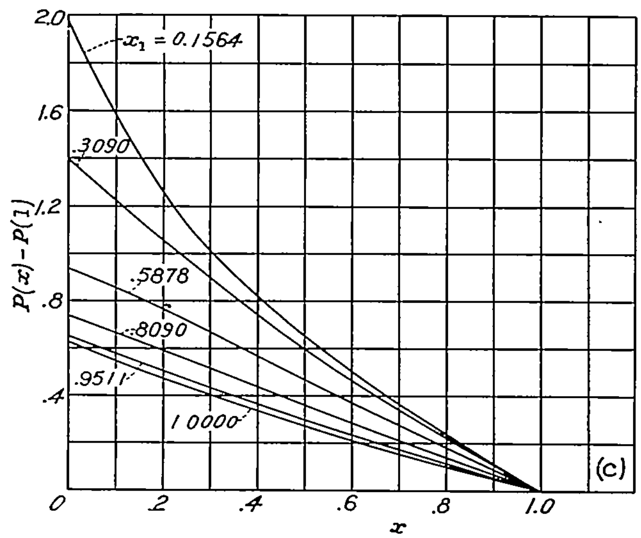
(a) $\lambda=\frac{1}{2}$.
FIGURE 5.—$P(x)-P(1)$ against x for $\tau=120°$.



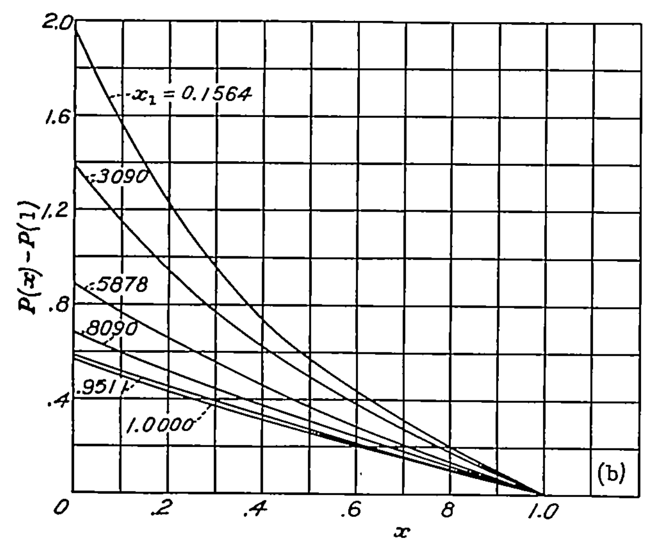
(b) $\lambda=1$—Concluded.
FIGURE 5.—Continued.



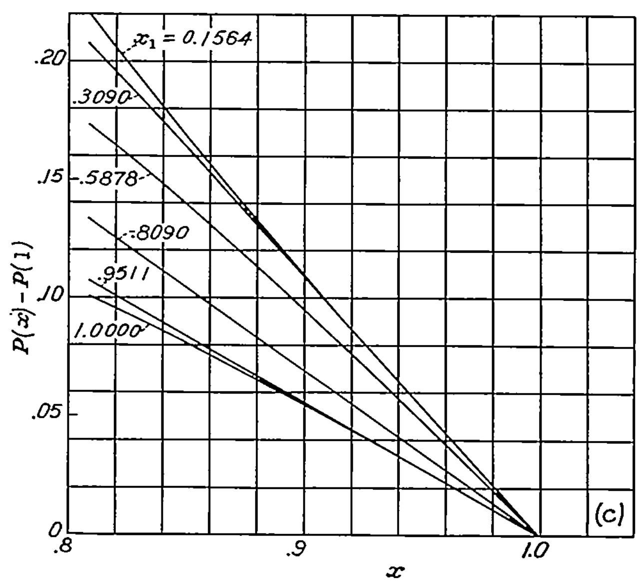
(a) $\lambda=\frac{1}{2}$—Concluded.
FIGURE 5.—Continued.



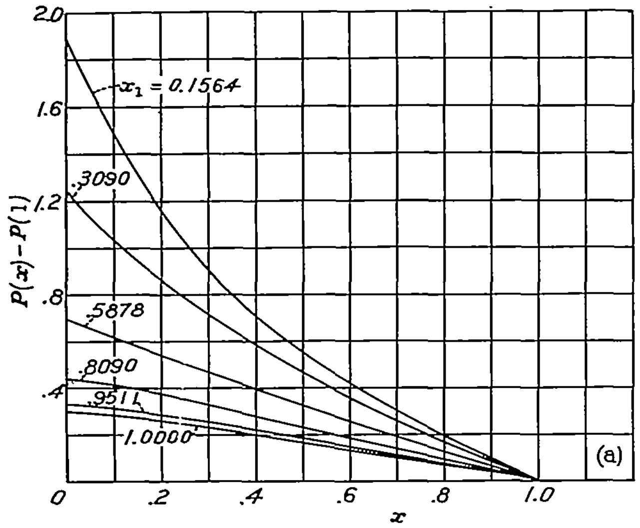
(c) $\lambda=2$.
FIGURE 5.—Continued.



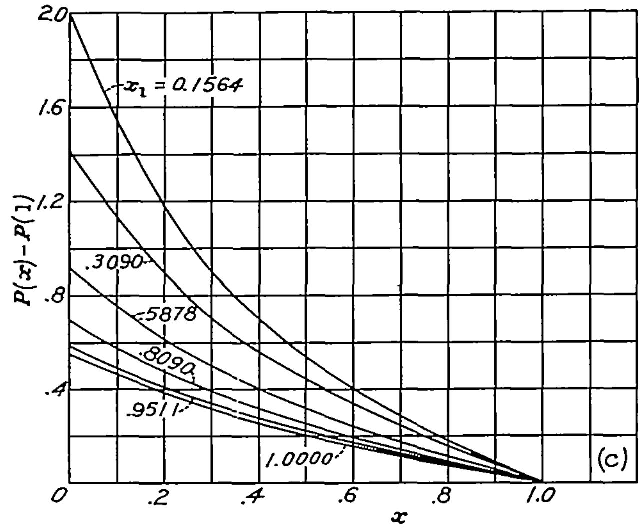
(b) $\lambda=1$.
FIGURE 5.—Continued.



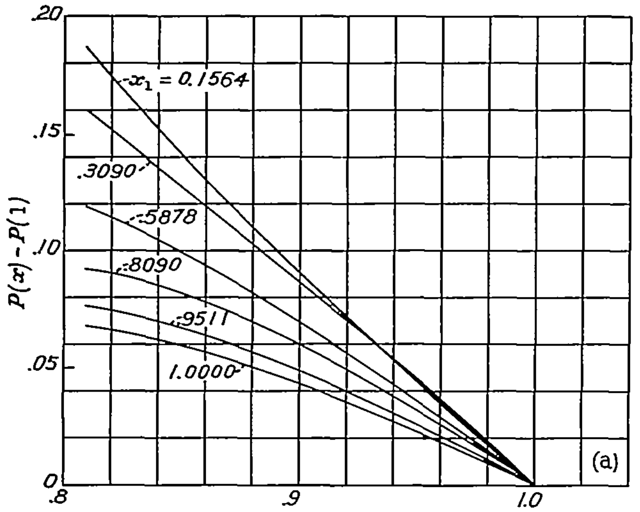
(c) $\lambda=2$—Concluded.
FIGURE 5.—Concluded.



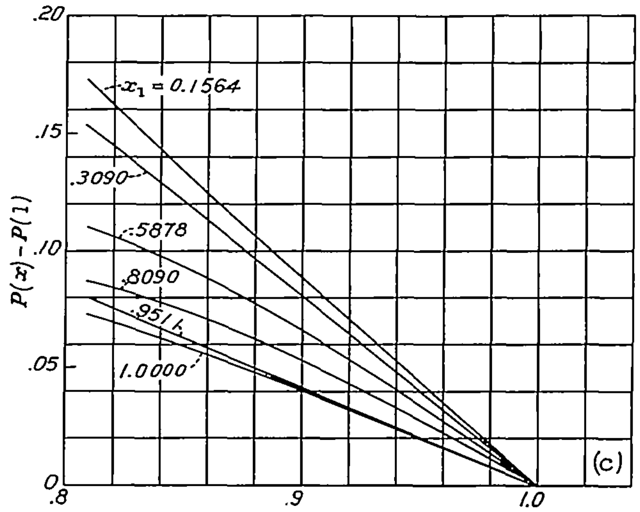
(a) $\lambda=\frac{1}{2}$.
FIGURE 6.—$P(x)-P(1)$ against x for $\tau=180°$.



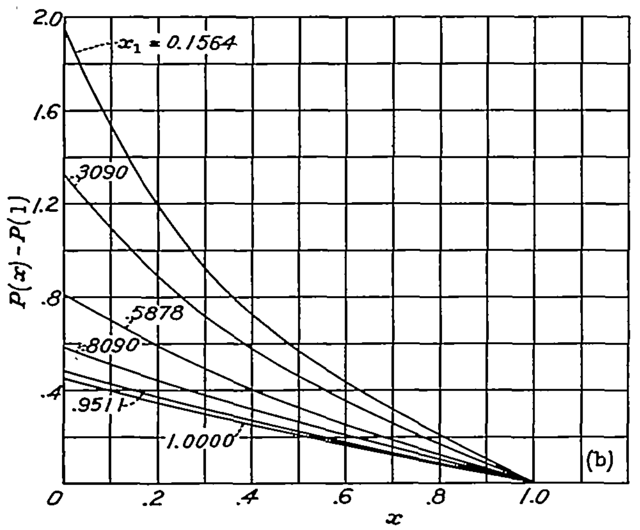
(c) $\lambda=2$.
FIGURE 6.—Continued.



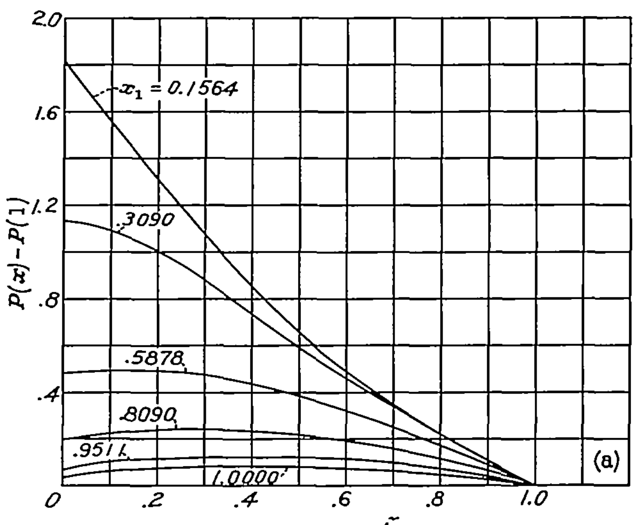
(a) $\lambda=\frac{1}{2}$—Concluded.
FIGURE 6.—Continued.



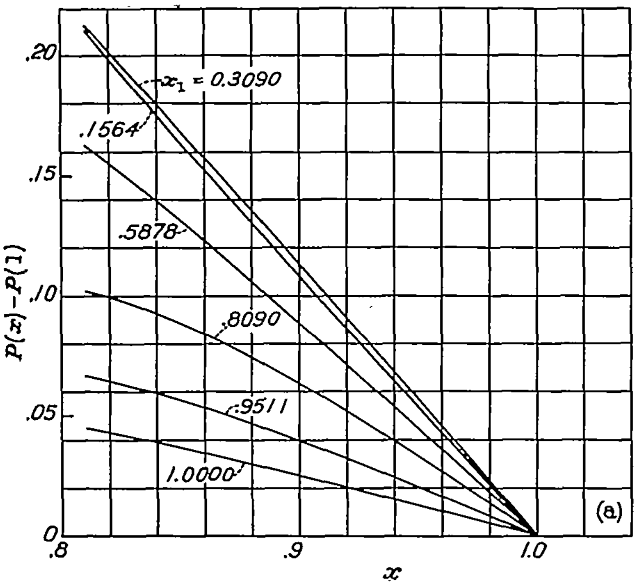
(c) $\lambda=2$—Concluded.
FIGURE 6.—Concluded.



