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INHOMOGENEOUS PRIMORDIAL NUCLEOSYNTHESIS AND NEW ABUNDANCE CONSTRAINTS ON $\Omega_b h^2$

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ABSTRACT

We discuss the upper limit to the baryonic contribution to the closure density. We consider effects of new observational and theoretical uncertainties in the primordial light element abundances, and the effects of fluctuation geometry on the inhomogeneous nucleosynthesis yields. We also consider implications of the possible detection of a high D/H abundance in a Lyman-α absorption cloud at high redshift and the implied chemical evolution effects of a high deuterium abundance. We show that there exists a region of the parameter space for inhomogeneous models in which a somewhat higher baryonic contribution to the closure density is possible than that allowed in standard homogeneous models. This result is contrary to some other recent studies and is due to both geometry and recently revised uncertainties in primordial light-element abundances, particularly $^7$Li. We find that the presently adopted abundance constraints are consistent with a contribution of baryons to the closure density as high as $\Omega_b h^2_{50} \leq 0.11$ ($\eta \leq 7 \times 10^{-10}$). This corresponds to a 20% increase over the limit from standard homogeneous models ($\Omega_b h^2_{50} \leq 0.08$ , $\eta \leq 5.8 \times 10^{-10}$). With a high deuterium abundance the upper limits for the inhomogeneous and homogeneous models would be $\Omega_b h^2_{50} \leq 0.04$ and 0.03 ($\eta \leq 2.6 \times 10^{-10}$ and 1.9 $\times 10^{-10}$), respectively. Even higher limits could be obtained by further relaxing the presently accepted primordial lithium abundance constraint as some have proposed.

Subject headings: early universe - abundances, nuclear reactions, nucleosynthesis, cosmology - dark matter, galactic evolution
1. Introduction

Calculations of standard homogeneous big bang nucleosynthesis (hereafter; HBBN) provide an important independent determination of the baryon content of the universe. Observed light-element abundances of $^2$H, $^3$He, $^4$He, and $^7$Li agree well with calculated primordial nucleosynthesis abundance yields for $\Omega_b^{\text{HBBN}} \approx 0.046 h_5^2 T_{2.75}^{-2}$ (Wagoner, Fowler, & Hoyle 1967; Wagoner 1973; Schramm & Wagoner 1977; Yang et al. 1984; Krauss & Romanelli 1990; Walker et al. 1991, Smith, Kawano, & Malaney 1993) (see however Hata et al. 1995). Here, $h_5$ is the Hubble constant in units of 50 km s$^{-1}$ Mpc$^{-1}$, and $T_{2.75}$ is the present microwave background temperature in units of 2.75 K.

When computational, observational, and nuclear reaction rate uncertainties are taken into account (Smith et al. 1993; Copi, Schramm & Turner 1995; Schramm & Mathews 1995) the allowed range for $\Omega_b^{\text{HBBN}}$ is

$$0.04 \lesssim \Omega_b^{\text{HBBN}} h_5^2 T_{2.75}^{-2} \lesssim 0.08,$$

where the lower limit on $\Omega_b^{\text{HBBN}}$ arises mainly from the upper limit on the deuterium plus $^3$He abundance (Walker et al. 1991; Smith et al. 1993) and the upper limit on $\Omega_b$ arises from the upper limit on the $^4$He abundance of $Y_p \lesssim 0.245$ and/or the lower limit on the deuterium abundance $D/H \geq 1.6 \times 10^{-5}$.

Current estimates of the Hubble constant range between $0.8 \lesssim h_5 \lesssim 1.7$ (cf. van den Bergh 1989) although a value greater than 1 is generally preferred. The present best determination of the microwave background temperature from the COBE satellite is $2.726 \pm 0.010$ K (Mather et al. 1990; 1994). The weighted mean of the COBE measurement with others at wavelengths greater than 1 mm is $2.76 \pm 0.10$ (Smith et al. 1993). Since the value of $T_{2.75}$ is so close to unity and its uncertainty so insignificant, we omit this factor in the subsequent discussion (although its presence is implied).

The fact that this range for $\Omega_b h_5^2$ is so much greater than the current upper limit to the contribution from luminous matter $\Omega_b^{\text{Lum}} \lesssim 0.01$ (Jedamzik Fuller & Mathews 1995) is one of the strongest arguments for the existence of baryonic dark matter.

