Modal Parameter Extraction Using Natural Excitation Response Data

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ABSTRACT

The use of natural excitation response data for the extraction of modal parameters has been an alluring idea for many years. The primary reason is that it offers the real world inputs (both spatial and temporal) and the associated responses of the system without the cost of a complex excitation system. The use of NExT allows for a linear representation of the system at operating levels, which is ideal for predictive linear simulation. The NExT parameter estimation methods have relied on using standard modal parameter extraction routines that do not exploit the special model form of NExT data. A parameter estimation method is developed here that is consistent with the form, thereby providing a more robust estimator in the presence of noise. This paper presents the basic methods used in NExT as well as some of the critical issues when using NExT.

NOMENCLATURE

This paper utilizes the IMAC standard nomenclature defined with the following additions:

NExT Natural Excitation Technique
RRD Random Decrement Technique
XP Cross Power Function
PE Parameter Estimation

INTRODUCTION

It is recognized that the modal parameters are important for understanding the system's response, but the operating responses offer the description of how the system responds (both linearly and nonlinearly) to the forces into the system (spatial and temporal) from normal operations. The Natural Excitation Technique (NExT) is a method that uses in-situ response data to estimate the modal parameters of a system.

When using NExT, the system's responses are monitored during normal operation and the cross powers are generated between the response and the selected reference sensors. This cross power (or the cross correlation) is then used directly as an input to a standard modal PE algorithm replacing the customary FRF (or IRF) of the system. The resulting output is the system's modal parameters.

Of course, in the implementation of any technique, there are details that cannot be overlooked. There are three significant issues that must be addressed when using NExT: the assumption of a pink noise input, the selection of independent reference signals, and the lack of a tailored PE algorithm. The first two problems are true for all in-situ techniques. The PE algorithm problem is unique to NExT and Random Decrement (RRD data converges to the same data set gathered by NExT). This paper presents the basic ideas of NExT, the problem areas, and a parameter estimation algorithm developed for NExT.

Data Acquisition and Reduction

The basic method of acquiring data for a NExT test is very similar to the techniques used in a standard modal test. Typically, a large number of accelerometers are placed throughout the test article. The complexity of the system and the acceptable fidelity of the model dictate the number of responses required. If it is expected that spatial filtering will be necessary to separate modes, then number of response transducers could easily be in the hundreds. The reference transducers are then selected as a subset of the response accelerometers. Next the test article is subjected to its normal operating regime, and the acquisition system gathers and stores the data. Given the typical nature of in-situ data, there is usually a high level of noise and possible nonlinear responses

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that will require long averaging times. Either in real
time or as a postprocessing effort, the selected
reference transducers are used as inputs to a cross-
power reduction routine. Alternatively a cross-
correlation routine may be used but fundamentally, the
results should be equivalent. Finally, the reduced data
is used in a parameter estimation algorithm to extract
estimates of the modal parameters. This data
sequence is shown in the following figure.

In-Situ
Excitation

In-Situ
Response

References

Figure 1 - Data path for NexT.

The basic data reduction is the generation of the cross
power spectra (XP) for the given reference signals. In
the case of multiple inputs, a MIMO XP is calculated.
Although the calculation of the XP is a common
practice, it is seldom used for direct input to the modal
parameter extraction algorithms which were
developed using the FRF data model. In order to
understand the subtle difference, Equation 1 presents
the relationship of the XP to the SISO FRF in the
frequency domain.

\[
G_{XY} = S_X S_Y^* \\
= \frac{S_X S_F^*}{S_F^*} \left( S_Y S_F^* \right)^* \tag{1}
\]

Two observations can be made from Equation 1:

1. The computation of the XP is directly affected by
   the auto-power of the system's true input.

2. The computation of the XP is directly affected by
   the FRF of channel 1 (the response) and the
   complex conjugate of the FRF of channel 2 (the
   reference).