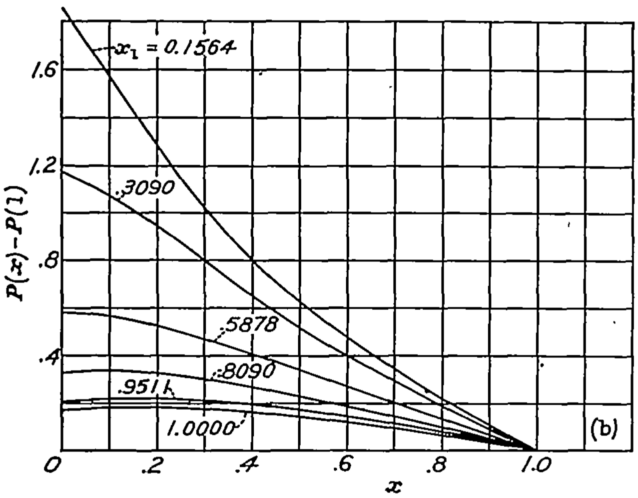
(b) $\lambda=1$.
FIGURE 6.—Continued.



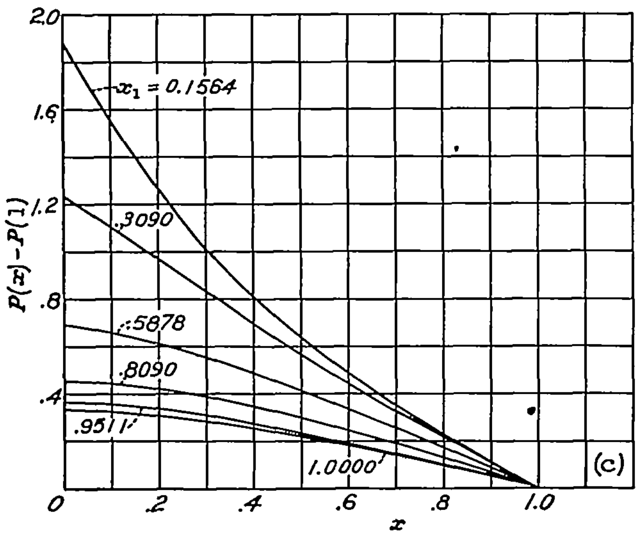
(a) $\lambda=\frac{1}{2}$.
FIGURE 7.—$P(x)-P(1)$ against x for $\tau=240°$.



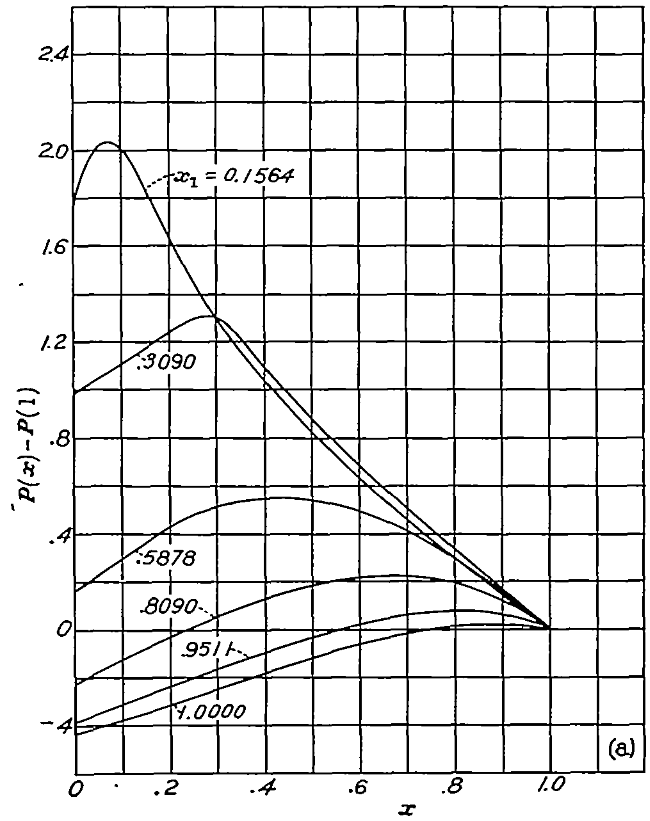
(a) $\lambda=\frac{1}{2}$—Concluded.
FIGURE 7.—Continued.



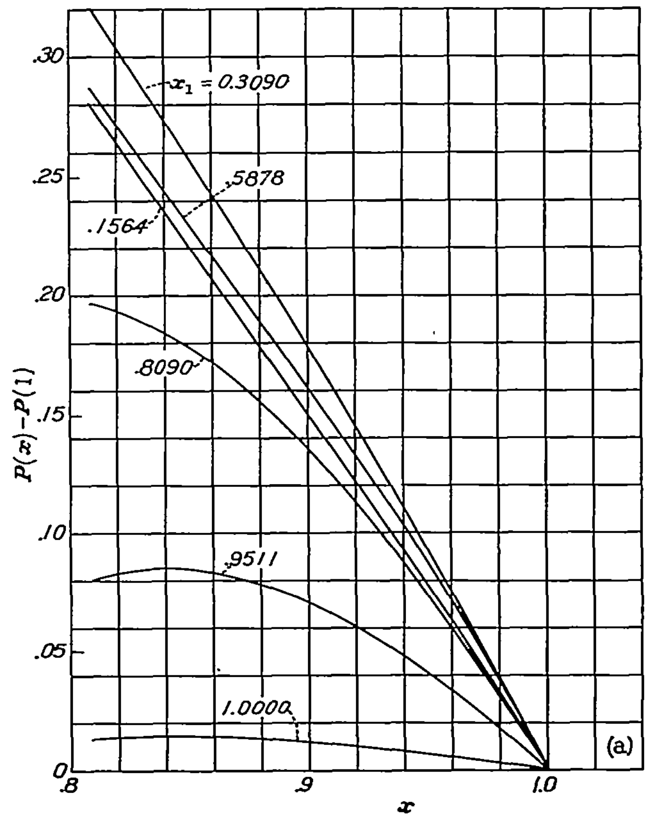
(b) $\lambda=1$.
FIGURE 7.—Continued.



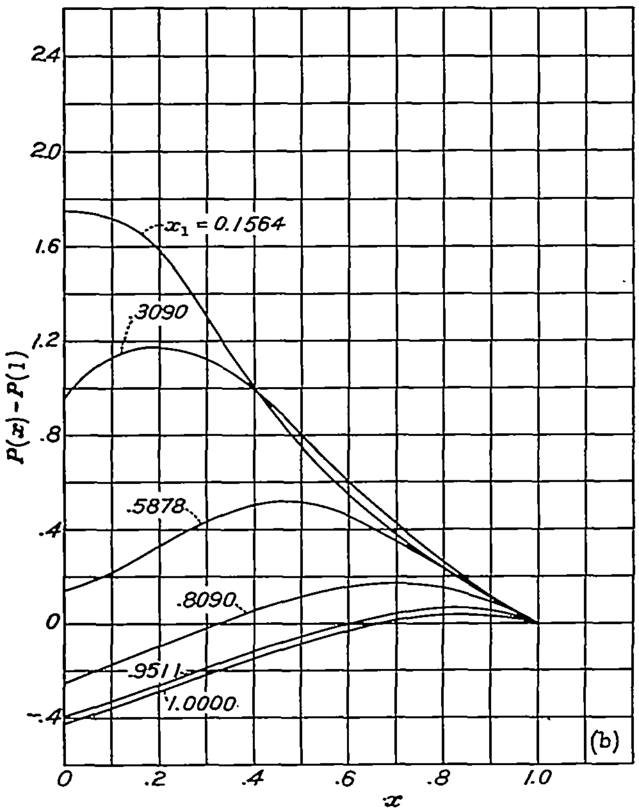
(c) $\lambda=2$.
FIGURE 7.—Concluded.



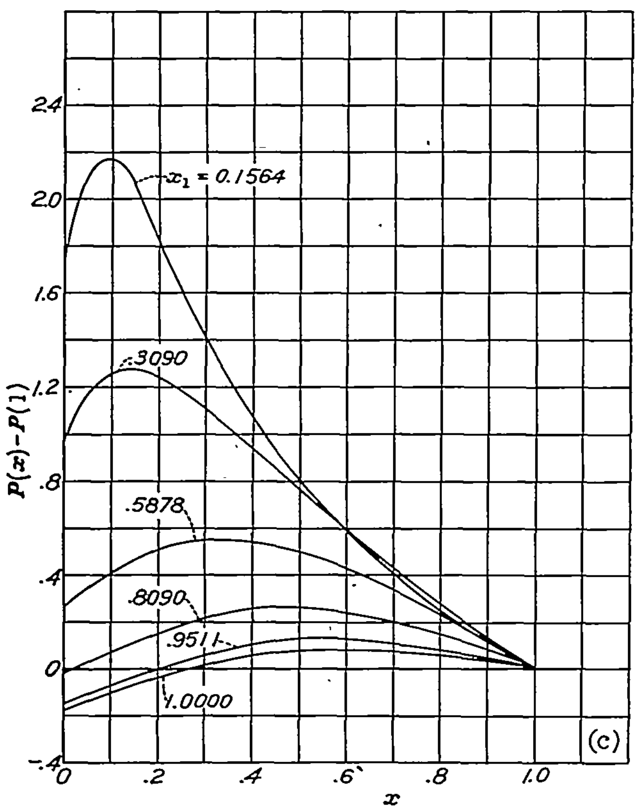
(a) $\lambda=\frac{1}{2}$.
FIGURE 8.—$P(x)-P(1)$ against x for $\tau=300°$.



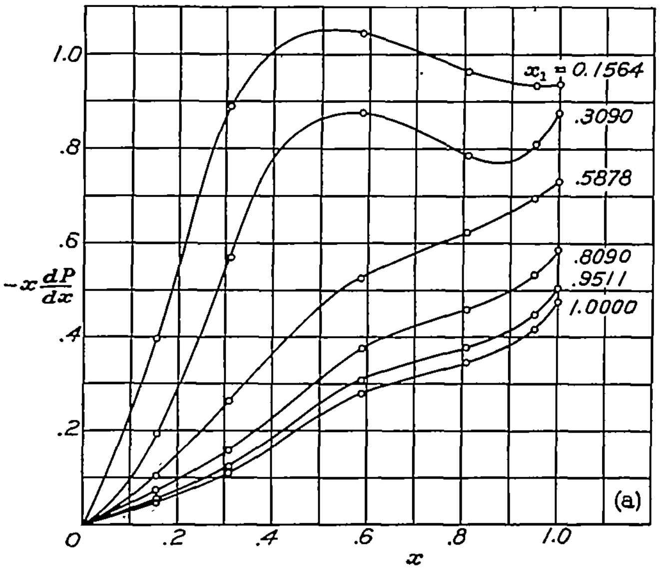
(a) $\lambda=\frac{1}{2}$—Concluded.
FIGURE 8.—Continued.



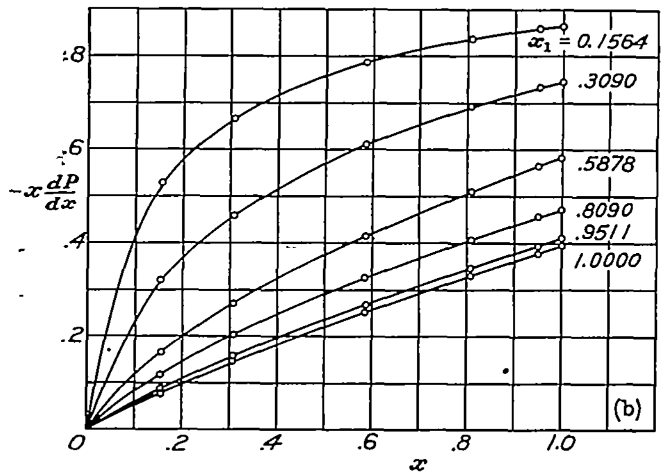
(b) λ=1.
FIGURE 8.—Continued.



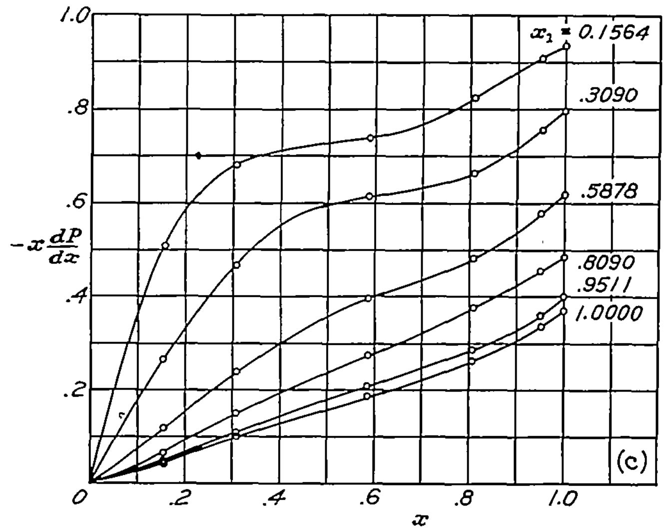
(c) λ=2.
FIGURE 8.—Concluded.