In this context, possible detections (e.g. Songaila et al. 1994; Carswell et al. 1994) of an isotope-shifted Lyman-$\alpha$ absorption line at high redshift along the line of sight to a quasar are of considerable interest.

These observations could imply a deuterium abundance of $1.9 \times 10^{-4} \lesssim (D/H) \lesssim 2.5 \times 10^{-4}$. If this value is interpreted as a primordial abundance then it is significantly larger than the previously accepted upper and lower limits on either $D/H$ or $[D + ^3\text{He}]/H$ (e.g. Walker et al. 1991). It is not yet clear, however, whether the new abundance for $(D/H)$ should be accepted because the probability of a systematic error from an intervening cloud at a lower redshift is significant.

If the primordial deuterium abundance were as large sd $1.9 \times 10^{-4} \lesssim (D/H) \lesssim 2.5 \times 10^{-4}$, then the allowed range of $\Omega_b$ inferred from HBBN changes to

$$0.022 \lesssim \Omega_b^{\text{HBBN}} h_5^2 \lesssim 0.026.$$

(Jedamzik, Fuller & Mathews 1994a; Krauss & Peters 1994; Vangioni-Flam & Casse 1995). In this case, particularly if $h_5$ is greater than $\sim 1.5$, then the big bang prediction could be so close to the the baryonic density in luminous matter that little or no baryonic dark matter is required (Jedamzik et al. 1995). This could be in contradiction with observation, particularly if the recently detected microlensing events (Alcock et al. 1993; Aubourg et al. 1993) are shown to be baryonic. This is also contrary to evidence (White et al. 1993) that baryons may contribute a large fraction of the closure density in the form of hot X-ray gas in dense galactic clusters.

With this in mind, it is worthwhile to consider alternative cosmologies in which a high primordial deuterium abundance can be maintained while allowing for a larger contribution to the closure density from baryonic matter. In this context, inhomogeneous big bang nucleosynthesis (hereafter IBBN) may offer an attractive possibility. It has been appreciated for some time (e.g. Zel'dovich 1975; Wagoner 1973; Applegate Hogan & Scherrer 1987; Mathews et al. 1990) that IBBN models might produce a high primordial deuterium abundance in a universe with a large $\Omega_b h^2$.

The purpose of this paper is, therefore, to discuss the recent issues surrounding IBBN models. Adopting the presently preferred (Copi et al. 1995) limits of $(D/H) \geq 1.6 \times 10^{-5}$ and $(\text{Li}/H) \leq 3.5 \times 10^{-10}$, we find that a baryonic contribution as high as $\Omega_b h_5^2 \lesssim 0.11$ ($\Omega_b \lesssim 0.15$) is possible. Even with a high primordial deuterium abundance, we find that a baryonic contribution as high as $\Omega_b h_5^2 \lesssim 0.040$ which is nearly twice the HBBN upper limit.

These upper limits to the baryonic contribution to closure in IBBN models are somewhat higher than
those quoted in other recent work (e.g. Kurki-Suonio et al. 1990; Thomas et al. 1994) where it was concluded that the limits on the baryon to photon ratio are not much different than those allowed in the standard HBBN model. The higher limits in the present work follow mainly from the recent evidence for a somewhat larger uncertainty in the primordial $^7$Li abundance than has been previously adopted. The present results also derive to some extent from considering other fluctuation geometries, and from the effects of the newest nuclear reaction rates. All of these effects tend to increase the upper limit to $\Omega_b h^2$.

2. Baryon Inhomogeneous Primordial Nucleosynthesis

Primordial nucleosynthesis in an environment with an inhomogeneous distribution of baryon-to-photon ratio has been the focus of considerable study in recent years (Alcock, Fuller, & Mathews 1987; Applegate et al. 1987; 1988; Fuller, Mathews, & Alcock 1988; Kurki-Suonio et al. 1988; 1990; Malaney & Fowler 1988; Boyd & Kajino 1989; Terasawa & Sato 1989abc; 1990; Kajino & Boyd 1990; Kurki-Suonio & Matzner 1989; 1990; Mathews et al. 1990; 1993; Kawano et al. 1991; Jedamzik et al. 1994a; Thomas et al. 1994). Such studies were originally motivated (Applegate and Hogan 1985; Applegate et al. 1987) from suggestions (e.g. Witten 1984) that a first-order cosmic QCD-phase transition in the early universe might lead to an inhomogeneous spatial distribution of baryons. Even though lattice QCD has not provided convincing evidence for a strongly first order QCD transition (e.g. Fukugita & Hogan 1991), the order of the transition must still be considered as uncertain (Gottlieb 1991; Petersson 1993). It depends sensitively on the number of light quark flavors. The transition is first order for 3 or more light flavors and second order for 2. Because the $s$-quark mass is so close to the transition temperature it has been difficult to determine the order. At least one recent calculation (Iwasaki et al. 1994) indicates a clear signature of a first order transition when realistic $u$, $d$, $s$ quark masses are included, but others indicate second order or not a phase transition at all.