Input Requirement

A basic assumption of NE\textsc{x}T is that the input to the
system is white noise. To the degree that it is not, the
parameter estimation can have significant errors. In
many cases a structure reacts with its boundary
conditions effectively coloring the input, whereas in
other cases there is relatively little impedance
matching of the excitation to the structure (i.e., a wind
turbine). In the case of a vehicle, although the road
input may be relatively white noise displacement, the
force input at the tires becomes colored through the
process described in this section. It should be pointed
out here that by measuring the forces into the tire, the
resulting system identification is for the free-free
system, rather than that for the vehicle with its tire
patch fixed to ground.

Figure 2 shows a cartoon of the vehicle suspension
system and the associated lumped mass
representation. The primary excitation of the vehicle is
the road surface. This displacement is fed through the
tire dynamics to the spindle to create a force through
the spindle to the vehicle. The spindle forces are a
combination of road excitation, tire stiffness, and
vehicle dynamics (applied as an impedance
backforce).

Figure 3 presents the flowchart of the way in which the
forces are colored through the system. The dynamics
of the vehicle play an important part in determining the
PSD of the input to the system; possibly a larger
influence than the type of road surface exciting the
system itself. There are system dynamics (the spindle
driving point FRF) that are fed back through the tire
stiffness to produce the forces through the spindle.
Depending on the character of the tire stiffness and
the vehicle dynamics, the force can be colored
significantly.
Equation 2 provides the mathematical description of the flowchart. The denominator of the equation tends to color the input noise. The extent of the coloring is determined by the relative stiffness of the tire to the spindle (impedance matching) as well as the dynamics of the spindle. If the stiffness of the tire is high, then the coloring is inversely proportional to the spindle dynamics (where the spindle dynamics are the driving point FRF at the spindle without the tire).

\[
F_{\text{spindle}} = \frac{K_{\text{Tire}}}{1 - K_{\text{Tire}} \ast H_{\text{Spindle}}} \tag{2}
\]

Obviously, the coupling of the system and the excitation is dependent on the impedance matching of the system with the mechanism for imparting the forces. In the case of a vehicle, there is a good match and therefore, the input forces are highly colored as shown in the flowchart. In the case of a wind turbine, the turbulence forces are reasonably uncoupled with the structure, and therefore, the input forces remain uncolored. It is important to analyze the system which is being tested to determine the extent of the coloring of the forces since successful analysis requires a white noise input.

An alternative viewpoint for a vehicle can be presented that has equivalent external forces at the spindle. In that case these virtual spindle forces turn out to be nominally the same character as the road displacements. The result of NEXT analysis using this view is the modal parameters of the vehicle with the tire dynamics as if excited at the spindle locations, with the tire patches tied to ground.

Selection of the Reference Transducers

The selection of the reference transducers is a critical step in the NEXT implementation. In typical modal analysis, the selection of reference transducers is very easy since the input is well known and controlled. The input for operational data is usually not well known nor is the path of those distributed forces. Therefore, it is often very difficult to determine the sensors which should be used as references.

In general, the references should include the sensors which observe all pertinent elements of the input (the chosen sensors should capture the space defined by the independent force inputs). The selection of the reference sensors can be very difficult and may require several iterations to converge on the proper set. A good alternative to performing XP reduction real time is to acquire raw data and use a postprocessing approach. By using the postprocessing alternative, a virtual force technique can be used to help determine the number of virtual forces and the optimal sensors to use as references or virtual references. Of course, the cost of postprocessing is the large quantity of stored raw data that is required for proper averaging.

The errors associated with undersampling the references (less references than independent inputs) would be similar to omitting a reference in a MIMO FRF test; that is, unaccounted energy in the system that will affect the quality of the data set used in the parameter estimation algorithm.

If the references are oversampled (more references than independent inputs), then the XP will create additional columns of information which have dependence upon other columns, thereby creating an ill-conditioned matrix to be inverted. Besides the additional computational cost (noise and time), it is unclear if there is any significant degradation in the analysis, since most MIMO PE algorithms will inherently condense this data down.

NEXT Model Form versus Modal Form

It was shown in Equation 1 that the XP format of NEXT was different from the FRF format used in typical modal analysis. Additional information can be gained by expanding Equation 1 as it relates to the system's modal parameters. Assuming that the system poles are global, the two FRF terms can be expanded as follows:
As can be seen by Equation 3, although the original SDOF system had only two system poles, the crosspower format adds two computational poles that are reflected about the imaginary axis. In effect this adds two computational poles, which have negative damping, which are never included in the typical modal PE algorithms. The addition of these computational poles is shown in Figure 4.