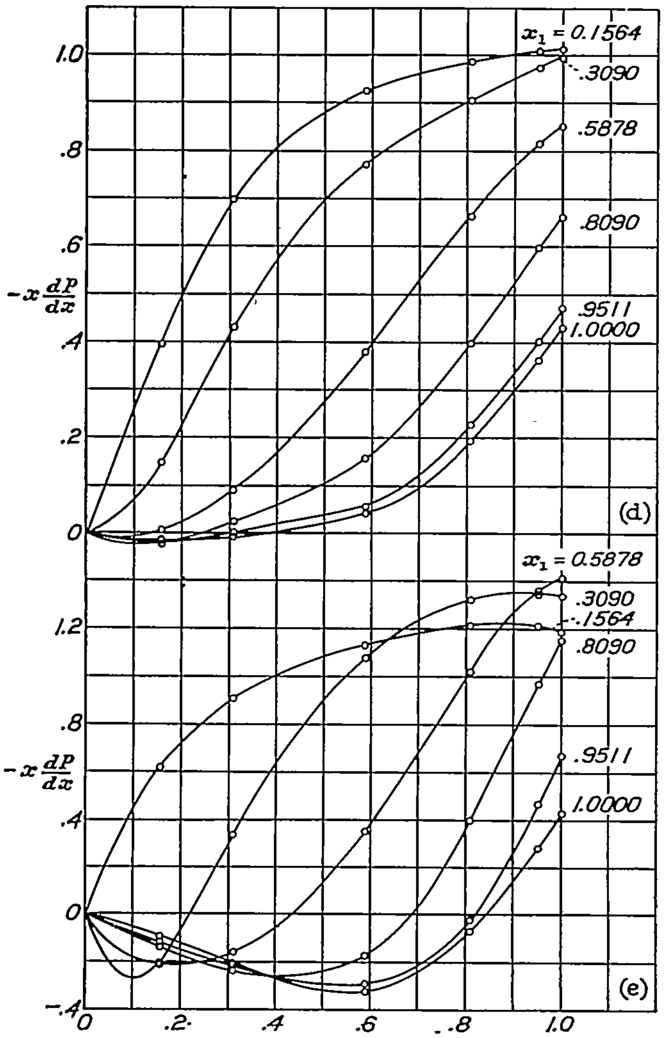
(a) τ=60°.
FIGURE 9.—Function $-x\frac{dP}{dx}$ against x for $\lambda=\frac{1}{2}$.



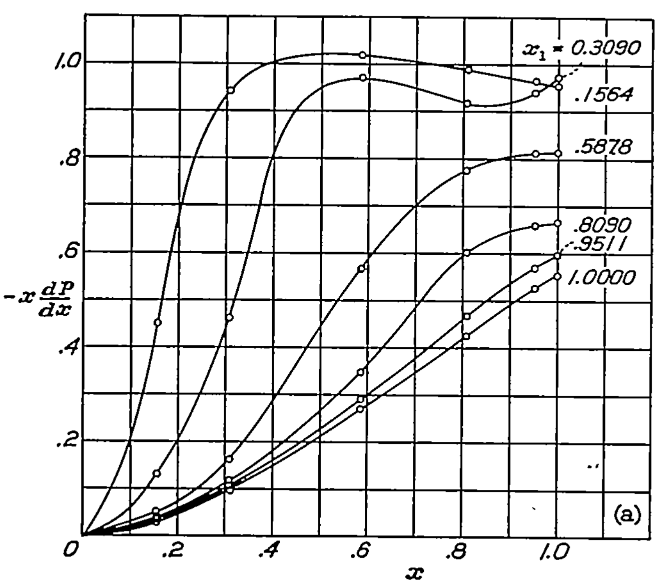
(b) τ=120°.
FIGURE 9.—Continued.



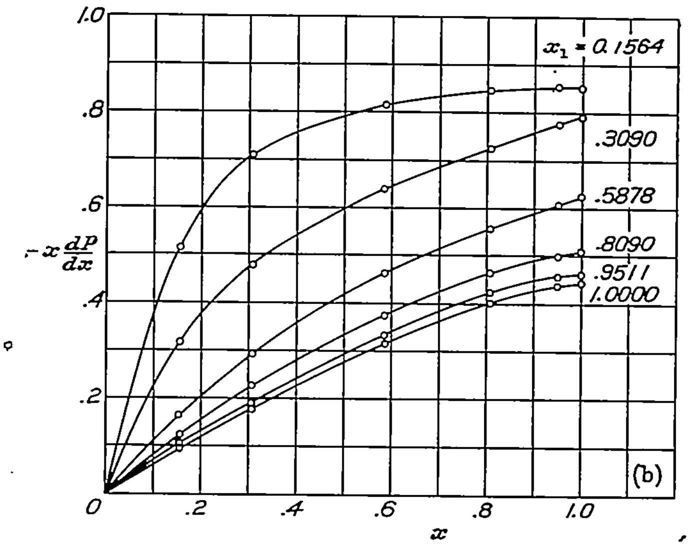
(c) τ=180°.
FIGURE 9.—Continued.



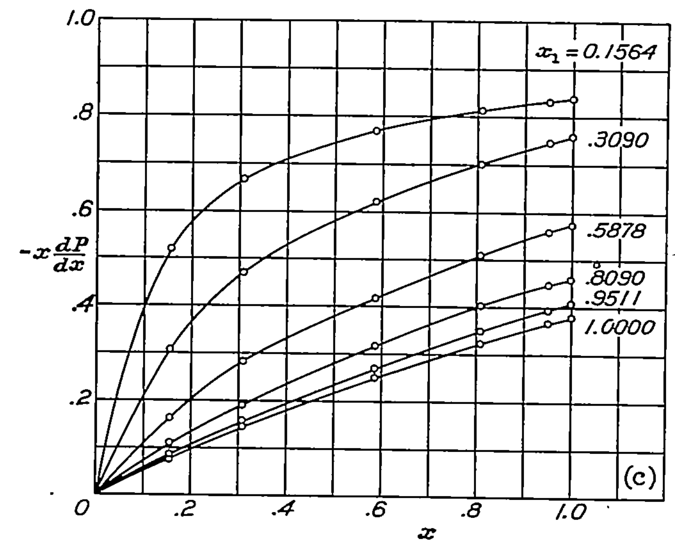
(d) τ=240°. (e) τ=300°.
FIGURE 9.—Concluded.



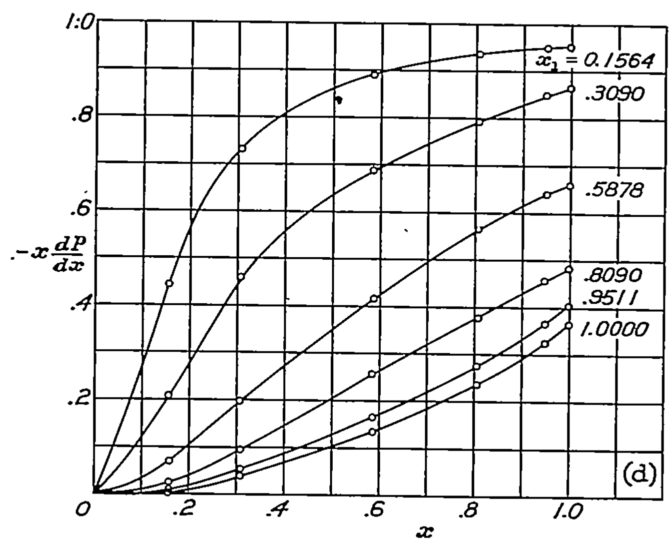
(a) τ=60°.
FIGURE 10.—Function $-x\frac{dP}{dx}$ against x for λ=1.



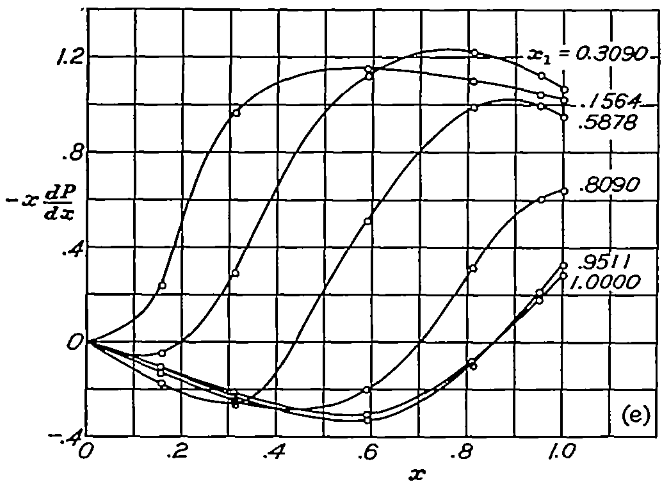
(b) τ=120°.
FIGURE 10.—Continued.



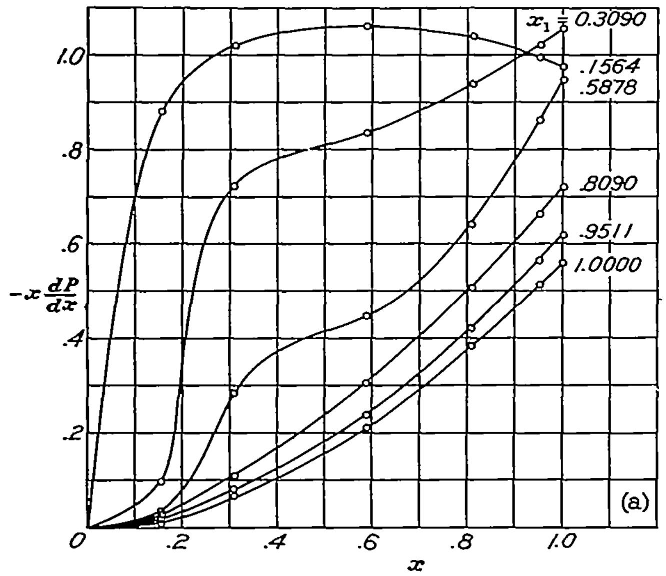
(c) τ=180°.
FIGURE 10.—Continued.



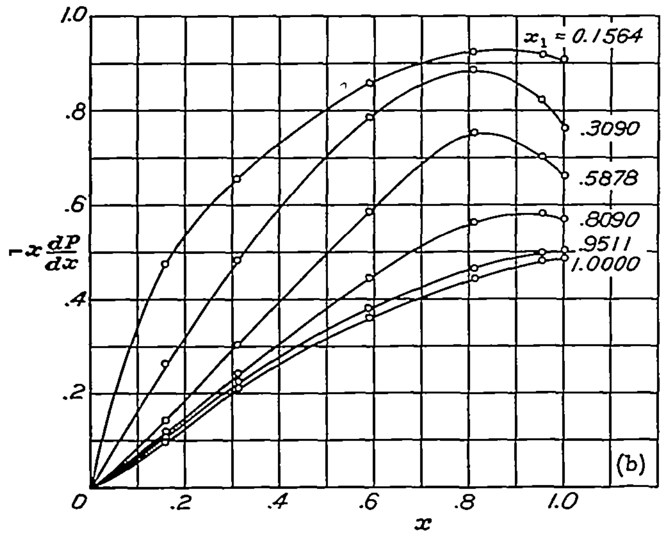
(d) τ=240°.
FIGURE 10.—Continued.



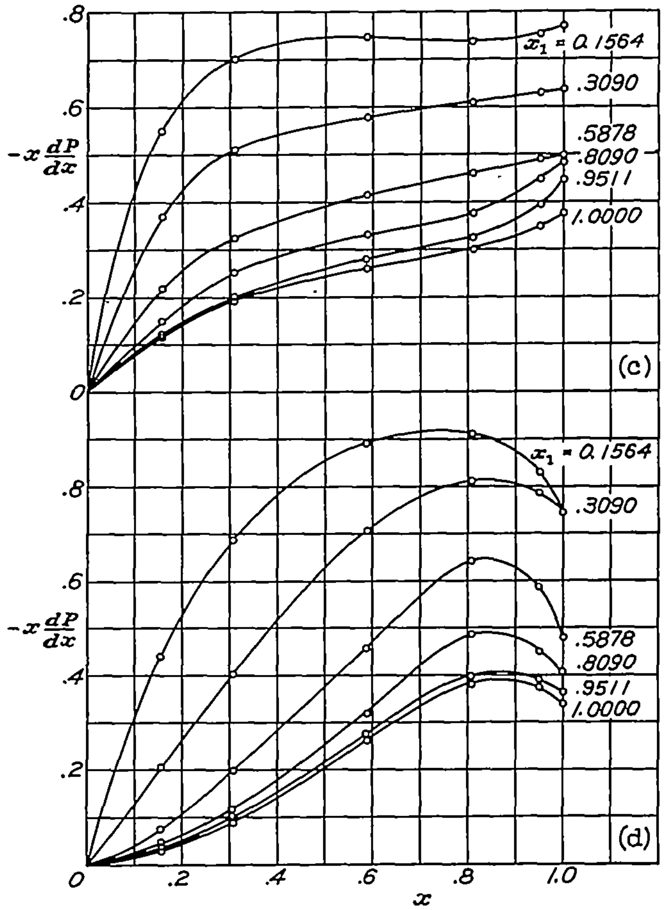
(e) $\tau=300°$.
FIGURE 10.—Concluded.