In view of this uncertainty, it seems to us to be worthwhile to explore the maximal cosmological impact that can occur. We do note, however, that this maximal impact may require a relatively strong first-order phase transition and sufficient surface tension of nucleated hadron bubbles to generate an optimum separation distance between baryon fluctuations (Fuller et al. 1988). Several recent lattice QCD calculations (e.g. Kajantie, Kärkkäinen & Rummukainen 1990; Brower et al. 1992) indicate that the surface tension is too small to allow sufficient fluctuation separation distance. However, such calculations are still far from the continuum and do not include effects of internal quark loops (Brower et al. 1992). Furthermore, even if the surface tension is low, the dynamics of coalescence and merger of hadron droplets may nevertheless lead to large separations between regions of shrinking quark-gluon plasma. Hence, we are of the opinion that it may be a bit premature to conclude (as some have, e.g. Reeves 1994) that a low value for the surface tension is well established.

Furthermore, even should the QCD transition be unable to generate baryon inhomogeneities, there remain a number of alternative mechanisms for generating them such as electroweak baryogenesis (Jedamzik et al. 1994b), inflation generated isocurvature fluctuations (Dolgov & Silk 1993), kaon condensation (Nelson 1990) or magnetic fields from superconducting cosmic strings (Malaney & Butler 1989), [see Malaney & Mathews (1993) for a recent review].

Therefore, independently of the source of baryon inhomogeneities, it is worthwhile to consider the limits on the baryon-to-photon ratio $\eta$ allowed in IBBN models. A number of papers have addressed this point (Alcock, et al. 1987; Applegate et al. 1987; 1988; Fuller et al. 1988; Kurki-Suonio et al. 1988; 1990; Malaney & Fowler 1988; Terasawa & Sato 1989abc; 1990; Kajino & Boyd 1990; Kurki-Suonio & Matzner 1989; 1990; Mathews et al. 1990; 1993; Jedamzik et al. 1994a; 1994b; Thomas et al. 1994). Most recent studies in which the coupling between the baryon diffusion and nucleosynthesis has been properly accounted for (e.g. Terasawa & Sato 1990; Kurki-Suonio et al. 1990; Mathews et al. 1990; 1993; Jedamzik et al. 1994a; 1994b) have concluded that, for spherically condensed fluctuations, the upper limit on $\Omega_b h^2$ is virtually unchanged when compared to the upper limit on $\Omega_b h^2$ derived from standard HBBN. It is also generally believed (e.g. Vangioni-Flam & Casse 1995) that the same holds true if the new high D/H abundance is adopted.

Here, however, we emphasize several points regarding the constraints on inhomogeneous models which are not widely appreciated. One is that the previously inferred constraints on $\eta$ and $\Omega_b h^2$ are largely
fixed by the $^7\text{Li}$ abundance. This constraint, however, is relatively weakly dependent upon the baryon-to-photon ratio (compared, for example, to the deuterium constraint). It has also been recently revised upward (Copi et al. 1994) and is subject to large stellar evolution uncertainties (Pinsonneault et al. 1992). Furthermore, new reaction rates for deuterium and $^3\text{He}$ imply a lower calculated $^7\text{Li}$ abundance (Smith et al. 1993) for large $\Omega_b$ models than in some previous studies. Taking all of the above factors into account, the allowed baryon density in IBBN models can be somewhat higher than that implied by the standard HBBN model.