To further illustrate the effect, a two degree of freedom (2-DOF) system is presented with stable modes at 0.5 and 2 Hertz. Figure 5 presents a typical set of FRFs and the cross power formatted data. The upper plot is the phase of the FRF, and the lower plot shows the associated magnitude. There are three functions in each plot: the driving point FRF, the cross point FRF, and the system relative cross power. From this plot the fact that the cross power format does not follow the FRF format is seen in the character of the phase (lack of phase change at the first and seconds mode).

Figure 6 presents the associated time domain data for the functions given in Figure 4. The upper plots are the impulse response functions for the stable system poles, the middle plots are the impulse response functions for the computational unstable system poles, and the lower plots are the impulse response functions for the XP (cross-correlation function). As can be seen in the upper plot of figure 6, the IRF associated with the stable poles are decaying exponentials. Alternatively the middle plots of the unstable computational poles is a growing exponential function. The resulting correlation function in the bottom plot is a 'bow tie' exponential.

As can be seen by comparing the stable system's response with the 'bow tie' correlation function, there is some data near the start of the impulse that appears to have little contamination affect from the
computational poles. Using this portion of the data in a time domain PE algorithm may result in a good estimate of the true system poles, since there is very little influence from the computational poles. As more data is required for the PE, then more errors will be incurred in the PE.

Figure 7 presents the same system as Figure 6 but using a slightly lower frequency resolution. In this case, the effects of the unstable poles map into the IRF throughout the entire data set. In fact, it is difficult to see any data that would be suitable for standard time domain PE that would not be severely affected by the computational modes.

Figure 7 – FRFs and XP for 2-DOF system.

By comparing the two figures, it is easy to see that if time domain algorithms are to be used for the analysis, then the data set should have very large time blocks to reduce the effects of the unstable poles or these unstable poles will have to be accounted for in the parameter estimation process.

Two additional comments should be noted:

- The frequency domain methods do not suffer from this lack of frequency resolution affects.
- The ‘mode’ shapes associated with the crosspower should be consistent with the system’s mode shapes.

Current Parameter Estimation Methods

The most obvious problem with using the traditional routines is that the form of the PE model is does not take full advantage of the form of the reduced data format. Fortunately, the modal model encompasses unstable poles; but it does not enforce symmetry about the imaginary axis. The standard PE could legitimately be used on the data, but it requires much more user interaction; and its result may be more artwork than science for many applications.

Any of the standard time domain algorithms (ERA, Ibrahim, PolyReference, etc.) could be used, but special attention should be paid to the data that is selected. It should be noted that depending on the selection of the data used for the analysis, there might be four (4) poles per mode instead of just two due to the XP poles. In the manual selection of which poles to keep, it should be expected that reflected unstable modes would appear in the computational pole list. If there is good reflection with the computational unstable poles, then this is a good indication of a true system pole as opposed to a standard computational pole. Again proper acquisition and data selection may reduce the error due to the reflected poles when using time domain algorithms.

All of the standard frequency domain algorithms (DPE, PolyReferenceFreq etc.) can also be used. Since the reflected poles will be unavoidable, the data selection is not as critical in this case; therefore, there will always be four (4) poles per mode, and the PE routine will ideally extract all four poles.

It is easy to imagine a Frequency Domain PE technique that will enforce the four pole-per-mode requirement: the technique could use the model directly in a linear fashion, or it could be iterative. In the time domain it is more difficult to imagine a method, since the contribution of the unstable pole is inconsistent depending on the selected data set.