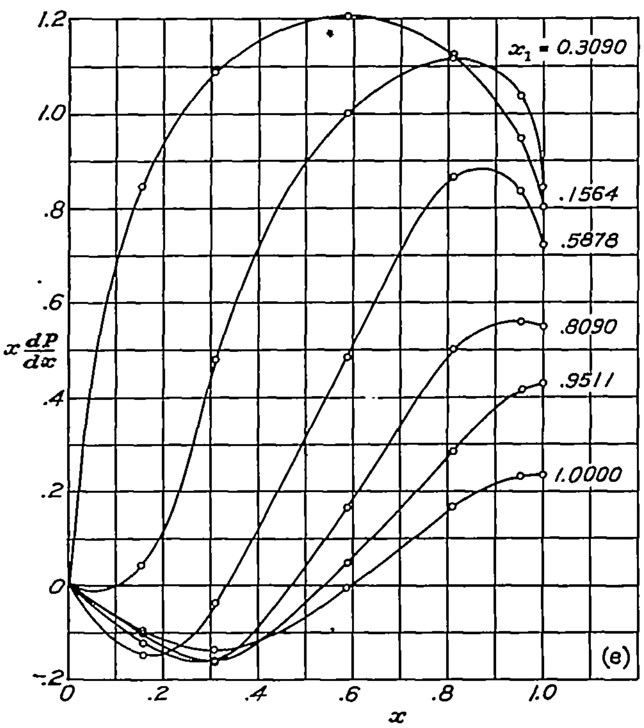
(a) $\tau=60°$.
FIGURE 11.—Function $-x\frac{dP}{dx}$ against x for $\lambda=2$



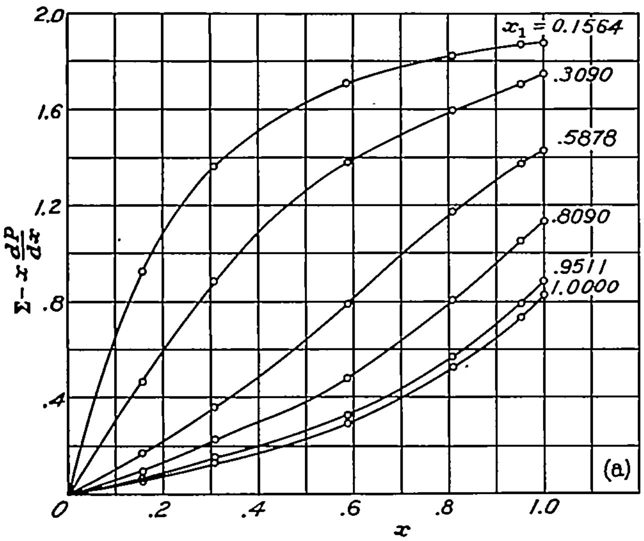
(b) $\tau=120°$.
FIGURE 11.—Continued.



(c) $\tau=180°$. (d) $\tau=240°$.
FIGURE 11.—Continued.

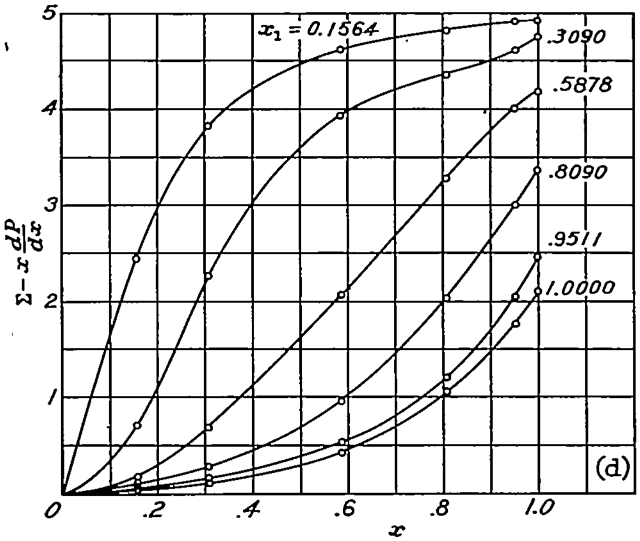


(e) $\tau=300°$.
FIGURE 11.—Concluded.



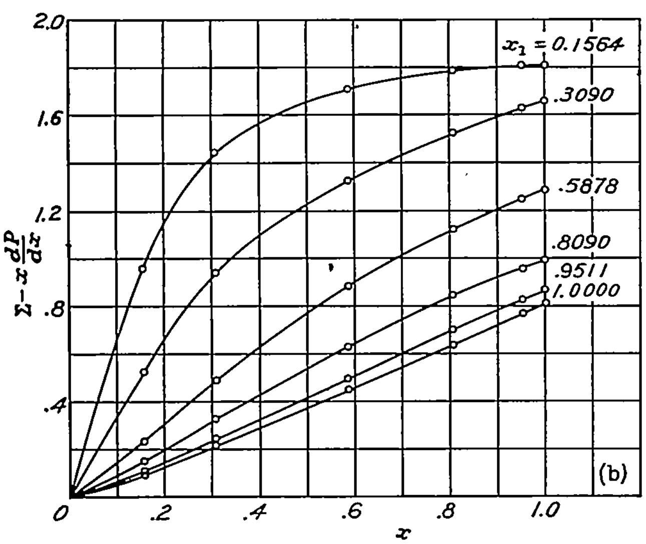
(a) Three-blade propeller; $\lambda=\frac{1}{2}$.

FIGURE 12.—Values of $\sum -x\frac{dP}{dx}$ against x for three-blade and six-blade propellers.



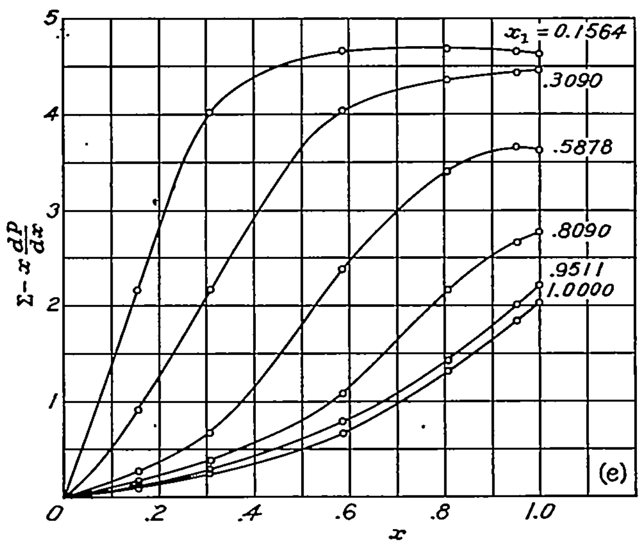
(d) Six-blade propeller; $\lambda=\frac{1}{2}$.

FIGURE 12.—Continued.



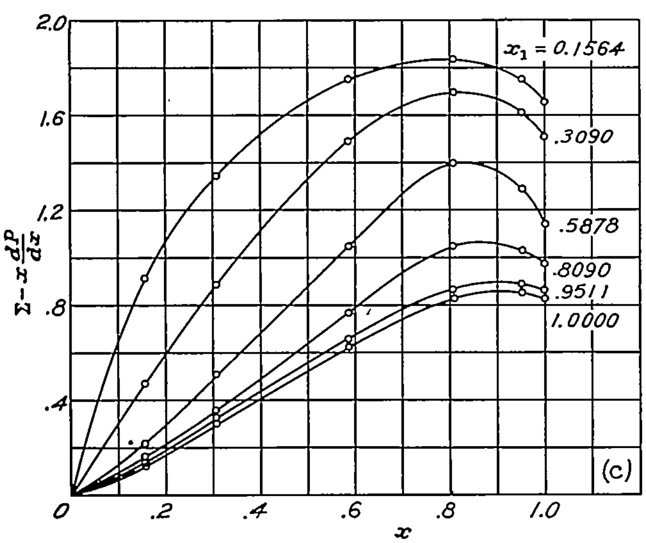
(b) Three-blade propeller; $\lambda=1$.

FIGURE 12.—Continued.



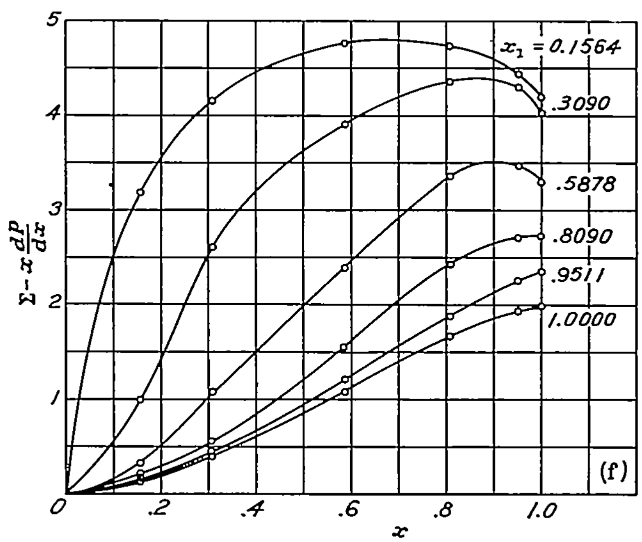
(e) Six-blade propeller; $\lambda=1$.

FIGURE 12.—Continued.



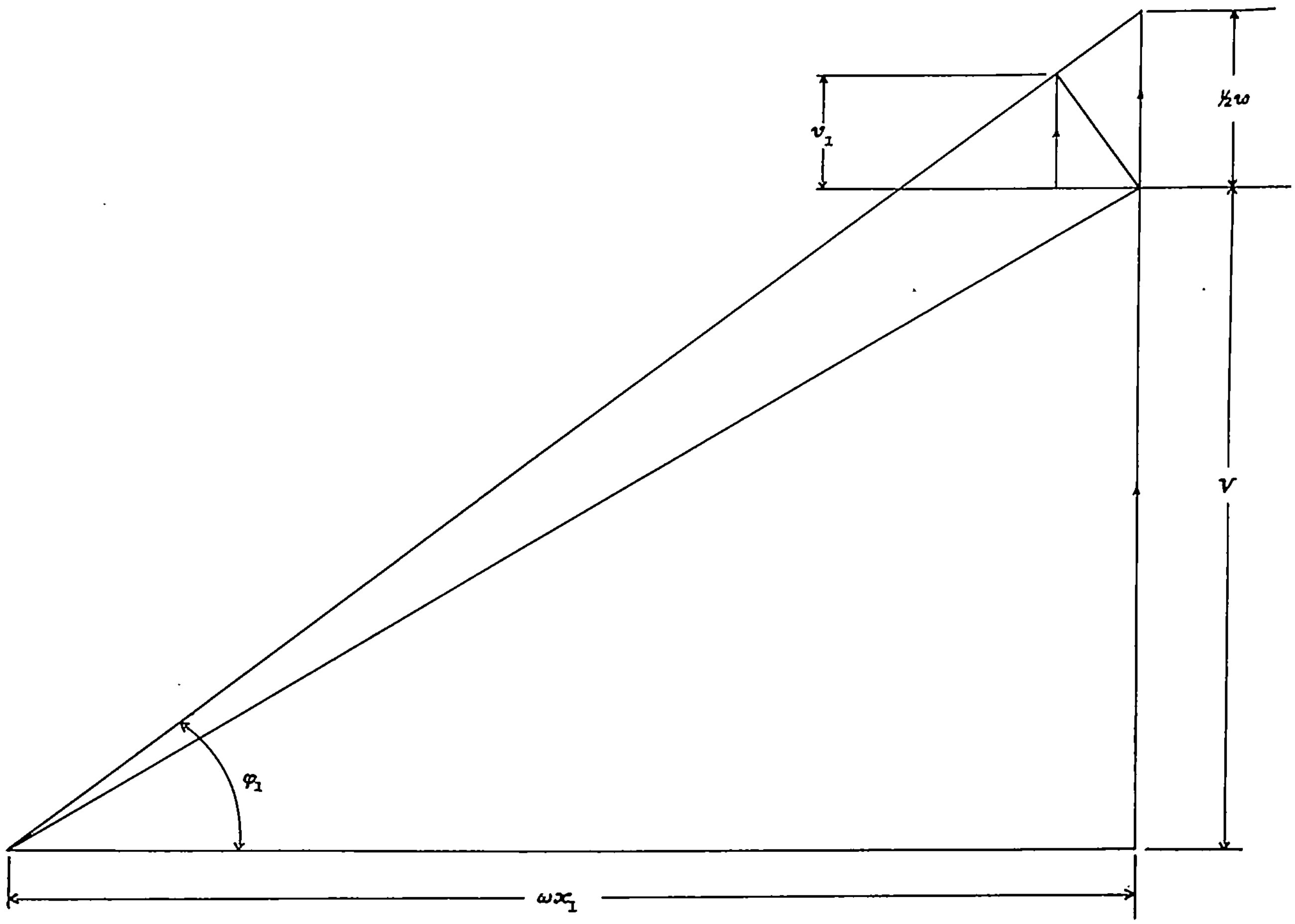
(c) Three-blade propeller; $\lambda=2$.