Another point which we consider here is the sensitivity of the upper limit of $\Omega_b$ in IBBN models to the geometry of the fluctuations. In Mathews et al. (1990) it was found that by placing the fluctuations in spherical shells rather than condensed spheres lower calculated abundances of $^4\text{He}$ and $^7\text{Li}$ were possible for the same $\Omega_b$. After all, a condensed spherical geometry is not necessarily the optimum or even the most physically motivated choice. Here we, therefore, also consider the possibility of spherical shells and cylindrical geometry (Orito et al. 1995) for the fluctuations in addition to condensed spheres. With the new reaction rates and new lithium abundance uncertainty, the spherical shell geometry geometry allows for a higher baryonic contribution to the closure density than the usually adopted condensed sphere geometry.

3. Primordial Lithium Abundance Constraint

A number of different values for the upper limit to the primordial lithium abundance have been adopted in the literature. It is, therefore, worthwhile to say a few words about them. It is convention in the literature to quote the lithium abundance relative to H = 10$^{12}$. Hence, one defines a quantity $[\text{Li}] = 12 + \log(\text{Li}/\text{H})$. One recently adopted primordial lithium abundance constraint (Walker et al. 1991) [also used in Thomas et al. (1994)] is $[\text{Li}] \leq 2.15$ ($\text{Li}/\text{H} \leq 1.4 \times 10^{-13}$). This limit is based upon a weighted mean of observations of 35 low metallicity halo stars with $T_{\text{eff}} \geq 5500$ K on the so called "lithium plateau" (Spite & Spite 1982). A limit of $[\text{Li}] \leq 2.15$ corresponds to the 2σ confidence limit above the mean value of 2.08. This upper limit was motivated somewhat by the standard main sequence models of Deliyannis et al. (1990) which imply little lithium depletion in low metallicity halo stars.

However, even in Deliyannis et al. (1990) it was pointed out that a higher limit to primordial lithium is more appropriate. By adopting conservative errors in abundance determinations for both cool and hot stars, and directly fitting a series of isochrones to the data, they obtained a 2σ upper limit of $[\text{Li}] \leq 2.21$. Including effects of diffusion into their stellar evolution code, increases this upper limit to $[\text{Li}] \leq 2.36$. This is the limit adopted in Smith et al. (1993). It represents the most conservative application of the Deliyannis et al. (1990) results. One important development since that limit was adopted is a reanalysis (Thorburn 1994) of the model atmospheres used to infer the lithium abundance which shifts $[\text{Li}]$ upward by 0.2. These data also indicate systematic variations in the lithium abundance with surface temperature, possibly indicating that some depletion has occurred. We also note another recent discussion of model atmospheres (Kurucz 1995) which suggests that as much as an order of magnitude upward shift in the primordial lithium abundance could be warranted due to the tendency of one-dimensional models to under estimate the ionization of lithium.

Related to the above it is also worth noting that when effects of rotational mixing have been added to stellar models (Pinsonneault et al. 1992) for lithium depletion, a much larger lithium depletion seems possible. This factor is largely independent of initial rotation for low metallicity stars. Furthermore, the predicted metallicity dependence of the dispersion in lithium depletion with rotation may even be necessary to account for the dispersion in the observed plateau lithium abundances. It is also noted in (Pinsonneault et al. 1992) that the rotational models with the same set of parameters and physical assumptions are capable of reproducing the very different lithium depletion patterns observed in both metal poor halo stars and population I stars in the disk which exhibit much greater lithium depletion and dispersion. This is a powerful argument for the validity of the rotational mixing models which should, perhaps, be taken seriously.

An objection to the possible large depletion factor for lithium, however, stems from recent possible detections (Smith, Lambert & Nissen 1992; Hobbs & Thorburn 1994) of $^6\text{Li}$ in two of the plateau halo stars. Since $^6\text{Li}$ should be destroyed much more rapidly than $^7\text{Li}$ (Brown & Schramm 1988), the presence of $^6\text{Li}$ argues against significant $^7\text{Li}$ destruction. On the other
hand, the $^6$Li detection is still consistent with as much as a factor of two $^6$Li destruction (Copi et al. 1995). Furthermore, it is possible (Yoshii, Mathews & Kawano 1995) that some of the $^6$Li is the result of more recent accretion of interstellar material which could occur as halo stars episodically plunge through the disk. Such a process could mask the earlier destruction of lithium. A possible way to distinguishing between accreted and primordial material might be the detections of a B/Be ratio which is consistent with IBBN or HBBN rather than the cosmic-ray ratio. The IBBN B/Be ratio from these calculations is discussed separately in Yoshii et al. (1995).