Using Equation 3 as a starting point, the model for the XP data set can be developed:

\[
H_x \times H_r^* = \frac{b}{a_1(j\omega)^2 + a_1(j\omega)^3 + a_0} \times \left( \frac{c}{a_2(j\omega)^2 + a_1(j\omega)^3 + a_0} \right)
\]

\[
= \frac{bc}{a_2(j\omega)^4 + a_2(j\omega)^3 + a_0}
\]
where:

\[ \alpha_2 = a_2^2 \]
\[ \alpha_1 = 2a_0a_2 - a_1^2 \]
\[ \alpha_0 = a_0^2 \]

Using this as a parameter estimation model in the frequency domain, the alphas can be solved for directly in a least squares sense similar to the method used in DPE. The alphas can then be transformed to the poles of the system which will result in four poles per system mode (reflected about the imaginary and real axes). The obvious advantage of this technique is that the PE model will then enforce the requirements imposed by the data form. The expected result would be an estimator that is more robust to noise in the measurements.

The implementation of this new model can be accomplished using a small perturbation of the standard frequency domain algorithms. A perturbation of the DPE algorithm was performed and tested on the example case. The example case consisted of the simple 2-DOF system, where the first mode was placed at 0.516 Hz with 1.0 percent damping, and the second mode was placed at 2.0 Hz with 2.0 percent damping. The table below shows the results of the standard DPE method compared to the NExT DPE method for first mode of the 2-DOF system.

Table 1 NExT PE algorithm results of a 2-DOF system.

<table>
<thead>
<tr>
<th>Method</th>
<th>Pole (Freq/Damp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DPE (4roots)</td>
<td>0.517 Hz, 0.9%</td>
</tr>
<tr>
<td>NExT-DPE (2 roots)</td>
<td>0.516 Hz, 1.1%</td>
</tr>
</tbody>
</table>

The use of the NExT-DPE solving for two roots showed the ability to extract the fully symmetric roots of the NExT formatted data (although there is some error that needs to be identified). Given that the mode for this modified routine conforms to the model of the data, it should be much less sensitive to noise on the system. The next step in development of this PE algorithm is to apply it directly to the MIMO/MDOF PE format.

**Uncertainty**

As with any testing and analysis method, NExT is subject to errors which can be introduced throughout the process, causing uncertainty in the results. Since NExT is based on experimental modal analysis, it shares many of the same sources of uncertainty. Within the data collection effort, errors can come from inaccurate calibrations of the sensors and the data acquisition system as well as in the signal processing parameters. Given good experimental practices, the cumulative error of the processed data (cross power) can be better than 5 percent in magnitude, 5 degrees in phase and 0.01 percent in frequency. Assuming that high quality processed data can be achieved, then the most significant uncertainties arise during the parameter estimation process. In typical modal parameter estimation, there are several parameters that the operator uses to tune his or her 'fit' of the data. The user defines the 'fit'. Given the same data set, this 'fit' can vary significantly and therefore the uncertainty in frequency may be 0.5 percent and 50 percent in damping. Of course, the variation in results is much less as the quality and quantity of the data increases or as the structure becomes simpler.

Since NExT utilizes crospower data instead of FRFs, there are a few additional sources of uncertainty: assumption of a white noise input, the selection of the reference set, and the additional poles created by the data form. It is practically impossible to quantify errors associated with the character of the input since the input is not measurable (by definition since FRFs would be generated if they were available). The colored noise input to the system will most likely affect the estimate of the damping. The selection of the reference transducers is critical in successfully accounting for all of the independent inputs to the system. If all inputs are not accounted for then the extraneous input energy will affect the quality of the parameter estimation. The doubling of poles for the system creates additional problems. The traditional parameter estimation methods do not utilize the
symmetry for the four poles per mode and thereby effectively has one-half the averages of the presented parameter estimation algorithm. Therefore, if all other pre-processing parameters are the same, the use of the new estimation procedure should statistically increase the confidence band by a factor of two for the parameter estimation task.

Conclusions

The use of NExT can be a very useful technique to help define the modal parameters of many systems when exposed to in-situ operations. The success of the analysis is determined by the character of the input forces, the selection of the reference transducers, the linearity of the system, the signal processing parameters, and the parameter estimation methods. If the application is suited to these constraints, then a data set can be obtained that contains the modal parameters of the system. The current methods for extracting modal parameters from NExT data appears to be limited to the traditional modal PE routines. A frequency-domain parameter estimation routine was presented here that was designed for NExT data. The fact that this method utilizes the fundamental form NExT data should make it a good estimator in the presence of noise.

References