FIGURE 12.—Continued



(f) Six-blade propeller; $\lambda=2$.

FIGURE 12.—Concluded.



w axial displacement velocity ($\frac{1}{2}w$ at propeller)
v_1 actual axial interference velocity ($\frac{1}{2}w\cos^2\varphi_1$ at propeller)
φ_1 helix angle at radius x

FIGURE 13.—Velocity diagram.

TABLE I.—FUNCTION Q AGAINST x

x_1 \ x	0	0.1564	0.3090	0.4540	0.5878	0.7071	0.8090	0.8910	0.9511	0.9877	1.00
					$\lambda=\frac{1}{2}$						
0.1564	0.11826	0.12420	0.11817	0.10898	0.09334	0.07351	0.06051	0.04820	0.03937	0.03429	0.03252
.3090	.11826	.11817	.10580	.08696	.06259	.03851	.01679	−.00264	−.01646	−.02407	−.02729
.4540	.11826	.10898	.08696	.06101	.02926	−.00072	−.02257	−.04633	−.06370	−.07355	−.07720
.5878	.11826	.09334	.06259	.02926	−.00169	−.03644	−.06191	−.08528	−.10315	−.11315	−.11755
.7071	.11826	.07351	.03851	.00072	−.03644	−.07081	−.09533	−.12238	−.14059	−.15029	−.15430
.8090	.11826	.06051	.01679	−.02257	−.06191	−.09533	−.12420	−.14934	−.16703	−.17761	−.18246
.8910	.11826	.04820	−.00264	−.04633	−.08528	−.12238	−.14934	−.16900	−.18837	−.20063	−.20496
.9511	.11826	.03937	−.01646	−.06370	−.10315	−.14059	−.16703	−.18837	−.20752	−.21634	−.22098
.9877	.11826	.03429	−.02407	−.07355	−.11315	−.15029	−.17761	−.20003	−.21634	−.22633	−.23283
1.00	.11826	.03252	−.02729	−.07720	−.11755	−.15430	−.18246	−.20496	−.22098	−.23233	−.23724
					$\lambda=1$						
0.1564	0.06390	0.05804	0.04915	0.04323	0.03635	0.02954	0.02318	0.01727	0.01236	0.01021	0.01111
.3090	.06390	.04915	.03428	.02244	.01054	−.00262	−.01301	−.02336	−.03059	−.03352	−.03425
.4540	.06390	.04323	.02244	.00444	−.01419	−.03213	−.04733	−.05956	−.06941	−.07376	−.07401
.5878	.06390	.03635	.01054	−.01419	−.03715	−.05761	−.07560	−.09053	−.10235	−.10847	−.10903
.7071	.06390	.02954	−.00262	−.03213	−.05761	−.08155	−.10030	−.11704	−.13087	−.13861	−.13974
.8090	.06390	.02318	−.01301	−.04733	−.07560	−.10030	−.12049	−.13758	−.15259	−.16012	−.16143
.8910	.06390	.01727	−.02336	−.05956	−.09053	−.11704	−.13758	−.15528	−.16920	−.17850	−.18026
.9511	.06390	.01236	−.03059	−.06941	−.10235	−.13087	−.15259	−.16920	−.18140	−.19344	−.19590
.9877	.06390	.01021	−.03352	−.07376	−.10847	−.13861	−.16012	−.17850	−.19344	−.20407	−.20496
1.00	.06390	.01111	−.03425	−.07401	−.10903	−.13974	−.16140	−.18026	−.19590	−.20496	−.20486
					$\lambda=2$						
0.1564	0.02794	0.02524	0.02008	0.01748	0.01426	0.01102	0.00792	0.00521	0.00301	0.00305	0.00310
.3090	.02794	.02008	.01370	.00697	−.00084	−.00624	−.01128	−.01605	−.01970	−.02072	−.02064
.4540	.02794	.01748	.00697	−.00182	−.01432	−.02246	−.02941	−.03612	−.04060	−.04210	−.04251
.5878	.02794	.01426	−.00084	−.01432	−.02785	−.03786	−.04536	−.05329	−.05897	−.06081	−.06145
.7071	.02794	.01102	−.00624	−.02246	−.03786	−.05035	−.05964	−.06815	−.07407	−.07713	−.07820
.8090	.02794	.00792	−.01128	−.02941	−.04536	−.05964	−.07314	−.08246	−.08781	−.09040	−.09199
.8910	.02794	.00521	−.01605	−.03612	−.05329	−.06815	−.08246	−.09277	−.10019	−.10284	−.10437
.9511	.02794	.00301	−.01970	−.04060	−.05897	−.07407	−.08781	−.10019	−.10970	−.11211	−.11318
.9877	.02794	.00305	−.02072	−.04210	−.06081	−.07713	−.09040	−.10284	−.11211	−.11582	−.11708
1.00	.02794	.00310	−.02064	−.04251	−.06145	−.07820	−.09199	−.10437	−.11318	−.11708	−.11779

TABLE II.—FUNCTION $\frac{dQ}{dx}$ AGAINST x

x_1 \ x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
					$\lambda=\frac{1}{2}$						
0.1564	0.05000	0.0175	−0.0150	−0.0470	−0.0800	−0.1100	−0.1340	−0.1470	−0.1520	−0.1568	−0.15974
.3090	.00250	−.0320	−.0672	−.1025	−.1396	−.1748	−.1984	−.2120	−.2184	−.2220	−.22254
.4540	−.06293	−.0975	−.1300	−.1648	−.1928	−.2168	−.2340	−.2448	−.2544	−.2578	−.25584
.5878	−.16149	−.1825	−.2000	−.2150	−.2300	−.2425	−.2555	−.2655	−.2720	−.2750	−.26922
.7071	−.28838	−.2648	−.2550	−.2548	−.2576	−.2665	−.2752	−.2840	−.2880	−.2875	−.27642
.8090	−.42088	−.3255	−.2880	−.2740	−.2700	−.2750	−.2840	−.2915	−.2952	−.2950	−.28349
.8910	−.53863	−.4000	−.3352	−.3140	−.3024	−.2970	−.2952	−.2978	−.3020	−.3068	−.31310
.9511	−.63559	−.4620	−.3744	−.3375	−.3160	−.3050	−.3020	−.3060	−.3100	−.3160	−.32400
.9877	−.69631	−.5100	−.4040	−.3525	−.3264	−.3150	−.3100	−.3135	−.3165	−.3248	−.33100
1.00	−.71759	−.5240	−.4144	−.3590	−.3320	−.3190	−.3150	−.3160	−.3220	−.3290	−.3360
					$\lambda=1$						
0.1564	−0.04690	−0.0475	−0.0480	−0.0500	−0.0520	−0.0530	−0.0550	−0.0575	−0.0600	−0.0630	−0.06727
.3090	−.08960	−.0900	−.0902	−.0920	−.0930	−.0955	−.0980	−.1014	−.1050	−.1090	−.11586
.4540	−.14818	−.1390	−.1360	−.1350	−.1370	−.1375	−.1400	−.1426	−.1465	−.1500	−.15343
.5878	−.19684	−.1800	−.1720	−.1680	−.1670	−.1675	−.1685	−.1712	−.1740	−.1770	−.17918
.7071	−.25298	−.2265	−.2120	−.2030	−.1980	−.1960	−.1950	−.1940	−.1936	−.1936	−.19592
.8090	−.30136	−.2620	−.2396	−.2270	−.2196	−.2140	−.2108	−.2090	−.2080	−.2100	−.21471
.8910	−.34255	−.2955	−.2712	−.2560	−.2450	−.2380	−.2280	−.2225	−.2200	−.2225	−.22800
.9511	−.37091	−.3187	−.2920	−.2748	−.2620	−.2508	−.2416	−.2350	−.2336	−.2370	−.24250
.9877	−.39041	−.3298	−.3010	−.2822	−.2680	−.2575	−.2480	−.2420	−.2390	−.2420	−.24700
1.00	−.39689	−.3322	−.3025	−.2837	−.2700	−.2600	−.2505	−.2438	−.2410	−.2435	−.24850
					$\lambda=2$						
0.1564	−0.02960	−0.0295	−0.0294	−0.0291	−0.0290	−0.0288	−0.0284	−0.0280	−0.0276	−0.0270	−0.02641
.3090	−.04857	−.0492	−.0499	−.0500	−.0500	−.0500	−.0500	−.0498	−.0491	−.0482	−.04712
.4540	−.07063	−.0712	−.0720	−.0725	−.0730	−.0730	−.0730	−.0730	−.0725	−.0715	−.07058
.5878	−.09199	−.0929	−.0937	−.0940	−.0940	−.0939	−.0936	−.0931	−.0930	−.0929	−.09270
.7071	−.11265	−.1114	−.1100	−.1090	−.1084	−.1080	−.1078	−.1078	−.1078	−.1078	−.10800
.8090	−.13155	−.1291	−.1269	−.1243	−.1220	−.1200	−.1188	−.1178	−.1170	−.1167	−.11700
.8910	−.14748	−.1450	−.1420	−.1387	−.1356	−.1327	−.1300	−.1281	−.1270	−.1266	−.12705
.9511	−.15874	−.1560	−.1534	−.1508	−.1480	−.1454	−.1430	−.1410	−.1396	−.1390	−.14034
.9877	−.16554	−.1600	−.1561	−.1530	−.1501	−.1478	−.1456	−.1434	−.1420	−.1416	−.14240
1.00	−.16757	−.1616	−.1571	−.1538	−.1510	−.1486	−.1460	−.1440	−.1428	−.1420	−.14320