In view of the above discussion, it is our opinion, that the most realistic upper limit to the lithium abundance is probably that adopted in Copi et al. (1995), i.e. $\text{Li}/H \leq 3.5 \times 10^{-10}$. This limit includes the systematic increase from the model atmospheres of Thorburn (1994) and the possibility of as much as a factor of 2 increase due to stellar destruction (consistent with the $^6$Li observations). This is the limit which we adopt here. For comparison, however, the most extreme conservative upper limit to the lithium abundance is probably that derived from the fits to the data by Pinsonneault et al. (1992) based upon models in which rotational mixing has been included. Using a fit their isochrones to the lithium plateau, they obtained an upper limit on the primordial population II lithium abundance of $[\text{Li}] \leq 3.1$ (Li/H $\leq 1.3 \times 10^{-9}$). We also show results from this more conservative upper limit, with the caveat that this limit may not be consistent with the observed $^6$Li abundance.

4. $D/H$ and $[D + ^3He]/H$ Constraints

The upper limit to $\Omega_d^H$, will come from a combination of the abundances of lithium, $^4$He and the sum of deuterium and $^3$He, it is worthwhile to review these primordial abundances.

To begin with, the primordial abundances of deuterium and $^3$He are particularly uncertain due to the unknown degree to which they have been destroyed in stars and (in the case of $^3$He) the possible production in stars. Previously, limits on these nuclides have been inferred from abundances in presolar material (e.g. Walker et al. 1991). It is reasonable to assume that deuterium was mostly converted into $^3$He by the time that gas-rich meteorites formed, but not until after the more primitive carbonaceous chondrites formed. One can then use the abundance of $^3$He in the gas rich meteorites to infer the presolar sum of $[D + ^3He]/H$, and the carbonaceous chondrite abundance to infer the abundance of presolar $^3$He alone. The difference between the $^4$He abundance for the two meteorite classes then gives a lower limit to the deuterium abundance alone. This lower limit can also be adopted as the lower limit to the primordial deuterium abundance since the process of galactic evolution up to the time of Solar system formation could only have decreased the initial primordial abundance. There are now also accurate HST measurements (Linsky et al. 1993) of deuterium in the present interstellar medium. These are consistent with the meteoritic limits.

Following the analyses Walker et al. (1991) and Copi et al. (1995) we adopt the following limits on the presolar abundances,

$$1.6 \leq 10^5 y_{^2} \leq 3.6 \quad (3a)$$
$$1.3 \leq 10^5 y_{^3} \leq 1.8 \quad (3b)$$
$$3.3 \leq 10^5 y_{^23} \leq 4.9 \quad (3c)$$

where we use the common notation that $y$ denotes the number abundance relative to hydrogen and the subscripts denote deuterium, $^3$He, or their sum in obvious notation. From this, a lower limit to the primordial deuterium abundance of $D/H \geq 1.8 \times 10^{-5}$ is inferred. Clearly, however, this is a number which could be quite uncertain.

In order to derive a lower limit to $\Omega_d^h$, it is more useful to consider the sum of deuterium plus $^3$He. This is because the deuterium destroyed in stars is largely converted into $^3$He. However, the determination of this upper limit is subject to the uncertainty in the degree to which $^3$He and D are destroyed and/or produced in stars.

In the context of a closed-box instantaneous recycling approximation it is straightforward (Olive et al. 1990) to show that the sum of primordial deuterium and $^3$He can be written,

$$y_{^23p} \leq A_\odot (3a)^{-1} y_{^23} \left( \frac{X_\odot}{X_p} \right) , \quad (4)$$

where, $A_\odot$ is the fraction of the initial primordial deuterium still present when the Solar system formed, $g_3$ is the fraction of $^3$He which survives incorporation into a single generation of stars, $y_{^23}$ is the presolar value of $[D + ^3He]/H$ inferred from the gas rich meteorites, and $X_\odot / X_p$ is the ratio of the presolar hydrogen mass fraction to the primordial value. These
factors together imply an upper limit (Walker et al. 1991; Copi et al. 1995) of \( y_{23p} \leq 1.1 \times 10^{-4} \).