TABLE II.—FUNCTION $\frac{dQ}{dx}$ AGAINST x—CONCLUDED

x_1 \ x	0	0.1564	0.3090	0.4540	0.5878	0.7071	0.8090	0.8910	0.9511	0.9877	1.00
$\lambda=\frac{1}{2}$											
0.1564	0.05000	−0.0010	−0.0501	−0.0970	−0.1315	−0.1475	−0.1540	−0.1565	−0.1595	−0.1600	−0.15974
.3090	.00250	−.0520	−.1060	−.1598	−.1960	−.2130	−.2200	−.2220	−.2240	−.2235	−.22254
.4540	−.06293	−.1165	−.1665	−.2065	−.2330	−.2470	−.2551	−.2580	−.2575	−.2565	−.25584
.5878	−.16149	−.1930	−.2170	−.2370	−.2545	−.2665	−.2740	−.2750	−.2735	−.2700	−.26922
.7071	−.28838	−.2580	−.2540	−.2630	−.2750	−.2840	−.2880	−.2890	−.2845	−.2780	−.27642
.8090	−.42088	−.3010	−.2735	−.2730	−.2830	−.2925	−.2965	−.2955	−.2915	−.2860	−.28349
.8910	−.53863	−.3575	−.3120	−.2990	−.2950	−.2980	−.3025	−.3060	−.3110	−.3115	−.31319
.9511	−.63559	−.4040	−.3350	−.3085	−.3020	−.3060	−.3100	−.3155	−.3200	−.3235	−.32400
.9877	−.69631	−.4420	−.3495	−.3190	−.3100	−.3135	−.3185	−.3240	−.3280	−.3301	−.33100
1.00	−.71759	−.4535	−.3560	−.3240	−.3150	−.3175	−.3230	−.3280	−.3325	−.3350	−.33600
$\lambda=1$											
0.1564	−0.04690	−0.0477	−0.0505	−0.0526	−0.0550	−0.0575	−0.0600	−0.0625	−0.0650	−0.0669	−0.06727
.3090	−.08960	−.0900	−.0922	−.0947	−.0978	−.1020	−.1051	−.1085	−.1123	−.1149	−.11580
.4540	−.14818	−.1368	−.1351	−.1373	−.1398	−.1430	−.1468	−.1498	−.1522	−.1530	−.15343
.5878	−.19684	−.1748	−.1678	−.1670	−.1683	−.1718	−.1742	−.1765	−.1776	−.1790	−.17918
.7071	−.25298	−.2170	−.2027	−.1972	−.1950	−.1940	−.1936	−.1936	−.1949	−.1952	−.19592
.8090	−.30136	−.2475	−.2265	−.2168	−.2110	−.2088	−.2080	−.2098	−.2120	−.2132	−.21471
.8910	−.34255	−.2805	−.2550	−.2395	−.2285	−.2225	−.2200	−.2223	−.2250	−.2275	−.22800
.9511	−.37091	−.3020	−.2732	−.2560	−.2423	−.2345	−.2336	−.2365	−.2400	−.2420	−.24250
.9877	−.39041	−.3123	−.2810	−.2625	−.2495	−.2418	−.2390	−.2412	−.2445	−.2465	−.24700
1.00	−.39689	−.3135	−.2822	−.2645	−.2518	−.2436	−.2410	−.2428	−.2465	−.2480	−.24850
$\lambda=2$											
0.1564	−0.02960	−0.0295	−0.0291	−0.0290	−0.0285	−0.0280	−0.0273	−0.0270	−0.0267	−0.0264	−0.02641
.3090	−.04857	−.0497	−.0500	−.0500	−.0500	−.0498	−.0490	−.0483	−.0478	−.0472	−.04712
.4540	−.07063	−.0718	−.0726	−.0730	−.0730	−.0730	−.0723	−.0718	−.0711	−.0708	−.07058
.5878	−.09199	−.0933	−.0940	−.0940	−.0937	−.0931	−.0930	−.0929	−.0928	−.0927	−.09270
.7071	−.11265	−.1108	−.1090	−.1081	−.1078	−.1078	−.1078	−.1078	−.1080	−.1080	−.10800
.8090	−.13155	−.1279	−.1241	−.1210	−.1189	−.1177	−.1169	−.1167	−.1168	−.1170	−.11700
.8910	−.14748	−.1432	−.1384	−.1340	−.1304	−.1280	−.1269	−.1266	−.1268	−.1270	−.12705
.9511	−.15874	−.1546	−.1505	−.1467	−.1432	−.1409	−.1396	−.1390	−.1393	−.1400	−.14034
.9877	−.16554	−.1578	−.1528	−.1490	−.1459	−.1433	−.1420	−.1416	−.1419	−.1422	−.14240
1.00	−.16757	−.1589	−.1534	−.1497	−.1463	−.1439	−.1428	−.1420	−.1424	−.1429	−.14320

TABLE III.—FUNCTION $-x\frac{dQ}{dx}$ AGAINST x

x_1 \ x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$\lambda=\frac{1}{2}$											
0.1564	0	−0.00175	0.00300	0.01410	0.03200	0.05500	0.08040	0.10290	0.12160	0.14112	0.15974
.3090	0	.00320	.01344	.03075	.05584	.08740	.11904	.14840	.17472	.19980	.22254
.4540	0	.00975	.02600	.04944	.07712	.10840	.14040	.17136	.20352	.23202	.25584
.5878	0	.01825	.04000	.06450	.09200	.12125	.15192	.18585	.21760	.24750	.26922
.7071	0	.02648	.05100	.07644	.10304	.13325	.16512	.19880	.23040	.25875	.27642
.8090	0	.03255	.05760	.08220	.10800	.13750	.17040	.20405	.23616	.26550	.28349
.8910	0	.04000	.06704	.09420	.12096	.14850	.17712	.20846	.24160	.27612	.31319
.9511	0	.04620	.07488	.10125	.12640	.15250	.18120	.21420	.24800	.28440	.32400
.9877	0	.05100	.08080	.10575	.13056	.15750	.18600	.21945	.25320	.29232	.33100
1.00	0	.05240	.08288	.10770	.13280	.15950	.18900	.22120	.25760	.29610	.33600
$\lambda=1$											
0.1564	0	0.00475	0.00960	0.01500	0.02080	0.02650	0.03300	0.04025	0.04800	0.05670	0.06727
.3090	0	.00900	.01804	.02760	.03720	.04775	.05880	.07098	.08400	.09810	.11586
.4540	0	.01390	.02720	.04050	.05480	.06375	.08400	.09982	.11720	.13500	.15343
.5878	0	.01800	.03440	.05040	.06680	.08375	.10110	.11984	.13920	.15930	.17918
.7071	0	.02265	.04240	.06090	.07920	.09800	.11700	.13580	.15488	.17424	.19592
.8090	0	.02620	.04792	.06810	.08784	.10700	.12648	.14630	.16640	.18900	.21471
.8910	0	.02955	.05424	.07680	.09800	.11900	.13680	.15575	.17600	.20025	.22800
.9511	0	.03187	.05840	.08244	.10480	.12540	.14496	.16450	.18688	.21330	.24250
.9877	0	.03298	.06020	.08466	.10720	.12875	.14880	.16940	.19120	.21780	.24700
1.00	0	.03322	.06050	.08511	.10800	.13000	.15030	.17066	.19280	.21915	.24850
$\lambda=2$											
0.1564	0	0.00295	0.00588	0.00873	0.01160	0.01440	0.01704	0.01960	0.02208	0.02430	0.02641
.3090	0	.00492	.00998	.01500	.02000	.02500	.03000	.03486	.03928	.04338	.04712
.4540	0	.00712	.01440	.02175	.02920	.03650	.04380	.05110	.05800	.06435	.07058
.5878	0	.00929	.01874	.02820	.03760	.04695	.05616	.06517	.07440	.08361	.09270
.7071	0	.01114	.02200	.03270	.04336	.05400	.06468	.07546	.08624	.09702	.10800
.8090	0	.01291	.02538	.03729	.04880	.06000	.07128	.08246	.09360	.10503	.11700
.8910	0	.01450	.02840	.04161	.05424	.06635	.07800	.08967	.10160	.11394	.12705
.9511	0	.01560	.03068	.04524	.05920	.07270	.08580	.09870	.11168	.12510	.14034
.9877	0	.01600	.03122	.04590	.06004	.07390	.08736	.10038	.11360	.12744	.14240
1.00	0	.01616	.03142	.04614	.06040	.07430	.08760	.10080	.11424	.12780	.14320

TABLE III.—FUNCTION $-x\frac{dQ}{dx}$ AGAINST x—Concluded

x_1 \ x	0	0.1564	0.3090	0.4540	0.5878	0.7071	0.8090	0.8910	0.9511	0.9877	1.00
$\lambda=\frac{1}{2}$											
0.1564	0	0.00016	0.01548	0.04404	0.07730	0.10430	0.12459	0.13944	0.15170	0.15803	0.15974
.3090	0	.00813	.03275	.07255	.11521	.15061	.17798	.19780	.21305	.22075	.22254
.4540	0	.01822	.05145	.09375	.13696	.17465	.20638	.22988	.24491	.25335	.25584
.5878	0	.03019	.06705	.10760	.14960	.18844	.22167	.24502	.26013	.26668	.26922
.7071	0	.04035	.07849	.11940	.16164	.20082	.23299	.25750	.27059	.27458	.27642
.8090	0	.04708	.08451	.12394	.16635	.20683	.23987	.26329	.27725	.28248	.28349
.8910	0	.05591	.09641	.13575	.17340	.21072	.24472	.27265	.29579	.30767	.31319
.9511	0	.06319	.10351	.14006	.17752	.21637	.25079	.28111	.30435	.31952	.32400
.9877	0	.06913	.10800	.14483	.18222	.22168	.25767	.28868	.31196	.32604	.33100
1.00	0	.07093	.11000	.14710	.18516	.22450	.26131	.29225	.31624	.33088	.33600
$\lambda=1$											
0.1564	0	0.00746	0.01560	0.02388	0.03233	0.04066	0.04854	0.05569	0.06182	0.06608	0.06727
.3090	0	.01408	.02849	.04299	.05749	.07212	.08503	.09667	.10681	.11349	.11586
.4540	0	.02140	.04175	.06233	.08217	.10112	.11876	.13347	.14476	.15112	.15343
.5878	0	.02734	.05185	.07582	.09893	.12148	.14093	.15726	.16892	.17680	.17918
.7071	0	.03394	.06263	.08953	.11462	.13718	.15662	.17250	.18537	.19280	.19592
.8090	0	.03871	.06999	.09843	.12403	.14764	.16827	.18693	.20163	.21058	.21471
.8910	0	.04387	.07880	.10873	.13431	.15733	.17798	.19807	.21400	.22470	.22800
.9511	0	.04723	.08442	.11622	.14242	.16581	.18898	.21072	.22826	.23902	.24250
.9877	0	.04884	.08683	.11917	.14666	.17098	.19335	.21491	.23254	.24347	.24700
1.00	0	.04903	.08720	.12008	.14801	.17225	.19497	.21633	.23445	.24495	.24850
$\lambda=2$											
0.1564	0	0.00461	0.00899	0.01317	0.01675	0.01980	0.02209	0.02406	0.02539	0.02608	0.02641
.3090	0	.00777	.01545	.02270	.02939	.03521	.03964	.04304	.04546	.04663	.04712
.4540	0	.01123	.02243	.03314	.04291	.05162	.05849	.06397	.06762	.06993	.07058
.5878	0	.01459	.02905	.04268	.05508	.06583	.07524	.08277	.08826	.09156	.09270
.7071	0	.01733	.03368	.04908	.06336	.07623	.08721	.09605	.10272	.10667	.10800
.8090	0	.02000	.03835	.05493	.06989	.08323	.09457	.10398	.11109	.11556	.11700
.8910	0	.02240	.04277	.06084	.07665	.09051	.10266	.11280	.12060	.12544	.12705
.9511	0	.02418	.04650	.06660	.08417	.09963	.11294	.12385	.13249	.13828	.14034
.9877	0	.02468	.04722	.06765	.08576	.10133	.11488	.12617	.13496	.14045	.14240
1.00	0	.02485	.04740	.06796	.08600	.10175	.11553	.12652	.13544	.14114	.14320

TABLE IV.—FUNCTION $x\frac{dF}{dx}$ AGAINST x

x_1 \ x	0	0.1564	0.3090	0.4540	0.5878	0.7071	0.8090	0.8910	0.9511	0.9877	1.00
$\lambda=\frac{1}{2}$											
0		−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00
.1564	0	±∞	−1.99303	−1.46458	−1.26170	−1.19621	−1.06355	−1.01614	−.98157	−.96187	−.95540
.3090	0	.79883	±∞	−2.78366	−1.80906	−1.45440	−1.26665	−1.15397	−1.03539	−1.04803	−1.03613
.4540	0	.34549	1.42781	±∞	−3.41645	−2.10410	−1.64634	−1.41821	−1.29261	−1.22783	−1.20769
.5878	0	.21336	.63897	1.99871	±∞	−4.03919	−2.43710	−1.89999	−1.64972	−1.53080	−1.49506
.7071	0	.16030	.41011	.93133	2.61274	±∞	−4.81641	−2.90544	−2.29684	−2.04745	−1.97675
.8090	0	.13685	.31447	.61779	1.26965	3.38854	±∞	−5.98509	−3.67916	−3.01834	−2.85105
.8910	0	.12638	.26841	.48398	.87241	1.72629	4.53529	±∞	−8.07131	−5.23205	−4.69950
.9511	0	.12189	.24512	.41901	.70492	1.24095	2.45465	6.55772	±∞	−12.97115	−9.87489
.9877	0	.12017	.23403	.38579	.63067	1.05355	1.88986	3.90597	11.31495	±∞	−37.32797
1.00	0	.11974	.23074	.37922	.60914	1.00201	1.75072	3.42125	8.32837	35.30113	∞
$\lambda=1$											
0		−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00
.1564	0	∞	−2.02000	−1.51292	−1.33779	−1.24658	−1.19054	−1.15399	−1.13052	−1.11731	−1.11302
.3090	0	.95868	∞	−3.04334	−2.02983	−1.68390	−1.50919	−1.40738	−1.34642	−1.31349	−1.30302
.4540	0	.46900	1.88440	∞	−4.09131	−2.72121	−2.06744	−1.82101	−1.68705	−1.61849	−1.59724
.5878	0	.31308	.92928	2.82641	∞	−5.50622	−3.21099	−2.54678	−2.24206	−2.09846	−2.05546
.7071	0	.24131	.62835	1.41541	3.87876	∞	−6.66344	−4.05123	−3.23335	−2.90145	−2.80771
.8090	0	.20322	.49077	.97474	1.98748	5.20307	∞	−8.68248	−5.35062	−4.40589	−4.16788
.8910	0	.18179	.41840	.77603	1.40438	2.75532	7.12155	∞	−12.11751	−7.84804	−7.05128
.9511	0	.16974	.37883	.67518	1.15039	2.02152	3.96099	10.43287	∞	−19.95323	−15.16694
.9877	0	.16360	.35885	.62606	1.03526	1.73314	3.09010	6.32119	18.08469	∞	−58.44076
1.00	0	.16171	.35272	.61123	1.00149	1.65306	2.87394	5.56464	13.39780	56.18133	∞
$\lambda=2$											
0		−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00
.1564	0	∞	−2.02393	−1.52266	−1.35649	−1.27453	−1.22695	−1.19742	−1.17919	−1.16922	−1.16602
.3090	0	1.00761	∞	−3.10970	−2.08915	−1.75281	−1.58947	−1.49766	−1.44428	−1.41600	−1.40710
.4540	0	.51046	2.06247	∞	−4.31441	−2.73708	−2.22262	−1.97925	−1.85005	−1.78491	−1.76488
.5878	0	.34911	1.05685	3.22439	∞	−5.73646	−3.53335	−2.82827	−2.51033	−2.36233	−2.31828
.7071	0	.27224	.73259	1.67673	4.59423	∞	−7.55951	−4.61041	−3.70046	−3.33496	−3.23228
.8090	0	.22951	.57977	1,18224	2.43642	6.35690	∞	−10.17771	−6.27367	−5.17695	−4.90194
.8910	0	.20419	.49669	.95382	1.75803	3.46676	8.90569	∞	−14.59886	−9.43808	−8.47927
.9511	0	.18920	.44984	.83532	1.45735	2.58738	5.06961	13.24395	∞	−24.56797	−18.64159
.9877	0	.18124	42561	.77664	1.31929	2.23798	4.00223	8.15130	23.10187	∞	−73.19713
1.00	0	.17874	.41808	.75876	1.27851	2.14041	3.73593	7.20978	17.22166	71.52164	∞