A key ingredient in previous estimates of the upper limit to \( y_{23} \) is that the astration factor be \( A_\odot \geq 1/3 \). This was based on the fact that the metallicity is also related to the astration factor in the simple one zone closed-box model, i.e.

\[
A_\odot = e^{-Z/y_Z},
\]

where \( Z \) is the metallicity and \( y_Z \) is the average metal yield for a generation of stars. Typically, \( y_Z \sim y_\odot \) which implies an astration factor of \( \geq 1/3 \) when the metallicity reaches the solar abundance.

Such an astration factor, however, cannot be consistent with a high Lyman-\( \alpha \) deuterium abundance. Adopting the presolar deuterium abundance of Eq. (3a) and a primordial deuterium abundance of \( y_{2p} = 1.9 - 2.5 \times 10^{-4} \) implies

\[
0.064 \leq A_\odot \leq 0.19
\]

Reconciling such an astration factor with the metallicity constraint in Eq. (5) requires some modification to the simple closed box with instantaneous recycling (Edmunds 1994; Vangioni-Flam & Casse 1995). For example, metallicity dependent yields (or equivalently a metallicity dependent initial mass function) such that \( y_Z \) is less at earlier times could increase the astration factor for a given metallicity. Similarly, a galactic wind (Edmunds 1995; Vangioni-Flam & Casse 1995) at early times could reduce the net metallicity enrichment for the same integrated star formation history. Neither of these additions to the simple closed-box model is particularly unrealistic, so one must not take the limit of Eq. (5) on the astration factor too seriously (see however Edmunds 1994).

It is interesting to apply the astration factor derived from the new deuterium observation to the deuterium plus \(^3\)He limit in Eq. (4), while keeping the other factors as in Walker et al. (1991), i.e. \( g_3 \geq 1/4 \) (Dearborn, Schramm, & Steigman 1986), \( y_{230}X_\odot/X_p \leq 0.422 \), this implies an upper limit of

\[
y_{230} \leq 3 \times 10^{-4}
\]

which is completely consistent with the new deuterium observation. We adopt this as the upper limit to the deuterium plus \(^3\)He abundance appropriate to a possible high Lyman-\( \alpha \) deuterium abundance.

5. \(^4\)He Constraint

The current status of \(^4\)He observations and potential systematic errors have been recently reviewed (Skillman et al. 1994; Schramm & Mathews 1995). The primordial helium abundance is generally inferred from the correlation of helium abundance with metallicity in the HII regions of compact blue irregular galaxies. The random errors in the correlation of helium with metallicity are very small due to multiple exposures, several standard stars, and good linear detectors. In principle, it is possible to obtain line ratios that are accurate to within \( \pm 2\% \) (Skillman et al. 1994). There is, however, a need for more high quality observations at low \([\text{O}/\text{H}]\). Also it is not known whether there are deviations in a linear regression at low metallicity. Such deviations might be expected from galactic chemical evolution models (Mathews et al. 1993; Balbes et al. 1993; Pagel et al. 1993). Most importantly, the uncertainties in theoretical recombination/cascade calculations are not well quantified.

Based upon an analysis (Steigman & Olive 1994) of preliminary data from Skillman, the presently inferred primordial value is \( Y_p = 0.232 \), with a statistical uncertainty of \( \pm 0.003 \), and possible systematic errors as much as \( \pm 0.005 \). This implies an upper limit of 0.245 to the primordial helium abundance which is adopted here as in other recent reviews (Copi et al. 1995; Schramm & Mathews 1995).

6. Results

The calculations described here are based upon the coupled diffusion and nucleosynthesis code of Mathews et al. (1990), but with a number of nuclear reaction rates updated. We also have implemented an improved numerical scheme which gives a more more accurate description the effects of proton diffusion, and Compton drag at late times. Although our approach does not explicitly include the effects of late time hydrodynamic expansion (Jedamzik et al. 1994a), it produces similar results for the parameters employed here. The neglect of this effect, however, may cause our \(^7\)Li abundance to be slightly overestimated implying that a slightly larger upper limit to \( \Omega_m h^2 \) may be possible than that reported here. We have also included all of the new nuclear reaction rates summarized in Smith et al. (1993) as well as those given in Thomas et al. (1993). We have found that the abundances of D, \(^3\)He, and \(^7\)Li are particularly affected by the new rates involving D and \(^3\)He which
are summarized in Table 4 of Smith et al. (1993). In that paper it was shown that standard HBBN models with high $\Omega_b h^2$ exhibit higher deuterium and lower $^7\text{Li}$ when the new reaction rates are included. We obtain the same result as Smith et al. (1993) for our IBBN model using these rates and homogeneous conditions. We also agree with results of Jedamzik et al. (1994a) for similar inhomogeneous conditions. However, our results do not agree with those of Thomas et al. (1994) in the HBBN limit or for the same IBBN conditions. We consistently find a lithium abundance which is 20-30% lower than that given in Figure 7 of the Thomas et al. (1994) paper. This discrepancy can be largely traced to differences between the more recent reaction rates for light nuclei given in Smith et al. (1993) compared to the older rates actually used in Thomas et al. (1994).

Calculations were performed in the geometry of both condensed spheres and spherical shells. The latter geometry, for example, approximates the kind of inhomogeneities which might occur in a first order QCD phase transition if the surface tension of shrinking bubbles of quark-gluon plasma is insufficient to sphericalize the bubbles. They might also approximate the kinds of fluctuations which could be induced by cosmic strings or electroweak baryogenesis (Jedamzik et al. 1994b).

In the calculations, the fluctuations are resolved into 16 zones of variable width as described in Mathews et al. (1990). We assume 3 neutrino flavors and square-wave fluctuations. Such fluctuation shapes are the most likely to emerge, for example, after neutrino-induced expansion (Jedamzik & Fuller 1994). The ratio $R$ of baryon densities in the high density to low density regions and the volume fraction $f_v$ occupied by the high density regions were optimized to allow for the highest values for $\Omega_b$ while still satisfying the light-element abundance constraints. For fluctuations represented by condensed spheres, optimum parameters are $R \sim 10^6$ and $f_v^{1/3} \sim 0.5$. For spherical shells the optimum parameters are $R \sim 10^5$ and $f_v^{1/3} = 0.125$ (Mathews et al. 1990) although there is not much sensitivity to $R$ once $R \gtrsim 10^3$.

The variable parameters in the calculation are then, the average separation distance between fluctuations $r$ and the total average baryon to photon ratio $\eta$ (or $\Omega_b h^2$), where $\eta = 6.6 \times 10^{-9} \Omega_b h^2$.

Figures 1 shows contours of allowed parameters in the $r$ vs. $\eta$ and $r$ vs. $\Omega_b h^2$ plane for condensed sphere fluctuations for the adopted light-element abundance constraints (Copi et al. 1995). The fluctuation cell radius $r$ is given in units of meters for a comoving length scale fixed at a temperature of $kT = 1\text{ MeV}$. The limits from various light-element abundance constraints (including both possible $^7\text{Li}$ limits) as discussed above are drawn as indicated.

Also, for illustration, Figure 2 shows the same contour plots for a possible high Lyman-$\alpha$ D/H and [D $+^3\text{He}]/\text{H}$ constraint. Figures 3 and 4 show the same contours for a spherical shell geometry. As in previous calculations (Mathews et al. 1990) the shell geometry models (shown in Figures 3 & 4) produce a slightly lower helium and lithium abundance than the condensed sphere geometry for the same value of $\Omega_b h^2$. One additional advantage of the spherical shell geometry is that the yields are largely independent of the fluctuation separation distance, which decreases the sensitivity of the calculation to that unknown parameter.

Calculations have also been performed with a condensed cylindrical geometry. These results will be given in Orito et al. (1995). They allow values of $\Omega_b h^2$ which are also slightly more than those produced by the spherical geometry.

Some points to note from figures 1 through 4 are: 1) that with the presently adopted primordial light-element abundances, the upper limits to $\eta$ and $\Omega_b h^2$ are now largely determined from D/H and $^7\text{Li}$ for condensed sphere geometry, but by $Y_e$ and $^7\text{Li}$ for spherical shells; 2) the range of allowable values for the baryon density are comparable to HBBN for small separation distances $r$, but there remain regions of the parameter space with optimum separation distances at which significantly higher values for $\eta$ or $\Omega_b h^2$ are allowed. This is true even for a high deuterium abundance; and 3) These limits can be increased even further if a higher (population I) primordial $^7\text{Li}$ abundance limit is adopted as some have proposed.