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TABLE V.—FUNCTION P AGAINST x FOR $\tau=60°$

x_1 \ x	0	0.1564	0.3090	0.5878	0.8090	0.9511	1.00
$\lambda=\frac{1}{2}$							
0.1564	2.2478	1.9613	1.4980	0.8830	0.5696	0.4113	0.3600
.3090	1.5673	1.4980	1.2413	.7646	.4963	.3580	.3046
.5878	.9259	.8830	.7646	.5146	.3280	.2230	.1866
.8090	.6084	.5696	.4963	.3280	.1946	.1146	.0826
.9511	.4482	.4113	.3580	.2230	.1146	.0446	.0213
1.00	.3986	.3600	.3046	.1866	.0826	.0213	0
$\lambda=1$							
0.1564	2.2463	2.0200	1.5000	0.8834	0.5700	0.4200	0.3734
.3090	1.5655	1.5000	1.3134	.8267	.5300	.3800	.3300
.5878	.9228	.8834	.8267	.6069	.3900	.2567	.2134
.8090	.6039	.5700	.5300	.3900	.2400	.1367	.1000
.9511	.4425	.4200	.3800	.2567	.1367	.0567	.0267
1.00	.3924	.3734	.3300	.2134	.1000	.0267	0
$\lambda=2$							
0.1564	2.1727	2.2333	1.5200	0.8667	0.5333	0.3667	0.3133
.3090	1.4928	1.5200	1.2667	.7700	.4900	.3367	.2867
.5878	.8488	.8667	.7700	.5367	.3667	.2500	.2067
.8090	.5295	.5333	.4900	.3667	.2433	.1467	.1067
.9511	.3678	.3667	.3367	.2500	.1467	.0600	.0267
1.00	.3177	.3133	.2867	.2067	.1067	.0267	0

TABLE VI.—FUNCTION P AGAINST x FOR $\tau=120°$

x_1 \ x	0	0.1564	0.3090	0.5878	0.8090	0.9511	1.00
$\lambda=\frac{1}{2}$							
0.1564	2.2825	1.6693	1.2393	0.8000	0.5377	0.3960	0.3547
.3090	1.6020	1.2393	.9860	.6460	.4360	.3160	.2813
.5878	.9606	.8000	.6460	.4280	.2777	.1893	.1640
.8090	.6431	.5377	.4360	.2777	.1693	.0977	.0777
.9511	.4829	.3960	.3160	.1893	.0977	.0360	.0167
1.00	.4333	.3547	.2813	.1640	.0777	.0167	0
$\lambda=1$							
0.1564	2.4163	1.8234	1.3900	0.9067	0.6500	0.5100	0.4600
.3090	1.7355	1.3934	1.1100	.7567	.5300	.4134	.3634
.5878	1.0928	.9067	.7567	.4967	.3434	.2534	.2067
.8090	.7739	.6500	.5300	.3434	.2067	.1300	.0934
.9511	.6125	.5100	.4134	.2534	.1300	.0634	.0300
1.00	.5624	.4600	.3634	.2067	.0934	.0300	0
$\lambda=2$							
0.1564	2.4794	1.8934	1.5133	1.0267	0.7267	0.5600	0.5067
.3090	1.7985	1.5133	1.2800	.8800	.6067	.4534	.4000
.5878	1.1555	1.0267	.8800	.6000	.3934	.2667	.2200
.8090	.8362	.7267	.6067	.3934	.2334	.1334	.1000
.9511	.6745	.5600	.4534	.2667	.1334	.0534	.0267
1.00	.6244	.5067	.4000	.2200	.1000	.0267	0

TABLE VII.—FUNCTION P AGAINST x FOR $\tau=180°$

x_1 \ x	0	0.1564	0.3090	0.5878	0.8090	0.9511	1.00
$\lambda=\frac{1}{2}$							
0.1564	2.1518	1.5686	1.1586	0.7106	0.4653	0.3220	0.2786
.3090	1.4713	1.1586	.9320	.5973	.3920	.2753	.2320
.5878	.8299	.7106	.5973	.3986	.2573	.1736	.1386
.8090	.5124	.4653	.3920	.2573	.1600	.0986	.0680
.9511	.3522	.3220	.2753	.1736	.0986	.0470	.0220
1.00	.3026	.2786	.2320	.1386	.0680	.0220	0
$\lambda=1$							
0.1564	2.3063	1.6867	1.2667	0.8100	0.5600	0.4100	0.3634
.3090	1.6255	1.2667	.9934	.6534	.4534	.3300	.2934
.5878	.9828	.8100	.6534	.4367	.2934	.2034	.1767
.8090	.6639	.5600	.4534	.2934	.1834	.1100	.0834
.9511	.5025	.4100	.3300	.2034	.1100	.0433	.0200
1.00	.4524	.3634	.2934	.1767	.0834	.0200	0
$\lambda=2$							
0.1564	2.4060	1.7433	1.3000	0.8400	0.5933	0.4033	0.4200
.3090	1.7251	1.3000	.9933	.6533	.4600	.3466	.3066
.5878	1.0821	.8400	.6533	.4200	.2733	.1966	.1633
.8090	.7628	.5933	.4600	.2733	.1600	.1000	.0733
.9511	.6011	.4633	.3466	.1966	.1000	.0400	.0200
1.00	.5511	.4200	.3066	.1633	.0733	.0200	0

TABLE VIII.—FUNCTION P AGAINST x FOR $\tau=240°$

x_1 \ x	0	0.1564	0.3090	0.5878	0.8090	0.9511	1.00
$\lambda=\frac{1}{2}$							
0.1564	1.8785	1.4720	1.1153	0.5693	0.2800	0.1220	0.0680
.3090	1.1980	1.1153	.9320	.5450	.2853	.1287	.0720
.5878	.5566	.5693	.5453	.4066	.2413	.1220	.0773
.8090	.2391	.2800	.2853	.2413	.1573	.0787	.0453
.9511	.0789	.1220	.1287	.1220	.0787	.0320	.0120
1.00	.0293	.0680	.0720	.0773	.0453	.0120	0
$\lambda=1$							
0.1564	2.0296	1.5867	1.1833	0.6767	0.3967	0.2400	0.1833
.3090	1.3482	1.1833	.9633	.5933	.3600	.2234	.1767
.5878	.7071	.6767	.5933	.4067	.2567	.1633	.1267
.8090	.3872	.3967	.3600	.2567	.1567	.0900	.0633
.9511	.2258	.2400	.2234	.1633	.0000	.0433	.0200
1.00	.1757	.1833	.1767	.1267	.0633	.0200	0
$\lambda=2$							
0.1564	2.1927	1.7033	1.3033	0.8167	0.5300	0.3700	0.3167
.3090	1.5118	1.3033	1.0967	.7300	.4767	.3300	.2767
.5878	.8688	.8167	.7300	.5200	.3433	.2200	.1800
.8090	.5495	.5300	.4767	.3433	.2167	.1267	.0933
.9511	.3878	.3700	.3300	.2200	.1267	.0567	.0267
1.00	.3377	.3167	.2767	.1800	.0933	.0267	0

TABLE IX.—FUNCTION P AGAINST x FOR τ=300°

x_1 \ x	0	0.1564	0.3090	0.5878	0.8090	0.9511	1.00
			$\lambda=\frac{1}{2}$				
0.1564	1.4198	1.4933	0.9400	0.3100	−0.0600	−0.2734	−0.3400
.3090	.7393	.9400	1.0566	.4566	.0766	−.1534	−.2434
.5878	.0979	.3100	.4566	.4400	.2233	.0233	−.0634
.8090	−.2196	−.0600	.0766	.2233	.2100	.0866	.0133
.9511	−.3798	−.2634	−.1534	.0233	.0866	.0466	.0066
1.00	−.4294	−.3400	−.2434	−.0634	.0133	.0066	0
			$\lambda=1$				
0.1564	1.4296	1.3467	0.9567	0.2433	−0.0967	−0.2700	−0.3200
.3090	.7488	.9567	.9067	.4100	.0333	−.1600	−.2133
.5878	.1061	.2433	.4100	.4333	.1833	.0167	−.0367
.8090	−.2128	−.0967	.0333	.1833	.1900	.0900	.0367
.9511	−.3742	−.2700	−.1600	.0167	.0900	.0567	.0200
1.00	−.4243	−.3200	−.2133	−.0367	−.0367	.0200	0
			$\lambda=2$				
0.1564	1.0794	1.9600	1.3000	0.5534	0.1734	−0.0100	−0.0600
.3090	.9985	1.3000	1.1200	.6334	.2834	.0867	.0234
.5878	.3555	.5534	.6334	.5234	.3000	.1434	.0800
.8090	.0362	.1734	.2834	.3000	.1434	.0967	.0534
.9511	−.1255	−.0100	.0867	.1434	.0967	.0467	.0200
1.00	−.1756	−.0600	.0234	.0800	.0534	.0200	0

TABLE X.—FUNCTION $\frac{dP}{dx}$ AGAINST τ FOR $\lambda=\frac{1}{2}$

x_1 \ τ (deg)	60	120	180	240	300
			$x=0$		
0.1564	2.2667	−3.3333	−6.0000	−2.2667	3.3333
.3090	.2667	−1.8667	−1.9333	−.2667	1.8667
.4540	--------	--------	--------	--------	--------
.5878	−.4133	−1.0267	−.8333	.4133	1.0267
.7071	--------	--------	--------	--------	--------
.8090	−.3300	−.7333	−.4800	.3300	.7333
.8910	--------	--------	--------	--------	--------
.9511	−.3017	−.5733	−.3000	.3017	.5733
.9877	--------	--------	--------	--------	--------
1.00	−.2600	−.5400	−.2600	.2600	.5400
			$x=1.00$		
0.1564	−0.9367	−0.8633	−0.9333	−1.0133	−1.190
.3090	−.8767	−.7467	−.7933	−.9967	−1.340
.4540	--------	--------	--------	--------	--------
.5878	−.7300	−.5800	−.6183	−.8583	−1.4133
.7071	--------	--------	--------	--------	--------
.8090	−.5850	−.4700	−.4867	−.6650	−1.1567
.8910	--------	--------	--------	--------	--------
.9511	−.5033	−.4100	−.4000	−.4733	−.6700
.9877	--------	--------	--------	--------	--------
1.00	−.4767	−.3967	−.3693	−.4333	−.4333

TABLE XI.—FUNCTION $\frac{dP}{dx}$ AGAINST τ FOR $\lambda=1$

x_1 \ τ (deg)	60	120	180	240	300
			$x=0$		
0.1564	5.0667	−8.6667	−10.3333	−5.0667	8.6667
.3090	.7667	−1.9333	−3.4667	−.7667	1.9333
.4540	--------	--------	--------	--------	--------
.5878	.2133	−.9667	−1.1000	−.2133	.9667
.7071	--------	--------	--------	--------	--------
.8090	−.0533	−.6933	−.7500	.0533	.6933
.8910	--------	--------	--------	--------	--------
.9511	−.1400	−.6400	−.5200	.1400	.6400
.9877	--------	--------	--------	--------	--------
1.00	−.1667	−.6200	−.4800	.1667	.6200
			$x=1.00$		
0.1564	−0.9567	−0.8533	−0.8400	−0.9533	−1.0300
.3090	−.9733	−.7933	−.7600	−.8667	−1.0733
.4540	--------	--------	--------	--------	--------
.5878	−.8133	−.6267	−.5767	−.6600	−.9533
.7071	--------	--------	--------	--------	--------
.8090	−.6633	−.5100	−.4600	−.4833	−.6467
.8910	--------	--------	--------	--------	--------
.9511	−.5967	−.4633	−.4100	−.4033	−.3333
.9877	--------	--------	--------	--------	--------
1.00	−.5533	−.4467	−.3800	−.3633	−.2900

TABLE XII.—FUNCTION $\frac{dP}{dx}$ AGAINST τ FOR $\lambda=2$

x_1 \ τ (deg)	60	120	180	240	300
			$x=0$		
0.1564	11.3333	−8.833	−16.3333	−11.3333	8.833
.3090	2.8000	−2.400	−5.6667	−2.8000	2.400
.4540	--------	--------	--------	--------	--------
.5878	.6833	−.9333	−1.2000	−.6833	.9333
.7071	--------	--------	--------	--------	--------
.8090	.2533	−.6667	−.7667	−.2533	.6667
.8910	--------	--------	--------	--------	--------
.9511	.2000	−.5867	−.6333	−.2000	.5867
.9877	--------	--------	--------	--------	--------
1.00	.1600	−.5467	−.5667	−.1600	.5467
			$x=1.00$		
0.1564	−0.9733	−0.9067	−0.7733	−0.7467	−0.8000
.3090	−1.0533	−.7600	−.6400	−.7200	−.8400
.4540	--------	--------	--------	--------	--------
.5878	−.9467	−.6600	−.5000	−.4800	−.7200
.7071	--------	--------	--------	--------	--------
.8090	−.7200	−.5667	−.4867	−.4067	−.5467
.8910	--------	--------	--------	--------	--------
.9511	−.6200	−.5000	−.4500	−.3600	−.4267
.9877	--------	--------	--------	--------	--------
1.00	−.5600	−.4867	−.3800	−.3400	−.2333

TABLE XIII.—FUNCTION $-x\frac{dP}{dx}$ AGAINST x FOR $\lambda=\frac{1}{2}$

x_1 \ x	0.1564	0.3090	0.5878	0.8090	0.9511	1.00
$\tau=60°$						
0.1564	0.399	0.890	1.044	0.964	0.933	0.9367
.3090	.194	.570	.877	.787	.810	.8767
.5878	.105	.261	.526	.623	.695	.7300
.8090	.074	.159	.375	.460	.532	.5850
.9511	.055	.124	.309	.379	.448	.5033
1.00	.050	.111	.280	.347	.417	.4767
$\tau=120°$						
0.1564	0.526	0.662	0.785	0.834	0.859	0.8633
.3090	.316	.457	.609	.690	.731	.7467
.5878	.162	.269	.411	.508	.562	.5800
.8090	.117	.200	.321	.406	.453	4700
.9511	.085	.154	.266	.342	.391	.4100
1.00	.075	.142	.249	.328	.375	.3967
$\tau=180°$						
0.1564	0.507	0.679	0.736	0.823	0.908	0.9333
.3090	.264	.465	.611	.663	.752	.7933
.5878	.118	.238	.394	.482	.579	.6183
.8090	.064	.149	.273	.377	.454	.4867
.9511	.044	.106	.208	.285	.358	.4000
1.00	.041	.101	.184	.260	.334	.3693
$\tau=240°$						
0.1564	0.397	0.6999	0.924	0.987	1.0090	1.0133
.3090	.147	.431	.771	.905	.973	.9967
.5878	.008	.091	.381	.663	.816	.8533
.8090	−.020	.025	.159	.400	.598	.6650
.9511	−.018	.002	.059	.228	.4015	.4733
1.00	−.014	−.008	.042	.194	.363	.4333
$\tau=300°$						
0.1564	0.6200	0.9070	1.1330	1.216	1.209	1.1900
.3090	−.2030	.3380	1.0800	1.3222	1.350	1.3400
.5878	−.2074	−.1591	.3590	1.021	1.360	1.4133
.8090	−.1372	−.2385	−.1734	.402	.970	1.1567
.9511	−.1175	−.2148	−.2933	−.025	.465	.6700
1.00	−.0940	−.2042	−.3245	−.067	.284	.4333

TABLE XIV.—FUNCTION $-x\frac{dP}{dx}$ AGAINST x FOR $\lambda=1$

x_1 \ x	0.1564	0.3090	0.5878	0.8090	0.9511	1.00
$\tau=60°$						
0.1564	0.450	0.943	1.020	0.990	0.964	0.9507
.3090	.130	.463	.970	.918	.940	.9733
.5878	.050	.163	.568	.778	.811	.8133
.8090	.038	.117	.348	.602	.660	.6033
.9511	.035	.105	.290	.468	.570	.5967
1.00	.029	.099	.270	.427	.528	.5533
$\tau=120°$						
0.1564	0.515	0.711	0.817	0.849	0.853	0.8533
.3090	.318	.480	.640	.7295	.778	.7933
.5878	.167	.292	.466	.559	.610	.6267
.8090	.124	.228	.375	.464	.501	.5100
.9511	.106	.191	.336	.426	.460	.4633
1.00	.095	.179	.318	.404	.440	.4467
$\tau=180°$						
0.1564	0.518	0.668	0.770	0.814	0.834	0.840
.3090	.307	.470	.622	.7105	.748	.760
.5878	.162	.283	.419	.510	.560	.577
.8090	.110	.191	.319	.403	.4490	.460
.9511	.087	.158	.2705	.350	.395	.410
1.00	.080	.147	.2505	.326	.369	.380
$\tau=240°$						
0.1564	0.442	0.733	0.890	0.937	0.9505	0.9533
.3090	.205	.460	.688	.794	.8500	.8667
.5878	.068	.198	.415	.565	.6400	.6600
.8090	.022	.095	.256	.379	.456	.4833
.9511	.002	.052	.163	.274	.364	.4033
1.00	.0006	.039	.134	.235	.324	.3633
$\tau=300°$						
0.1564	0.241	0.967	1.1580	1.1033	1.0533	1.0300
.3090	−.051	.298	1.1266	1.2224	1.1300	1.0733
.5878	−.175	−.259	.516	.995	1.0330	.9533
.8090	−.128	−.244	−.205	.314	.6067	.6407
.9511	−.108	−.230	−.330	−.087	.215	.3333
1.00	−.109	−.214	−.308	−.080	.185	.2900

TABLE XV.—FUNCTION $-x\frac{dP}{dx}$ AGAINST x FOR $\lambda=2$

x_1 \ x	0.1564	0.3090	0.5878	0.8090	0.9511	1.00
$\tau=60°$						
0.1564	0.880	1.019	1.061	1.040	0.9985	0.9733
.3090	.097	.723	.834	.939	1.023	1.0533
.5878	.035	.282	.448	.642	.861	.9467
.8090	.027	.110	.305	.509	.6615	.7200
.9511	.021	.081	.235	.422	.568	.6200
1.00	.017	.067	.210	.384	.514	.5600
$\tau=120°$						
0.1564	0.473	0.655	0.856	0.922	0.918	0.9067
.3090	.260	.480	.781	.882	.8195	.7600
.5878	.142	.301	.583	.750	.700	.6600
.8090	.117	.239	.442	.560	.5775	.5667
.9511	.1035	.221	.379	.462	.495	.5000
1.00	.0942	.210	.358	.441	.477	.4867
$\tau=180°$						
0.1564	0.551	0.707	0.752	0.740	0.7550	0.7733
.3090	.373	.516	.581	.612	.631	.6400
.5878	.220	.327	.419	.465	.491	.5000
.8090	.153	.256	.332	.379	.451	.4867
.9511	.124	.200	.281	.327	.394	.4500
1.00	.122	.197	.262	.301	.351	.3800
$\tau=240°$						
0.1564	0.439	0.689	0.893	0.914	0.832	0.7467
.3090	.208	.406	.708	.814	.788	.7200
.5878	.077	.200	.460	.646	.585	.4800
.8090	.043	.119	.322	.489	.452	.4067
.9511	.033	.100	.278	.3996	.391	.3600
1.00	.029	.091	.262	.381	.376	.3400
$\tau=300°$						
0.1564	0.847	1.087	1.202	1.121	0.943	0.8000
.3090	.046	.482	1.000	1.118	1.039	.8400
.5878	−.149	−.040	.481	.862	.837	.7200
.8090	−.127	−.161	.162	.498	.5535	.5467
.9511	−.107	−.160	.045	.281	.412	.4267
1.00	−.100	−.140	−.008	.166	.230	.2333

TABLE XVI.—VALUES OF $\sum -x\frac{dP}{dx}$ AGAINST x FOR 3-BLADE AND 6-BLADE PROPELLERS

[For 3-blade propeller, $\tau=120°$ and 240°; for 6-blade propeller, $\tau=60°$, 120°, 180°, 240°, and 300°]

x_1 \ x	0.1564	0.3090	0.5878	0.8090	0.9511	1.00
3-blade propeller; $\lambda=\frac{1}{2}$						
0	0	0	0	0	0	0
.1564	.923	1.361	1.709	1.821	1.868	1.8766
.3090	.463	.888	1.380	1.595	1.704	1.7434
.5878	.170	.360	.792	1.171	1.378	1.4333
.8090	.097	.225	.480	.806	1.051	1.135
.9511	.067	.156	.325	.570	.7925	.8833
1.00	.061	.134	.291	.522	.738	.8300
3-blade propeller; $\lambda=1$						
0.1564	0.957	1.444	1.707	1.786	1.8035	1.8066
.3090	.523	.940	1.328	1.5235	1.628	1.660
.5878	.235	.490	.881	1.124	1.250	1.2867
.8090	.146	.323	.631	.843	.957	.9933
.9511	.108	.243	.499	.700	.824	.8666
1.00	.0956	.218	.452	.639	.764	.8100
3-blade propeller; $\lambda=2$						
0	0	0	0	0	0	0
.1564	.912	1.344	1.749	1.836	1.750	1.6534
.3090	.468	.886	1.489	1.696	1.6075	1.5067
.5878	.219	.501	1.043	1.396	1.285	1.1400
.8090	.160	.358	.764	1.049	1.0295	.9734
.9511	.1365	.321	.657	.8616	.886	.8600
1.00	.1232	.301	.620	.822	.853	.8267
6-blade propeller; $\lambda=\frac{1}{2}$						
0	0	0	0	0	0	0
.1564	2.4490	3.837	4.622	4.824	4.918	4.9366
.3090	.718	2.261	3.948	4.367	4.616	4.7534
.5878	.1856	.6999	2.071	3.297	4.012	4.1949
.8090	.0978	.2945	.9546	2.045	3.007	3.3634
.9511	.0485	.1712	.5487	1.209	2.0635	2.4566
1.00	.0580	.1418	.4305	1.062	1.773	2.1093
6-blade propeller; $\lambda=1$						
0	0	0	0	0	0	0
.1564	2.166	4.022	4.655	4.6933	4.6548	4.6333
.3090	.909	2.171	4.0466	4.3654	4.446	4.4666
.5878	.272	.677	2.384	3.407	3.654	3.6303
.8090	.166	.387	1.093	2.162	2.6733	2.7633
.9511	.122	.276	.7295	1.431	2.004	2.2066
1.00	.0956	.250	.6645	1.312	1.846	2.0333
6-blade propeller; $\lambda=2$						
0	0	0	0	0	0	0
.1564	3.190	4.157	4.764	4.737	4.4465	4.200
.3090	.984	2.607	3.904	4.365	4.3005	4.040
.5878	.325	1.070	2.391	3.365	3.474	3.3067
.8090	.213	.563	1.563	2.435	2.7005	2.7268
.9511	.1745	.442	1.218	1.8916	2.260	2.3567
1.00	.1622	.425	1.084	1.673	1.948	2.000