The optimum separation distance in each case roughly corresponds to a neutron diffusion length during nucleosynthesis (Mathews et al. 1990). Allowing for this possibility increases the maximum allowable values of the baryonic contribution to the closure density to $\Omega_b h^2 \leq 0.11$ ($\eta \leq 7 \times 10^{-10}$) for the spherical shell geometry and the adopted limits. The condensed sphere limits, however are unchanged from the HBBN model. On the other hand, if the primordial $^7\text{Li}$ abundance could be as high as $\text{Li}/\text{H} \leq 1.3 \times 10^{-9}$, then the upper limits for and condensed sphere geometry could
be as high as $\Omega_b h_\text{50}^2 \leq 0.13 (\eta \leq 8.6 \times 10^{-10})$ with similar values for the spherical shells. With a possible high deuterium abundance, the maximum allowable baryonic contribution decreases to $\Omega_b h_\text{50}^2 \leq 0.04 (\eta \leq 2.6 \times 10^{-10})$ for spherical shell geometry, or $\Omega_b h_\text{50}^2 \leq 0.03 (\eta \leq 2.1 \times 10^{-10})$ for condensed spheres. A high primordial lithium abundance would increase both of these limits to $\Omega_b h_\text{50}^2 \leq 0.06$.

7. Conclusions

We have reexamined the upper limits to $\eta$ and $\Omega_b h_\text{50}^2$ in inhomogeneous primordial nucleosynthesis models with different geometries and incorporating recently revised light-element abundance constraints. We have also considered implications of the possible detection (Songaila et al. 1994) of a high deuterium abundance in a Lyman-$\alpha$ absorption system. We have shown that with the presently adopted light-element abundance constraints (Copi et al. 1995), values of $\Omega_b h_\text{50}^2$ as large as 0.11 are possible in IBBN models. If one allows a possible high Lyman-$\alpha$ deuterium abundance, then $\Omega_b h_\text{50}$ as large as 0.04 could be allowed in the inhomogeneous models. These upper limits are higher than in standard homogeneous models or some other recent IBBN studies. The reason that we find a higher value for $\Omega_b$ and $\eta$ than in other recent IBBN studies (e.g. Kurki-Suonio et al. 1990; Thomas et al. 1994) is primarily due to the fact that, in addition to new reaction rates, we have allowed for the larger presently accepted (Copi et al. 1994) upper limit to the lithium abundance in population II halo stars due to systematic errors in the model atmospheres and possible lithium destruction in stars consistent with recent $^7\text{Li}$ detections.

We conclude that as long as the observationally inferred upper limit to the primordial lithium abundance remains uncertain, and fluctuations in baryon density of the optimum characteristics are not ruled out, and values of $h_{50}$ as small as 0.8 are possible, then the upper limit to the baryonic contribution to the closure density remains as large as $\Omega_b \leq 0.17$ with the presently accepted light-element abundance constraints. If a high primordial (Population I) $^7\text{Li}$ abundance limit is allowed, then $\Omega_b$ as large as 0.20 is possible. If the possible high Lyman-$\alpha$ deuterium abundance should prove to be correct then these limits reduce to $\Omega_b \leq 0.06$ and 0.10, respectively. These higher upper limits relative to HBBN are of interest since they are consistent with the inferred baryonic mass in the form of hot X-ray gas (White et al. 1993) in dense galactic clusters. They are, however, below the inferred dynamical mass of galactic halos (Trimble 1987; Ashman 1992) hence some form on nonbaryonic dark matter is still required, even in the IBBN scenario.

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Fig. 1.— Contours of allowed values for baryon-to-photon ratio $\eta$ (or $\Omega_b h^2$) and fluctuation separation radius $r$ based upon the various light-element abundance constraints as indicated. The separation $r$ is given in units of meters comoving at $kT = 1$ MeV. This calculation is based upon baryon density fluctuations represented by condensed spheres. The double cross hatched region corresponds to the allowed region based upon the adopted primordial abundance limits (Copi et al. 1994). The single cross hatched region depicts the allowed parameters if an extreme $^7\text{Li}$ upper limit is allowed.

Fig. 2.— Same as Figure 1, but with possible higher limits on D/H and [D + $^3\text{He}$]/H.

Fig. 3.— Same as Figure 1, but for fluctuations represented by spherical shells.

Fig. 4.— Same as Figure 2, but for fluctuations represented by spherical shells.